汕头大学 ACM 模板

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```

动态规划

背包问题 #

状态: 从前 n 个背包中组成重量为 m 的......

01背包

O(VN)

滚动数组,从右往左。

完全背包

O(VN)

滚动数组,从左往右。

多重背包

 $O(V \sum \log M_i)$

每种物品分解成多个01背包中的物品,每个物品个数分别为1、2、4......个。

可行性多重背包

O(VN)

状态:用了前;种物品填满容量为j的背包后,最多还剩下几个第i种物品可用。

$$dp\left[i
ight]\left[j
ight] = \left\{egin{aligned} M\left[i
ight], & ext{if } dp\left[i-1
ight]\left[j
ight] \geq 0 \ & ext{max}\{dp\left[i
ight]\left[j-C\left[i
ight]
ight] - 1\}(C\left[i
ight] \leq j \leq V), & ext{if } dp\left[i-1
ight]\left[j
ight] < 0 \end{aligned}
ight.$$

```
// HDU 2844
int n, m;
while (cin >> n >> m && n)  {
    for (int i = 1; i <= n; ++i) cin >> a[i];
    for (int i = 1; i \le n; ++i) cin >> c[i];
    memset(dp, -1, sizeof(dp));
    dp[0] = 0;
    for (int i = 1; i \le n; ++i) {
        for (int j = 0; j <= m; ++j) {
            if (dp[j] \ge 0) dp[j] = c[i];
            else if (j \ge a[i]) dp[j] = max(dp[j], dp[j - a[i]] - 1);
        }
    }
    int ans = 0;
    for (int i = 1; i \le m; ++i)
       if (dp[i] >= 0) ++ans;
    cout << ans << '\n';
}
```

LIS #

占坑(

技巧 #

1. 某些 dp 的状态可看作分层图,此时转移可用邻接矩阵,并使用矩阵快速幂进行状态转移。(在无权图中,设A 为图的邻接矩阵, A^n 为矩阵的 n 次幂,则 A^n 中的元素(i,j)表示顶点 i 到顶点 j 的长度为 n 的路径数)

数学

数值分析 #

大数运算

```
import java.util.Scanner;
import java.math.*;
public class Main{
   public static void main(String args[]){
       Scanner cin = new Scanner(System.in);
       //使用Sacnner类创建cin对象
       BigInteger a, b;//创建大数对象
       while(cin.hasNext()){
           a = cin.nextBigInteger();
           b = cin.nextBigInteger();
           System.out.println("a+b=" + a.add(b));
           System.out.println("a-b=" + a.subtract(b));
           System.out.println("a*b=" + a.multiply(b));
           System.out.println("a/b=" + a.divide(b));
           System.out.println("a%b=" + a.remainder(b));
           if(a.compareTo(b) == 0) //比较两数的大小
               System.out.println("a==b");
           else if(a.compareTo(b) > 0)
               System.out.println("a>b");
               System.out.println("a<b");</pre>
           System.out.println(a.abs());//取绝对值
           int e = 10;
           System.out.println(a.pow(e));//求a^e
           System.out.println(a.toString()); //将大数a转字符串输出
           int p = 8;
           System.out.println(a.toString(p)); //将大数a转换成p进制后 按字符串输出
```

占坑(

矩阵

```
struct matrix {
    typedef LL ele_type;
    int r, c;
    vector<vector<ele_type>> ma;
    matrix(int r, int c): r(r), c(c), ma(r, vector<ele_type>(c)) {}
    matrix(matrix \&\&rhs) noexcept : r(rhs.r), c(rhs.c) {
        ma = move(rhs.ma);
    matrix(const matrix &rhs) = default;
    matrix& operator= (matrix &&rhs) noexcept {
        if (this != &rhs) {
            r = rhs.r;
            c = rhs.c;
            ma = move(rhs.ma);
        return *this;
    }
    matrix operator* (const matrix &rhs) const {
        matrix res(r, rhs.c);
        for (int i = 0; i < r; ++i) {
            for (int j = 0; j < c; ++j) {
                for (int k = 0; k < rhs.c; ++k) {
                    res.ma[i][k] += ma[i][j] * rhs.ma[j][k];
                    res.ma[i][k] %= MOD;
//
                      if (res.ma[i][k] >= MOD2) res.ma[i][k] -= MOD2; // MOD2 == MOD * MOD
                }
            }
        return res;
    void setUnit() {
        for (int i = 0; i < r; ++i) ma[i][i] = 1;
    }
};
```

快速傅里叶变换

 $O(n \log n)$

```
complex<double> a[N << 1];
complex<double> b[N << 1];
void init_pos(complex<double> *a, int n) { // 逆向加法
  for (int i = 0, j = 0; i != n; i++) {
    if (i > j) swap(a[i], a[j]);
    for (int l = n >> 1; (j ^= 1) < l; l >>= 1);
}
```

```
void FFT(int I, complex<double> *a, int n) { // 傅里叶变换(I==1) or 插值
   init_pos(a, n);
   for (int i = 2, mid = 1; i \le n; i \le 1, mid i \le 1) {
        complex<double> wn(cos(2.0 * PI / i), sin(I * 2.0 * PI / i));
       for (int j = 0; j < n; j += i) {
           complex<double> w(1, 0);
           for (int k = j; k < j + mid; k++, w = w * wn) {
               complex<double> 1 = a[k], r = w * a[k + mid];
               a[k] = 1 + r;
               a[k + mid] = 1 - r;
           }
       }
   }
   if (I == 1) return;
   for (int i = 0; i < n; i++) {
       a[i] /= n;
   }
```

数论 #

一些性质

```
1. (a - b) \% k == 0 <=> a \% k == b \% k
```

- 2. a % b < a / 2 (a >= b)
- 3. 完全平方数有奇数个约数。1到n中有 $\lfloor \sqrt{n} \rfloor$ 个完全平方数。
- 4. 有 n 位的数的平方根有 $\left[\frac{n}{2}\right]$ 位。

GCD & LCM

 $O(\log \max(a, b))$

```
inline int gcd(int a, int b) {
    int tmp;
    while (b != 0) {
        tmp = b;
        b = a % b;
        a = tmp;
    }
    return a;
}

inline int lcm(int a, int b) {
    return a / gcd(a, b) * b;
}
```

Extened GCD

 $O(\log \max(a, b))$

```
// solve ax + by = gcd(a, b)
int extgcd(int a, int b, int &x, int &y) { // return gcd(a, b)
```

```
int d = a;
if (b != 0) {
    d = extgcd(b, a % b, y, x);
    y -= (a / b) * x;
}
else {
    x = 1; y = 0;
}
return d;
}

// get inverse number of a modulo m
int mod_inv(int a, int m) {
    int x, k;
    extgcd(a, m, x, k);
    return (x % m + m) % m;
}
```

素数筛法

 $O(n \log \log n)$

O(n)

```
// Euler筛法
int C[N]; // 该数的最小质因数
int prime[N], pn; // 素数表
void sieve(int size) {
    for (int i = 2; i < size; ++i) { //
        if (!C[i]) C[i] = prime[pn++] = i;
        for (int j = 0; i * prime[j] < size; ++j) {
            C[i * prime[j]] = prime[j];
            if (i % prime[j] == 0) break; // smallest prime factor of i is prime[j], so we need
break
        }
    }
}
```

欧拉函数

欧拉函数 $\phi(n)$ 定义为: $1 \le k \le n$, 并且 $k \ne n$ 互质的 k 的个数。

```
// use linear sieve to compute phi(x) for all x up to n
```

```
int phi[N];
int C[N];
int p[N], pn;
void getPhi(int n) {
   phi[1] = 1;
   for (int i = 2; i < n; ++i) {
       if (!C[i]) {
            p[pn++] = i;
            phi[i] = i - 1; // (1) i is prime
        for (int j = 0; i * p[j] < n; ++j) {
            C[i * p[j]] = p[j];
            if (i % p[j] == 0) {
                phi[i * p[j]] = phi[i] * p[j]; // ② phi(ip) = phi(i) * p, if p divides i
                break; // smallest prime factor of i is p[j], so we need break
            }
            else {
                phi[i * p[j]] = phi[i] * phi[p[j]]; // ③ p does not divides i, i.e. they are co-
prime, so phi(ip) = phi(i) * phi(p) due to phi is a multiplicative function
       }
   }
}
```

快速幂

```
\begin{aligned} & \text{mul:} & O(1) \\ & \text{quickPow:} & O(\log n) \end{aligned}
```

```
LL mul(LL a, LL b, LL mod) { // 带模乘法
    if (mod <= 20000000000) return a * b % mod;
    LL d = llround((long double)a * b / mod);
    LL ans = a * b - d * mod;
    if (ans < 0) ans += mod;
    return ans;
}

// 求a在模MOD下的逆元 | 费马小定理 | inv(a, MOD - 2, MOD)

LL quickPow(LL a, LL n, LL mod) { // 带模快速幂
    LL ans = 1;
    while (n) {
        if (n & 1) ans = mul(ans, a, mod);
        a = mul(a, a, mod);
        n >>= 1;
    }
    return ans;
}
```

素性检测

 $O(\sqrt{n})$

```
// 试除法
bool isprime(int n) {
    for (int i = 2; i * i <= n; ++i) {
        if (n % i == 0) return false;
    }
    return true;
}
```

 $O(\log n)$

```
// Miller-Rabin 素性检测
bool isprime(LL n, int S = 7) {
   if (n == 2) return true;
   if (n < 2 || n % 2 == 0) return false;
   LL u = n - 1, t = 0;
   while (u \% 2 == 0) {
       u >>= 1;
       ++t;
   }
   const int SPRP[7] = {2, 325, 9375, 28178, 450775, 9780504, 1795265022};
   // cover all numbers < 2^64
   for (int k = 0; k < S; ++k) {
       LL a = SPRP[k] % n;
       if (a == 0 || a == 1 || a == n - 1) continue;
       LL x = qpow(a, u, n); // 带模快速幂
       if (x == 1 || x == n - 1) continue;
       for (int i = 0; i < t - 1; ++i) {
           x = mul(x, x, n); // 带模乘法
           if (x == 1) return false;
           if (x == n - 1) break;
       if (x == n - 1) continue;
       return false;
   return true;
```

质因数分解

 $O(\sqrt{n})$

```
// 直接小除到大
vector<int> p; // 存放分解的质因数
void factorize(int n) {
    for (int i = 2; i * i <= n; ++i) {
        while (n % i == 0) {
            p.push_back(i);
            n /= i;
        }
        if (n == 1) return;
    }
    if (n > 1) p.push_back(n);
}
```

 $O(\log n)$

```
// 预处理出各数的最小质因数,然后按最小质因数除
int C[N]; // 最小质因数,可用欧拉筛获得
vector<int> p;
void factorize(int n) {
    for (int t = n; t > 1; ) {
        int x = C[t]; // t的最小质因数
        while (C[t] == x) {
            p.push_back(x);
            t /= C[t];
        }
    }
}
```

Pollard's p: $O(n^{\frac{1}{4}})$ (大约)

factorize: O(玄学)

```
// Pollard's ρ 算法 POJ 1811
inline LL g(LL x, int c, LL mod) { // (伪) 随机数生成
   return (mul(x, x, mod) + c) \% mod;
LL pollard_rho(LL n, int c) {
   LL x = 2, y = 2, i = 1, k = 2;
   while (true) {
       x = g(x, c, n);
       // if (x == y) return n;
       LL d = \_gcd(llabs(x - y), n);
       if (d > 1 & d <= n) return d;
       if (++i == k) {
           y = x;
           k <<= 1;
      }
  }
}
vector<LL> factor; // 刚执行完后是无序的
void factorize(LL n) { // 能够承受10000个数左右的分解 (?)
   if (n == 1) return;
```

```
if (isprime(n)) {
    factor.push_back(n);
    return;
}

LL d = n;
for (int c = 1; d == n; ++c) {
    d = pollard_rho(n, c);
}

factorize(d);
factorize(n / d);
}
```

反素数

对于任何正整数x, 其约数的个数记作g(x)。例如g(1) = 1、g(6) = 4。

如果某个正整数x满足: g(x) > g(i)(0 < i < x),则称x为反质数。

性质一:一个反素数的质因子必然是从2开始连续的质数

性质二:分解过后的形式(如 $p=2^{t_1}*3^{t_2}*5^{t_3}*\cdots$),必然有 $t1 \geq t2 \geq t3 \geq \cdots$

```
int anti_prime[] =
```

 $\{1396755360, 1102701600, 735134400, 698377680, 551350800, 367567200, 294053760, 245044800, 183783600, 147026880, 122522400, 110270160, 73513440, 61261200, 43243200, 36756720, 32432400, 21621600, 17297280, 14414400, 10810800, 8648640, 7207200, 6486480, 4324320, 3603600, 2882880, 2162160, 1441440, 1081080, 720720, 665280, 554400, 498960, 332640, 277200, 221760, 166320, 110880, 83160, 55440, 50400, 45360, 27720, 25200, 20160, 15120, 10880, 7560, 50400, 2520, 1680, 1260, 840, 720, 360, 240, 180, 120, 60, 48, 36, 24, 12, 6, 4, 2, 1, 0\};$

图论

一些性质 #

- 1. 先从任意一个节点u开始深搜找到一个最远点v,再从v深搜找到最远点w,则v->w就是树的直径。
- 2. 握手定理:任何一个无向图都有偶数个度数为奇数的顶点。

拓扑排序 #

```
// dfs版拓扑排序,可判断是否为有向无环图,若不是则返回 false
vector<int> topo; // 拓扑序的倒序
bool dfs(int u) {
    vis[u] = -1; // -1为正在处理的节点,0为未处理的节点,1为已经处理完的节点
    for (int v = 1; v <= n; ++v) { // 对于每一个u的后继v
        if (g[u][v]) {
            if (vis[v] < 0) return false;
            if (!vis[v] && !dfs(v)) return false;
            // 若v没被处理并且v中发现环,则停止深搜,返回false
```

```
// 若v是已被处理完的节点,则忽略

}

vis[u] = 1;

topo.push_back(u);

return true;
}

bool toposort() {

    memset(vis, 0, sizeof(vis));
    topo.clear();
    for (int u = 1; u <= n; ++u) {

        if (!vis[u]) {

            if (!dfs(u)) return false;
        }
    }

    return true;
}
```

最短路 #

Bellman-Ford (SPFA)

O(VE) (对于大部分图 SPFA 还是很快的)

```
// 单纯判环 UVA 11090
double d[N];
bool vis[N];
bool SPFA(int u) {
    if (vis[u]) return true;
    vis[u] = true;
    for (int i = g[u]; ~i; i = es[i].next) {
        double tmp = d[u] + es[i].v;
        if (tmp < d[es[i].b]) {
            d[es[i].b] = tmp;
```

```
if (SPFA(es[i].b)) return true;
}

}

vis[u] = false;
return false;
}

bool detect() {
  for (int i = 1; i <= n; ++i) {
    vis[i] = false;
    d[i] = 0;
}
  for (int i = 1; i <= n; ++i) if (SPFA(i)) return true;
return false;
}</pre>
```

```
// 求最短路并判环
double d[N]; // 最短路长度
int L[N]; // 源点到各个节点的最短路的边数
bool inq[N];
bool SPFA(int s) { // true表示无环, false表示有环
   memset(d, 0x3f, sizeof(d));
   memset(inq, 0, sizeof(inq));
   memset(L, 0, sizeof(L));
   queue<int> q;
   d[s] = 0;
   q.push(s);
   while (!q.empty()) {
       int u = q.front(); q.pop();
       inq[u] = false;
       for (int i = g[u]; \sim i; i = es[i].next) {
           int v = es[i].b; double val = es[i].v;
           double tmp = d[u] + val;
           if (tmp < d[v]) {
               L[v] = L[u] + 1;
               if (L[v] >= n) return false; // 最短路长度大于等于顶点数即有环
               d[v] = tmp;
               if (!inq[v]) {
                   q.push(v);
                   inq[v] = true;
               }
           }
       }
   return true;
}
```

Dijkstra

 $O(E \log E)$

```
int d[N];
```

```
bool vis[N];
void dijkstra(int s) {
   memset(d, 0x3f, sizeof(d));
   memset(vis, 0, sizeof(vis));
   priority_queue<pair<int, int>> q;
   d[s] = 0;
   q.push({0, s});
   while (!q.empty()) {
        int u = q.top().second; q.pop();
        if (vis[u]) continue;
       vis[u] = true;
        for (auto \&e : g[u]) {
            int v = e.first, w = e.second;
            int tmp = d[u] + w;
            if (tmp < d[v]) {
                d[v] = tmp;
                q.push({-tmp, v});
       }
}
```

Floyd

 $O(V^3)$

```
int d[N][N];
int g[N][N];
for (int i = 1; i <= n; i++) {
    for (int j = 1; j <= n; j++) {
        if (i == j) d[i][j] = 0;
        else if (g[i][j]) d[i][j] = g[i][j];
        else d[i][j] = INF;
    }
}
for (int k = 1; k <= n; ++k) {
    for (int i = 1; i <= n; ++i) {
        for (int j = 1; j <= n; ++j) {
            d[i][j] = min(d[i][j], d[i][k] + d[k][j]);
        }
    }
}</pre>
```

K短路

O(玄学)

```
// POJ 2449
#include <bits/stdc++.h>

using namespace std;

const int INF = 0x3f3f3f3f;
const double EPS = 1e-8;
```

```
//const double PI = acos(-1.0);
//const int MOD = 1000000007;
typedef long long LL;
typedef unsigned long long uLL;
const int N = 1000+10;
int S, T, K;
vector<pair<int, int> > g[N];
vector<pair<int, int> > rev[N]; // 将所有边反向的逆图
int h[N];
int cnt[N]; // 每个节点的访问次数
bool vis[N];
void dij() { // 求出最优的h, 即每个节点到终点的最短路长度
   memset(h, 0x3f, sizeof(h));
   priority_queue<pair<int, int> > q;
   h[T] = 0;
   q.push({0, T});
   while (!q.empty()) {
        int u = q.top().second; q.pop();
        if (vis[u]) continue;
        vis[u] = true;
        for (int i = 0; i < rev[u].size(); ++i) {</pre>
           int tmp = h[u] + rev[u][i].second;
           if (tmp < h[rev[u][i].first]) {</pre>
                h[rev[u][i].first] = tmp;
                q.push({-tmp, rev[u][i].first});
        }
   }
}
int astar() {
   priority_queue<pair<int, int> > q;
   q.push({-h[S], S});
    while (!q.empty()) {
        int u = q.top().second, val = -q.top().first; q.pop();
        if (++cnt[u] == K && u == T) return val; // 访问K次即K短路
        if (cnt[u] > K) continue;
        for (int i = 0; i < g[u].size(); ++i) {
           int v = g[u][i].first, t = g[u][i].second;
           if (cnt[v] < K) {
                q.push({-val + h[u] - t - h[v], v});
        }
    return -1;
}
int main() {
   ios::sync_with_stdio(false);
   cin.tie(0);
```

```
int n, m;
scanf("%d%d", &n, &m);
while (m--) {
    int a, b, t;
        scanf("%d%d%d", &a, &b, &t);
        g[a].push_back({b, t});
        rev[b].push_back({a, t});
}
scanf("%d%d%d", &S, &T, &K);
if (S == T) ++K;
dij();
printf("%d\n", astar());
return 0;
}
```

生成树 #

Kruskal

对边排序然后贪心

 $O(E \log E)$

Prim

和Dijkstra相似

 $O(E \log E)$

次小生成树

 $O(E \log E + V^2)$ or $O(E \log E + V \log V)$

```
// POJ 1679
#include <bits/stdc++.h>
using namespace std;
const int INF = 0x3f3f3f3f;
const double EPS = 1e-8;
// const double PI = acos(-1.0);
// const int MOD = 1000000007;
typedef long long LL;
typedef unsigned long long uLL;
const int N = 100 + 10;
const int M = 2000000 + 10;
int n, m;
struct edge {
    int a, b, w;
    bool operator<(const edge &r) const { return w < r.w; }</pre>
};
```

```
vector<edge> es;
int fa[N];
int find(int u) { return (fa[u] < 0 ? u : (fa[u] = find(fa[u]))); }
vector<pair<int, int> > spt[N]; // 最小生成树
vector<int> candi; // 次小生成树的候选边 (即不在最小生成树中的边)
void setUnion(int a, int b) {
   int r1 = find(a), r2 = find(b);
   if (r1 == r2) return;
   if (fa[r1] < fa[r2]) {</pre>
       fa[r2] = r1;
       return;
   }
   if (fa[r1] == fa[r2]) --fa[r2];
   fa[r1] = r2;
}
int kruskal() {
   memset(fa, -1, sizeof(fa));
   candi.clear();
   sort(es.begin(), es.end());
   int ans = 0;
   for (int i = 0; i < es.size(); ++i) {</pre>
       edge \&e = es[i];
       if (find(e.a) != find(e.b)) {
           setUnion(e.a, e.b);
           spt[e.a].push_back({e.b, e.w});
           spt[e.b].push_back({e.a, e.w});
           ans += e.w;
       } else {
           candi.push_back(i);
       }
   return ans;
}
int F[N][N]; // 最小生成树中任意两点路径中最大边权
bool vis[N];
void dfs(int u, int fa, int ans, int a) {
   for (int i = 0; i < spt[u].size(); ++i) {</pre>
       int v = spt[u][i].first;
       if (v != fa) {
           dfs(v, u, F[a][v] = max(ans, spt[u][i].second), a);
       }
   }
}
void pre() { // 求出F, 可利用LCA与区间查询将复杂度将为0(nlogn)
   for (int i = 1; i \le n; ++i) dfs(i, -1, -1, i);
}
```

```
void addedge(int u, int v, int w) { es.push_back({u, v, w}); }
int solve(int ori) {
   int ans = INF;
   for (int i = 0; i < candi.size(); ++i) { // 枚举每条候选边
       edge &e = es[candi[i]];
       ans = min(ori - F[e.a][e.b] + e.w, ans); // 将它与F[e.a][e.b]的边替换
   return ans;
}
int main() {
   ios::sync_with_stdio(false);
   cin.tie(0);
   int T;
   scanf("%d", &T);
   while (T--) {
       scanf("%d%d", &n, &m);
       es.clear();
       for (int i = 1; i <= n; ++i) spt[i].clear();
       for (int i = 0; i < m; ++i) {
           int u, v, w;
           scanf("%d%d%d", &u, &v, &w);
           addedge(u, v, w);
       int ans = kruskal();
       pre();
       int tmp = solve(ans);
       if (tmp == ans) // 若次小生成树权值与最小生成树相同
           printf("Not Unique!\n");
       else
           printf("%d\n", ans);
   return 0;
}
```

$O(E \log E)$

```
// codeforces contest 1108 F
// improved kruskal
// return the number of alternative edges of MST
int kruskal() {
    int ret = 0;
    memset(fa, -1, sizeof(fa));
    sort(es.begin(), es.end());
    map<int, set<pii>> mp;
    int cnt = 0;
    for (int i = 0; i < es.size(); ++i) {
        auto &e = es[i];
        if (i + 1 == es.size() || es[i + 1].w != e.w) {
            for (int j = i; j >= 0 && es[j].w == e.w; --j) {
                if (find(es[j].u) != find(es[j].v)) ++cnt;
        }
}
```

```
}
for (int j = i; j >= 0 && es[j].w == e.w; --j) {
    if (find(es[j].u) != find(es[j].v)) {
        unite(find(es[j].u), find(es[j].v));
        --cnt;
    }
}
ret += cnt;
cnt = 0;
}
return ret;
}
```

网络流

Edmonds-Karp

 $O(VE^2)$

```
// UVA 820
struct edge {
    int a, b, cap;
};
vector<edge> es;
vector<int> g[N];
int p[N]; // 每次增广的路径记录
int flows[N]; // 记录每次增广流大小顺便用于bfs (
inline void addedge(int a, int b, int cap) {
    es.push_back({a, b, cap});
    es.push_back({b, a, 0});
   int i = es.size();
    g[a].push_back(i - 2);
    g[b].push_back(i - 1);
}
int bfs(int s, int d) {
    memset(flows, 0, sizeof(flows));
    queue<int> q;
    q.push(s);
    flows[s] = INF;
    while (!q.empty()) {
        int u = q.front(); q.pop();
        for (int ee = 0; ee < g[u].size(); ++ee) {
           const int &e = g[u][ee];
           const edge &eg = es[e];
           if (!flows[eg.b] && eg.cap > 0) {
               flows[eg.b] = min(flows[u], eg.cap);
               p[eg.b] = e; // 记录此次增广的路径, 用于更新
               if (eg.b == d) return flows[d];
               q.push(eg.b);
```

```
}
}
return flows[d];

int ekalgo(int s, int d) {
    int maxflow = 0;
    while (true) {
        if (!bfs(s, d)) break;
        for (int u = d; u != s; u = es[p[u]].a) {
            es[p[u]].cap -= flows[d];
            es[p[u]^1].cap += flows[d];
        }
        maxflow += flows[d];
}
return maxflow;
}
```

Scaling

 $O(E^2 \log c)$ (c is the initial threshold)

```
// UVA 820
struct edge {
   int a, b, cap;
};
vector<edge> es;
vector<int> g[N];
int p[N]; // 用于dfs的标识数组
inline void addedge(int a, int b, int cap) {
   es.push_back({a, b, cap});
   es.push_back({b, a, 0});
   int i = es.size();
   g[a].push_back(i - 2);
   g[b].push_back(i - 1);
}
int dfs(int u, int f) { // 每次只增广大小大于等于阈值的流, 若无则返回0
   if (u == d) {
       return f;
   p[u] = 1;
   for (auto ee : g[u]) {
       edge &e = es[ee];
       if (!p[e.b] \&\& e.cap >= thre) {
           int flow = dfs(e.b, min(f, e.cap));
           if (flow >= thre) {
               e.cap -= flow;
               es[ee^1].cap += flow;
               return flow;
       }
```

Dinic

 $O(V^2E)$

```
int d[N];
bool bfs() {
    memset(d, -1, sizeof(d));
    queue<int> q;
    q.push(0);
    d[0] = 0;
    while (!q.empty()) {
        int u = q.front(); q.pop();
        for (int v = 1; v \le n; ++v) {
            if (d[v] < 0 \& g[u][v] > 0) {
                d[v] = d[u] + 1;
                q.push(v);
            }
        }
    return d[n] >= 0;
}
bool vis[N];
int dfs(int u, int f) {
    if (u == n) return f;
   int ans = 0;
    vis[u] = true;
    for (int v = 1; v \le n; ++v) {
        if (!vis[v] && d[v] == d[u] + 1 && g[u][v] > 0) {
            int flow = dfs(v, min(f, g[u][v]));
            if (flow > 0) {
                g[u][v] -= flow;
                g[v][u] += flow;
                ans += flow;
                f -= flow;
```

```
if (f <= 0) break;
}
}
return ans;
}

int dinic() {
  int ans = 0;
  while (bfs()) {
    int flow;
    while (memset(vis, 0, sizeof(vis)), flow = dfs(0, INF)) ans += flow; // O(EV)
}
return ans;
}</pre>
```

上下界可行流

建一个特殊的图然后跑网络流(具体参见"图论总结 by amber.pdf")

连通分量 #

强连通分量 (Kosaraju)

```
// POJ 2186:分解强连通分量,如果图中任何一个点都能到最后一个连通分量,则输出最后一个连通分量所含节点数
int n, m;
vector<int> g[N];
vector<int> rev[N];
bool vis[N];
int s[N], stop; // 第一次dfs得到的dfs序列
int ord[N]; // 节点所在强连通分量的拓扑序
void dfs1(int u) {
   vis[u] = true;
   for (int i = 0; i < g[u].size(); ++i) {
       if (!vis[g[u][i]]) dfs1(g[u][i]);
   s[stop++] = u;
}
void dfs2(int u, int k) {
   ord[u] = k;
   for (int i = 0; i < rev[u].size(); ++i) {
       if (!ord[rev[u][i]]) dfs2(rev[u][i], k);
   }
}
int cnt;
void check(int u) {
   vis[u] = true;
   ++cnt;
```

```
for (int i = 0; i < rev[u].size(); ++i) {</pre>
       if (!vis[rev[u][i]]) check(rev[u][i]);
}
int main() {
   ios::sync_with_stdio(false);
   cin.tie(0);
   scanf("%d%d", &n, &m);
   while (m--) {
       int a, b;
       scanf("%d%d", &a, &b);
       g[a].push_back(b);
       rev[b].push_back(a);
   for (int i = 1; i <= n; ++i)
       if (!vis[i]) dfs1(i);
   int k = 0;
   while (stop > 0) { // Kosaraju here
       int u = s[--stop];
       if (ord[u]) continue;
       dfs2(u, ++k); // 按栈中的顺序处理结点得到的是拓扑序
   }
   int ans = 0;
   int u = 0;
   for (int i = 1; i \le n; ++i) {
       if (ord[i] == k) { // 拓扑序中最后一个连通分量, 即无出边的分量
           u = i;
           ++ans;
       }
   }
   memset(vis, 0, sizeof(vis));
   check(u); // 查看是否任意节点都可达该分量
   if (cnt != n) printf("0\n");
   else printf("%d\n", ans);
   return 0;
}
```

强连通分量 (Tarjan)

```
// POJ 2186: 分解强连通分量,如果图中任何一个点都能到最后一个连通分量,则输出最后一个连通分量所含节点数 #include <bits/stdc++.h>

using namespace std;

const int INF = 0x3f3f3f3f;
const double EPS = 1e-8;
//const double PI = acos(-1.0);
//const int MOD = 10000000007;

typedef long long LL;
```

```
typedef unsigned long long uLL;
const int N = 10000 + 10;
int n, m;
vector<int> g[N];
int cnt; // dfs访问过程中的时间标记
int s[N], stop; // 存放还未被决定连通分量的节点,在栈中的节点一定有路径到当前访问的节点
int ord[N]; // 节点属于哪个连通分量,同时连通分量是拓扑有序的
bool ins[N]: // 是否在栈中
int dfn[N]; // 时间戳
int low[N]; // 各个节点能到达的时间戳最早的节点
int cpn;
void dfs(int u) { // 本质上就是一个模板 dfs 而已
   s[stop++] = u;
   ins[u] = true;
   dfn[u] = low[u] = ++cnt;
   for (int i = 0; i < g[u].size(); ++i) {</pre>
       if (!dfn[g[u][i]]) { // 如果该节点没有被访问
           dfs(g[u][i]);
           low[u] = min(low[u], low[g[u][i]]);
       else if (ins[g[u][i]]) // 如果该节点在栈中
           low[u] = min(low[u], dfn[g[u][i]]);
   if (dfn[u] == low[u]) { // 新连通分量, u是该连通分量的"根"
       ++cpn;
       int t; // 存放栈顶元素
       do {
           t = s[--stop];
           ins[t] = false;
           ord[t] = cpn;
       } while (t != u); // 栈顶到u的顶点都属于该连通分量
}
int out[N];
int main() {
   ios::sync_with_stdio(false);
   cin.tie(0);
   scanf("%d%d", &n, &m);
   while (m--) {
       int a, b;
       scanf("%d%d", &a, &b);
       g[a].push_back(b);
   for (int i = 1; i \le n; ++i) // Tarjan here
       if (!dfn[i]) dfs(i);
   for (int i = 1; i \le n; ++i) {
       for (int j = 0; j < g[i].size(); ++j) {
```

```
if (ord[i] != ord[g[i][j]]) ++out[ord[i]];
       }
    }
   int cnt = 0;
   for (int i = 1; i <= cpn; ++i)
       if (!out[i]) {
           ++cnt;
        }
   if (cnt > 1) printf("0\n");
   else {
       int ans = 0;
       for (int i = 1; i \le n; ++i)
           if (!out[ord[i]]) ++ans;
       printf("%d\n", ans);
   return 0;
}
```

双连通分量 (Tarjan)

性质: 双连通图的所有节点都在一个环上, 所有节点都在一个环上的图是双连通的

```
// POJ 2942
// 双连通图就是没有任何一个单独的点可以成为割点的图
#include <bits/stdc++.h>
using namespace std;
const int INF = 0x3f3f3f3f;
const double EPS = 1e-8;
//const double PI = acos(-1.0);
//const int MOD = 1000000007;
typedef long long LL;
typedef unsigned long long uLL;
const int N = 1000+10;
int n, m;
vector<int> g[N];
bool hate[N][N];
int cnt; // dfs访问过程中的时间戳
int s[N], stop;
bool ins[N];
int dfn[N], low[N];
int ord[N];
int cpn;
int tmp[N];
int color[N];
bool expelled[N];
inline void init() {
```

```
for (int i = 1; i <= n; ++i) g[i].clear();</pre>
   memset(hate, 0, sizeof(hate));
   cnt = stop = cpn = 0;
   memset(ins, 0, sizeof(ins));
   memset(dfn, 0, sizeof(dfn));
   memset(ord, 0, sizeof(ord));
   memset(expelled, 1, sizeof(expelled));
}
bool bipartite(int u) { // 二分图判定
   for (int i = 0; i < g[u].size(); ++i) {</pre>
       int v = g[u][i];
       if (ord[v] == cpn) {
           if (!color[v] && !bipartite((color[v] = -color[u], v))) return false;
           if (color[v] && color[v] == color[u]) return false;
       }
   }
   return true;
}
void dfs(int u, int fa) { // 本质上就是一个模板 dfs 而已
   s[stop++] = u;
   ins[u] = true; // 原始算法中栈中存的是边
   dfn[u] = low[u] = ++cnt;
   for (int i = 0; i < g[u].size(); ++i) {</pre>
       int v = g[u][i];
       if (!dfn[v]) {
           dfs(v, u);
           low[u] = min(low[u], low[v]);
           if (low[v] >= dfn[u]) {
               ++cpn; // 新的双连通分量
               int cnt = 0;
               int t; // 存放栈顶元素
               do {
                   t = s[--stop]:
                   ins[t] = false;
                   tmp[cnt++] = t; // 该数组中的元素为同一个连通分量中的点,用于后面的二分图判断
                   ord[t] = cpn;
               } while (t != v);
               tmp[cnt++] = u; // u也属于该连通分量
               memset(color, 0, sizeof(color));
               color[u] = 1;
               if (cnt >= 3 && !bipartite(u)) { // 不是二分图即有奇圈
                   for (int i = 0; i < cnt; ++i) expelled[tmp[i]] = false;</pre>
           }
       else if (ins[v] && v != fa)
           low[u] = min(low[u], dfn[v]);
   }
}
```

```
int solve() {
    int ans = 0;
    for (int i = 1; i \le n; ++i) if (expelled[i]) ++ans;
    return ans;
}
int main() {
    ios::sync_with_stdio(false);
    cin.tie(0);
    while (scanf("%d%d", &n, &m) && n) {
        init();
        for (int i = 0; i < m; ++i) {
            int a, b;
            scanf("%d%d", &a, &b);
            hate[a][b] = hate[b][a] = true;
        }
        for (int i = 1; i \le n; ++i) {
            for (int j = i + 1; j \le n; ++j) {
                if (!hate[i][j]) {
                    g[i].push_back(j);
                    g[j].push_back(i);
                }
            }
        }
        for (int i = 1; i \le n; ++i) // Tarjan here
            if (!dfn[i]) dfs(i, 0);
        printf("%d\n", solve());
    return 0;
```

LCA #

Tarjan 离线算法

 $O((V+E)+(V+Q)\log V)$ (Q is the number of queries)

```
// POJ 1470
// call: tarjan(root);
int fa[N];
bool vis[N];
vector<int> g[N];
vector<int> qs[N];
int ans[N];
int root;

int find(int u) {
    if (fa[u] < 0)
        return u;
}</pre>
```

```
return fa[u] = find(fa[u]);
}
inline void setUnion(int a, int b) {
   int r1 = find(a), r2 = find(b);
   if (r1 == r2) return;
   fa[r1] = r2;
}
void tarjan(int u) {
   vis[u] = true;
   for (int i = 0; i < qs[u].size(); ++i) { // 处理该节点对应的查询
       int q = qs[u][i];
       if (vis[q]) ++ans[find(q)]; // 如果对应的节点已被访问,则其所在集合的代表就是lca
   for (int i = 0; i < g[u].size(); ++i) { // dfs}
       int ch = g[u][i];
       tarjan(ch);
       setUnion(ch, u); // 处理完一个儿子后就将其与父亲合并
   }
}
inline void init() {
   memset(fa, -1, sizeof(fa));
   memset(vis, 0, sizeof(vis));
   memset(ans, 0, sizeof(ans));
```

倍增法

```
init: O(V \log V) getLCA: O(\log V)
```

```
// 预处理后任意两点的LCA
int dep[N];
vector<int> g[N];
int ans[N];
int root;
int anc[20][N];
int n;
void dfs(int u, int fa, int w) {
    anc[0][u] = fa;
    maxe[0][u] = w;
    for (int k = 1; k < LOGN; ++k) {
        if ((1 << k) > dep[u]) break;
        anc[k][u] = anc[k-1][anc[k-1][u]];
        maxe[k][u] = max(maxe[k-1][anc[k-1][u]], maxe[k-1][u]);
    }
    for (auto &e : spt[u]) {
        int v = e.first;
        if (v == fa) continue;
        dep[v] = dep[u] + 1;
```

```
dfs(v, u, e.second);
  }
void init() { // 预处理
   dep[root] = 0;
   dfs(root, root, 0);
}
inline int getk(int u, int k) { // 得到节点u的第k个祖先
   if (k > dep[u]) return root;
   for (int i = 0; (1<<i) <= k; ++i) {
       if ((1 << i) \& k) u = anc[i][u];
   return u;
}
int getLCA(int u, int v) {
   if (dep[u] > dep[v]) swap(u, v);
   v = getk(v, dep[v] - dep[u]); // 先让v走到和u相同深度的节点
   // if (u == v) return u;
   // for (int k = LOG_V; k >= 0; --k) {
   // if (anc[k][u] != anc[k][v]) {
   // u = anc[k][u];
// v = anc[k][v];
   // }
   // }
   // return anc[0][u];
   int l = 0, r = dep[u];
   while (l < r) { // 二分搜索找到u、v最低的公共祖先
       int mid = 1 + (r - 1) / 2;
       if (getk(u, mid) == getk(v, mid)) {
          r = mid;
       }
       else {
          1 = mid + 1;
   return getk(u, 1);
```

```
匹配与覆盖
```

占坑(

Floyd判圈算法

TODO

欧拉通路/回路

定义:图上每条边都经过且只经过一次。

存在性:

- 无向图: 所有点连通且
 - 。 每个点的度都是偶数 (回路)
 - · 恰好两个点的度是奇数,其他都是偶数 (通路) (奇数点即通路的起点与终点)
- 有向图: 所有点连通且
 - 。 每个点的入度都等于出度 (回路)
 - 。 一个点的入度比出度大1(终点),另一个点的出度比入度大1(起点),其他点入度等于出度(通路)

字符串算法

字符串哈希算法 #

O(n)

Karp-Rabin #

preprocess: O(n) getval: O(1)

```
const int A = 911382323;
const int B = 972663749;
int h[N]; // h[k] contains the hash value of the prefix s[0...k]
int p[N]; // p[k] == Ak mod B

void preprocess(const char s[]) {
   int len = strlen(s);
   h[0] = s[0];
   p[0] = 1;
   for (int i = 1; i < len; ++i) {
      h[i] = ((LL)h[i-1] * A + s[i]) % B;
      p[i] = ((LL)p[i-1] * A) % B;
   }
}
int getval(int a, int b) { // 子串[a,b]的哈希值</pre>
```

```
if (a == 0) return h[b];
int tmp = (h[b] - ((LL)h[a - 1] * p[b - a + 1]) % B) % B;
return (tmp < 0 ? tmp + B : tmp);
}</pre>
```

Z-algorithm #

Z数组的使用:把模式串 P 加在被匹配串 S 前,中间隔一个无干扰字符,如 P#S ,再对这个字符串计算 Z-array,则 Z-array 中值为 P 的长度的位置即是模式串出现的位置。

O(n)

```
int z[N]; // z[i]为以位置i为起点的是s的前缀的最长子串的长度
void Z(const char s[]) { // compute z-array
    int len = strlen(s);
    int x = 0, y = 0;
    for (int i = 1; i < len; ++i) {
        z[i] = max(0, min(z[i-x], y - i + 1)); // 若i在[x, y]之外,则z[i]为0, 否则为min(z[i-x], y - i + 1)

        while (i + z[i] < len && s[z[i]] == s[i + z[i]]) {
            x = i; y = i + z[i]; ++z[i];
        }
    }
}</pre>
```

KMP #

KMP可用来求串的最大循环节。

O(n)

```
template<typename T>
vector<int> kmp(const T &a) { // fail[i]是为a[0...i]的后缀的最长前缀
   int n = a.size();
   vector<int> fail(n);
   fail[0] = -1;
   for (int i = 1; i < n; ++i) {
       fail[i] = fail[i - 1];
       while (~fail[i] && a[fail[i] + 1] != a[i]) {
           fail[i] = fail[fail[i]];
       }
       if (a[fail[i] + 1] == a[i]) {
           ++fail[i];
       }
   }
   return fail;
vector<int> KMP(const string &s, const string &p) {
   auto \&fail = kmp(p);
   vector<int> res;
   for (int i = 0, j = 0; i < s.size(); ++i) {
       while (s[i] != p[j] \&\& j > 0) j = fail[j - 1] + (fail[j - 1] != j - 1);
       if (s[i] == p[j]) {
```

```
if (++j == p.size()) {
    res.push_back(i - p.size() + 2);
    j = fail[j - 1] + (fail[j - 1] != j - 1);
}

return res;
}
```

AC**自动机** #

AC自动机就是把pattern从一个字符串变成了一个trie树 (即有多个pattern串,即字典)的KMP

建字典: O(cS) (c is the size of the charset and S is the total length of all words in the trie)

匹配: O(n+z) (n is the length of the original string and z is the total time that the patterns appear)

```
// HDU 2222 2896
// 失配后回溯到是字典中的某个词的前缀的后缀的位置
struct ACautomaton {
   int trie[N][26];
   int fail[N];
   int cnt[N];
   int tol = 0;
   int newnode() {
       for (int i = 0; i < 26; ++i) trie[tol][i] = 0;</pre>
       fail[tol] = 0;
       cnt[tol] = 0;
       return tol++;
   void init() {
       tol = 0;
       newnode();
   void add(const char s[]) { // 在字典中新加入一个词
       int cur = 0;
       for (int i = 0; s[i]; ++i) {
           int c = s[i] - 'a';
           if (trie[cur][c] == 0) trie[cur][c] = newnode();
           cur = trie[cur][c];
       ++cnt[cur];
   void build() { // bfs
       queue<int> q;
       q.push(0);
       while (!q.empty()) {
           int u = q.front(); q.pop();
           for (int i = 0; i < 26; ++i) {
               int &v = trie[u][i];
               if (v) {
                   if (u != 0) fail[v] = trie[fail[u]][i]; // 非根节点失配才回溯
                   // 如果trie[fail[u]][i]存在,即该位置就是fail节点v的失配指针
                   // 如果不存在,则它保存着节点fail[u]失配时的失配指针
```

```
q.push(v);
               }
               else { // 不存在
                  v = trie[fail[u]][i]; // 保存u的失配指针
           }
       }
   int count(const char s[]) { // 返回字典的词在s中总共出现了多少次 (有重复)
       int ans = 0;
       int u = 0;
       for (int i = 0; s[i]; ++i) {
           int c = s[i] - 'a';
           u = trie[u][c];
           int p = u;
           while (p && cnt[p]) { // 后缀
               ans += cnt[p];
               cnt[p] = 0;
               p = fail[p];
       return ans;
} ac;
```

后缀数组

LCP: Longest Common Prefix

很大。后缀数组也一样,它的含义是S的各个后缀的排列SA,它满足:

$$Suffix(SA[i]) < Suffix(SA[i+1]) \quad (1 \leq i < n)$$

注意这里不用等号,任何两个后缀的长度不同,因此不可能相等。为了方便,我们另外定义一个名次数组 $Rank=SA^{-1}$,即若SA[i]=j,则Rank[j]=i,因此Rank[i]保存的是后缀Suffix(i)在所有后缀中从小到大的"名次"。

```
倍增法构造: O(n \log n) 计算LCP: O(n) LCP定理: 设i < j, 则LCP(i,j) = \min\{LCP(k-1,k) \mid i+1 \le k \le j\}
```

```
// ACM-ICPC 2018 焦作赛区网络预赛 H: 子串出现次数
struct SuffixArray {
    int n;
    int arr[N];
    int sa[N];
    int rank[N];
    int lcp[N]; // LCP between sa[i] and sa[i-1]
    int t1[N], t2[N], cnt[N];
```

```
void init (const string &s) {
       n = 0:
       for (auto i : s) arr[n++] = i;
       arr[n++] = 0;
       build_sa(127);
        build();
//
   }
   // m为最大字符集数加1
   void build_sa(int m) {
       int *x = t1, *y = t2;
       for (int i = 0; i < m; ++i) cnt[i] = 0;
       for (int i = 0; i < n; ++i) ++cnt[x[i] = arr[i]];
       for (int i = 1; i < m; ++i) cnt[i] += cnt[i-1];
       for (int i = n-1; i \ge 0; --i) sa[--cnt[x[i]]] = i;
       for (int k = 1; k <= n; k <<= 1) {
           int p = 0;
           // 按照次关键字进行排序
           for (int i = n - k; i < n; i++) y[p++] = i;
           for (int i = 0; i < n; i++) if (sa[i] >= k) y[p++] = sa[i] - k;
           // 按照主关键字进行排序
           for (int i = 0; i < m; i++) cnt[i] = 0;
           for (int i = 0; i < n; i++) ++cnt[x[y[i]]];
           for (int i = 0; i < m; i++) cnt[i] += cnt[i - 1];
           for (int i = n - 1; i \ge 0; i--) sa[--cnt[x[y[i]]]] = y[i];
           swap(x, y);
           // 计算rank
           p = 1;
           x[sa[0]] = 0;
           for (int i = 1; i < n; i++)
               x[sa[i]] = (y[sa[i-1]] == y[sa[i]] && y[sa[i-1] + k] == y[sa[i] + k]
                           ? p - 1 : p++);
           if (p >= n) break;
           m = p;
       }
       for (int i = 0; i < n; ++i) rank[sa[i]] = i;
   void build_lcp() {
       int k = 0;
       for(int i = 0; i < n; i++) {
           if(k) k--;
           if(rank[i] - 1 < 0) continue; // rank[i]可能是0
           int j = sa[rank[i]-1];
           while(arr[i+k] == arr[j+k]) k++;
           lcp[rank[i]] = k;
       }
   }
   // 所有height值之和为重复子串的个数
```

```
// 同一子串会在后缀数组中连续出现
} sa;
```

区间问题

Sparse Table (稀疏表)

#

构建: $O(n \log n)$ 查询: O(1)

```
int minima[17][N];
int a[N];
int n;
void init() {
    for (int i = 0; i < n; ++i) {
        minima[0][i] = a[i];
    for (int i = 1; (1<<i) <= n; ++i) {
        for (int j = 0; j < n; ++j) {
            if (j + (1 << i) > n) break;
            minima[i][j] = min(minima[i-1][j], minima[i-1][j+(1<<(i-1))]);
    }
}
inline int getmin(int a, int b) { // return minimum in [a, b]
    int len = b - a + 1;
    int k = 0;
    while ((1 << (k+1)) <= len) ++k;
    return min(minima[k][a], minima[k][b-(1<< k)+1]);
}
```

树状数组 #

建树: O(n)

查询、修改: $O(\log n)$

```
// 求区间和
int binary_indexed_tree[N]; // [0] is redundant
// tree[i]'s length is the largest power of two that divides i and that ends at position i.
int n;
int arr[N]; // original array

int sum(int k) { // get sum of arr[1, k]
    int s = 0;
    while (k >= 1) {
        s += binary_indexed_tree[k];
        k -= k & -k; // lowest 1-bit, the largest power of 2 that divides k
    }
    return s;
```

```
}
inline void add(int k, int v) {
    while (k <= n) {
        binary_indexed_tree[k] += v;
        k += k & -k;
    }
}
inline void init() {
    for (int i = 1; i <= n; ++i) add(i, arr[i]);
}

int inversion() { // 求逆序数 (权值树状数组)
    int ans = 0;
    for (int i = 0; i < n; ++i) {
        ans += i - sum(arr[i]);
        add(arr[i], 1);
    }
    return ans;
}
```

线段树 #

建树: O(n)

查询、修改: $O(\log n)$

```
int segTree[N<<2];</pre>
int n;
/* --- from bottom to top --- */
// minimum query
int getMin(int a, int b) {
    a += n; b += n;
    int ans = INF;
    while (a <= b) {
       if (a & 1) ans = min(ans, segTree[a++]);
        if (!(b & 1)) ans = min(ans, segTree[b--]);
        a >>= 1; b >>= 1;
    return ans;
}
void modifyMin(int k, int v) { // modified arr[k] value
    k += n;
    segTree[k] = v;
    for (k >>= 1; k >= 1; k >>= 1) {
        segTree[k] = min(segTree[2 * k], segTree[2 * k + 1]);
    }
}
/* -- from top to bottom --- */
// range sum update and query
inline void pushdown(int k, int x, int y) {
    int inc = segTree[k].inc;
```

```
if (inc) {
        segTree[k].v += (y - x + 1) * inc;
        segTree[k].inc = 0;
        segTree[k << 1].inc += inc;</pre>
        segTree[k << 1 | 1].inc += inc;</pre>
    }
}
int sum(int a, int b, int k, int x, int y) {
    if (b < x \mid | a > y) return 0;
    if (a \le x \&\& b \ge y) return segTree[k].v + segTree[k].inc * <math>(y - x + 1);
    pushdown(k, x, y);
    int half = (x + y) >> 1;
    return sum(a, b, k << 1, x, half) + sum(a, b, k << 1 | 1, half + 1, y);
void add(int a, int b, int k, int x, int y, int v) {
    if (b < x \mid \mid a > y) return;
    if (a \le x \& b \ge y) {
        segTree[k].inc += v; // 将该区间内的元素都加v
        return;
    }
    pushdown(k, x, y);
    int half = (x + y) >> 1;
    int len = b - a + 1; // 相交区间的长度 (min(y, b) - max(x, a) + 1)
    if (a < x \&\& b >= x) len = b - x + 1;
    if (b > y && a <= y) len = y - a + 1;
    segTree[k].v += len * v;
    add(a, b, k << 1, x, half, v); add(a, b, k << 1 | 1, half + 1, y, v);
}
// rmq and rMq
pii tree[N<<2];</pre>
int lazy[N<<2];</pre>
int a[N];
int n. m;
inline int lch(int k) { return k << 1; }</pre>
inline int rch(int k) { return k << 1 | 1; }
void build(int k, int x, int y) {
    if (x == y) {
        tree[k].first = tree[k].second = a[x];
        return;
    }
    int h = (x + y) / 2;
    build(lch(k), x, h);
    build(rch(k), h + 1, y);
    tree[k].first = min(tree[lch(k)].first, tree[rch(k)].first);
    tree[k].second = max(tree[lch(k)].second, tree[rch(k)].second);
}
void pushdown(int k) {
```

```
tree[k].first += lazy[k];
    tree[k].second += lazy[k];
    lazy[lch(k)] += lazy[k];
    lazy[rch(k)] += lazy[k];
    lazy[k] = 0;
}
inline int getm(int k) {
    return tree[k].first + lazy[k];
inline int getM(int k) {
    return tree[k].second + lazy[k];
}
void add(int a, int b, int k, int x, int y, int v) {
    if (a \le x \& b \ge y) {
        lazy[k] += v;
        return;
    }
    pushdown(k);
    int h = (x + y) / 2;
    if (a \le h) add(a, b, lch(k), x, h, v);
    if (b > h) add(a, b, rch(k), h + 1, y, v);
    tree[k].first = min(getm(lch(k)), getm(rch(k)));
    tree[k].second = max(getM(lch(k)), getM(rch(k)));
}
int rmq(int a, int b, int k, int x, int y) {
    if (a \le x \& b \ge y) {
        return getm(k);
    pushdown(k);
    int h = (x + y) / 2;
    int ret = INF;
    if (a \le h) ret = min(ret, rmq(a, b, lch(k), x, h));
    if (b > h) ret = min(ret, rmq(a, b, rch(k), h + 1, y));
    return ret;
}
int rMq(int a, int b, int k, int x, int y) {
    if (a \le x \& b \ge y) {
        return getM(k);
    }
    pushdown(k);
    int h = (x + y) / 2;
    int ret = -INF;
    if (a \le h) ret = max(ret, rMq(a, b, lch(k), x, h));
    if (b > h) ret = max(ret, rMq(a, b, rch(k), h + 1, y));
    return ret;
```

主席树 #

维护着区间 $[1,i](i\in[1,n])$ 的信息(一段与区间有关的信息(如权值线段树什么的),主席树本质上就是n棵权值线段树,不过利用了重复信息压缩空间而已)

适用主席树的题目:满足与树状数组相似的区间减法

时间:与原生线段树相同

空间:每次修改增加 $O(\log n)$ 的空间

```
// HDU 2665: 求区间第k大的值
#include <bits/stdc++.h>
using namespace std;
const int INF = 0x3f3f3f3f;
const double EPS = 1e-8;
// const double PI = acos(-1.0);
// const int MOD = 1000000007;
typedef long long LL;
typedef unsigned long long uLL;
const int N = 100000 + 10;
const int M = 100000 + 10;
struct node {
    int 1, r; // pointers to 1 & r subtree
    int cnt;
    node() : 1(-1), r(-1), cnt(0) \{ \}
} pool[N * 20]; // 节点静态内存池
int n, m;
int w[N]; // 原序列的值
pair<int, int> ans[N]; // 离散后的值对原值的映射
int cnt; // 内存池的指针
int roots[N]; // 可持久化线段树
int query(int 1, int r, int x, int y, int k) {
    if (x == y) return x;
    int dif = (pool[pool[r].1].cnt - pool[pool[1].1].cnt); // 原序列[1,r]区间中的该节点元素个数
    int h = (x + y) >> 1;
    if (k <= dif) return query(pool[1].1, pool[r].1, x, h, k); // 说明第k小的值在左子树
    return query(pool[1].r, pool[r].r, h + 1, y, k - dif); // 否则在右子树, 此时k要减去dif
}
void add(int s, int t, int x, int y, int v) {
    // s代表上一版本的节点, t代表当前版本的节点
    pool[t] = pool[s]; // 复制上一版本的节点
    ++pool[t].cnt; // 修改
    if (x == y) return;
    int h = (x + y) >> 1;
    if (v \le h)
        add(pool[s].1, pool[t].1 = ++cnt, x, h, v); // 要修改的值属于左子树, 更新该版本的节点的左子树
    else
        add(pool[s].r, pool[t].r = ++cnt, h + 1, y, v);
}
```

```
void build(int u, int x, int y) {
   pool[u] = node();
   if (x == y) return;
   int h = (x + y) >> 1;
   build(pool[u].l = ++cnt, x, h);
   build(pool[u].r = ++cnt, h + 1, y);
}
void init() {
   build(roots[0] = cnt = 0, 1, n);
}
int main() {
   ios::sync_with_stdio(false);
   cin.tie(0);
   int t;
   cin >> t;
   while (t--) {
       cin >> n >> m;
       init();
       for (int i = 1; i \le n; ++i) {
           cin >> w[i];
           ans[i].first = w[i];
           ans[i].second = i;
        sort(ans + 1, ans + 1 + n); // 排序好后下标即为离散后的值, first即为原值
       for (int i = 1; i <= n; ++i) w[ans[i].second] = i; // 原序列值改为离散后的值
        for (int i = 1; i \le n; ++i) {
            add(roots[i - 1], roots[i] = ++cnt, 1, n, w[i]);
       for (int i = 0; i < m; ++i) {
           int s, t, k;
           cin >> s >> t >> k;
           cout << ans[query(roots[s - 1], roots[t], 1, n, k)].first << '\n';
       }
   }
   return 0;
```

莫队算法 #

莫队算法学习笔记 | Sengxian's Blog

普通莫队

离线算法,将询问分为 \sqrt{n} 个块后,按照 $(\lfloor \frac{l}{\sqrt{n}} \rfloor, r)$ 进行排序,然后依序处理。

 $O(n\sqrt{n})$

```
// 树上莫队 SPOJ COT2
#include <bits/stdc++.h>
```

```
using namespace std;
const int INF = 0x3f3f3f3f;
const double EPS = 1e-8;
//const double PI = acos(-1.0);
//const int MOD = 1000000007;
typedef long long LL;
typedef unsigned long long uLL;
const int N = 10000*4+10;
const int M = 100000 + 10;
int n. m:
vector<int> g[N];
int stk[N], sz; // 分块所用辅助栈
int block_size, block_cnt;
int w[N]; // 节点离散化后的权值
int bel[N]; // 每个节点所属的块
int dep[N]; // 节点深度
int pa[N]; // 节点老爸
struct qry{
    int u, v, id;
    bool operator< (const qry &b) const {</pre>
       return bel[u] < bel[b.u] \mid | (bel[u] == bel[b.u] && bel[v] < bel[b.v]);
    }
} query[M];
int ans[M];
bool vis[N]; // 节点是否在查询集合中
int nowAns = 0; // 每一轮的查询结果
int wcnt[N]; // 每个权值的计数
int anc[N][20]; // LCA
void LCAinit() {
    for (int u = 1; u \le n; ++u) anc[u][0] = pa[u];
    bool ok = false;
    for (int k = 1; !ok; ++k) { // 初始化anc表
        ok = true;
        for (int u = 1; u \le n; ++u) {
           if ((1 << k) > dep[u]) continue;
           anc[u][k] = anc[anc[u][k-1]][k-1];
           ok = false;
       }
   }
}
inline int getk(int u, int k) { // 得到节点u的第k个祖先
    if (k > dep[u]) return 1;
    for (int i = 0; (1<<i) <= k; ++i) {
       if ((1 << i) & k) u = anc[u][i];
    }
    return u;
}
```

```
int getLCA(int u, int v) {
   if (dep[u] > dep[v]) swap(u, v);
   v = getk(v, dep[v] - dep[u]); // 先让v走到和u相同深度的节点
   int l = 0, r = dep[u];
   while (1 < r) { // 二分搜索找到u、v最低的公共祖先
       int mid = 1 + (r - 1) / 2;
       if (getk(u, mid) == getk(v, mid)) {
           r = mid;
       }
       else {
           l = mid + 1;
       }
   return getk(u, 1);
}
void dfs(int u, int fa) { // 分块
   pa[u] = fa;
   dep[u] = dep[fa] + 1;
   int bottom = sz;
   for (int i = 0; i < (int)g[u].size(); ++i) {
       int v = g[u][i];
       if (v == fa) continue;
       dfs(v, u);
       if (sz - bottom >= block_size) {
           ++block_cnt;
           while (sz != bottom) {
               bel[stk[--sz]] = block_cnt;
       }
   stk[sz++] = u;
}
void discrete(vector<pair<int, int>> &vs) { // 离散化
   sort(vs.begin(), vs.end());
   w[vs[0].second] = 1;
   for (int i = 1; i < vs.size(); ++i) {</pre>
       if (vs[i].first == vs[i-1].first) w[vs[i].second] = w[vs[i-1].second];
       else w[vs[i].second] = i + 1;
   }
}
void flip(int u) { // "异或"操作
   if (vis[u]) {
       if (--wcnt[w[u]] == 0) --nowAns;
       vis[u] = false;
   }
   else {
       if (++wcnt[w[u]] == 1) ++nowAns;
       vis[u] = true;
   }
```

```
void moveto(int u, int v) { // 节点 u 向节点 v 转移
   if (dep[u] < dep[v]) swap(u, v);
   while (dep[u] > dep[v]) {
        flip(u);
        u = pa[u];
   while (u != v) {
       flip(u); u = pa[u];
       flip(v); v = pa[v];
   }
}
void solve() {
   LCAinit();
   int u = 1, v = 1;
   int LCA = 1;
   flip(1);
   for (int i = 0; i < m; ++i) {
        const qry &q = query[i];
       flip(LCA);
       moveto(u, q.u); moveto(v, q.v); // 转移区间
       flip(LCA = getLCA(q.u, q.v));
       ans[q.id] = nowAns;
        u = q.u; v = q.v;
   }
   for (int i = 0; i < m; ++i) printf("%d\n", ans[i]);
}
int main() {
   ios::sync_with_stdio(false);
   cin.tie(0);
   vector<pair<int, int>> vs;
   scanf("%d%d", &n, &m);
   block_size = int(ceil(sqrt(n)));
   for (int i = 1; i <= n; ++i) {
       int w;
        scanf("%d", &w);
        vs.emplace_back(w, i);
   discrete(vs);
   for (int i = 1; i < n; ++i) {
       int u, v;
        scanf("%d%d", &u, &v);
        g[u].push_back(v);
       g[v].push_back(u);
   for (int i = 0; i < m; ++i) {
```

```
scanf("%d%d", &query[i].u, &query[i].v);
query[i].id = i;
}

dep[0] = -1;
dfs(1, 0); // 分块
while (sz) bel[stk[--sz]] = block_cnt; // 分块操作的最后一块
sort(query, query + m);

solve();
return 0;
}
```

带修改莫队

将在线变为离线,按照 $(\lfloor \frac{l}{n^{\frac{2}{3}}} \rfloor, \lfloor \frac{r}{n^{\frac{2}{3}}} \rfloor, t)$ 三元组从小到大排序,t表示这个询问之前经过了多少次修改。

```
O(n^{\frac{5}{3}})
```

```
// BZ0J 2120
void udTime(int x, int op) {
    int pos = ms[x].a, val = ms[x].b, pre = ms[x].p;
    if (op > 0) {
        if (pos <= R && pos >= L) {
            if (--cnt[a[pos]] == 0) --now;
           if (++cnt[val] == 1) ++now;
        }
        a[pos] = val;
    } else {
        if (pos <= R && pos >= L) {
            if (--cnt[a[pos]] == 0) --now;
            if (++cnt[pre] == 1) ++now;
        a[pos] = pre;
    }
}
void update(int x, int op) {
    if (x == 0) return;
    cnt[a[x]] += op;
    if (cnt[a[x]] == 0 \&\& op < 0) --now;
    if (cnt[a[x]] == 1 \&\& op > 0) ++now;
}
void moveto(int 1, int r, int t) {
    while (T < t) udTime(++T, 1);
    while (T > t) udTime(T--, -1);
    while (L < 1) update(L++, -1);
    while (L > 1) update(--L, 1);
    while (R < r) update(++R, 1);
   while (R > r) update(R--, -1);
}
```

```
void solve() {
    S = pow(n, 2.0 / 3);
    sort(qs, qs + qn);
    now = 0;
    L = R = 0;
    T = mn;
    for (int i = 0; i < qn; ++i) {
        const qry &q = qs[i];
        moveto(q.1, q.r, q.t);
        ans[q.i] = now;
    }
}</pre>
```

其他

枚举技巧 #

枚举排列

```
// 比一般的生成法稍快,但生成的排列不按字典序
int num[N] = {1,2,3,4,5,6,7,8,9};
void go(int i) {
    // 函数开始时会得到一个新排列

    for (; i < N; ++i) {
        for (int j = i + 1; j < N; ++j) {
            swap(num[i], num[j]);
            go(i + 1);
            swap(num[i], num[j]);
        }
    }
}
```

枚举子集

```
inline void genSub(int s) { // s可为任意的集合
    for (int i = s; i; i = (i - 1) & s) {
        // i 即为一个子集
    }
}
inline void genRsub(int n, int r) {
    // n 为原始集合大小,枚举大小为 r 的子集
    for (int s = (1 << r) - 1; s < (1 << n); ) {
        // s 即为一个子集

        int x = s & -s, y = s + x;
        int t = s & ~y;
        s = ((t / x) >> 1) | y;
        // s = (t >> (__builtin_ctz(t) + 1)) | y; // optimization
}
```

}

```
inline bool isPowerOf2(unsigned n) {
   return (n \& (n-1)) == 0; // n \& (n-1) sets the last 1 bit of n to 0
}
inline int getLargestPowerOf2(unsigned v) {
   v |= v >> 1;
   v = v >> 2;
   v = v >> 4;
   v |= v >> 8;
   v = v >> 16;
   return v ^ (v>>1);
}
inline int getRoundUpPowerOf2(unsigned v) {
   v--;
   v |= v >> 1;
   v |= v >> 2;
   v |= v >> 4;
   v = v >> 8;
   V = V >> 16;
   v++;
   return v;
```

并查集 #

 $O(\log n)$

```
// 按秩求并的并查集
int find(int u) {
   if (fa[u] < 0) // 只有一个节点时高度为-1, 因为有可能节点编号从0开始
       return u;
   return fa[u] = find(fa[u]);
}
inline void setUnion(int a, int b) {
   int r1 = find(a), r2 = find(b);
   if (r1 == r2) return;
   if (fa[r1] < fa[r2]) {</pre>
       fa[r2] = r1;
       return;
   }
   if (fa[r1] == fa[r2]) {
       --fa[r2];
   }
   fa[r1] = r2;
```

表达式树 #

```
// ACM-ICPC 2018 沈阳赛区网络预赛 B
struct node {
    int op;
    int 1, r;
} tree[N];
int cnt;
int build(string &s, int x, int y) {
    bool ok = true;
    for (int i = x; i < y; ++i)
       if (!isdigit(s[i])) {
           ok = false;
           break;
       }
    if (ok) {
       auto &u = tree[++cnt];
       u.1 = u.r = 0;
       u.op = stoi(s.substr(x, y - x));
       return cnt;
    int p1 = -1, p2 = -1, p3 = -1, p = 0;
    for (int i = x; i < y; ++i) {
        switch (s[i]) {
           case '(':
               p++;
               break;
           case ')':
               p--;
               break;
           case '+':
           case '-':
               if (!p) p1 = i;
               break;
           case '*':
               if (!p) p2 = i;
               break;
           case 'd':
               if (!p \&\& p3 < 0) p3 = i;
               break;
        }
    if (p1 < 0) p1 = p2; // 没有括号外的加减号
    if (p1 < 0) p1 = p3; // 也没有乘号
   if (p1 < 0) return build(s, x + 1, y - 1); // 也没有d,则整个表达式被括号包着
    int tmp = ++cnt;
    auto &u = tree[tmp];
    u.l = build(s, x, p1);
    u.r = build(s, p1 + 1, y);
    u.op = s[p1];
    return tmp;
```

整体二分 #

```
// BZ0J 3110: 求区间第K大, 支持插入(修改), 查询。
// 对于每个查询,二分之前的修改(限定修改的值的范围,若不在范围内则不执行该修改)
// 由于需要对每个查询进行二分,所以需要依二分结果将操作分成两类
void solve(int ql, int qr, int L, int R) {
   if (ql > qr) return;
   if (L == R) {
       for (int i = ql; i <= qr; ++i) {
           if (qs[i].op == 2) ans[qs[i].id] = L;
       }
       return;
   int t1 = 0, t2 = 0, mid = L + (R - L) / 2;
   for (int i = ql; i <= qr; ++i) {
       if (qs[i].op == 1) {
           if (qs[i].c <= mid)</pre>
               q1[t1++] = qs[i];
           else
               q2[t2++] = qs[i],
               add(qs[i].a, qs[i].b, 1, -n, n, 1); // interval add
       } else {
           LL tmp = sum(qs[i].a, qs[i].b, 1, -n, n); // interval sum
           if (tmp < qs[i].c)
               qs[i].c = tmp, q1[t1++] = qs[i];
           else
               q2[t2++] = qs[i];
       }
   }
   // sort queries and clear tree
   int qmid = ql + t1;
   for (int i = ql; i < qmid; ++i) {</pre>
       qs[i] = q1[i - q1];
       if (qs[i].op == 1 && qs[i].c > mid) add(qs[i].a, qs[i].b, 1, -n, n, -1);
   for (int i = qmid; i <= qr; ++i) {
       qs[i] = q2[i - qmid];
       if (qs[i].op == 1 \& qs[i].c > mid) add(qs[i].a, qs[i].b, 1, -n, n, -1);
   solve(ql, qmid - 1, L, mid);
   solve(qmid, qr, mid + 1, R);
}
```

一些杂七杂八的东西

1. 某个元素在冒泡排序中被交换的次数等于其在逆序对中出现的次数。

2. $\left\lceil \frac{a}{b} \right\rceil = \frac{a-1}{b} + 1$

.vimrc

```
syntax on
set number " line number
set cursorline " show line cursor
set shiftwidth=4
```

```
set softtabstop=4
set tabstop=4
set expandtab " change tab to blank
set autoindent
set smartindent
set cindent
set showmatch " show match braces
set matchtime=3 " time for vim to wait for showmatch
set ruler

nnoremap <F4> :w <CR> :!g++ % -o %< --std=c++11 -Wall -Wshadow -g -fsanitize=address -
fsanitize=undefined && for i in ./in*; do echo $i; ./%< < $i; done <CR>
```

对拍相关 #

随机数生成

```
mt19937 rng(chrono::steady_clock::now().time_since_epoch().count());
int r(int s, int e) { return uniform_int_distribution<int>(s, e)(rng); }
```

脚本

```
Pecho off

:loop

data > .\cmake-build-debug\input.txt

main < .\cmake-build-debug\input.txt > .\cmake-build-debug\output.txt

test < .\cmake-build-debug\input.txt > .\cmake-build-debug\answer.txt

fc .\cmake-build-debug\output.txt .\cmake-build-debug\answer.txt > nul

if not errorlevel 1 goto loop

// checker

@echo off

:loop

data > .\cmake-build-debug\input.txt

main < .\cmake-build-debug\input.txt > .\cmake-build-debug\output.txt

test < .\cmake-build-debug\output.txt

if not errorlevel 1 goto loop
```

树数据生成

```
int main() {
    int n = r(2, 20);
    printf("%d\n", n);
    for(int i = 2; i <= n; ++i) {
        printf("%d %d\n", r(1, i - 1), i); // 前者是后者的父节点
    }
    return 0;
}</pre>
```