By the formula given on slide 7 of Lesson 4, "Linear Discriminant Analysis and Bayes' Rule":

$$P(Y=k|X=x)=p_k(x)=\frac{\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{1}{2}\left(\frac{x-\mu_k}{\sigma}\right)^2}\pi_k}{\sum_{m=1}^{K}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{1}{2}\left(\frac{x-\mu_m}{\sigma}\right)^2}\pi_m}$$

For this problem:

- K = 2 categories
- $\pi_0 = 0.2$ and $\pi_1 = 0.8$, because 80% of companies issue dividends (category 1).
- $\sigma = 6$, the joint standard deviation
- $\mu_0 = 0$ and $\mu_1 = 10$, the mean of X for companies that don't and do issue dividends.

So we can write

$$P(Y=0|X=x)=p_0(x)=\frac{\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{1}{2}\left(\frac{x-\mu_0}{\sigma}\right)^2}\pi_0}{\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{1}{2}\left(\frac{x-\mu_0}{\sigma}\right)^2}\pi_0+\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{1}{2}\left(\frac{x-\mu_1}{\sigma}\right)^2}\pi_1}$$

$$=\frac{\frac{1}{\sqrt{2\pi}\cdot 6}e^{-\frac{1}{2}\left(\frac{x-0}{6}\right)^2}0.2}{\frac{1}{\sqrt{2\pi}\cdot 6}e^{-\frac{1}{2}\left(\frac{x-0}{6}\right)^2}\cdot 0.2+\frac{1}{\sqrt{2\pi}\cdot 6}e^{-\frac{1}{2}\left(\frac{x-10}{6}\right)^2}\cdot 0.8}$$

We can factor $\frac{1}{\sqrt{2\pi}\cdot 6}$ out of the numerator and denominator and cancel it to get

$$=\frac{e^{-\frac{1}{2}\left(\frac{x-0}{6}\right)^2}0.2}{e^{-\frac{1}{2}\left(\frac{x-0}{6}\right)^2}\cdot 0.2+e^{-\frac{1}{2}\left(\frac{x-10}{6}\right)^2}\cdot 0.8}$$

$$=\frac{e^{-\frac{1}{2}\left(\frac{x}{6}\right)^2}0.2}{e^{-\frac{1}{2}\left(\frac{x}{6}\right)^2}\cdot 0.2+e^{-\frac{1}{2}\left(\frac{x-10}{6}\right)^2}\cdot 0.8}$$

Then factor 0.2 out of the numerator and denominator to get

$$=\frac{e^{-\frac{1}{2}\left(\frac{x}{6}\right)^2}}{e^{-\frac{1}{2}\left(\frac{x}{6}\right)^2}+4e^{-\frac{1}{2}\left(\frac{x-10}{6}\right)^2}}$$

Then factor $e^{-\frac{1}{2}\left(\frac{x}{6}\right)^2}$ out of the numerator and denominator to get

$$= \frac{1}{1 + 4e^{-\frac{1}{2}\left(\frac{x-10}{6}\right)^2 - \left[-\frac{1}{2}\left(\frac{x}{6}\right)^2\right]}}$$

(Recall that dividing e^a/e^b is the same as e^{a-b}.)

Now, let's simplify the exponent of $e^{-\frac{1}{2}\left(\frac{x-10}{6}\right)^2 - \left[-\frac{1}{2}\left(\frac{x}{6}\right)^2\right]}$.

$$-\frac{1}{2}\left(\frac{x-10}{6}\right)^2 - \left[-\frac{1}{2}\left(\frac{x}{6}\right)^2\right] = -\frac{1}{2}\left(\frac{x-10}{6}\right)^2 + \frac{1}{2}\left(\frac{x}{6}\right)^2$$

$$= -\frac{1}{2}\left(\frac{x^2 - 2 \cdot 10x + 100}{6^2}\right) + \frac{1}{2}\left(\frac{x^2}{6^2}\right)$$

$$= \frac{1}{2}\left(\frac{x^2}{6^2}\right) - \frac{1}{2}\left(\frac{x^2 - 2 \cdot 10x + 100}{6^2}\right)$$

$$= \frac{1}{2 \cdot 36}\left(x^2 - (x^2 - 20x + 100)\right) =$$

So our original conditional probability becomes

$$P(Y = 0|X = x) = \frac{1}{1 + 4e^{\frac{1}{72}(20x-100)}}$$

The derivation for $P(Y = 1|X = x)$ is similar.