Homework: Lambda Calculus

Learning Objectives:

1. Understand evaluation order
2. Understand church encoding
3. Learn to perform β-reduction

Instructions:

- Total points: 49 pt
- Early deadline: Oct 30 (Wed) 2019 at 11:59 PM; Regular deadline: Nov 1 (Fri) 2019 at 11:59 PM (you can continue working on the homework till TA starts to grade the homework).
- Submit one pdf file on Canvas under Assignments, Homework 6 submission. You are encouraged to use latex. But we will accept a scanned copy as well.

Questions:

1. (9 pt) Perform β-reduction for the following λ expressions.
   (a) (3 pt) $(((\lambda(x)(x\ x))(\lambda(y)(y\ x)))\ z)$
   (b) (3 pt) $(((\lambda(a)(\lambda(b)(a\ b)))((\lambda(c)\ c)\ x))\ y)$
   (c) (3 pt) $(((\lambda(x)(x\ x))(\lambda(y)\ y))(\lambda(y)\ y))$
2. (6 pt) The goal of this problem is to help you understand the evaluation order of lambda calculus. In the following, show the steps of β-reduction for the lambda expression using two types of evaluation orders
   $$((\lambda(x)\ p)((\lambda(y)(y\ y))(\lambda(z)(z\ z))))$$
3. (4 pt) Encode the logic Boolean operations of *and a b* using *true*, *false* and *ite* given in the lecture.
4. (18 pt) Using *zero*, *one* and *two* as well as *succ*, *true* and *false* provided in the lecture, answer the following two questions:
   (a) (4 pt) What is the result of $((\lambda(z)((two\ f)\ z))\ (succ\ zero))$?
   (b) Suppose we define *unknown*: $(\lambda(x)(\lambda(y)(\lambda(z)\ y)))$ and *g*: $(\lambda(n)((n\ unknown)\ false))$, what is the result of:
      i. (4 pt) $(g\ zero)$
      ii. (3 pt) $(g\ one)$
      iii. (3 pt) $(g\ two)$

iv. (4 pt) What mathematical/logical operation is computed by g?

5. (12 pt) Given:
   g: $(\lambda(a)(\lambda(b)(\lambda(c)((a\ b)\ ((a\ b)\ c)))))$
   $zero$: $(\lambda(f)(\lambda(x)x))$
   one: $(\lambda(f)(\lambda(x)(f\ x)))$.
   two: $(\lambda(f)(\lambda(x)(f\ (f\ x))))$.
   $three$: $(\lambda(f)(\lambda(x)(f\ (f\ (f\ x)))))$.
   $four$: $(\lambda(f)(\lambda(x)(f\ (f\ (f\ (f\ x))))))$.

   (a) (4 pt) What is the result of (g one)?

   (b) (4 pt) What is the result of (g two)?

   (c) (4 pt) What computation does g performs?