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Evaluating the cosmos Equation Detection and eqdec Equation Decoding Models

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ABSTRACT

We test the robustness of Wisconsin's equation detection pipeline (`cosmos`) and the AutoMATES equation decoding model (`eqdec`) against a small sample of papers. The dataset includes 4 papers from a single domain, which have been submitted to different journals over a 90 year period, ensuring a wide diversity of fonts, layouts, and resolution. We find that both `cosmos` and `eqdec` model performs the best on newer papers, with `eqdec` always performing better with the manually-generated input, holding equation length as a constant. We find that `eqdec`'s main weakness is its handling of multi-lined equations; the decoded LaTeX of most multi-lined equations either fail to render or drop a significant fraction of lines. By breaking multi-lined equations into smaller 1- and 2-lined equations, we find a performance increase with the `eqdec` model.

1. DATA

To test `cosmos` and `eqdec`'s performance, we select a subset of papers under a single domain with diverse characteristics as shown in Figure 1. Due to a surge in interest in modeling infectious diseases brought on by the recent COVID-19 pandemic, we perform a literature search for Susceptible, Infectious, or Recovered (SIR) models. The following subsections describe the PDF sources that were found in the literature search which we included in our dataset. Table 1 summarizes the page and equation counts of each paper.

1.1. *Wikipedia Excerpt*

To evaluate model performance, we include a excerpt from a Wikipedia article on the bio-mathematical deterministic treatment of the SIR model as our baseline example. This article is formatted using a modern LaTeX journal article template to test both `cosmos` and `eqdec` using the most ideal example: a short, high-resolution paper with a clear formatting scheme and relatively simple equations. Notably, all 13 equations are represented in their own equation environments. (i.e., There are no matrices or multi-line equations.) The most complex equations contain fractions and variable subscripts in exponents.

Table 1. Equations Detected

Paper	Page Count	Equation Env. Count
Wikipedia Excerpt	3	13
Kermack & McKendrick (1927)	22	86
Anderson & May (1979)	7	14
Hethcote (2000)	55	85
Miller (2017)	21	52
Total	108	248

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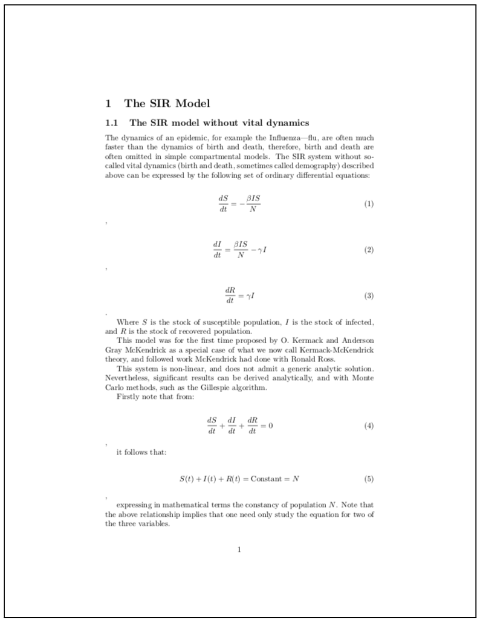
1.2. *Kermack & McKendrick (1927)*

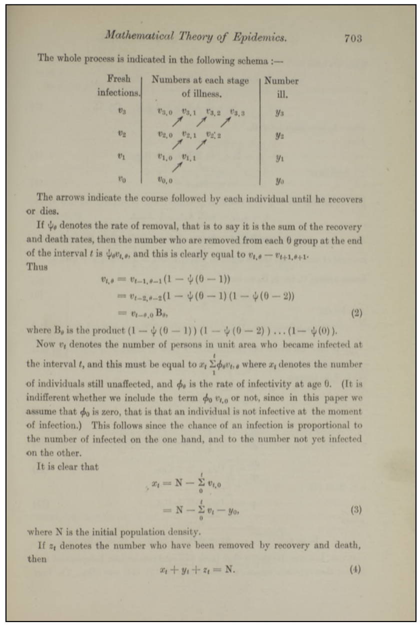
The Kermack & McKendrick (1927) is the seminal work formalizing the mathematical description of the spread of infectious diseases. Kermack & McKendrick's work is only available in digital form as a scanned copy of the original work. While the paper and equations are well-formatted, the slight rotation, non-white background color, low-resolution, and image artifacts from the scanning process pose a serious challenge for both the `cosmos` equation detection and `eqdec` equation decoding pipelines. As such, this example is the document that poses the greatest challenge for both pipelines within the dataset. Both pipelines must be able to handle this document if the pipelines are to handle photographs of documents in the future. We count a total of 84 separate equations environments in this 22-page paper. Many equation environments contain multi-line equations, the longest spanning 6-lines. In the most complex examples, there are multi-line equations with bounded integrals in both numerators and denominators of fractions that span the entire page width.

1.3. *Anderson & May (1979)*

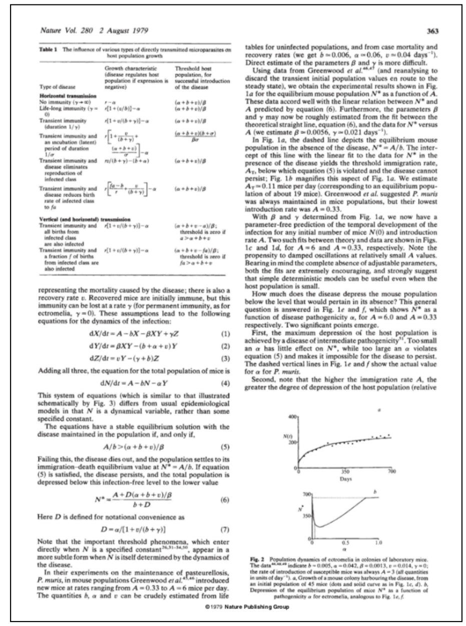
This short, 7-page Nature review article from Anderson & May (1979) is notably the only paper with a double-column format in our sample dataset. Because this older article has been digitized with OCR, the resolution of this paper is low compared to more



(a) Wikipedia Excerpt

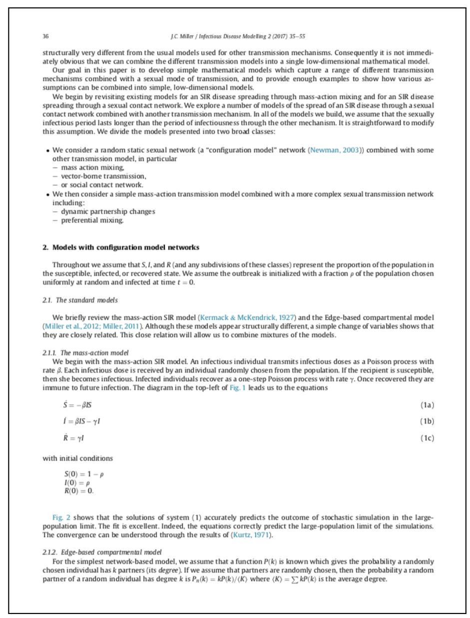


(b) Kermack & McKendrick (1927)



(c) Anderson & May (1979)

(d) Hethcote (2000)



(e) Miller (2016)

Figure 1. We test the robustness of the Wisconsin's equation detection model (`cosmos`) and the AutoMATES equation decoding model (`eqdec`) on a variety of papers. We include a screenshot of the first page of each paper containing multiple equations to show the variety of formats we test.

recent LaTeX-rendered documents. However, this article does not exhibit the rotation, non-white background color, and image artifacts present in the Kermack & McKendrick (1927) example. We expect Anderson & May (1979) to be representative of most older articles that our models would encounter. All 18 equations are numbered and grouped into 14 unique equation environments–the smallest equation environment count amongst the 4 published papers. While these equations are fairly isolated from the text and, on average, feature the simplest equation within the published papers, we anticipate a potential complication with equation detection due to one of the tables present in the paper. As seen in Figure 1c, Table 1 in the paper features multiple inline equations.

1.4. *Hethcote (2000)*

Hethcote (2000) is a more recent review article, featured as both the longest and the only LaTeX-rendered PDF in our dataset, excluding the Wikipedia excerpt. At 55-pages, the document hosts 85 unique equation environments. A large fraction of these equations span multiple lines, with the longest multi-equation environment spanning a total of 9-lines and longest single equation broken into 3 lines. In addition to the long, multi-lined equations, there are many instances of multiple equations occupying the same line in the same equation environment, separated by small horizontal white space. Furthermore, the equations in this paper often break standard practices of limiting LaTeX-equations to 3-levels; the most notable example features variables with exponents and subscripts in fractions nested inside the numerators and denominators of other fractions.

1.5. *Miller (2017)*

Unlike the rest of the articles in the dataset, Miller (2017) is distinctly formatted in a standard word processing program (e.g., Microsoft Word). The non-LaTeX-generated equations exhibit unique spacing and misaligned, multi-line equation environments.

2. METHODS

2.1. *Manual Equation Detection*

To quantify detection and decoding statistics, we first need to delineate a set of conditions to determine an equation environment. For LaTeX-rendered documents, we define an equation environment as a single or group of non-inline equations that could be generated in one of the following display, math environments:

- `\begin{equation} ... \end{equation}`
- `\begin{align} ... \end{align}`
- `\begin{gather} ... \end{gather}`
- `\[ ... \]`

Equation environments containing single equations are easy to identify due to its relative isolation from the surrounding text in the document. However, when multiple consecutive equations are found, it maybe harder to determine whether the set of equations was generated from a single, multi-line, math environment or multiple math environments. Figure 2 shows a sample of 14 manually-determined, equation environments. Each equation environment possesses the following traits:

1. Contains 1 or more equations on the same or separate, consecutive lines.
2. Does not contain breaks or in-line text between equations.
3. Spans multiple lines if (a) 1 or multiple breaks occur in a long equation or (b) multiple equations occupy successive lines.
4. May consist of nested environments within the math environment, including but not limited to `cases`, `aligned`, and `array` environments.

To render the equations, we use the `standalone` document class for our LaTeX template. (See Lines 5-12 in the `render_image_from_latex.py` script, which will be further discuss in Section §2.4.) Because the standalone document class requires all equations to be in in-text format (`$ ... $`) when generated, the manually-generated LaTeX tokens must compile and render when placed within the `\begin{equation} ... \end{equation}` math environment. This ensures that there will be no compilation errors when inserting the same tokens in the in-text equation format (`$ ... $`). More explicitly, manually-generatd LaTeX tokens will not compile and will fail to render if they are written using the `align` or `gather` environments, posing a challenge for multi-lined equations. However, to mimic the `align` environment within an in-lined equation, we can nest the `aligned` environment inside the `equation` environment. While this method enables us to create multi-lined equations aligned on specified characters, we are currently unable to reproduce the center alignment generated by the `gather` environment.

All equations in a document are organized into a single input file, with a single equation environment occupying a single line in the input. While multi-lined equation environments are often written on multiple lines for clarity, these extra lines must be removed as the `render_eq_img_from_latex.py` script expects 1 equation environment per line. Lastly, the leading and trailing `\begin{equation} ... \end{equation}` markers must be removed, leaving any nested equation environments in tact.

Furthermore, we edit the `standalone` template for this specific dataset, which includes papers that uses the Dirac Bra-ket notation for expectation values and symbols from Ralph Smith's Formal Script (RSFS) font. The following is the editted version of the template:

```
def standalone_eq_template(eqn):
    template = '\\documentclass{standalone}\n' \
        '\\usepackage{amsmath}\n' \
        '\\usepackage{amssymb}\n' \
        '\\usepackage{braket}\n'  \
        '\\usepackage{mathrsfs}'  \
        '\\begin{document}\n'     \
        f'$\\displaystyle {{{{ {eqn} }}}} $\n' \
        '\\end{document}'
    return template
```

Once the above changes have been implemented to the template and input file containing the manually-

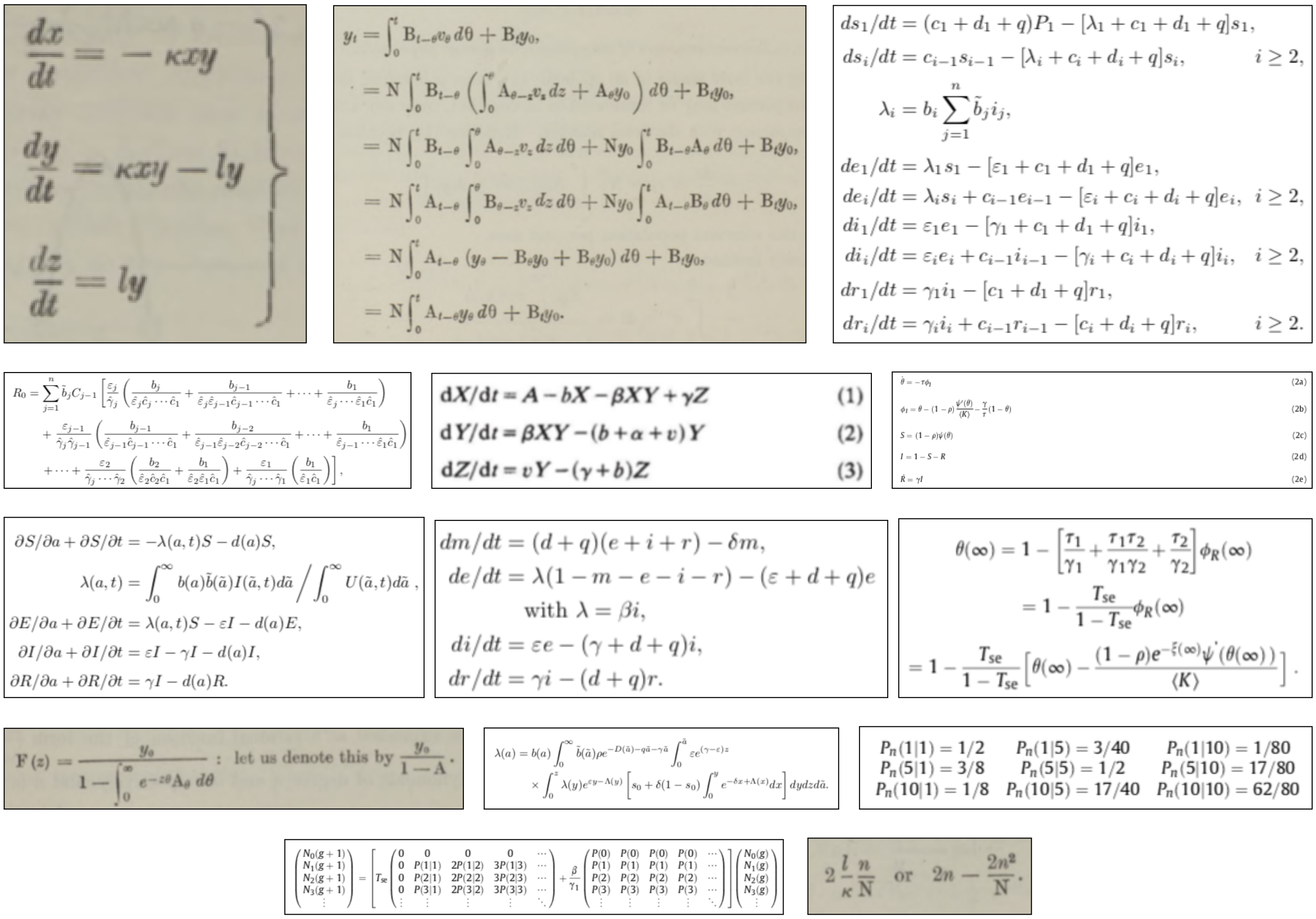


Figure 2. 14 examples of multi-lined equations considered as a single equation environment.

generated LaTeX tokens is created, follow the directions outlined in Section §2.4.

2.2. *Docker Application for* `cosmos`

With Docker unavailable on the University of Arizona's HPC system, we opted to use Clara to run the `cosmos` equation detection pipeline. However, we could not make use of Clara's GPU due to the unavailability of `docker-compose` (Version 2.3). However, the older Version 2.2 was sufficient to run the Docker application on Clara's compute nodes. Once on Clara, we cloned the `cosmos`'s stand-alone CPU `docker-compose` application from the `w2v` branch of the `cosmos`'s GitHub repository. To run this application on Clara, we changed the first line of `docker-compose-standalone-CPU.yml` script to `version: "2.2"`. The script builds the docker image and runs the applications with the following command:

```
OUTPUT_DIR=./output INPUT_DIR=./input \
DEVICE=cpu docker-compose -f \
docker-compose-standalone-CPU.yml up
```

providing an `input` directory with all 5 PDF documents to be processed. If an `output` directory exists prior to running the `docker-compose` script, the permission settings on the directory must be changed to allow access for everyone. Otherwise, the script will hang and will fail to produce an output. Note that the Docker image requires 9 GB of free space. Once the image is built, it will be available for future runs, saving roughly 30 minutes of computing time. We do not recommend running the `cosmos` application on a personal computer. The job processed the 5 documents within 1-day on Clara; however, we could not complete the job in 3 days on a mid-2012 MacBook Pro with a 2.6-GHz Intel i7 processor and 8 GB of memory. The process appeared to hang due to limited memory.

2.3. *Docker Image for* `eqdec`

To build the `eqdec` Docker image, we clone the `master` branch of the `AutoMATES` github repository. While instructions to build and launch the image can be found in the `README.md` in the `automates/src/equation_reading/equation_translation/eqdec` directory, important `README.md` instructions are missing. Namely, Line 12 of the Dockerfile requires an unmentioned model checkpoint `arxiv2018-downsample-aug-model_step_`

80000.pt. This checkpoint can be retrieved from `Clara` in the directory /data/nlp/corpora/arxiv/opennmt. Once this file is placed in the same directory as the Dockerfile, the `eqdec` image can be built using the following command:

```
docker build -t eqdec .
```

2.4. *Running the Equation Decoding Pipeline*

Detailed directions and code to run the `eqdec` equation decoding pipeline can be found in the automates/scripts/eqn_reading/eqn_translation directory in the `eqn_clay` branch of the `AutoMATES` repository. In short, the equation decoding pipeline is broken down into 3 scripts. We describe the basic functions, bugs, and helpful hints to run these scripts:

- render_eq_img_from_latex.py: This script converts manually produced LaTeX tokens into cropped images of equations. (Skip this step and continue to the batch decoding step when using the `cosmos` equation detection pipeline.) As mentioned in Section §2.1, an input file named <paper-id>_equations.txt with each line containing the contents of a single equation environment (in order of appearance in a document) must be created. This script calls render_image_from_latex.py from the automates/src/equation_reading/equation_extraction directory to render each equation by inserting each equation into the template and running `pdflatex`. The template from this secondary script can be altered as outlined in Section §2.1 to include any additional packages as needed. This script generates two directories: (1) manual_latex which contains a `tex` subdirectory with `.tex` files and their corresponding `.pdf` renderings for each equation (using the 0-indexed equation number as its filename) and (2) manual_eqn_images which contains `.png` images of the `.pdf` renderings.

- batch_decode_eq_images.py: The script calls img_translator.py in the automates/equation_reading/equation_translation directory, which runs the the `AutoMATES`'s equation decoding model pipeline, to decode cropped images of equations into LaTeX tokens. To run this script, the `eqdec` application must be running. See Section §2.3 for installation instructions. Once installed, use the following command to launch the server:

```
docker run -d -p 8000:8000 -t eqdec
```

The script requires an input directory manual_eqn_images (which contains rendered `.png` images of equations) and produces the decoded_images directory. In the `json` subdirectory of decoded_images, the script outputs tokenized LaTeX from the `.png` images in manual_eqn_images using the `OpenNMT` model. The generated LaTeX for each equation image are stored in `.json` files.

At this point in the pipeline, an innocuous bug occurs in the file naming convention of the tokenized LaTeX output; the `.json` files have the file extension `.json` appended to the respective decoded equation image without first removing the previous `.png` file extension. This bug occurs in Line 25 of img_translator.py script, and can be fixed by altering Line 25 to:

```
img_basename = input_path[dir_path_index:
file_ending_index]
```

After the `.json` output files are produced, all `eqdec`-generated LaTeX tokens from all equations in a single document are stored into a single decoded_equations.txt file, formatted in the a same fashion as <paper-id>_equations.txt. Each line from decoded_equations.txt is then rendered using `pdflatex`, which is called from the render_image_from_latex.py script. This generates another `tex` subdirectory which contains both the `.tex` and rendered `.pdf` files for each equation. The rendered `.pdf` files are then converted to `.png` images in the decoded_images directory.

We have identified several issues in running this section of the `eqdec` pipeline after the `OpenNMT` model decodes the images. The main caveat is the model's inability to guarantee compilable LaTeX syntax. In the cases of long, multi-lined equations, the model may produce a respectable output that misses an ending brace, drops parts of the equation, and/or repeats series of numbers that fail to converge. In the case of older papers, low-resolution cutouts of equations are often coupled with a non-white background, rotation, and/or image artifacts, preventing the model from understanding the input image and resulting in unintelligible LaTeX output. In both such cases, the model often produces uncompilable LaTeX tokens causing `pdflatex` to hang while attempting to render. The script halts and waits for user input for all such equations, and in rare cases, hangs until the script is terminated.

While we have yet to determine the definitive cause for the hanging response, we outline a series of recommendations for future improvements to the pipeline to alleviate the burden of human intervention at each step.

1. Run `pdflatex` in "nonstopmode" to reduce the number of interactions with the pipeline

by altering Line 63 in render_image_from_latex.py.

```
command_args=['pdflatex'
,'-interaction','nonstopmode'
,'-output-directory',eqn_tex_dst_
root,eqn_tex_file]
```

2. Skip uncompilable equations by incorporating Line 68 of render_image_from_latex.py into a try/except block, passing over uncompilable equations.
3. Render "None" for all equations that fail to compile. Currently, the pipeline creates an .png image with the text "None" for a small fraction of equations. When "None" is not produced, no .pdf or .png files are generated for the respective equation. While we have yet to identify why only a small subset of uncompilable equations produce this "None" output, a quick remedy would be to generate this image in the above try/except block when pdflatex fails to render the equation.

- compare_decoded.py: This script generates a two-column, side-by-side comparison between (1) the renderings of the manually-generated LaTeX tokens or cosmos-detected equation cutouts with (2) renderings generated by eqdec. We can evaluate eqdec's performance using this side-by-side comparison.

  A ordering error occurs when eqdec fails to produce a compilable LaTeX tokens, resulting in an unrendered equation. The compare_decoded.py script retrieves the next properly rendered equation image, creating an equation numbering offset between the first column with either the rendered images of the manually-generated LaTeX tokens or the images of detected equations and the second column with the rendered images of eqdec-generated tokens. While this problem would be fixed when "None" images are produced as intended for all uncompilable equations, we recommend implementing a second fail-safe to check that every row in the comparison table are from matching equations. When such a mismatch occurs, the comparison .HTML file cuts off the table to the last pair of images occurring in the same row. Every unpaired equation thereafter is not included in the comparison table. Altering the script to include all unpaired equations will help identify any bugs which may occur further upstream in the pipeline. We manually corrected this offset error for this analysis in a post-process step by (1) searching for missing rendered images for each equation environment in a document and (2) copying the "None" .png image for every missing rendering with the appropriate equation number in its filename.

  Furthermore, we recommend one additional change to allow for a more descriptive labeling between the comparison tables in the .HTML file. As it stands, the script uses the parameter model_name to search for relevant files to be used by the script and to label each table in the comparison .HTML output, as define in Lines 39-40 in compare_decoded.py. Adding an additional entry in the dictionary for labeling each table will enable for more descriptive information to be added (as opposed to the current 1-word scheme) to clarify between the columns being compared, given that the same pipeline is being used to compare the eqdec results with manually-generated LaTeX tokens and with image cutouts from the paper. Adding additional context will help distinguish between the two different comparisons.

2.5. *Combining the cosmos & eqdec Pipelines*

We first build and run the cosmos Docker application (described in Section §2.3) on Clara with an input directory containing all PDF documents. The output directory produced contains a README.md document, 3 .csv files for figures, output, and tables, and 4 subdirectories (html, html_out, images, and xml). Every page from all inputted documents are processed and extraction results from each page are deposited in a new subdirectory named <paper_id>.pdf_<page_number> in output/html/img. The identified extractions are labelled as a "body text," "equation," "figure," "figure caption," "page header," "section header," "table," "table caption," or "other." We manually inspect all extracted images from the cosmos detection pipeline as the model often fails to recognize equations, mislabels other types of extractions as equations, and misidentifies equations as other types of extractions. When the model successfully segments an equation environment, we consider this a detection regardless if the model mislabels the type of extraction.

For each unique document, we manually create a directory named manual_eqn_images. We aggregate all successful detections in each document and take screenshots of the remaining, undetected equations, renaming them (in order of their appearance in the document) with their zero-indexed equation number. This simulates the output of render_eq_img_from_latex.py to run the eqdec batch decoding script. The remainder of the eqdec pipeline remains the same and instructions from Section §2.4 should be used.

3. RESULTS

We analyze the performance of (1) the cosmos equation detection pipeline and (2) the eqdec decoding pipeline for both the manually-generated LaTeX and the cosmos-identified equations. We note that this analysis is purely observational and no metrics have been used to quantify these results.

In total, we analyze 4 documents (Kermack & McKendrick 1927; Anderson & May 1979; Hethcote 2000; Miller 2017) for the manual LaTeX decoding task. We add a fifth document (i.e., the Wikipedia excerpt) for the combined `cosmos` extraction and `eqdec` decoding task.

3.1. *Decoding Manually-generated LaTeX Renderings*

Decoding renderings of manually-generated LaTeX tokens enables the identification of prominent equation features for which the `eqdec` decoding model has trouble deciphering. This task of decoding manually-generated LaTeX tokens is an idealized version the combined equation detection and decoding task by approximating the output of the detection model with the renderings of the manually-generated LaTeX. This approximation is only valid with the availability of high resolution, LaTeX-rendered documents with standardized fonts and a nearly-perfect equation detection model. We show the limitations of these assumptions in Section §3.2.

The results of the `eqdec` decoding task for manually-generated LaTeX renderings are included in the Appendix. Tables 3, 4, 5, and 6 show a summary of this task for each equation environment analyzed in the Kermack & McKendrick (1927), Anderson & May (1979), Hethcote (2000), and Miller (2017) documents, respectively. Every equation environment is identified by a zero-indexed number assigned in order of appearance in each document, denoted as "Equation Environment" in the tables. To assess how well the model handles multi-line equations, we include a "Line Count," which counts the number of lines occupied by a single equation environment, regardless of whether the environment is composed of a single, long equation broken up on separate lines or a collection of multiple, shorter equations. The "`eqdec` LaTeX Rendered?" column denotes whether the `eqdec` model produced a compilable LaTeX equation, resulting in a `.png` image rendering of the equation. The "Lines Rendered?" column denotes the number of lines in the rendered equation. If the `eqdec` renderings do not replicated the format or spacing present in the manually-generated renderings (which was generated to mimic those present in the document equations), we make note of this in the "Alignment Issues?" column. Lastly, we note the presence of any anomalies in the "Additional Notes" column.

We briefly summarize the results of Tables 3, 4, 5, and 6 to highlight key problems with the `eqdec` model:

- Kermack & McKendrick (1927) has 86 equation environments spanning 22-pages. This document includes some of the most challenging equations to decipher due to the equation length and complexity. Of the 86 environments, 20 equation environments span multiple lines. The `eqdec` model decoded all 66 single line equation environments without any error. However, of the 20 multi-lined environments (with 11 spanning 2-lines, 4 spanning 3-lines, 2 spanning 4-lines, another 2 spanning 5-lines, and the final one spanning 6-lines), (1) just under half of all 2-lined environments failed to render (18.2%), rendered partial equations (9.1%), or experienced alignment problems (18.2%); (2) half of all 3-lined environments failed to render (25%) or rendered partial equations with alignment problems (25%); and (3) most alarmingly, all environments with 4-lines or greater either failed to render (40%) or produced a partial render (60%) with some alignment problems. In nearly all but one case where the model yielded a partial render, only the first 2-lines of the environment were reproduced. The exception, while also producing a 2-line environment, decoded the first and last lines of a 3-lined equation environment. We note that of the equations that were decoded, the `eqdec` model only made 3 decoding errors: 2 errors involved dropping a brace around multiple equations and 1 error misidentifying a brace for a parenthesis. All decoding errors occur in multi-lined equations.

- Across 7 pages, Anderson & May (1979) contains a total of 14 equation environments: 12 of which are single-lined and 2 of which have 3-lines. In Anderson & May (1979), the `eqdec` pipeline achieves excellent performance by accurately decoding all 14 equations with one small exception of duplicating β at the end of Equation Environment #13 (a single-lined equation environment).

- The longest of all documents, Hethcote (2000) contains 85 equations environments over 55-pages. 23 out of 85 equation environments are multi-lined with 8 spanning 2-lines, 7 spanning 3-lines, 2 spanning 4-lines, another 2 spanning 5-lines, 3 spanning 5-lines, and the last spanning 9-lines. All 10 environments that fail to render are multi-lined equations. Two-thirds of all equations composed of 2- or 3-lines either fail to render (46.7%), render part of the equation (6.7%), or has alignment issues (13.3%). All of the equations composed of 4- or more lines do not render (37.5%) or yield a partial render with majority containing mistakes (62.5%). Of the remaining 62 single-lined equations, 7 contain mistakes (see #22, 46, 53, 69, 74, 75, 76). In all but one case, terms were dropped (57.1%) or multiple terms were combined (28.5%) in unusually long, complex, single-lined equations with fractions.

- Miller (2017) has 52 equation environments spanning 21-pages, 25 of which are multi-lined equations. There are 9 instances of 2-lined equations, 5 with 3-lines, 4 with 4-lines, 2 with 10-lines, and single instances of 5-, 6-, 11-, 12-, and 14-lined equations. Of these 52 equation environments, eqdec fails to produce compilable LaTeX for 8, 7 of which are multi-lined. In the cases of the 2- and 3-lined equations that do not render, all are fairly long with unique alignment or equation structure. The lone, uncompilable, single-lined equation is a long matrix equation with matrices of rank 5. All equations spanning more than 5-lines compile, but only include the first 3-4 lines with some exhibiting minor mistakes. The eqdec model performs fairly well with majority of the mistakes stemming from dropped lines in multi-lined equations and dropped terms in long equations with complex structure.

3.2. *Equation Detection Task*

The Wikipedia excerpt is an test case for the most ideal document for the cosmos extraction pipeline; the document is short (3-pages), well-formatted, contains no figures or tables, contains only single-lined equations, and is rendered in LaTeX with high resolution. We use the results of the Wikipedia excerpt as the best possible model performance achievable with the cosmos pipeline. The model successfully detects 12 out of 13 equations, correctly labelling all but one as an "equation." The last detected equation is labeled as a "figure." The one undetected equation on the last page of the PDF was identified with surrounding text as a "page header." The other 9 objects identified as "equations" are all equation numbers.

- Wikipedia excerpt serves as the optimal document for the extraction pipeline. In the first 12 of the total 13 equations environments present, the detection pipeline accurately extracts all equations. 11 of the 12 are correctly identified as "equations" with the remaining instance labeled as a "figure" with its equation number included. The last equation, occurring on the last page of the document, is extracted as a "page header" along with all surrounding text. We suspect that the model identified this object as a "page header" because the last page of the document is relatively short and contains 1 paragraph located near the top of the page. The cosmos model also extracts equation numbers as separate "equation" objects, so roughly half (55%) of all "equation" objects are true instances of the equation. To streamline the process of running the eqdec model after the cosmos detection pipeline, the cosmos model can be improved upon by separately labeling equation numbers into a different category from the document equations.

- Kermack & McKendrick (1927) serves as the most challenging document to perform extraction because the appearance of the scanned document starkly contrasts the training set on which the cosmos extraction pipeline was likely trained. Not surprisingly, cosmos fails to detect all 86 equation environments. All 22-pages in the document were detected as single objects containing the entire span of the page. While each object appears similar, the cosmos model detected these objects as "tables" in 10 cases, "figures" in 3 cases, "page headers" in 2 cases, "others" in 6 cases, and as a "reference text" in the last case. Aside from its inability to extract anything meaningful in this particular document, this variation in labeling these page-long extractions demonstrates the models inability to accurately identify its extractions. This appears to be a common phenomenon through most documents we test, and we must manually analyze each extraction as a majority of extractions are incorrectly labeled.

- Of the 14 equations environments in Anderson & May (1979), the cosmos pipeline accurately detects 4, with cosmos identifying 3 page headers and 1 figure as the remaining 4 equations in its database. The system fails to pick up the additional 10 equation environments. In 2 instances, equation environments are grouped with small segments of in-line text and labelled as "body text." (e.g., 1 equation environment consisting of 3 equations is grouped in one "body text." 2 equation environments, consisting of 1 equation each, are grouped with 2 segments of text in another "body text.") In another 2 instances, equation environments are grouped together with surrounding text and other page elements and labelled as "figures." One of the figures is indistinguishable with inaccurately labelled "body text" denoted above. In the 2nd instance, the pipeline fails to detect individual elements in a particularly challenging page consisting of tables, figures, text, and equations. The cosmos pipeline instead identifies the entire page, including 5 equation environments (4 single equation environments and 1 multi-lined environment) as a "figure."

- The cosmos pipeline failed to detect all 85 equation environments in Hethcote (2000). All equations were embedded in "table" and "body text" objects. The images of these objects contained the cropped image of all the contents of the entire page, including text, equations, tables, and figures, excluding the 1" bordering margins on all sides. The only objects consistently detected correctly in this document were "page headers." As such, all equations were manually acquired by hand.

- The Miller (2017) document provides unique insight into potential performance gains for the eqdec model by breaking single, multi-lined equation environments into multiple, smaller equations. Of the 52 equation environments, 48 environments are successfully detected in its entirety in 92 separate detections. The `cosmos` model breaks long environments into 1- and 2-lined equations to be processed by `eqdec`. The 4 undetected equations are all long and single-lined. They are all detected along with a single-line of adjacent text as "figure" objects. Of the total 96 unique detections containing equation environments, 20 are "figure" objects, 7 are "table" objects, 1 is a "page header" object, 1 is labeled "other", and all remaining are correctly classified as equations. Of the 80 "equations" detected by the `cosmos` model, 9 are equation numbers and 4 are segments of figures which include mathematical symbols; the remaining 67 are equations.

3.3. *Decoding `cosmos`-detected LaTeX*

- The `eqdec` model performed perfectly on the Wikipedia excerpt, decoding all equation environments without error.

- Of the 86 equation environments in Kermack & McKendrick (1927), the `eqdec` model produced compilable LaTeX for 9 equations (10.5%): 3 (3.7%) of which are multi-lined and 6 (7.0%) of which are multi-lined. However, all the compilable LaTeX produced were nonsensical outputs. All rendered equations showed similar features to one another, including (1) one or multiple boxes surrounding the generated equation, (2) multiple nested fractions, (3) repeating variables and numbers, (4) bars over variables, and, in some cases, (5) square roots in the equation. While we should not read too deeply in the generated output, we speculate that the boxes were generated due to the background color of the equations, which sharply contrasts with the arXiv dataset used to train the `eqdec` model. Perhaps preprocessing such PDFs to remove the non-white background would help the model handle such documents.

- We used the cropped images provided by `cosmos` for the 4 equations detected in Anderson & May (1979) and captured screenshots of the remaining 14 equation environments. We achieved decent results with the `eqdec` decoding of all 4 detected equations. Minor errors were present in 3 of the 4, including a missing slash, a change from a left parenthesis to a curly brace, and changes in font and bold-style in variables. The results from the manual screenshots provide a stark contrast to this picture. None of the tokenized LaTeX output from the 14 equations with screenshots yielded intelligible equations. 3 of the 14 failed to render. In many cases, the screenshots and the rendered `eqdec` output appear to have similar features; the decoding pipeline understands the general structure of the equation and places fractions, comparison/assignment operators, and exponents in the proper locations. However, decoding variables from a downsampled screenshot often falters. Namely, downsampled screenshots appear to produce a significant number of blackboard bold-style variables and variables that look similar to those used in the original equation. For example, in Equation Environment #1, A is decoded as $\mathcal{A}$ and N is decoded as $\mathbb{N}$. Lastly, we note that in 2 cases, decoding downsampled screenshots produces a repetitive series of variables–a feature occasionally seen when the model decodes equations with numbers.

- After manually acquiring the screenshots of all 85 equations environments in Hethcote (2000), we find that the `eqdec model` fails to generate compilable LaTeXin 33 cases (38.8%). 20 of the 33 instances are multi-lined equations, with multi-lined equation composing 23 of the total 85 environments in the document. Of the remaining 52 instances in which `eqdec` produced compilable results, the model failed to produce accurate LaTeX decodings for all but 5 instances (see #0, 60, 62, 63, 64). Due to the sheer amount of mistakes, we opt not to record these as notes in Table 5. The 5 environments with the accurate decoded LaTeX output were all short, single-lined equations. While the decoded LaTeX of many others instances were nearly correct, we observed the model repeating the same mistakes. Common mistakes included mistaking letters and symbols for look-alikes (e.g., s_0 for 8_0 and a for d), randomly inserting line-like (e.g., l, $|$, $/$, and $\int$) symbols, randomly adding small characters (e.g., *), and interchanging parenthesis, brackets, and braces for one another. Often, we can understand why the `eqdec` model may make small mistakes. We suspect these issues arise due to the resolution quality of the input images in the `eqdec` model or due to the font this paper uses. Expanding the training set to include older papers or augmenting the current papers by rendering them with different fonts or resolutions may help improve the results.

- Of the 52 equation environments in Miller (2017), roughly 40 produce compilable LaTeX. In comparison to the results from the `eqdec`-only task with this same input document, the `eqdec` model produces a greater number of decoded environments with uncompilable LaTeX tokens, as expected due to using cropped images of detected equations from the `cosmos` model as opposed to the standardized .png images we create in the `eqdec`-only task. This finding is consistent across all other documents tested. However, even with the presence of this handicap of having a greater number of uncompilable equations, we find that this combined task renders more lines from all equation environments. We attribute this gain to the `cosmos` model; the model detects long, multi-lined environments as multiple, 1- to 2-lined equations. Feeding the smaller input equations to the `eqdec` model results in a 11.1% gain in the number of lines recovered.

4. CONCLUSIONS

From analyzing the results of all documents, we draw the following conclusions:

1. Both the `cosmos` and `eqdec` models perform better on newer papers. This is likely a reflection of the data used to train both models.

2. The `eqdec` model tends to perform worse on screenshots of equations compared to the detected outputs of the `cosmos` model. While one can argue that equations with screenshots tend to be more complex than equations with `cosmos`-detections resulting in worse `eqdec` model performance, we must not exclude the different properties between both images. We suspect that there may be down-sampling in the screenshots, resulting in images with lower resolution. If this is the case, we recommend augmenting the `eqdec` training set with lower resolution images. This may result in higher performances for older papers.

3. The `eqdec` model performance is always higher for the manually-generated LaTeX equations as opposed to the `cosmos`-detected equations (holding equation length constant) due to the standardized format of the document equations.

4. There is an anti-correlation between the number of lines in an equation environment and (1) the probability that `eqdec` will produce compilable LaTeX and (2) the fraction of lines decoded in the environment. However, by breaking up multi-lined equations, we see an improvement in the recovery of individual lines in equation environments.

Table 2. Wikipedia Excerpt

		cosmos-detected Equations				
Equation Environment	Line Count	Equation Detected?	eqdec LaTeX Rendered?	Lines Rendered?	Alignment Issues?	Additional Notes
0	1	.	.	.	.	.
1	1	.	.	.	.	.
2	1	.	.	.	.	Identified as a "figure." Equation number included in detection.
3	1	.	.	.	.	.
4	1	.	.	.	.	.
5	1	.	.	.	.	.
6	1	.	.	.	.	.
7	1	.	.	.	.	.
8	1	.	.	.	.	.
9	1	.	.	.	.	.
10	1	.	.	.	.	.
11	1	.	.	.	.	.
12	1	N	-	-	.	Identified with surrounding text as a "page header."
Total: 13	13	12Y/1N	13Y	13	13N	.

Table 3. Kermack & McKendrick (1927)

		Manually-generated Equations			cosmos-detected Equations				
Equation Environment	Line Count	eqdec LaTeX Rendered?	Lines Rendered?	Alignment Issues?	Equation Detected?	eqdec LaTeX Rendered?	Lines Rendered?	Alignment Issues?	Additional Manual LaTeX Decoding Notes
0	1	.	.	.	N	.	.	.	cosmos: Gibberish. Many repeating letters.
1	3	.	.	.	N	N	-	-	.
2	2	.	.	.	N	N	-	-	.
3	1	.	.	.	N	N	-	-	.
4	2	.	.	Y	N	N	-	-	.
5	1	.	.	.	N	N	-	-	.
6	1	.	.	.	N	N	-	-	.
7	1	.	.	.	N	.	.	.	cosmos: Gibberish. Many repeating letters.
8	1	.	.	.	N	N	-	-	.
9	1	.	.	.	N	N	-	-	.
10	5	.	2	Y	N	.	.	.	1st 2 Lines Rendered. cosmos: Gibberish. Many repeating letters.
11	1	.	.	.	N	N	-	-	.
12	3	N	-	-	N	N	-	-	Medium-Length Equations.
13	2	N	-	-	N	N	-	-	Long Equations.
14	1	.	.	.	N	.	.	.	cosmos: Gibberish. Many repeating letters.
15	2	.	1	.	N	N	-	-	1st 2 Lines Rendered. Uses parenthesis instead of brace.
16	1	.	.	.	N	N	-	-	.
17	1	.	.	.	N	N	-	-	.
18	1	.	.	.	N	N	-	-	.
19	1	.	.	.	N	N	-	-	.
20	3	.	2	Y	N	N	-	-	1st & Last Lines Rendered.
21	1	.	.	.	N	N	-	-	.
22	1	.	.	.	N	N	-	-	.

Table 3 *continued*

Table 3 *(continued)*

		Manually-generated Equations			cosmos-detected Equations				
Equation Environment	Line Count	eqdec LaTeX Rendered?	Lines Rendered?	Alignment Issues?	Equation Detected?	eqdec LaTeX Rendered?	Lines Rendered?	Alignment Issues?	Additional Manual LaTeX Decoding Notes
23	1	.	.	.	N	N	-	-	.
24	2	.	.	.	N	N	-	-	.
25	1	.	.	.	N	N	-	-	.
26	1	.	.	.	N	N	-	-	.
27	1	.	.	.	N	N	-	-	.
28	2	.	.	.	N	N	-	-	.
29	1	.	.	.	N	N	-	-	.
30	1	.	.	.	N	.	.	.	cosmos: Gibberish. Many repeating letters.
31	1	.	.	.	N	N	-	-	.
32	1	.	.	.	N	N	-	-	.
33	1	.	.	.	N	N	-	-	.
34	1	.	.	.	N	N	-	-	.
35	1	.	.	.	N	N	-	-	.
36	2	N	-	-	N	N	-	-	Unusually Long Equations.
37	1	.	.	.	N	N	-	-	.
38	1	.	.	.	N	.	.	.	cosmos: Gibberish. Many repeating letters.
39	1	.	.	.	N	N	-	-	.
40	1	.	.	.	N	N	-	-	.
41	1	.	.	.	N	.	.	.	cosmos: Gibberish. Many repeating letters.
42	1	.	.	.	N	N	-	-	.
43	1	.	.	.	N	N	-	-	.
44	1	.	.	.	N	N	-	-	.
45	4	N	-	-	N	N	-	-	Long, Complex Equations.
46	1	.	.	.	N	N	-	-	.
47	6	.	2	Y	N	.	.	.	1st 2 Lines Rendered. cosmos: Gibberish. Many repeating letters.
48	1	.	.	.	N	N	-	-	.

Table 3 *continued*

Table 3 *(continued)*

		Manually-generated Equations			cosmos-detected Equations				
Equation Environment	Line Count	eqdec LaTeX Rendered?	Lines Rendered?	Alignment Issues?	Equation Detected?	eqdec LaTeX Rendered?	Lines Rendered?	Alignment Issues?	Additional Manual LaTeX Decoding Notes
49	1	.	.	.	N	N	-	-	.
50	4	.	2	.	N	.	.	.	1st 2 Lines Rendered. `cosmos`: Gibberish. Many repeating letters.
51	1	.	.	.	N	N	-	-	.
52	1	.	.	.	N	N	-	-	.
53	1	.	.	.	N	N	-	-	.
54	1	.	.	.	N	N	-	-	.
55	1	.	.	.	N	N	-	-	.
56	1	.	.	.	N	N	-	-	.
57	1	.	.	.	N	N	-	-	.
58	1	.	.	.	N	N	-	-	.
59	3	.	.	.	N	N	-	-	Missing brace around all 3-lines.
60	1	.	.	.	N	N	-	-	.
61	1	.	.	.	N	N	-	-	.
62	1	.	.	.	N	N	-	-	.
63	1	.	.	.	N	N	-	-	.
64	1	.	.	.	N	N	-	-	.
65	1	.	.	.	N	N	-	-	.
66	1	.	.	.	N	N	-	-	.
67	1	.	.	.	N	N	-	-	.
68	1	.	.	.	N	N	-	-	.
69	1	.	.	.	N	N	-	-	.
70	1	.	.	.	N	N	-	-	.
71	1	.	.	.	N	N	-	-	.
72	2	.	.	Y	N	N	-	-	.
73	1	.	.	.	N	N	-	-	.
74	1	.	.	.	N	N	-	-	.

Table 3 *continued*

Table 3 *(continued)*

Equation Environment	Line Count	Manually-generated Equations: `eqdec` LaTeX Rendered?	Manually-generated Equations: Lines Rendered?	Manually-generated Equations: Alignment Issues?	`cosmos`-detected Equations: Equation Detected?	`cosmos`-detected Equations: `eqdec` LaTeX Rendered?	`cosmos`-detected Equations: Lines Rendered?	`cosmos`-detected Equations: Alignment Issues?	Additional Manual LaTeX Decoding Notes
75	1	.	.	.	N	N	-	-	.
76	2	.	.	.	N	N	-	-	.
77	2	.	.	.	N	N	-	-	Missing brace around both equations.
78	1	.	.	.	N	N	-	-	#78 in the manual side-by-side comparison table.
79	1	.	.	.	N	N	-	-	#78 “ ... ”
80	1	.	.	.	N	N	-	-	#79 “ ... ”
81	1	.	.	.	N	N	-	-	#80 “ ... ”
82	1	.	.	.	N	N	-	-	#80 “ ... ”
83	2	.	.	.	N	N	-	-	#81 “ ... ”
84	5	N	-	-	N	N	-	-	#82 “ ... ”; Long Equations.
85	1	.	.	.	N	N	-	-	#83 “ ... ”
Total: 86	124	81Y/5N	97	5Y/76N	0Y/86N	9Y/77N	?	?	

Table 4. Anderson & May (1979)

		Manually-generated Equations			`cosmos`-detected Equations				
Equation Environment	Line Count	`eqdec` LaTeX Rendered?	Lines Rendered?	Alignment Issues?	Equation Detected?	`eqdec` LaTeX Rendered?	Lines Rendered?	Alignment Issues?	Additional Notes
0	3	.	.	.	N	N	-	-	.
1	1	.	.	.	N	.	.	.	`cosmos`: Gibberish output.
2	1	.	.	.	N	.	.	.	`cosmos`: Gibberish. Many repeating letters.
3	1	.	.	.	N	.	2	-	`cosmos`: Gibberish. Fraction $\rightarrow$ 2 lines.
4	1	.	.	.	N	.	.	.	`cosmos`: Gibberish. Many repeating letters.
5	3	.	.	.	N	N	-	-	.
6	1	.	.	.	.	.	.	.	`cosmos` detection includes Equation #. `cosmos`: Font & style changes. Missing /.
7	1	.	.	.	.	.	.	.	`cosmos` detection includes Equation #. `cosmos`: Font & style changes. Missing /.
8	1	.	.	.	N	.	.	.	`cosmos`: Gibberish.
9	1	.	.	.	N	.	.	.	`cosmos`: Gibberish.
10	1	.	.	.	N	.	.	.	`cosmos`: Gibberish.
11	1	.	.	.	N	N	-	-	.
12	1	.	.	.	.	.	.	.	`cosmos` detection includes Equation #.
13	1	.	.	.	.	.	.	.	Manual: β duplicated at the end. `cosmos` detection includes Equation #. `cosmos`: ($\rightarrow$ {. Non-italicized variable.
Total: 14	18	14Y	18	14N	4Y/10N	11Y/3N	12	11N	

Table 5. Hethcote (2000)

		Manually-generated Equations			cosmos-detected Equations				
Equation Environment	Line Count	eqdec LaTeX Rendered?	Lines Rendered?	Alignment Issues?	Equation Detected?	eqdec LaTeX Rendered?	Lines Rendered?	Alignment Issues?	Additional Notes
0	1	.	.	.	N	.	.	.	.
1	3	.	.	Y	N	N	-	-	2 column format.
2	2	.	.	.	N	N	-	-	2 column format.
3	1	.	.	.	N	.	.	.	.
4	1	.	.	.	N	.	.	.	.
5	1	.	.	.	N	.	2	.	.
6	3	.	.	Y	N	N	-	-	2 column format.
7	2	.	.	.	N	N	-	-	2 column format.
8	1	.	.	.	N	.	.	.	.
9	6	.	4	.	N	N	-	-	1st 4 equations rendered. 3rd eqn's RHS repeats 4th equation's RHS.
10	5	.	4	.	N	N	-	-	Skips 3rd & 5th equations. 4th eqn repeated twice.
11	1	.	.	.	N	.	.	.	.
12	1	.	.	.	N	N	-	-	.
13	4	.	2	Y	N	N	-	-	1st 2 equations rendered. $\delta \to \varepsilon$.
14	1	.	.	.	N	.	.	.	.
15	1	.	.	.	N	.	.	.	.
16	1	.	.	.	N	.	.	.	.
17	1	.	.	.	N	.	.	.	.
18	1	.	.	.	N	.	.	.	.
19	1	.	.	.	N	.	.	.	.
20	1	.	.	.	N	.	.	.	.
21	1	.	.	.	N	.	.	.	.
22	1	.	.	.	N	.	.	.	.] duplicated at the end.

Table 5 *continued*

Table 5 *(continued)*

		Manually-generated Equations			cosmos-detected Equations				
Equation Environment	Line Count	eqdec LaTeX Rendered?	Lines Rendered?	Alignment Issues?	Equation Detected?	eqdec LaTeX Rendered?	Lines Rendered?	Alignment Issues?	Additional Notes
23	1	.	.	.	N	.	.	.	.
24	1	.	.	.	N	.	.	.	.
25	3	N	-	-	N	.	.	.	Long equations. 2 column format.
26	1	.	.	.	N	.	.	.	.
27	1	.	.	.	N	.	.	.	.
28	1	.	.	.	N	.	.	.	.
29	6	N	-	-	N	N	-	-	Long equations.
30	2	.	.	.	N	.	.	Y	.
31	6	.	3	.	N	N	-	-	1st 3 equations rendered.
32	2	.	.	.	N	N	-	-	.
33	4	N	-	-	N	N	-	-	Unusually long equations.
34	2	N	-	-	N	N	-	-	Unusually long equations.
35	1	.	.	.	N	.	.	.	.
36	1	.	.	.	N	.	.	.	.
37	2	N	-	-	N	N	-	-	Unusually long equations.
38	1	.	.	.	N	.	.	.	.
39	1	.	.	.	N	.	.	.	.
40	2	.	.	.	N	.	.	.	.
41	1	.	.	.	N	.	.	0	.
42	2	.	1	.	N	N	-	-	1st line of split equation rendered.
43	1	.	.	.	N	N	-	-	.
44	1	.	.	.	N	.	.	.	.
45	1	.	.	.	N	.	.	0	.
46	1	.	.	.	N	N	-	-	Unusually long. Missing period. Dropped terms in denominator.
47	1	.	.	.	N	.	.	.	.
48	1	.	.	.	N	N	-	-	.

Table 5 *continued*

Table 5 *(continued)*

		Manually-generated Equations			cosmos-detected Equations				
Equation Environment	Line Count	`eqdec` LaTeX Rendered?	Lines Rendered?	Alignment Issues?	Equation Detected?	`eqdec` LaTeX Rendered?	Lines Rendered?	Alignment Issues?	Additional Notes
49	1	.	.	.	N	.	.	.	.
50	1	.	.	.	N	N	-	-	.
51	1	.	.	.	N	.	.	.	.
52	1	.	.	.	N	.	.	.	.
53	1	.	.	.	N	N	-	-	Unusually long. Dropped most terms in nested fractions.
54	1	.	.	.	N	.	.	.	.
55	1	.	.	.	N	.	.	.	.
56	1	.	.	.	N	.	.	2	.
57	1	.	.	.	N	.	.	.	.
58	1	.	.	.	N	N	-	-	.
59	1	.	.	.	N	.	.	.	.
60	1	.	.	.	N	.	.	.	.
61	1	.	.	.	N	.	.	.	.
62	1	.	.	.	N	.	.	.	.
63	1	.	.	.	N	.	.	.	.
64	1	.	.	.	N	.	.	.	.
65	1	.	.	.	N	.	.	.	.
66	5	.	3	.	N	N	-	-	1st 3 equations rendered. 2nd eqn missing last integral term.
67	9	N	-	-	N	N	-	-	Long equations; 2 column format.
68	3	N	-	-	N	N	-	-	Short equations; 3 column format.
69	1	.	.	.	N	N	-	-	Long equation; Missing last 1/3.
70	3	N	-	-	N	N	-	-	Unusually long equations.
71	3	N	-	-	N	N	-	-	Unusually long equations.
72	3	N	-	-	N	N	-	-	Unusually long equations.
73	1	.	.	.	N	.	.	.	.

Table 5 *continued*

Table 5 *(continued)*

		Manually-generated Equations			cosmos-detected Equations				
Equation Environment	Line Count	eqdec LaTeX Rendered?	Lines Rendered?	Alignment Issues?	Equation Detected?	eqdec LaTeX Rendered?	Lines Rendered?	Alignment Issues?	Additional Notes
74	1	.	.	.	N	.	.	.	Unusually long. Combined 2nd-to-last & last terms.
75	1	.	.	.	N	N	-	-	Unusually long. Combined 2nd-to-last & last terms.
76	1	.	.	.	N	.	.	.	Unusually long. Missing period. Dropped terms in denominator.
77	1	.	.	.	N	N	-	-	.
78	1	.	.	.	N	N	-	-	.
79	1	.	.	.	N	N	-	-	.
80	1	.	.	.	N	.	.	.	.
81	1	.	.	.	N	.	.	.	.
82	1	.	.	.	N	N	-	-	.
83	1	.	.	.	N	.	.	.	.
84	1	.	.	.	N	.	.	.	.
Total: 85	144	75Y/10N	96	3Y/72N	0Y/85N	52Y/33N	56	1Y/51N	

Table 6. Miller (2017)

Equation Environment	Line Count	Manually-generated Equations: eqdec LaTeX Rendered?	Lines Rendered?	Alignment Issues?	cosmos-detected Equations: Equation Detected?	eqdec LaTeX Rendered?	Lines Rendered?	Alignment Issues?	Additional Notes
0	3	.	.	.	.	2Y	2,1	.	#0-1 in the `cosmos` side-by-side comparison table. `cosmos`: 0 detected as "Figure."
1	3	.	.	.	.	.	2	.	#2
2	5	.	3	.	.	4Y	1,1,1,2	.	#3-6; 1st 3 equations rendered. `cosmos`: #6 detected as "Figure."
3	2	.	.	.	.	.	2	.	#7
4	1	.	.	.	.	.	1	.	#8
5	1	.	.	.	.	.	1	.	#9
6	2	.	.	Y	.	.	2	.	#10; Center-aligned.
7	1	.	.	.	.	.	1	.	#11
8	1	.	.	.	.	.	1	.	#12; Awkward format; Position of limits. Different size parenthesis. $Se^{\beta} \to \int_S^t$. `cosmos`: `cosmos`: Detected as "Figure."
9	1	.	.	.	.	.	1	.	#13
10	4	.	3	.	.	4Y	1,1,1,1	.	#14-17; 1st 3 equations rendered. `cosmos`: #14 detected as "Figure."
11	1	.	.	.	.	.	1	.	#18
12	1	.	.	.	.	.	1	.	#19; $\mathscr{R}_0 \to \beta_0$
13	2	.	.	.	.	2Y	1,1	.	#20-21; $\mathscr{R}_0 \to \partial_0$
14	1	.	.	.	.	.	1	.	#22; $\mathscr{R}_0 \to \partial\bar{\partial}_0$
15	1	.	.	.	.	.	1	.	#23
16	10	.	4	.	.	5Y/1N	1,-,1, 1,1,1	.	#24-29; 1st 3 equations rendered. Repeats 3rd equation. `cosmos`: #25 detected twice as a "Table."
17	1	.	.	.	.	N	-	-	#30
18	2	N	-	-	N	N	-	-	#31; Long equations. Center-aligned. `cosmos`: Detected as "Figure."

Table 6 *continued*

Table 6 *(continued)*

Equation Environment	Line Count	Manually-generated Equations: `eqdec` LaTeX Rendered?	Manually-generated Equations: Lines Rendered?	Manually-generated Equations: Alignment Issues?	`cosmos`-detected Equations: Equation Detected?	`cosmos`-detected Equations: `eqdec` LaTeX Rendered?	`cosmos`-detected Equations: Lines Rendered?	`cosmos`-detected Equations: Alignment Issues?	Additional Notes
19	1	.	.	.	.	N	-	-	#32
20	1	.	.	.	.	.	1	.	#33; $\mathscr{R}_0 \to \mathscr{Z}_0$ and $\mathscr{O} \to \ell$. Missing). Missing rest of denominator for last term.
21	3	N	-	-	.	N	-	-	#34; Long equations. Right-aligned. `cosmos`: Detected as "Figure."
22	2	.	.	.	.	.	2	.	#35; `cosmos`: Detected as "Figure."
23	1	.	.	.	.	.	1	.	#36
24	1	.	.	.	.	.	1	.	#37; Position of limits.
25	1	.	.	.	.	.	1	.	#38; Position of limits.
26	1	.	.	.	.	.	1	.	#39
27	3	N	-	-	.	.	2	.	#40; Long equations. Center-aligned. `cosmos`: Detected as "Figure."
28	2	.	.	.	.	N	-	-	#41; `cosmos`: Detected as "Figure."
29	1	.	.	.	.	.	1	.	#42
30	2	.	.	.	.	2Y	1,1	.	#43-44
31	12	.	4	.	.	11Y	1,1,1,1,2, 1,1,1,1,1,1	.	#45-55; 1st 3 equations rendered. Repeats 3rd equation with $\gamma_1 \to \gamma_2$. `cosmos`: #48, 50, 54 detected as "Table" & #49 as "Other."
32	1	.	.	.	.	.	1	.	#56
33	11	.	4	.	.	11Y	1,1,1,1,1, 1,1,1,1,1,1	.	#57-67; 1st 4 equations rendered. `cosmos`: #60 detected as "Figure."
34	4	N	-	-	.	2Y	2,1	.	#68-69; Short equations. `cosmos`: #68 detected as "Figure."
35	1	.	.	.	.	.	1	.	#70
36	2	N	-	-	N	N	-	-	#71; Long equations. `cosmos`: Detected as "Figure."
37	1	.	.	.	.	.	1	.	#72
38	2	N	-	-	.	N	-	-	#73; Long equations. `cosmos`: Detected as "Page Header."

Table 6 *continued*

Table 6 *(continued)*

Equation Environment	Line Count	Manually-generated Equations: `eqdec` LaTeX Rendered?	Lines Rendered?	Alignment Issues?	`cosmos`-detected Equations: Equation Detected?	`eqdec` LaTeX Rendered?	Lines Rendered?	Alignment Issues?	Additional Notes
39	14	.	4	.	.	N	-	-	#74; 1st 4 equations rendered. $\dot{I}_2 \rightarrow \dot{R}$ `cosmos`: Detected as "Figure."
40	6	.	4	.	.	.	3	.	#75; 1st 4 equations rendered. `cosmos`: Detected as "Figure."
41	1	.	.	.	.	.	1	.	#76
42	1	.	.	.	.	.	1	.	#77
43	1	.	.	.	N	N	-	-	#78; `cosmos`: Detected as "Figure."
44	10	.	3	.	.	10Y	1,1,1,1,1, 1,1,1,1,1	.	#79-88; 1st 3 equations rendered. $I_1 \rightarrow \dot{I}_1$ `cosmos`: #82, 84 detected as "Figure."
45	4	.	3	.	.	.	3	.	#89; 1st 3 equations rendered. `cosmos`: Detected as "Figure."
46	3	N	-	-	.	N	-	-	#90; Short equations. 3 column format. Center-aligned. `cosmos`: Detected as "Table."
47	1	.	.	.	.	.	1	.	#91; Parentheses sizes.
48	1	N	-	-	N	N	-	-	#92; Unusually long matrix equation. `cosmos`: Detected as "Figure."
49	1	.	.	.	.	N	-	-	#93; `cosmos`: Detected as "Table."
50	1	.	.	.	.	.	1	.	#94; `cosmos`: Detected as "Figure."
51	1	.	.	.	.	.	1	.	#95; `cosmos`: Detected as "Table."
Total: 52	144	44Y/8N	80	1Y/43N	48Y/4N	39.83Y/12.17N	96		
(Detections)					92Y/4N	83Y/13N			

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1927 (Manually-generated LaTeX)

Eqn Num	Original Image	Decoded Image
0	$v_{0,0} = v_0 + y_0$	$v_{0,0} = v_0 + y_0$
1	$\begin{aligned} v_{t,\theta} &= v_{t-1,\theta-1}(1-\psi(\theta-1)) \\ &= v_{t-2,\theta-2}(1-\psi(\theta-1)(1-\psi(\theta-2)) \\ &= v_{t-\theta,0}B_\theta, \end{aligned}$	$\begin{aligned} v_{t,\theta} &= v_{t-1,\theta-1}(1-\psi(\theta-1)) \\ &= v_{t-2,\theta-2}(1-\psi(\theta-1)(1-\psi(\theta-2))) \\ &= v_{t-\theta,0}B_\theta, \end{aligned}$
2	$\begin{aligned} x_t &= N - \sum_0^t v_{t,0} \\ &= N - \sum_0^t v_t - y_0, \end{aligned}$	$\begin{aligned} x_t &= N - \sum_0^t v_{t,0} \\ &= N - \sum_0^t v_t - y_0, \end{aligned}$
3	$x_t + y_t + z_t = N.$	$x_t + y_t + z_t = N.$
4	$\begin{aligned} v_t = x_t \sum_1^t \phi_\theta v_{t,\theta} &= x_t \sum_1^t \phi_\theta B_\theta v_{t-\theta,0} \qquad \text{(by 2)} \\ &= x_t(\sum_1^t A_\theta v_{t-\theta} + A_t y_0) \qquad \text{(by 1)}, \end{aligned}$	$\begin{aligned} v_t = x_t \sum_1^t \phi_\theta v_{t,\theta} = x_t \sum_1^t \phi_\theta B_\theta v_{t-\theta,0} \qquad \text{(by 2)} \\ = x_t(\sum_1^t A_\theta v_{t-\theta} + A_t y_0) \qquad \text{(by 1)}, \end{aligned}$
5	$y_t = \sum_0^t v_{t,0} = \sum_0^t B_\theta v_{t-\theta} + B_t y_0.$	$y_t = \sum_0^t v_{t,0} = \sum_0^t B_\theta v_{t-\theta} + B_t y_0.$
6	$-v_t = x_{t+1} - x_t,$	$-v_t = x_{t+1} - x_t,$
7	$x_t - x_{t+1} = x_t(\sum_1^t A_\theta v_{t-\theta} + A_t y_0).$	$x_t - x_{t+1} = x_t(\sum_1^t A_\theta v_{t-\theta} + A_t y_0).$
8	$z_{t+1} - z_t = \sum_1^t C_\theta v_{t-\theta} + C_t y_0.$	$z_{t+1} - z_t = \sum_1^t C_\theta v_{t-\theta} + C_t y_0.$
9	$y_{t+1} - y_t = x_t[\sum_1^t A_\theta v_{t-\theta} + A_t y_0] - [\sum_1^t C_\theta v_{t-\theta} + C_t y_0].$	$y_{t+1} - y_t = x_t[\sum_1^t A_\theta v_{t-\theta} + A_t y_0] - [\sum_1^t C_\theta v_{t-\theta} + C_t y_0].$

10	$\begin{aligned} & x_t + y_t + z_t = N. \\ & v_t = -\frac{dx_t}{dt}, \\ & \frac{dx_t}{dt} = -x_t \left[\int_0^t A_\theta v_{t-\theta} d\theta + A_t y_0 \right], \\ & \frac{dz_t}{dt} = \int_0^t C_\theta v_{t-\theta} d\theta + C_t y_0, \\ & y_t = \int_0^t B_\theta v_{t-\theta} d\theta + B_t y_0, \end{aligned}$	$\begin{aligned} & x_t + y_t + z_t = N. \\ & v_t = -\frac{dx_t}{dt}, \end{aligned}$
11	$B_\theta = e^{-\int_0^\infty \psi(\alpha) d\alpha}, \quad A_\theta = \phi_\theta B_\theta, \quad \text{and} \quad C_\theta = \psi(\theta) B_\theta.$	$B_\theta = e^{-\int_0^\infty \psi(\alpha) d\alpha}, \quad A_\theta = \phi_\theta B_\theta, \quad \text{and} \quad C_\theta = \psi(\theta) B_\theta.$
12	$\begin{aligned} \frac{dx}{dt} &= -x \left[\int_0^t A_\theta v_{t-\theta} d\theta + A_t y_0 \right], \\ &= -x \left[\int_0^t A_{t-\theta} v_\theta d\theta + A_t y_0, \right] \\ &= x \left[\int_0^t A_{t-\theta} \frac{dx_\theta}{d\theta} d\theta - A_t y_0 \right], \end{aligned}$	$None$
13	$\begin{aligned} \frac{d \log x}{dt} &= A_{t-\theta} x_\theta \Big\|_0^t - \int_0^t x_\theta \frac{dA_{t-\theta}}{d\theta} d\theta - A_t y_0, \\ &= A_0 x_t - A_t x_0 + \int_0^t x_\theta A'_{t-\theta} d\theta - A_t y_0, \end{aligned}$	$None$
14	$A'_{t-\theta} = \frac{dA_{t-\theta}}{d(t-\theta)} = -\frac{dA_{t-\theta}}{d\theta}$	$A'_{t-\theta} = \frac{dA_{t-\theta}}{d(t-\theta)} = -\frac{dA_{t-\theta}}{d\theta}$
15	$\left. \begin{aligned} \frac{d \log x}{dt} &= -A_t(x_0 + y_0) + \int_0^t x_\theta A'_{t-\theta} d\theta, \\ &= -A_t N + \int_0^t A'_\theta x_{t-\theta} d\theta. \end{aligned} \right\}$	$\frac{d \log x}{dt} = -A_t(x_0 + y_0) + \int_0^t x_\theta A'_{t-\theta} d\theta, \Big)$
16	$f(t) = \phi(t) + \int_0^t N(t, \theta) \phi(\theta) d\theta,$	$f(t) = \phi(t) + \int_0^t N(t, \theta) \phi(\theta) d\theta,$
17	$\frac{d \log x}{dt} = A_t + \lambda \int_0^t N(t, \theta) x(\theta) d\theta,$	$\frac{d \log x}{dt} = A_t + \lambda \int_0^t N(t, \theta) x(\theta) d\theta,$
18	$x = f_0(t) + \lambda f_1(t) + \lambda^2 f_2(t) + \text{etc.}$	$x = f_0(t) + \lambda f_1(t) + \lambda^2 f_2(t) + \text{etc}$
19	$\frac{dx}{dt} = x \left[A_t + \lambda \int_0^t N(t, \theta) x(\theta) d\theta \right],$	$\frac{dx}{dt} = x \left[A_t + \lambda \int_0^t N(t, \theta) x(\theta) d\theta \right],$

20	$\frac{d}{dt}f_n(t) = f_n(t)A_t + f_{n-1}(t)\int_0^t N(t,\theta)f_0(\theta)d\theta + f_{n-2}(t)\int_0^t N(t,\theta)f_1(\theta)d\theta + \ldots + f_0(t)\int_0^t N(t,\theta)f_{n-1}(\theta)d\theta = L_{n-1}(t) \quad \text{say.}$	$\frac{d}{dt}f_n(t) = f_n(t)A_t + f_{n-1}(t)\int_0^t N(t,\theta)f_0(\theta)d\theta + f_{n-2}(t)\int_0^t N(t,\theta)f_1(\theta)d\theta = L_{n-1}(t) \quad \text{say.}$
21	$f_n(t)e^{-\int_0^t A_t dt} = \int_0^t L_{n-1}(t)e^{-\int_0^t A_t dt}dt + \text{constant},$	$f_n(t)e^{-\int_0^t A_t dt} = \int_0^t L_{n-1}(t)e^{-\int_0^t A_t dt}dt + \text{constant},$
22	$\frac{df_0(t)}{dt} = f_0(t)A_t,$	$\frac{df_0(t)}{dt} = f_0(t)A_t,$
23	$f_0(t) = f_0(0)e^{\int_0^t A_t dt},$	$f_0(t) = f_0(0)e^{\int_0^t A_t dt},$
24	$x = x_0E_t + \sum_{n=1}^{\infty}\lambda^n E_t\int_0^t \frac{L_{n-1}(t)}{E_t}dt, = E_t\left[x_0 + \sum_1^{\infty}\lambda^n\int_0^t \frac{L_{n-1}(t)}{E_t}dt\right],$	$x = x_0E_t + \sum_{n=1}^{\infty}\lambda^n E_t\int_0^t \frac{L_{n-1}(t)}{E_t}dt, = E_t\left[x_0 + \sum_1^{\infty}\lambda^n\int_0^t \frac{L_{n-1}(t)}{E_t}dt\right],$
25	$x = E_t\left[x_0 + \sum_1^{\infty}\int_0^t \frac{L_{n-1}(t)}{E_t}dt\right].$	$x = E_t\left[x_0 + \sum_1^{\infty}\int_0^t \frac{L_{n-1}(t)}{E_t}dt\right].$
26	$\frac{d\log x}{dt} = A_t + \int_0^t Q_{t-\theta}x_\theta d\theta.$	$\frac{d\log x}{dt} = A_t + \int_0^t Q_{t-\theta}x_\theta d\theta.$
27	$\int_0^\infty e^{-zt}\frac{d\log x}{dt}dt = \int_0^\infty e^{-zt}A_t dt + \int_0^\infty e^{-zt}\int_0^t Q_{t-\theta}x_\theta d\theta dt,$	$\int_0^\infty e^{-zt}\frac{d\log x}{dt}dt = \int_0^\infty e^{-zt}A_t dt + \int_0^\infty e^{-zt}\int_0^t Q_{t-\theta}x_\theta d\theta dt,$
28	$-\log x_0 + \int_0^\infty ze^{-zt}\log x dt = F(z) + \int_0^\infty e^{-z\theta}Q_\theta d\theta\int_0^\infty e^{-zt}x_t dt, = F(z) + F_1(z)\int_0^\infty e^{-zt}x_t dt,$	$-\log x_0 + \int_0^\infty ze^{-zt}\log x dt = F(z) + \int_0^\infty e^{-z\theta}Q_\theta d\theta\int_0^\infty e^{-zt}x_t dt, = F(z) + F_1(z)\int_0^\infty e^{-zt}x_t dt,$
29	$\int_0^\infty e^{-zt}(z\log x - F_1(z)x)dt = F(z) + \log x_0.$	$\int_0^\infty e^{-zt}(z\log x - F_1(z)x)dt = F(z) + \log x_0.$
30	$\int_0^\infty \phi(x,z)\psi(z,t)dt = \chi(z),$	$\int_0^\infty \phi(x,z)\psi(z,t)dt = \chi(z),$
31	$-\int_0^\infty \frac{d\log x}{dt}dt = \int_0^\infty\int_0^t A_\theta v_{t-\theta}d\theta dt + y_0\int_0^\infty A_t dt,$	$-\int_0^\infty \frac{d\log x}{dt}dt = \int_0^\infty\int_0^t A_\theta v_{t-\theta}d\theta dt + y_0\int_0^\infty A_t dt,$
32	$\log\frac{x_0}{x_\infty} = \int_0^\infty A_\theta d\theta\int_0^\infty v_t dt + y_0\int_0^\infty A_t dt.$	$\log\frac{x_0}{x_\infty} = \int_0^\infty A_\theta d\theta\int_0^\infty v_t dt + y_0\int_0^\infty A_t dt.$
33	$\log\frac{x_0}{x_\infty} = A(x_0 - x_\infty) + Ay_0 = A(N - x_\infty).$	$\log\frac{x_0}{x_\infty} = A(x_0 - x_\infty) + Ay_0 = A(N - x_\infty).$
34		

	$-\log\frac{1-p}{1-\frac{y_0}{N}} = ANp.$	$-\log\frac{1-p}{1-\frac{y_0}{N}} = ANp.$
35	$\int_0^\infty y_t dt = Np\int_0^\infty B_\theta d\theta.$	$\int_0^\infty y_t dt = Np\int_0^\infty B_\theta d\theta.$
36	$\begin{aligned} -\int_0^\infty e^{-zt}\frac{d\log x}{dt}dt &= \int_0^\infty e^{-zt}\int_0^t A_\theta v_{t-\theta}d\theta dt + y_0\int_0^\infty e^{-zt}A_t dt, \\ &= \int_0^\infty e^{-zt}A_\theta d\theta\int_0^\infty e^{-zt}v_t dt + y_0\int_0^\infty e^{-zt}A_t dt, \end{aligned}$	$None$
37	$\int_0^\infty e^{-zt}A_t dt = \frac{-\int_0^\infty e^{-zt}\frac{d\log x}{dt}dt}{y_0+\int_0^\infty e^{-zt}v_t dt},$	$\int_0^\infty e^{-zt}A_t dt = \frac{-\int_0^\infty e^{-zt}\frac{d\log x}{dt}dt}{y_0+\int_0^\infty e^{-zt}v_t dt},$
38	$A_\theta = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} e^{zt}F_2(z)dz.$	$A_\theta = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} e^{zt}F_2(z)dz.$
39	$\int_0^\infty e^{-zt}y_t dt = \int_0^\infty e^{-zt}\int_0^t B_\theta v_{t-\theta}d\theta dt + y_0\int_0^\infty e^{-zt}B_t dt,$	$\int_0^\infty e^{-zt}y_t dt = \int_0^\infty e^{-zt}\int_0^t B_\theta v_{t-\theta}d\theta dt + y_0\int_0^\infty e^{-zt}B_t dt,$
40	$\int_0^\infty e^{-zt}B_t dt = \frac{\int_0^\infty e^{-zt}y_t dt}{y_0+\int_0^\infty e^{-zt}v_t dt},$	$\int_0^\infty e^{-zt}B_t dt = \frac{\int_0^\infty e^{-zt}y_t dt}{y_0+\int_0^\infty e^{-zt}v_t dt},$
41	$B_\theta = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} e^{zt}F_3(z)dt.$	$B_\theta = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} e^{zt}F_3(z)dt.$
42	$-\frac{dx}{dt} = v_t = N\left[\int_0^\infty A_\theta v_{t-\theta}d\theta + A_t y_0\right],$	$-\frac{dx}{dt} = v_t = N\left[\int_0^\infty A_\theta v_{t-\theta}d\theta + A_t y_0\right],$
43	$\int_0^\infty e^{-zt}v_t dt = \frac{Ny_0\int_0^\infty e^{-zt}A_t dt}{1-N\int_0^\infty e^{-zt}A_t dt}$	$\int_0^\infty e^{-zt}v_t dt = \frac{Ny_0\int_0^\infty e^{-zt}A_t dt}{1-N\int_0^\infty e^{-zt}A_t dt}$
44	$v_t = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} e^{zt}F_4(z)dz.$	$v_t = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} e^{zt}F_4(z)dz.$
45	$\begin{aligned} \int_0^\infty e^{-zt}y_t dt &= \int_0^\infty e^{-zt}\int_0^t B_\theta v_{t-\theta}d\theta dt + y_0\int_0^\infty e^{-zt}B_t dt, \\ &= \int_0^\infty e^{-zt}v_t dt\int_0^\infty e^{-z\theta}B_\theta d\theta + y_0\int_0^\infty e^{-zt}B_t dt, \\ &= \frac{Ny_0\int_0^\infty e^{-zt}A_t dt\int_0^\infty e^{-zt}B_t dt}{1-N\int_0^\infty e^{-zt}A_t dt} + y_0\int_0^\infty e^{-zt}B_t dt, \\ &= \frac{y_0\int_0^\infty e^{-zt}B_t dt}{1-N\int_0^\infty e^{-zt}A_t dt} \end{aligned}$	$None$
46		

	$y_t = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} e^{zt}F_5(z)dz.$	$y_t = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} e^{zt}F_5(z)dz.$
47	$\begin{aligned} y_t &= \int_0^t B_{t-\theta}v_\theta d\theta + B_t y_0, \\ &= N\int_0^t B_{t-\theta}\left(\int_0^\theta A_{\theta-z}v_z dz + A_\theta y_0\right)d\theta + B_t y_0, \\ &= N\int_0^t B_{t-\theta}\int_0^\theta A_{\theta-z}v_z dz d\theta + N y_0\int_0^t B_{t-\theta}A_\theta d\theta + B_t y_0, \\ &= N\int_0^t A_{t-\theta}\int_0^\theta B_{\theta-z}v_z dz d\theta + N y_0\int_0^t A_{t-\theta}B_\theta d\theta + B_t y_0, \\ &= N\int_0^t A_{t-\theta}(y_\theta - B_\theta y_0 + B_\theta y_0)d\theta + B_t y_0, \\ &= N\int_0^t A_{t-\theta}y_\theta d\theta + B_t y_0. \end{aligned}$	$\begin{aligned} y_t &= \int_0^t B_{t-\theta}v_\theta d\theta + B_t y_0, \\ &= N\int_0^t B_{t-\theta}\left(\int_0^\theta A_{\theta-z}v_z dz + A_\theta y_0\right)d\theta + B_t y_0, \end{aligned}$
48	$v_{t,0} = \int_0^t A_\theta v_{t-\theta,0}d\theta,$	$v_{t,0} = \int_0^t A_\theta v_{t-\theta,0}d\theta,$
49	$v_{t,0} = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} e^{zt}\frac{N_\theta}{1-\int_0^\infty e^{-z\theta}A_\theta d\theta}dz.$	$v_{t,0} = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} e^{zt}\frac{N_\theta}{1-\int_0^\infty e^{-z\theta}A_\theta d\theta}dz.$
50	$\begin{aligned} v_{t,0} &= v_{t,0} - v_{\epsilon,0} + v_{\epsilon,0}, \\ &= \int_\epsilon^t A_{t-\theta}v_{\theta,0}d\theta + \int_0^\epsilon A_{t-\theta}v_{\theta,0}d\theta, \\ &= \int_0^t A_{t-\theta}v_\theta d\theta + A_{t-\epsilon'}\int_0^\epsilon v_{\theta,0}d\theta, \quad \text{where} \quad 0<\epsilon'<\epsilon, \\ &= \int_0^t A_{t-\theta}v_\theta d\theta + A_t y_0. \end{aligned}$	$\begin{aligned} v_{t,0} &= v_{t,0} - v_{\epsilon,0} + v_{\epsilon,0}, \\ &= \int_0^t A_{t-\theta}v_{\theta,0}d\theta + \int_0^\epsilon A_{t-\theta}v_{\theta,0}d\theta, \end{aligned}$
51	$v_{t,0} = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} e^{zt}F(z)dz,$	$v_{t,0} = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} e^{zt}F(z)dz,$
52	$F(z) = \frac{y_0}{1-\int_0^\infty e^{-z\theta}A_\theta d\theta}: \quad \text{let us denote this by } \frac{y_0}{1-A}.$	$F(z) = \frac{y_0}{1-\int_0^\infty e^{-z\theta}A_\theta d\theta}: \quad \text{let us denote this by } \frac{y_0}{1-A}.$
53	$F_4(z) = -y_0 + \frac{y_0}{1-A} = \frac{Ay_0}{1-A},$	$F_4(z) = -y_0 + \frac{y_0}{1-A} = \frac{Ay_0}{1-A},$
54	$A_\theta = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} e^{zt}\frac{\int_0^\infty e^{-zt}v_t dt}{Ny_0 + N\int_0^\infty e^{-zt}v_t dt}dz,$	$A_\theta = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} e^{zt}\frac{\int_0^\infty e^{-zt}v_t dt}{Ny_0 + N\int_0^\infty e^{-zt}v_t dt}dz,$
55	$B_\theta = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} e^{zt}\frac{\int_0^\infty e^{-zt}y_t dt}{y_0 + \int_0^\infty e^{-zt}v_t dt}dz.$	$B_\theta = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} e^{zt}\frac{\int_0^\infty e^{-zt}y_t dt}{y_0 + \int_0^\infty e^{-zt}v_t dt}dz.$
56	$\int_0^\infty e^{-zt}e^{\alpha t}t^c dt = \frac{c!}{(z-\alpha)^{c+1}},$	$\int_0^\infty e^{-zt}e^{\alpha t}t^c dt = \frac{c!}{(z-\alpha)^{c+1}},$
57		

	$\int_0^\infty e^{-zt}\phi(t)dt = \Sigma\Sigma\frac{A_{r,s}}{(z-\alpha_r)^s}$	$\int_0^\infty e^{-zt}\phi(t)dt = \Sigma\Sigma\frac{A_{r,s}}{(z-\alpha_r)^s}$
58	$\phi(t) = \Sigma\Sigma\frac{A_{r,s}}{(s-1)!}t^{s-1}e^{\alpha_r t}$: see Fock (*loc. cit.*).	$\phi(t) = \Sigma\Sigma\frac{A_{r,s}}{(s-1)!}t^{s-1}e^{\alpha_r t}$: see Fock (loc. cit.).
59	$\left.\begin{aligned} \frac{dx}{dt} &= -\kappa xy \\ \frac{dy}{dt} &= \kappa xy - ly \\ \frac{dz}{dt} &= ly \end{aligned}\right\}$	$\begin{aligned} \frac{dx}{dt} &= -\kappa xy \\ \frac{dy}{dt} &= \kappa xy - ly \\ \frac{dz}{dt} &= ly \end{aligned}$
60	$\frac{dz}{dt} = l(N-x-z),$	$\frac{dz}{dt} = l(N-x-z),$
61	$\frac{dz}{dt} = l\left(N - x_0 e^{-\frac{\kappa}{l}z} - z\right).$	$\frac{dz}{dt} = l\left(N - x_0 e^{-\frac{\kappa}{l}z} - z\right).$
62	$\frac{dz}{dt} = l\left\{N - x_0 + \left(\frac{\kappa}{l}x_0 - 1\right)z - \frac{x_0\kappa^2 z^2}{2l^2}\right\}.$	$\frac{dz}{dt} = l\left\{N - x_0 + \left(\frac{\kappa}{l}x_0 - 1\right)z - \frac{x_0\kappa^2 z^2}{2l^2}\right\}.$
63	$z = \frac{l^2}{\kappa^2 x_0}\left\{\frac{\kappa}{l}x_0 - 1 + \sqrt{-q}\tanh\left(\frac{\sqrt{-q}}{2}lt - \phi\right)\right\}$	$z = \frac{l^2}{\kappa^2 x_0}\left\{\frac{\kappa}{l}x_0 - 1 + \sqrt{-q}\tanh\left(\frac{\sqrt{-q}}{2}lt - \phi\right)\right\}$
64	$\phi = \tanh^{-1}\frac{\frac{\kappa}{l}x_0 - 1}{\sqrt{-q}},$	$\phi = \tanh^{-1}\frac{\frac{\kappa}{l}x_0 - 1}{\sqrt{-q}},$
65	$\sqrt{-q} = \left\{\left(\frac{\kappa}{l}x_0 - 1\right)^2 + 2x_0 y_0\frac{\kappa^2}{l^2}\right\}^{\frac{1}{2}}.$	$\sqrt{-q} = \left\{\left(\frac{\kappa}{l}x_0 - 1\right)^2 + 2x_0 y_0\frac{\kappa^2}{l^2}\right\}^{\frac{1}{2}}.$
66	$\frac{dz}{dt} = \frac{l^3}{2x_0\kappa^2}\sqrt{-q}\ \mathrm{sech}^2\left(\frac{\sqrt{-q}}{2}lt - \phi\right).$	$\frac{dz}{dt} = \frac{l^3}{2x_0\kappa^2}\sqrt{-q}\ \mathrm{sech}^2\left(\frac{\sqrt{-q}}{2}lt - \phi\right).$

67	$\frac{dz}{dt} = 890\ \mathrm{sech}^2(0\cdot 2t - 3\cdot 4).$	$\frac{dz}{dt} = 890\ \mathrm{sech}^2(0\cdot 2t - 3\cdot 4).$
68	$z = \frac{2l}{\kappa x_0}\left(x_0 - \frac{l}{\kappa}\right)$	$z = \frac{2l}{\kappa x_0}\left(x_0 - \frac{l}{\kappa}\right)$
69	$2\frac{l}{\kappa}\frac{n}{N} \quad \text{or} \quad 2n - \frac{2n^2}{N}.$	$2\frac{l}{\kappa}\frac{n}{N} \quad \text{or} \quad 2n - \frac{2n^2}{N}.$
70	$-\log\frac{1-p}{1-\frac{y_0}{N}} = ApN,$	$-\log\frac{1-p}{1-\frac{y_0}{N}} = ApN,$
71	$A = \int_0^\infty A_\theta d\theta = \int_0^\infty \phi_\theta e^{-\int_0^\theta \psi_\alpha d\alpha} d\theta.$	$A = \int_0^\infty A_\theta d\theta = \int_0^\infty \phi_\theta e^{-\int_0^\theta \psi_\alpha d\alpha} d\theta.$
72	$\begin{aligned} p + \frac{p^2}{2} + \frac{p^3}{3} + \ldots &= ApN \\ &= p\left(1 + \frac{n}{N_0}\right), \end{aligned}$	$\begin{aligned} p + \frac{p^2}{2} + \frac{p^3}{3} + \ldots &= ApN \\ &= p\left(1 + \frac{n}{N_0}\right), \end{aligned}$
73	$\frac{p}{2} + \frac{p^2}{3} + \ldots = \frac{n}{N_0},$	$\frac{p}{2} + \frac{p^2}{3} + \ldots = \frac{n}{N_0},$
74	$pN = 2n\frac{N}{N_0} = 2n\left(1 + \frac{n}{N_0}\right) = 2n,$	$pN = 2n\frac{N}{N_0} = 2n\left(1 + \frac{n}{N_0}\right) = 2n,$
75	$A = \int_0^\infty \kappa e^{-\int_0^\theta l d\alpha} d\theta = \kappa\int_0^\infty e^{-l\theta} d\theta = \frac{\kappa}{l}.$	$A = \int_0^\infty \kappa e^{-\int_0^\theta l d\alpha} d\theta = \kappa\int_0^\infty e^{-l\theta} d\theta = \frac{\kappa}{l}.$
76	$\left.\begin{aligned} \frac{d\log x}{dt} &= \int_0^t A'_\theta v'_{t-\theta} d\theta + A'_t y'_0 \\ \frac{d\log x'}{dt} &= \int_0^t A_\theta v_{t-\theta} d\theta + A_t y_0 \end{aligned}\right\},$	$\left.\begin{aligned} \frac{d\log x}{dt} &= \int_0^t A'_\theta v'_{t-\theta} d\theta + A'_t y'_0 \\ \frac{d\log x'}{dt} &= \int_0^t A_\theta v_{t-\theta} d\theta + A_t y_0 \end{aligned}\right\},$
77		

	$\left.\begin{array}{l} -\log \dfrac{1-p}{1-\frac{y_0}{N}} = A'p'N' \\ -\log \dfrac{1-p'}{1-\frac{y_0'}{N'}} = ApN \end{array}\right\}.$	$\begin{array}{l} -\log \dfrac{1-p}{1-\frac{y_0}{N}} = A'p'N' \\ -\log \dfrac{1-p'}{1-\frac{y_0'}{N}} = ApN \end{array}.$
78	$p\left(1+\frac{p}{2}\right)p'\left(1+\frac{p'}{2}\right) = AA'pp'NN', \quad \frac{p}{2}+\frac{p'}{2} = AA'NN'-1.$	$p\left(1+\frac{p}{2}\right)p'\left(1+\frac{p'}{2}\right) = AA'pp'NN', \quad \frac{p}{2}+\frac{p'}{2} = AA'NN'-1.$
79	$p = \dfrac{2n}{N_0}\dfrac{A'N_0'}{1+A'N_0'}.$	$p = \dfrac{2n}{N_0}\dfrac{A'N_0'}{1+A'N_0'}.$
80	$N_0'pN_0 + N_0p'N_0' = 2N_0N_0'(AA'NN'-1). \quad N_0N_0' = 1/AA' = \pi_0,$	$N_0'pN_0 + N_0p'N_0' = 2N_0N_0'(AA'NN'-1). \quad N_0N_0' = 1/AA' = \pi_0,$
81	$N_0'pN_0 + N_0p'N_0' = 2(NN'-N_0N_0') = 2(\pi-\pi_0),$	$N_0'pN_0 + N_0p'N_0' = 2(NN'-N_0N_0') = 2(\pi-\pi_0),$
82	$\pi-\bar{\pi} = NN'-(N-\Delta N)(N'-\Delta N'), = N\Delta N' + N'\Delta N - \Delta N\Delta N', = Np'N' + N'pN - pNp'N', = NN'(p+p'-pp'), = N_0N_0'(p+p'-pp') + (NN'-N_0N_0')(p+p'-pp').$	$None$
83	$\pi-\bar{\pi} = N_0N_0'(p+p') = 2(\pi-\pi_0).$	$\pi-\bar{\pi} = N_0N_0'(p+p') = 2(\pi-\pi_0).$

1979 (Manually-generated LaTeX)

Eqn Num	Original Image	Decoded Image
0	$dX/dt = A - bX - \beta XY + \gamma Z \quad dY/dt = \beta XY - (b+\alpha+v)Y \quad dZ/dt = vY - (\gamma+b)Z$	$dX/dt = A - bX - \beta XY + \gamma Z \quad dY/dt = \beta XY - (b+\alpha+v)Y \quad dZ/dt = vY - (\gamma+b)Z$
1	$dN/dt = A - bN - \alpha Y$	$dN/dt = A - bN - \alpha Y$
2	$A/b > (\alpha+b+v)/\beta$	$A/b > (\alpha+b+v)/\beta$

3	$N^* = \frac{A + D(\alpha + b + v)/\beta}{b + D}$	$N^* = \frac{A + D(\alpha + b + v)/\beta}{b + D}$
4	$D = \alpha/[1 + v/(b + \gamma)]$	$D = \alpha/[1 + v/(b + \gamma)]$
5	$dX/dt = a(X + Y + Z) - bX - \beta XY + \gamma Z$ $dY/dt = \beta XY - (\alpha + b + v)Y$ $dZ/dt = vY - (b + \gamma)Z$	$dX/dt = a(X + Y + Z) - bX - \beta XY + \gamma Z$ $dY/dt = \beta XY - (\alpha + b + v)Y$ $dZ/dt = vY - (b + \gamma)Z$
6	$dN/dt = (a - b)N - \alpha Y$	$dN/dt = (a - b)N - \alpha Y$
7	$dN/dt = (r - \alpha y)N$	$dN/dt = (r - \alpha y)N$
8	$\alpha > r\left[1 + \frac{v}{b + \gamma}\right]$	$\alpha > r\left[1 + \frac{v}{b + \gamma}\right]$
9	$N^* = \frac{\alpha(\alpha + b + v)}{\beta[\alpha - r(l + v/\{b + \gamma\})]}$	$N^* = \frac{\alpha(\alpha + b + v)}{\beta[\alpha - r(l + v/\{b + \gamma\})]}$
10	$y^* = r/\alpha$	$y^* = r/\alpha$
11	$\rho = [B^2 - (b + \gamma)(\alpha - r) + rv]^{1/2} - B$	$\rho = [B^2 - (b + \gamma)(\alpha - r) + rv]^{1/2} - B$
12	$y \equiv Y/N \to (r - \rho)/\alpha$	$y \equiv Y/N \to (r - \rho)/\alpha$
13	$N_T = (\alpha + b + v)/\beta$	$N_T = (\alpha + b + v)/\beta\beta$

2000 (Manually-generated LaTeX)

Eqn Num	Original Image	Decoded Image
0	$R_0 \geq \sigma \geq R$	$R_0 > \sigma > R$
1	$dS/dt = -\beta IS/N, \quad S(0) = S_o \geq 0,$ $dI/dt = \beta IS/N - \gamma I, \quad I(0) = I_o \geq 0,$ $dR/dt = \gamma I, \quad R(0) = R_o \geq 0,$	$dS/dt = -\beta IS/N, \quad S(0) = S_o \geq 0,$ $dI/dt = \beta IS/N - \gamma I, \quad I(0) = I_o \geq 0,$ $dR/dt = \gamma I, \quad R(0) = R_o \geq 0,$
2		

	$\begin{aligned} ds/dt &= -\beta is, & s(0) &= s_o \geq 0 \\ di/dt &= \beta is - \gamma i, & i(0) &= i_o \geq 0, \end{aligned}$	$\begin{aligned} ds/dt &= -\beta is, & s(0) &= s_o \geq 0 \\ di/dt &= \beta is - \gamma i, & i(0) &= i_o \geq 0, \end{aligned}$
3	$T = \{(s,i) \mid s \geq 0, i \geq 0, s+i \leq 1\}$	$T = \{(s,i) \mid s \geq 0, i \geq 0, s+i \leq 1\}$
4	$i_o + s_o - s_\infty + \ln(s_\infty/s_o)/\sigma = 0.$	$i_o + s_o - s_\infty + \ln(s_\infty/s_o)/\sigma = 0.$
5	$i(t) + s(t) - [\ln s(t)]/\sigma = i_o + s_o - [\ln s_o]/\sigma$	$i(t) + s(t) - [\ln s(t)]/\sigma = i_o + s_o - [\ln s_o]/\sigma$
6	$\begin{aligned} dS/dt &= \mu N - \mu S - \beta IS/N, & S(0) &= S_o \geq 0, \\ dI/dt &= \beta IS/N - \gamma I - \mu I, & I(0) &= I_o \geq 0, \\ dR/dt &= \gamma I - \mu R, & R(0) &= R_o \geq 0, \end{aligned}$	$\begin{aligned} dS/dt &= \mu N - \mu S - \beta IS/N, & S(0) &= S_o \geq 0, \\ dI/dt &= \beta IS/N - \gamma I - \mu I, & I(0) &= I_o \geq 0, \\ dR/dt &= \gamma I - \mu R, & R(0) &= R_o \geq 0, \end{aligned}$
7	$\begin{aligned} ds/dt &= -\beta is + \mu - \mu s, & s(0) &= s_o \geq 0, \\ di/dt &= \beta is - (\gamma + \mu)i, & i(0) &= i_o \geq 0, \end{aligned}$	$\begin{aligned} ds/dt &= -\beta is + \mu - \mu s, & s(0) &= s_o \geq 0, \\ di/dt &= \beta is - (\gamma + \mu)i, & i(0) &= i_o \geq 0, \end{aligned}$
8	$\sigma \approx \dfrac{\ln(s_o/s_\infty)}{s_o - s_\infty}$	$\sigma \approx \dfrac{\ln(s_o/s_\infty)}{s_o - s_\infty}$
9	$\begin{aligned} dM/dt &= b(N-S) - (\delta + d)M, \\ dS/dt &= bS + \delta M - \beta SI/N - dS, \\ dE/dt &= \beta SI/N - (\varepsilon + d)E, \\ dI/dt &= \varepsilon E - (\gamma + d)I, \\ dR/dt &= \gamma I - dR, \\ dN/dt &= (b-d)N. \end{aligned}$	$\begin{aligned} dM/dt &= b(N-S) - (\delta + d)M, \\ dS/dt &= bS + \delta M - \beta SI/N - dS, \\ dE/dt &= \varepsilon E - (\gamma + d)I, \\ dI/dt &= \varepsilon E - (\gamma + d)I, \end{aligned}$
10	$\begin{aligned} dm/dt &= (d+q)(e+i+r) - \delta m, \\ de/dt &= \lambda(1-m-e-i-r) - (\varepsilon + d + q)e \\ &\quad \text{with } \lambda = \beta i, \\ di/dt &= \varepsilon e - (\gamma + d + q)i, \\ dr/dt &= \gamma i - (d+q)r. \end{aligned}$	$\begin{aligned} dm/dt &= (d+q)(e+i+r) - \delta m, \\ de/dt &= \lambda(1-m-e-i-r) - (\varepsilon + d + q)e \\ di/dt &= \varepsilon e - (\gamma + d + q)i, \\ di/dt &= \varepsilon e - (\gamma + d + q)i, \end{aligned}$
11	$\mathfrak{D} = \{(m,e,i,r) : m \geq 0, e \geq 0, i \geq 0, r \geq 0, m+e+i+r \leq 1\}.$	$\mathfrak{D} = \{(m,e,i,r) : m \geq 0, e \geq 0, i \geq 0, r \geq 0, m+e+i+r \leq 1\}.$
12	$R_0 = \sigma = \dfrac{\beta\varepsilon}{(\gamma + d + q)(\varepsilon + d + q)}.$	$R_0 = \sigma = \dfrac{\beta\varepsilon}{(\gamma + d + q)(\varepsilon + d + q)}.$
13		$\begin{aligned} m_e &= \dfrac{d+q}{\delta + d + q}\left(1 - \dfrac{1}{R_0}\right), \\ e_e &= \dfrac{\varepsilon(d+q)}{(\delta + d + q)(\varepsilon + d + q)}\left(1 - \dfrac{1}{R_0}\right), \end{aligned}$

	$m_e = \frac{d+q}{\delta+d+q}\left(1-\frac{1}{R_0}\right), \quad e_e = \frac{\delta(d+q)}{(\delta+d+q)(\varepsilon+d+q)}\left(1-\frac{1}{R_0}\right), \quad i_e = \frac{\varepsilon\delta(d+q)}{(\varepsilon+d+q)(\delta+d+q)(\gamma+d+q)}\left(1-\frac{1}{R_0}\right), \quad r_e = \frac{\varepsilon\delta\gamma}{(\varepsilon+d+q)(\delta+d+q)(\gamma+d+q)}\left(1-\frac{1}{R_0}\right),$	
14	$\lambda = \delta(d+q)(R_0-1)/(\delta+d+q),$	$\lambda = \delta(d+q)(R_0-1)/(\delta+d+q),$
15	$\frac{\partial U}{\partial a} + \frac{\partial U}{\partial t} = -d(a)U,$	$\frac{\partial U}{\partial a} + \frac{\partial U}{\partial t} = -d(a)U,$
16	$B(t) = U(0,t) = \int_0^\infty f(a)U(a,t)da.$	$B(t) = U(0,t) = \int_0^\infty f(a)U(a,t)da.$
17	$B(t) = U(0,t) = \int_0^t f(a)B(t-a)e^{-\int_0^a d(v)dv}da + \int_t^\infty f(a)U_0(a)e^{-\int_{a-t}^a d(v)dv}da.$	$B(t) = U(0,t) = \int_0^t f(a)B(t-a)e^{-\int_0^a d(v)dv}da + \int_t^\infty f(a)U_0(a)e^{-\int_{a-t}^a d(v)dv}da.$
18	$1 = \int_0^\infty f(a)\exp[-D(a)-qa]da.$	$1 = \int_0^\infty f(a)\exp[-D(a)-qa]da.$
19	$R_{pop} = \int_0^\infty f(a)\exp[-D(a)]da$	$R_{pop} = \int_0^\infty f(a)\exp[-D(a)]da$
20	$U(a,t) = \rho e^{qt}e^{-D(a)-qa} \quad \text{with } \rho = 1\Big/\int_0^\infty e^{-D(a)-qa}da\,.$	$U(a,t) = \rho e^{qt}e^{-D(a)-qa} \quad \text{with } \rho = 1\Big/\int_0^\infty e^{-D(a)-qa}da\,.$
21	$N_i(t) = \int_{a_{i-1}}^{a_i} U(a,t)da = e^{qt}\int_{a_{i-1}}^{a_i} A(a)da = e^{qt}P_i,$	$N_i(t) = \int_{a_{i-1}}^{a_i} U(a,t)da = e^{qt}\int_{a_{i-1}}^{a_i} A(a)da = e^{qt}P_i,$
22	$A(a) = A(a_{i-1})\exp[-(d_i+q)(a-a_{i-1})].$	$A(a) = A(a_{i-1})\exp[-(d_i+q)(a-a_{i-1})].].$
23	$P_i = A(a_{i-1})\{1-\exp[-(d_i+q)(a_i-a_{i-1})]\}/(d_i+q).$	$P_i = A(a_{i-1})\{1-\exp[-(d_i+q)(a_i-a_{i-1})]\}/(d_i+q).$
24	$c_i = \frac{A(a_i)}{P_i} = \frac{d_i+q}{\exp[(d_i+q)(a_i-a_{i-1})]-1}.$	$c_i = \frac{A(a_i)}{P_i} = \frac{d_i+q}{\exp[(d_i+q)(a_i-a_{i-1})]-1}.$
25	$dN_1/dt = \sum_{j=1}^n f_jN_j - (c_1+d_1)N_1, \quad dN_i/dt = c_{i-1}N_{i-1} - (c_i+d_i)Ni, \quad 2 \le i \le n-1, \quad dN_n/dt = c_{n-1}N_{n-1} - d_nN_n.$	$None$
26	$P_i = \frac{c_{i-1}\cdots c_1P_1}{(c_i+d_i+q)\cdots(c_2+d_2+q)}.$	$P_i = \frac{c_{i-1}\cdots c_1P_1}{(c_i+d_i+q)\cdots(c_2+d_2+q)}.$
27	$1 = \frac{f_1 + f_2\frac{c_1}{(c_2+d_2+q)} + \cdots + f_n\frac{c_{n-1}\cdots c_1}{(c_n+d_n+q)\cdots(c_2+d_2+q)}}{(c_1+d_1+q)}.$	$1 = \frac{f_1 + f_2\frac{c_1}{(c_2+d_2+q)} + \cdots + f_n\frac{c_{n-1}\cdots c_1}{(c_1+d_n+q)\cdots(c_2+d_2+q)}}{(c_1+d_1+q)}.$
28	$R_{pop} = f_1\frac{1}{(c_1+d_1)} + f_2\frac{c_1}{(c_2+d_2)(c_1+d_1)} + \cdots + f_n\frac{c_{n-1}\cdots c_1}{(c_n+d_n+q)\cdots(c_1+d_1)}.$	$R_{pop} = f_1\frac{1}{(c_1+d_1)} + f_2\frac{c_1}{(c_2+d_2)(c_1+d_1)} + \cdots + f_n\frac{c_{n-1}\cdots c_1}{(c_n+d_n+q)\cdots(c_1+d_1)}.$
29		

	$\partial M/\partial a+\partial M/\partial t=-(\delta+d(a))M,$ $\partial S/\partial a+\partial S/\partial t=\delta M-(\lambda(a,t)+d(a))S$ $\text{with } \lambda(a,t)=\int_0^\infty b(a)\tilde{b}(\tilde{a})I(\tilde{a},t)d\tilde{a}\int_0^\infty U(\tilde{a},t)d\tilde{a},$ $\partial E/\partial a+\partial E/\partial t=\lambda(a,t)S-(\varepsilon+d(a))E,$ $\partial I/\partial a+\partial I/\partial t=\varepsilon E-(\gamma+d(a))I,$ $\partial R/\partial a+\partial R/\partial t=\gamma I-d(a)R.$	$None$
30	$M(0,t)=\int_0^\infty f(a)[M+E+I+R]da,$ $S(0,t)=\int_0^\infty f(a)Sda,$	$M(0,t)=\int_0^\infty f(a)[M+E+I+R]da,$ $S(0,t)=\int_0^\infty f(a)Sda,$
31	$\partial m/\partial a+\partial m/\partial t=-\delta m,$ $\partial s/\partial a+\partial s/\partial t=\delta m-\lambda(a,t)s$ $\text{with } \lambda(a,t)=b(a)\int_0^\infty \tilde{b}(\tilde{a})i(\tilde{a},t)\rho e^{-D(\tilde{a})-q\tilde{a}}d\tilde{a},$ $\partial e/\partial a+\partial e/\partial t=\lambda(a,t)s-\varepsilon e,$ $\partial i/\partial a+\partial i/\partial t=\varepsilon e-\gamma i,$ $\partial r/\partial a+\partial r/\partial t=\gamma i,$	$\partial m/\partial a+\partial m/\partial t=-\delta m,$ $\partial s/\partial a+\partial s/\partial t=\delta m-\lambda(a,t)s$ $\text{with } \lambda(a,t)=b(a)\int_0^\infty \tilde{b}(\tilde{a})i(\tilde{a},t)\rho e^{-D(\tilde{a})-q\tilde{a}}d\tilde{a},$
32	$m(0,t)=\int_0^\infty f(a)[1-s(a,t)]e^{-D(a)-qa}da,$ $s(0,t)=\int_0^\infty f(a)s(a,t)e^{-D(a)-qa}da,$	$m(0,t)=\int_0^\infty f(a)[1-s(a,t)]e^{-D(a)-qa}da,$ $s(0,t)=\int_0^\infty f(a)s(a,t)e^{-D(a)-qa}da,$
33	$m(a)=(1-s_0)e^{-\delta a},$ $s(a)=e^{-\Lambda(a)}\left[s_0+\delta(1-s_0)a_0e^{-\delta x+\Lambda(x)}dx\right],$ $e(a)=e^{-\varepsilon a}\int_0^a \lambda(y)e^{\varepsilon y-\Lambda(y)}\left[s_0+\delta(1-s_0)\int_0^y e^{-\delta x+\Lambda(x)}dx\right]dy,$ $i(a)=e^{-\gamma a}\int_0^a \varepsilon e^{(\gamma-\varepsilon)z}\int_0^z \lambda(y)e^{\varepsilon y-\Lambda(y)}\left[s_0+\delta(1-s_0)\int_0^y e^{-\delta x+\Lambda(x)}dx\right]dydz,$	$None$
34	$\lambda(a)=b(a)\int_0^\infty \tilde{b}(\tilde{a})\rho e^{-D(\tilde{a})-q\tilde{a}-\gamma\tilde{a}}\int_0^{\tilde{a}} \varepsilon e^{(\gamma-\varepsilon)z}$ $\times\int_0^z \lambda(y)e^{\varepsilon y-\Lambda(y)}\left[s_0+\delta(1-s_0)\int_0^y e^{-\delta x+\Lambda(x)}dx\right]dydzd\tilde{a}.$	$None$
35	$s_0=s_0F_\lambda+\delta(1-s_0)F_*,$	$s_0=s_0F_\lambda+\delta(1-s_0)F_*,$
36	$F_*=\int_0^\infty f(a)e^{-\Lambda(a)-D(a)-qa}\int_0^a e^{-\delta x+\Lambda(x)}dxda.$	$F_*=\int_0^\infty f(a)e^{-\Lambda(a)-D(a)-qa}\int_0^a e^{-\delta x+\Lambda(x)}dxda.$
37	$1=\int_0^\infty \tilde{b}(\tilde{a})\rho e^{-D(\tilde{a})-q\tilde{a}-\gamma\tilde{a}}\int_0^{\tilde{a}} \varepsilon e^{(\gamma-\varepsilon)z}\int_0^z b(y)e^{\varepsilon y}$ $\times\left[\delta F_*e^{-k\int_0^y b(\alpha)d\alpha}+\delta(1-F_\lambda)\int_0^y e^{-\delta x-k\int_x^y b(\alpha)d\alpha}dx\right]\Big/(\delta F_*+1-F_\lambda)dydzd\tilde{a}.$	$None$
38	$R_0=\int_0^\infty \tilde{b}(\tilde{a})\rho e^{-D(\tilde{a})-q\tilde{a}-\gamma\tilde{a}}\int_0^{\tilde{a}} \varepsilon e^{(\gamma-\varepsilon)z}\int_0^z b(y)e^{\varepsilon y}dydzd\tilde{a}.$	$R_0=\int_0^\infty \tilde{b}(\tilde{a})\rho e^{-D(\tilde{a})-q\tilde{a}-\gamma\tilde{a}}\int_0^{\tilde{a}} \varepsilon e^{(\gamma-\varepsilon)z}\int_0^z b(y)e^{\varepsilon y}dydzd\tilde{a}.$
39		

	$V = \int_0^\infty [\alpha(a)e(a,t) + \beta(a)i(a,t)]da,$	$V = \int_0^\infty [\alpha(a)e(a,t) + \beta(a)i(a,t)]da,$
40	$\dot{V} = \int_0^\infty \{\alpha(a)[\lambda s - \varepsilon e - \partial e/\partial a] + \beta(a)[\varepsilon e - \gamma i - \partial i/\partial a]\}da$ $= \int_0^\infty \{\lambda s \alpha(a) + e[\alpha'(a) - \varepsilon\alpha(a) + \varepsilon\beta(a)] + [\beta'(a) - \gamma\beta(a)]i\}da.$	$\dot{V} = \int_0^\infty \{\alpha(a)[\lambda s - \varepsilon e - \partial e/\partial a] + \beta(a)[\varepsilon e - \gamma i - \partial i/\partial a]\}da$ $= \int_0^\infty \{\lambda s \alpha(a) + e[\alpha'(a) - \varepsilon\alpha(a) + \varepsilon\beta(a)] + [\beta'(a) - \gamma\beta(a)]i\}da.$
41	$\dot{V} = \int_0^\infty sb(a)\varepsilon e^{\varepsilon a} \int_a^\infty e^{-\varepsilon z}\beta(z)dzda \int_0^\infty \tilde{b}(\tilde{a})i\rho e^{-D(\tilde{a})-q\tilde{a}}d\tilde{a} + \int_0^\infty [\beta' - \gamma\beta]ida.$	$\dot{V} = \int_0^\infty sb(a)\varepsilon e^{\varepsilon a} \int_a^\infty e^{-\varepsilon z}\beta(z)dzda \int_0^\infty \tilde{b}(\tilde{a})i\rho e^{-D(\tilde{a})-q\tilde{a}}d\tilde{a} + \int_0^\infty [\beta' - \gamma\beta]ida.$
42	$\dot{V} = \left[\int_0^\infty sb(a)\varepsilon e^{\varepsilon a} \int_a^\infty e^{(\gamma-\varepsilon)z} \int_z^\infty \tilde{b}(x)\rho e^{-D(x)-qx-\gamma x}dxdzda - 1\right]$ $\times \int_0^\infty \tilde{b}(\tilde{a})i(\tilde{a},t)\rho e^{-D(\tilde{a})-q\tilde{a}}d\tilde{a}.$	$\dot{V} = \left[\int_0^\infty sb(a)\varepsilon e^{\varepsilon a} \int_a^\infty e^{(\gamma-\varepsilon)z} \int_z^\infty \tilde{b}(x)\rho e^{-D(x)-qx-\gamma x}dxdzda - 1\right]$
43	$\dot{V} \le (R_0 - 1)\int_0^\infty \tilde{b}(\tilde{a})i(\tilde{a},t)\rho e^{-D(\tilde{a})-q\tilde{a}}d\tilde{a} \le 0 \quad \text{if } R_0 \le 1.$	$\dot{V} \le (R_0 - 1)\int_0^\infty \tilde{b}(\tilde{a})i(\tilde{a},t)\rho e^{-D(\tilde{a})-q\tilde{a}}d\tilde{a} \le 0 \quad \text{if } R_0 \le 1.$
44	$1 = \int_0^\infty \tilde{b}(\tilde{a})\rho e^{-D(\tilde{a})-q\tilde{a}-\gamma\tilde{a}} \int_0^{\tilde{a}} b(y)e^{\gamma y - k\int_0^y b(\alpha)d\alpha}dyd\tilde{a}$	$1 = \int_0^\infty \tilde{b}(\tilde{a})\rho e^{-D(\tilde{a})-q\tilde{a}-\gamma\tilde{a}} \int_0^{\tilde{a}} b(y)e^{\gamma y - k\int_0^y b(\alpha)d\alpha}dyd\tilde{a}$
45	$R_0 = \int_0^\infty \tilde{b}(\tilde{a})\rho e^{-D(\tilde{a})-q\tilde{a}-\gamma\tilde{a}} \int_0^{\tilde{a}} b(y)e^{\gamma y}dyd\tilde{a}.$	$R_0 = \int_0^\infty \tilde{b}(\tilde{a})\rho e^{-D(\tilde{a})-q\tilde{a}-\gamma\tilde{a}} \int_0^{\tilde{a}} b(y)e^{\gamma y}dyd\tilde{a}.$
46	$A = E[a] = \frac{\int_0^\infty a\lambda(a)e^{-D(a)}[\delta F_* e^{-\Lambda(a)} + \delta(1-F_\lambda)\int_0^a e^{-\delta x - \Lambda(a) + \Lambda(x)}dx]da}{\int_0^\infty \lambda(a)e^{-D(a)}[\delta F_* e^{-\Lambda(a)} + \delta(1-F_\lambda)\int_0^a e^{-\delta x - \Lambda(a) + \Lambda(x)}dx]da}.$	$A = E[a] = \frac{\int_0^\infty a\lambda(a)e^{-D(a)}[\delta F_* e^{-\Lambda(a)} + \delta(1-F_\lambda)\int_0^a e^{-\delta x - \Lambda(a) + \Lambda(x)}dx]da}{\int_0^\infty \lambda(a)e^{-D(a)}[\delta F_* e^{-\Lambda(a)} + \delta(1-F_\lambda)\int_0^a}$
47	$A = E[a] = \frac{\int_0^\infty a\lambda(a)e^{-\Lambda(a)-D(a)}da}{\int_0^\infty \lambda(a)e^{-\Lambda(a)-D(a)}da}.$	$A = E[a] = \frac{\int_0^\infty a\lambda(a)e^{-\Lambda(a)-D(a)}da}{\int_0^\infty \lambda(a)e^{-\Lambda(a)-D(a)}da}.$
48	$R_0 = \beta\varepsilon/[(\gamma + d + q)(\varepsilon + d + q)],$	$R_0 = \beta\varepsilon/[(\gamma + d + q)(\varepsilon + d + q)],$
49	$1 = \frac{(d+q)R_0}{\lambda + d + q}\left[s_0 + \frac{\delta(1-s_0)}{\delta + d + q}\right].$	$1 = \frac{(d+q)R_0}{\lambda + d + q}\left[s_0 + \frac{\delta(1-s_0)}{\delta + d + q}\right].$
50	$s_0 = \frac{\delta - \lambda s_0}{\delta - \lambda}F_\lambda - \frac{\delta(1-s0)}{\delta - \lambda}F_\delta,$	$s_0 = \frac{\delta - \lambda s_0}{\delta - \lambda}F_\lambda - \frac{\delta(1-s0)}{\delta - \lambda}F_\delta,$
51	$s_0 = \delta(F_\lambda - F_\delta)/[\delta(1-F_\delta) - \lambda(1-F_\lambda)].$	$s_0 = \delta(F_\lambda - F_\delta)/[\delta(1-F_\delta) - \lambda(1-F_\lambda)].$
52	$1 = \frac{R_0(d+q)\delta\left[F_\lambda - F_\delta + \frac{(\delta-\lambda)(1-F_\lambda)}{\delta+d+q}\right]}{(\lambda+d+q)[\delta(1-F_\delta) - \lambda(1-F_\lambda)]},$	$1 = \frac{R_0(d+q)\delta\left[F_\lambda - F_\delta + \frac{(\delta-\lambda)(1-F_\lambda)}{\delta+d+q}\right]}{(\lambda+d+q)[\delta(1-F_\delta) - \lambda(1-F_\lambda)]},$
53	$A = E[a] = \frac{\lambda d\int_0^\infty a[c_1e^{-(\lambda+d)a} + c_2e^{-(\delta+d)a}]da}{\lambda d\int_0^\infty [c_1e^{-(\lambda+d)a} + c_2e^{-(\delta+d)a}]da} = \frac{\frac{\delta-\lambda s_0}{(\lambda+d)^2} - \frac{\delta(1-s_0)}{(\delta+d)^2}}{\frac{\delta-\lambda s_0}{(\lambda+d)} - \frac{\delta(1-s0)}{(\delta+d)}}.$	$A = E[a] = \frac{\lambda d\int_0^\infty a[c_1e^{-(\lambda+d)a} + c_2e^{-(\delta+d)a}]da}{\lambda d\int_0^\infty [c_1e^{-(\lambda+d)a} + c_2e^{-(\delta+d)a}]da} = \frac{\frac{\delta-\lambda s_0}{(\lambda+d)^2} - \frac{\delta(1-s_0)}{(}}{}$
54	$R_0 = \frac{[q + 1/(A-p)](1+pq)}{(q + 1/L)(1 - p/L)}.$	$R_0 = \frac{[q + 1/(A-p)](1+pq)}{(q + 1/L)(1 - p/L)}.$
55	$1 = \frac{R_0(d+q)}{\lambda+d+q}\left[s_0 + \frac{\delta(1-s0)}{\delta+d+q} - g[c_1e^{-(\lambda+d+q)A_v} + c_2e^{-(\delta+d+q)A_v}]\right],$	$1 = \frac{R_0(d+q)}{\lambda+d+q}\left[s_0 + \frac{\delta(1-s0)}{\delta+d+q} - g[c_1e^{-(\lambda+d+q)A_v} + c_2e^{-(\delta+d+q)A_v}]\right],$
56	$s_0 = c_1F_\lambda + c_2F_\delta - g\left[c_1 + c_2e^{(\lambda-\delta)A_v}\right]F_{A_v},$	$s_0 = c_1F_\lambda + c_2F_\delta - g\left[c_1 + c_2e^{(\lambda-\delta)A_v}\right]F_{A_v},$
57	$1 = \frac{R_0(d+q)}{\lambda+d+q}\left[1 - ge^{-(\lambda+d+q)A_v}\right].$	$1 = \frac{R_0(d+q)}{\lambda+d+q}\left[1 - ge^{-(\lambda+d+q)A_v}\right].$

58	$g e^{-(d+q)A_v} \geq 1 - 1/R_0,$	$g e^{-(d+q)A_v} \geq 1 - 1/R_0,$
59	$A = \frac{1}{\lambda+d} - \frac{gA_v[c_1 e^{-(\lambda+d)A_v} + c_2 e^{-(\delta+d)A_v}] + c_2 \frac{\delta-\lambda}{(\delta+d)^2}}{c_1[1 - g e^{-(\lambda+d)A_v}] + c_2[\frac{\lambda+d}{\delta+d} - g e^{-(\delta+d)A_v}]}.$	$A = \frac{1}{\lambda+d} - \frac{gA_v[c_1 e^{-(\lambda+d)A_v} + c_2 e^{-(\delta+d)A_v}] + c_2 \frac{\delta-\lambda}{(\delta+d)^2}}{c_1[1 - g e^{-(\lambda+d)A_v}] + c_2[\frac{\lambda+d}{\delta+d} - g e^{-(\delta+d)A_v}]}$
60	$R_0 = \frac{\beta}{\gamma}\left[1 + \frac{\gamma}{\varepsilon-\gamma}\frac{1-e^{-\varepsilon L}}{\varepsilon L} - \frac{\varepsilon}{(\varepsilon-\gamma)}\frac{1-e^{-\gamma L}}{\gamma L}\right].$	$R_0 = \frac{\beta}{\gamma}\left[1 + \frac{\gamma}{\varepsilon-\gamma}\frac{1-e^{-\varepsilon L}}{\varepsilon L} - \frac{\varepsilon}{(\varepsilon-\gamma)}\frac{1-e^{-\gamma L}}{\gamma L}\right].$
61	$1 = \beta\varepsilon\left[\frac{1-e^{-\lambda L}}{(\gamma-\lambda)(\varepsilon-\lambda)\lambda L} + \frac{1-e^{-\varepsilon L}}{(\varepsilon-\lambda)(\varepsilon-\gamma)\varepsilon L} - \frac{1-e^{-\gamma L}}{(\gamma-\lambda)(\varepsilon-\gamma)\gamma L}\right].$	$1 = \beta\varepsilon\left[\frac{1-e^{-\lambda L}}{(\gamma-\lambda)(\varepsilon-\lambda)\lambda L} + \frac{1-e^{-\varepsilon L}}{(\varepsilon-\lambda)(\varepsilon-\gamma)\varepsilon L} - \frac{1-e^{-\gamma L}}{(\gamma-\lambda)(\varepsilon-\gamma)\gamma L}\right].$
62	$R_0 = \frac{\beta}{\gamma}\left[1 - \frac{1-e^{-\gamma L}}{\gamma L}\right],$	$R_0 = \frac{\beta}{\gamma}\left[1 - \frac{1-e^{-\gamma L}}{\gamma L}\right],$
63	$1 = \frac{\beta}{\gamma-\lambda}\left[\frac{1-e^{-\lambda L}}{\lambda L} - \frac{1-e^{-\gamma L}}{\gamma L}\right].$	$1 = \frac{\beta}{\gamma-\lambda}\left[\frac{1-e^{-\lambda L}}{\lambda L} - \frac{1-e^{-\gamma L}}{\gamma L}\right].$
64	$A = \frac{1}{\lambda} - \frac{Le^{-\lambda L}}{1-e^{-\lambda L}}$	$A = \frac{1}{\lambda} - \frac{Le^{-\lambda L}}{1-e^{-\lambda L}}$
65	$\bar{s} = \int_0^L \frac{e^{-\lambda a}}{L} da = \frac{1-e^{-\lambda L}}{\lambda L}.$	$\bar{s} = \int_0^L \frac{e^{-\lambda a}}{L} da = \frac{1-e^{-\lambda L}}{\lambda L}.$
66	$\begin{aligned} \partial S/\partial a + \partial S/\partial t &= -\lambda(a,t)S - d(a)S, \\ \lambda(a,t) &= \int_0^\infty b(a)\tilde{b}(\tilde{a})I(\tilde{a},t)d\tilde{a} \Big/ \int_0^\infty U(\tilde{a},t)d\tilde{a}, \\ \partial E/\partial a + \partial E/\partial t &= \lambda(a,t)S - \varepsilon I - d(a)E, \\ \partial I/\partial a + \partial I/\partial t &= \varepsilon I - \gamma I - d(a)I, \\ \partial R/\partial a + \partial R/\partial t &= \gamma I - d(a)R. \end{aligned}$	$\begin{aligned} \partial S/\partial a + \partial S/\partial t &= -\lambda(a,t)S - d(a)S, \\ \lambda(a,t) &= \int_0^\infty b(a)\tilde{b}(\tilde{a})I(\tilde{a},t)d\tilde{a} \\ \partial E/\partial a + \partial E/\partial t &= \lambda(a,t)S - \varepsilon I - d(a)E, \end{aligned}$
67	$\begin{aligned} ds_1/dt &= (c_1+d_1+q)P_1 - [\lambda_1+c_1+d_1+q]s_1, \\ ds_i/dt &= c_{i-1}s_{i-1} - [\lambda_i+c_i+d_i+q]s_i, && i \geq 2, \\ \lambda_i &= b_i \sum_{j=1}^n \tilde{b}_j i_j, \\ de_1/dt &= \lambda_1 s_1 - [\varepsilon_1+c_1+d_1+q]e_1, \\ de_i/dt &= \lambda_i s_i + c_{i-1}e_{i-1} - [\varepsilon_i+c_i+d_i+q]e_i, && i \geq 2, \\ di_1/dt &= \varepsilon_1 e_1 - [\gamma_1+c_1+d_1+q]i_1, \\ di_i/dt &= \varepsilon_i e_i + c_{i-1}i_{i-1} - [\gamma_i+c_i+d_i+q]i_i, && i \geq 2, \\ dr_1/dt &= \gamma_1 i_1 - [c_1+d_1+q]r_1, \\ dr_i/dt &= \gamma_i i_i + c_{i-1}r_{i-1} - [c_i+d_i+q]r_i, && i \geq 2. \end{aligned}$	$None$
68	$\begin{aligned} s_1 &= \hat{c}_1 P_1/\hat{\lambda}_1, \quad s_i = c_{i-1}s_{i-1}/\hat{\lambda}_i && \text{for } i \geq 2, \\ e_1 &= \lambda_1 s_1/\hat{\varepsilon}_1, \quad e_i = (\lambda_i s_i + c_{i-1}e_{i-1})/\hat{\varepsilon}_i && \text{for } i \geq 2, \\ i_1 &= \varepsilon_1 e_1/\hat{\gamma}_1, \quad i_i = (\varepsilon_i e_i + c_{i-1}i_{i-1})/\hat{\gamma}_i && \text{for } i \geq 2, \end{aligned}$	$None$
69	$e_i = \frac{\lambda_i C_{i-1}}{\hat{\varepsilon}_i\hat{\lambda}_i\cdots\hat{\lambda}_1} + \frac{\lambda_{i-1}C_{i-1}}{\hat{\varepsilon}_i\hat{\varepsilon}_{i-1}\hat{\lambda}_{i-1}\cdots\hat{\lambda}_1} + \frac{\lambda_{i-2}C_{i-1}}{\hat{\varepsilon}_i\hat{\varepsilon}_{i-1}\hat{\varepsilon}_{i-2}\hat{\lambda}_{i-2}\cdots\hat{\lambda}_1} + \cdots + \frac{\lambda_1 C_{i-1}}{\hat{\varepsilon}_i\cdots\hat{\varepsilon}_1\hat{\lambda}_1}$	

		$e_i = \frac{\lambda_i C_{i-1}}{\hat\varepsilon_i\hat\lambda_i\cdots\hat\lambda_1} + \frac{\lambda_{i-1}C_{i-1}}{\hat\varepsilon_i\hat\varepsilon_{i-1}\hat\lambda_{i-1}\cdots\hat\lambda_1} + \frac{\lambda_{i-2}C_{i-1}}{\hat\varepsilon_i\hat\varepsilon_{i-1}\hat\varepsilon_{i-}}$
70	$\frac{i_i}{C_{i-1}} = \frac{\varepsilon_i}{\hat\gamma_i}\left(\frac{\lambda i}{\hat\varepsilon_i\hat\lambda_i\cdots\hat\lambda_1} + \frac{\lambda_{i-1}}{\hat\varepsilon_i\hat\varepsilon_{i-1}\hat\lambda_{i-1}\cdots\hat\lambda_1}+\cdots+\frac{\lambda_1}{\hat\varepsilon_i\cdots\hat\varepsilon_1\hat\lambda_1}\right) + \frac{\varepsilon_{i-1}}{\hat\gamma_i\hat\gamma_{i-1}}\left(\frac{\lambda_{i-1}}{\hat\varepsilon_{i-1}\hat\lambda_{i-1}\cdots\hat\lambda_1}+\frac{\lambda_{i-2}}{\hat\varepsilon_{i-1}\hat\varepsilon_{i-2}\hat\lambda_{i-2}\cdots\hat\lambda_1}+\cdots+\frac{\lambda_1}{\hat\varepsilon_{i-1}\cdots\hat\varepsilon_1\hat\lambda_1}\right)+\cdots+\frac{\varepsilon_2}{\hat\gamma_i\cdots\hat\gamma_2}\left(\frac{\lambda_2}{\hat\varepsilon_2\hat\lambda_2\hat\lambda_1}+\frac{\lambda_1}{\hat\varepsilon_2\hat\varepsilon_1\hat\lambda_1}\right)+\frac{\varepsilon_1}{\hat\gamma_i\cdots\hat\gamma_1}\left(\frac{\lambda_1}{\hat\varepsilon_1\hat\lambda_1}\right).$	None
71	$1=\sum_{j=1}^n \tilde b_j C_{j-1}\left[\frac{\varepsilon_j}{\hat\gamma_j}\left(\frac{b_j}{\hat\varepsilon_j\hat b_j\cdots\hat b_1}+\frac{b_{j-1}}{\hat\varepsilon_j\hat\varepsilon_{j-1}\hat b_{j-1}\cdots\hat b_1}+\cdots+\frac{b_1}{\hat\varepsilon_j\cdots\hat\varepsilon_1\hat b_1}\right)+\frac{\varepsilon_{j-1}}{\hat\gamma_j\hat\gamma_{j-1}}\left(\frac{b_{j-1}}{\hat\varepsilon_{j-1}\hat b_{j-1}\cdots\hat b_1}+\frac{b_{j-2}}{\hat\varepsilon_{j-1}\hat\varepsilon_{j-2}\hat b_{j-2}\cdots\hat b_1}+\cdots+\frac{b_1}{\hat\varepsilon_{j-1}\cdots\hat\varepsilon_1\hat b_1}\right)+\cdots+\frac{\varepsilon_2}{\hat\gamma_j\cdots\hat\gamma_2}\left(\frac{b_2}{\hat\varepsilon_2\hat b_2\hat b_1}+\frac{b_1}{\hat\varepsilon_2\hat\varepsilon_1\hat b_1}\right)+\frac{\varepsilon_1}{\hat\gamma_j\cdots\hat\gamma_1}\left(\frac{b_1}{\hat\varepsilon_1\hat b_1}\right)\right],$	None
72	$R_0=\sum_{j=1}^n \tilde b_j C_{j-1}\left[\frac{\varepsilon_j}{\hat\gamma_j}\left(\frac{b_j}{\hat\varepsilon_j\hat c_j\cdots\hat c_1}+\frac{b_{j-1}}{\hat\varepsilon_j\hat\varepsilon_{j-1}\hat c_{j-1}\cdots\hat c_1}+\cdots+\frac{b_1}{\hat\varepsilon_j\cdots\hat\varepsilon_1\hat c_1}\right)+\frac{\varepsilon_{j-1}}{\hat\gamma_j\hat\gamma_{j-1}}\left(\frac{b_{j-1}}{\hat\varepsilon_{j-1}\hat c_{j-1}\cdots\hat c_1}+\frac{b_{j-2}}{\hat\varepsilon_{j-1}\hat\varepsilon_{j-2}\hat c_{j-2}\cdots\hat c_1}+\cdots+\frac{b_1}{\hat\varepsilon_{j-1}\cdots\hat\varepsilon_1\hat c_1}\right)+\cdots+\frac{\varepsilon_2}{\hat\gamma_j\cdots\hat\gamma_2}\left(\frac{b_2}{\hat\varepsilon_2\hat c_2\hat c_1}+\frac{b_1}{\hat\varepsilon_2\hat\varepsilon_1\hat c_1}\right)+\frac{\varepsilon_1}{\hat\gamma_j\cdots\hat\gamma_1}\left(\frac{b_1}{\hat\varepsilon_1\hat c_1}\right)\right],$	None
73	$\dot V = \sum \alpha_i b_i s_i \sum \tilde b_j i_j - (\beta_1\hat\gamma_1 - \beta_2 c_1) i_1 - \cdots - (\beta_{n-1}\hat\gamma_{n-1} - \beta_n c_{n-1}) i_{n-1} - \beta_n\hat\gamma_n i_n.$	$\dot V = \sum \alpha_i b_i s_i \sum \tilde b_j i_j - (\beta_1\hat\gamma_1 - \beta_2 c_1) i_1 - \cdots - (\beta_{n-1}\hat\gamma_{n-1} - \beta_n c_{n-1}) i_{n-1} - \beta_n\hat\gamma_n i_n.$
74	$1=\sum_{j=1}^n\tilde b_jC_{j-1}\left[\frac{b_j}{\hat\gamma_j\hat b_j\cdots\hat b_1}+\frac{b_{j-1}}{\hat\gamma_j\hat\gamma_{j-1}\hat b_{j-1}\cdots\hat b_1}+\cdots+\frac{b_2}{\hat\gamma_j\cdots\hat\gamma_2\hat b_2\hat b_1}+\frac{b_1}{\hat\gamma_j\cdots\hat\gamma_1\hat b_1}\right],$	$1=\sum_{j=1}^n\tilde b_jC_{j-1}\left[\frac{b_j}{\hat\gamma_j\hat b_j\cdots\hat b_1}+\frac{b_{j-1}}{\hat\gamma_j\hat\gamma_{j-1}\hat b_{j-1}\cdots\hat b_1}+\cdots+\frac{b_2}{\hat\gamma_j\cdots\hat\gamma_2\hat b_2}\right.$
75	$R_0=\sum_{j=1}^n\tilde b_jC_{j-1}\left[\frac{b_j}{\hat\gamma_j\hat c_j\cdots\hat c_1}+\frac{b_{j-1}}{\hat\gamma_j\hat\gamma_{j-1}\hat c_{j-1}\cdots\hat c_1}+\cdots+\frac{b_2}{\hat\gamma_j\cdots\hat\gamma_2\hat c_2\hat c_1}+\frac{b_1}{\hat\gamma_j\cdots\hat\gamma_1\hat c_1}\right].$	$R_0=\sum_{j=1}^n\tilde b_jC_{j-1}\left[\frac{b_j}{\hat\gamma_j\hat c_j\cdots\hat c_1}+\frac{b_{j-1}}{\hat\gamma_j\hat\gamma_{j-1}\hat c_{j-1}\cdots\hat c_1}+\cdots+\frac{b_2}{\hat\gamma_j\cdots\hat\gamma_2\hat c}\right.$
76	$A=E[a]=\frac{\int_0^\infty a\lambda(a)s(a)e^{-D(a)}da}{\int_0^\infty\lambda(a)s(a)e^{-D(a)}da}=\frac{\sum_{i=1}^n\int_{a_{i-1}}^{a_i}a\lambda(a)s(a)e^{-D(a)}da}{\sum_{i=1}^n\int_{a_{i-1}}^{a_i}\lambda(a)s(a)e^{-D(a)}da}.$	$A=E[a]=\frac{\int_0^\infty a\lambda(a)s(a)e^{-D(a)}da}{\int_0^\infty\lambda(a)s(a)e^{-D(a)}da}=\frac{\sum_{i=1}^n\int_{a_{i-1}}^{a_i}a\lambda(a)s(a)e^{-D(a)}da}{\sum_{i=1}^n\int_{a_{i-1}}^{a}}$
77	$A=\frac{\sum_{i=1}^n b_is_i\pi_{i-1}[1+d_ia_{i-1}-(1+d_ia_i)e^{-d_i\Delta_i}]/d_i^2}{\sum_{i=1}^n b_is_i\pi_{i-1}[1-e^{-d_i\Delta_i}]/d_i}.$	$A=\frac{\sum_{i=1}^n b_is_i\pi_{i-1}[1+d_ia_{i-1}-(1+d_ia_i)e^{-d_i\Delta_i}]/d_i^2}{\sum_{i=1}^n b_is_i\pi_{i-1}[1-e^{-d_i\Delta_i}]/d_i}.$
78	$R\cong\frac{\sum_{j=1}^{16}\lambda_js_jP_j\left(\frac{1}{\gamma+d_j+q}\right)\left(\frac{\varepsilon}{\varepsilon+d_j+q}\right)}{\sum_{j=1}^{16}i_jP_j}.$	$R\cong\frac{\sum_{j=1}^{16}\lambda_js_jP_j\left(\frac{1}{\gamma+d_j+q}\right)\left(\frac{\varepsilon}{\varepsilon+d_j+q}\right)}{\sum_{j=1}^{16}i_jP_j}.$
79	$R_0=\sigma\cong\frac{\sum_{j=1}^{16}\lambda_jP_j\left(\frac{1}{\gamma+d_j+q}\right)\left(\frac{\varepsilon}{\varepsilon+d_j+q}\right)}{\sum_{j=1}^{16}i_jP_j}.$	$R_0=\sigma\cong\frac{\sum_{j=1}^{16}\lambda_jP_j\left(\frac{1}{\gamma+d_j+q}\right)\left(\frac{\varepsilon}{\varepsilon+d_j+q}\right)}{\sum_{j=1}^{16}i_jP_j}.$
80	$A\cong\frac{\sum_{j=1}^{16}[\frac{a_{j-1}+a_j}{2}]\lambda_js_jP_j}{\sum_{j=1}^{16}\lambda_js_jP_j}.$	$A\cong\frac{\sum_{j=1}^{16}[\frac{a_{j-1}+a_j}{2}]\lambda_js_jP_j}{\sum_{j=1}^{16}\lambda_js_jP_j}.$
81	$R_0=[1+\lambda/(d+q)][1+(d+q)/\delta],$	$R_0=[1+\lambda/(d+q)][1+(d+q)/\delta],$
82	$R\cong\frac{\sum_{j=1}^{32}\lambda_j(s_j+r_{1j}+r_{2j})P_j/(\gamma+d_j)}{\sum_{j=1}^{32}(i_j+i_{mj}+i_{wj})P_j},$	$R\cong\frac{\sum_{j=1}^{32}\lambda_j(s_j+r_{1j}+r_{2j})P_j/(\gamma+d_j)}{\sum_{j=1}^{32}(i_j+i_{mj}+i_{wj})P_j},$
83		

	$\sigma \cong \frac{\sum_{j=1}^{32} \lambda_j P_j/(\gamma + d_j)}{\sum_{j=1}^{32}(i_j + i_{mj} + i_{wj})P_j},$	$\sigma \cong \frac{\sum_{j=1}^{32} \lambda_j P_j/(\gamma + d_j)}{\sum_{j=1}^{32}(i_j + i_{mj} + i_{wj})P_j},$
84	$\sigma = R_0[I + \rho_m I_m + \rho_w I_w]/[I + I_m + I_w] < R_0,$	$\sigma = R_0[I + \rho_m I_m + \rho_w I_w]/[I + I_m + I_w] < R_0,$

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Eqn Num	Original Image	Decoded Image
0	$\dot{S} = -\beta IS$ $\dot{I} = \beta IS - \gamma I$ $\dot{R} = \gamma I$	$\dot{S} = -\beta IS$ $\dot{I} = \beta IS - \gamma I$ $\dot{R} = \gamma I$
1	$S(0) = 1 - \rho$ $I(0) = \rho$ $R(0) = 0.$	$S(0) = 1 - \rho$ $I(0) = \rho$ $R(0) = 0.$
2	$\dot{\theta} = -\tau\phi_I$ $\phi_I = \theta - (1-\rho)\frac{\psi'(\theta)}{\langle K \rangle} - \frac{\gamma}{\tau}(1-\theta)$ $S = (1-\rho)\psi(\theta)$ $I = 1 - S - R$ $\dot{R} = \gamma I$	$\dot{\theta} = -\tau\phi_I$ $\phi_I = \theta - (1-\rho)\frac{\psi'(\theta)}{\langle K \rangle} - \frac{\gamma}{\tau}(1-\theta)$ $S = (1-\rho)\psi(\theta)$

3	$\theta(0) = 1$ $R(0) = 0$	$\theta(0) = 1$ $R(0) = 0$
4	$\phi_S = (1-\rho)\psi'(\theta)/\langle K\rangle$	$\phi_S = (1-\rho)\psi'(\theta)/\langle K\rangle$
5	$\phi_R = \gamma(1-\theta)/\tau.$	$\phi_R = \gamma(1-\theta)/\tau.$
6	$\phi_I = \theta - \phi_S - \phi_R$ $= \theta - (1-\rho)\frac{\psi'(\theta)}{\langle K\rangle} - \frac{\gamma}{\tau}(1-\theta).$	$\phi_I = \theta - \phi_S - \phi_R$ $= \theta - (1-\rho)\frac{\psi'(\theta)}{\langle K\rangle} - \frac{\gamma}{\tau}(1-\theta).$
7	$\dot{S} + \beta I S = 0$	$\dot{S} + \beta I S = 0$
8	$\frac{d}{dt}\left(S e^{\beta \int_0^t I d\hat{t}} \right) = 0.$	$\frac{d}{dt}\left(\int_S^t \int_0^t I d\hat{t} \right) = 0.$
9	$S(t) = S(0)e^{-\xi(t)} = (1-\rho)e^{-\xi(t)}.$	$S(t) = S(0)e^{-\xi(t)} = (1-\rho)e^{-\xi(t)}.$
10		$S = (1-\rho)e^{-\xi}$ $I = 1 - S - R = 1 - (1-\rho)e^{-\xi} - \frac{\gamma\xi}{\beta}$ $R = \frac{\gamma\xi}{\beta}$

	$S = (1-\rho)e^{-\xi}$ $I = 1 - S - R = 1 - (1-\rho)e^{-\xi} - \frac{\gamma\xi}{\beta}$ $R = \frac{\gamma\xi}{\beta}$ $\dot{\xi} = \beta I = \beta\left[1 - (1-\rho)e^{-\xi} - \frac{\gamma\xi}{\beta}\right]$	
11	$\xi(0) = 0.$	$\xi(0) = 0.$
12	$\mathscr{R}_0 = \beta/\gamma.$	$\beta_0 = \beta/\gamma$
13	$\mathscr{R}_0 = \sum_k \frac{kP(k)}{\langle K \rangle}(k-1)\frac{\tau}{\tau+\gamma}$ $= \frac{\tau\langle K^2 - K\rangle}{(\tau+\gamma)\langle K\rangle}.$	$\partial_0 = \sum_k \frac{kP(k)}{\langle K \rangle}(k-1)\frac{\tau}{\tau+\gamma}$ $= \frac{\tau\langle K^2 - K\rangle}{(\tau+\gamma)\langle K\rangle}.$
14	$R(\infty) = 1 - (1-\rho)e^{-\mathscr{R}_0 R(\infty)}$	$R(\infty) = 1 - (1-\rho)e^{-\partial\bar{\partial}_0 R(\infty)}$
15	$\theta(\infty) = (1-\rho)\frac{\psi'(\theta(\infty))}{\langle K\rangle} = \frac{\gamma(1-\theta(\infty))}{\tau}.$	$\theta(\infty) = (1-\rho)\frac{\psi'(\theta(\infty))}{\langle K\rangle} = \frac{\gamma(1-\theta(\infty))}{\tau}.$
16		$S = (1-\rho)e^{-\xi}\psi(\theta)$ $\dot{I}_1 = 1 - S - I_2 - R$ $\dot{I}_2 = \gamma_1 I_1 - \gamma_2 I_2$ $\dot{I}_2 = \gamma_2 I_1 - \gamma_2 I_2$

	$\begin{aligned} S &= (1-\rho)e^{-\xi}\psi(\theta) \\ I_1 &= 1 - S - I_2 - R \\ \dot{I}_2 &= \gamma_1 I_1 - \gamma_2 I_2 \\ \dot{R} &= \gamma_2 I_2 \\ \dot{\xi} &= \beta I_1 \\ \dot{\theta} &= -\tau_1 \phi_{I,1} - \tau_2 \phi_{I,2} \\ \phi_S &= \frac{(1-\rho)e^{-\xi}\psi'(\theta)}{\langle K \rangle} \\ \phi_{I,1} &= \theta - \phi_S - \phi_{I,2} - \phi_R \\ \dot{\phi}_{I,2} &= \gamma_1 \phi_{I,1} - (\gamma_2 + \tau_2)\phi_{I,2} \\ \dot{\phi}_R &= \gamma_2 \phi_{I,2}. \end{aligned}$	
17	$\begin{pmatrix} N_{\mathrm{ma}}(g+1) \\ N_{\mathrm{se}}(g+1) \end{pmatrix} = \begin{pmatrix} R_{\mathrm{ma}} & R_{\mathrm{ma}} \\ R_{\mathrm{se\|ma}} & R_{\mathrm{se\|se}} \end{pmatrix} \begin{pmatrix} N_{\mathrm{ma}}(g) \\ N_{\mathrm{se}}(g) \end{pmatrix}.$	$\begin{pmatrix} N_{\mathrm{ma}}(g+1) \\ N_{\mathrm{se}}(g+1) \end{pmatrix} = \begin{pmatrix} R_{\mathrm{ma}} & R_{\mathrm{ma}} \\ R_{\mathrm{se\|ma}} & R_{\mathrm{se\|se}} \end{pmatrix} \begin{pmatrix} N_{\mathrm{ma}}(g) \\ N_{\mathrm{se}}(g) \end{pmatrix}.$
18	$\begin{aligned} \mathscr{R}_0 &= \frac{R_{\mathrm{ma}} + R_{\mathrm{se\|se}} + \sqrt{\left(R_{\mathrm{ma}} + R_{\mathrm{se\|se}}\right)^2 - 4\left(R_{\mathrm{ma}}R_{\mathrm{se\|se}} - R_{\mathrm{se\|ma}}R_{\mathrm{ma}}\right)}}{2} \\ &= \frac{R_{\mathrm{ma}} + R_{\mathrm{se\|se}} + \sqrt{\left(R_{\mathrm{ma}} + R_{\mathrm{se\|se}}\right)^2 + 4R_{\mathrm{se\|ma}}R_{\mathrm{ma}}}}{2}. \end{aligned}$	None
19	$\begin{pmatrix} N(g+1) \\ y(g+1) \end{pmatrix} = \begin{pmatrix} R_{\mathrm{ma}} & R_{\mathrm{se\|ma}}/\langle K \rangle \\ R_{\mathrm{ma}}\langle K \rangle & R_{\mathrm{se\|se}} \end{pmatrix} \begin{pmatrix} N(g) \\ y(g) \end{pmatrix}.$	$\begin{pmatrix} N(g+1) \\ y(g+1) \end{pmatrix} = \begin{pmatrix} R_{\mathrm{ma}} & R_{\mathrm{se\|ma}}/\langle K \rangle \\ R_{\mathrm{ma}}\langle K \rangle & R_{\mathrm{se\|se}} \end{pmatrix} \begin{pmatrix} N(g) \\ y(g) \end{pmatrix}.$
20	$\mathscr{R}_0 = \frac{R_{\mathrm{ma}} + R_{\mathrm{se\|se}} + \left\|R_{\mathrm{ma}} - R_{\mathrm{se\|se}}\right\|}{2} + \frac{R_{\mathrm{se\|ma}}R_{\mathrm{ma}}}{\left\|R_{\mathrm{ma}} - R_{\mathrm{se\|se}}\right\|} + \mathscr{O}\left(\frac{\left(R_{\mathrm{se\|ma}}R_{\mathrm{ma}}\right)^2}{\left\|R_{\mathrm{ma}} - R_{\mathrm{se\|se}}\right\|^3}\right).$	$\mathcal{Z}_0 = \frac{R_{\mathrm{ma}} + R_{\mathrm{se\|se}} + \left\|R_{\mathrm{ma}} - R_{\mathrm{se\|se}}\right\|}{2} + \frac{R_{\mathrm{se\|ma}}R_{\mathrm{ma}}}{\left\|R_{\mathrm{ma}} - R_{\mathrm{se\|se}}\right\|} + \ell\left(\frac{\left(R_{\mathrm{se\|ma}}R_{\mathrm{ma}}\right)^2}{\|R}\right.$
21	$\begin{aligned} \mathscr{R}_0 &= \frac{2R_{\mathrm{se\|se}} + \varepsilon + \sqrt{\left(2R_{\mathrm{se\|se}} + \varepsilon\right)^2 - 4R_{\mathrm{se\|se}}^4 - 4\varepsilon R_{\mathrm{se\|se}} + 4R_{\mathrm{se\|ma}}R_{\mathrm{ma}}}}{2} \\ &= R_{\mathrm{se\|se}} + \frac{\varepsilon + \sqrt{4R_{\mathrm{se\|ma}}R_{\mathrm{ma}} + \varepsilon^2}}{2} \\ &\approx R_{\mathrm{se\|se}} + \sqrt{R_{\mathrm{se\|ma}}R_{\mathrm{ma}}}. \end{aligned}$	None
22	$\begin{aligned} S(\infty) &= (1-\rho)e^{-\xi(\infty)}\psi(\theta(\infty)) \\ \phi_S(\infty) &= (1-\rho)e^{-\xi(\infty)}\frac{\psi'(\theta(\infty))}{\langle K \rangle}. \end{aligned}$	$\begin{aligned} S(\infty) &= (1-\rho)e^{-\xi(\infty)}\psi(\theta(\infty)) \\ \phi_S(\infty) &= (1-\rho)e^{-\xi(\infty)}\frac{\psi'(\theta(\infty))}{\langle K \rangle}. \end{aligned}$
23	$\xi(\infty) = \frac{\beta}{\gamma_1}R(\infty) = \frac{\beta}{\gamma_1}[1 - S(\infty)] = \frac{\beta}{\gamma_1}[1 - (1-\rho)e^{-\xi(\infty)}\psi(\theta(\infty))].$	$\xi(\infty) = \frac{\beta}{\gamma_1}R(\infty) = \frac{\beta}{\gamma_1}[1 - S(\infty)] = \frac{\beta}{\gamma_1}[1 - (1-\rho)e^{-\xi(\infty)}\psi(\theta(\infty))].$
24		

	$\theta(\infty) = 1 - \tau_1 \int\limits_0^\infty \phi_{I,1} dt - \tau_2 \int\limits_0^\infty \phi_{I,2} dt.$	$\theta(\infty) = 1 - \tau_1 \int_0^\infty \phi_{I,1} dt - \tau_2 \int_0^\infty \phi_{I,2} dt.$
25	$\gamma_1 \int\limits_0^\infty \phi_{I,1} dt = \phi_R(\infty) + \tau_2 \int\limits_0^\infty \phi_{I,2} dt.$	$\gamma_1 \int_0^\infty \phi_{I,1} dt = \phi_R(\infty) + \tau_2 \int_0^\infty \phi_{I,2} dt.$
26	$\theta(\infty) = 1 - \frac{\tau_1}{\gamma_1}\phi_R(\infty) - \left(\frac{\tau_1 \tau_2}{\gamma_1} + \tau_2\right) \int\limits_0^\infty \phi_{I,2} dt.$	$\theta(\infty) = 1 - \frac{\tau_1}{\gamma_1}\phi_R(\infty) - \left(\frac{\tau_1 \tau_2}{\gamma_1} + \tau_2\right) \int_0^\infty \phi_{I,2} dt.$
27	$\begin{aligned} \theta(\infty) =& 1 - \left[\frac{\tau_1}{\gamma_1} + \frac{\tau_1}{\gamma_1}\frac{\tau_2}{\gamma_2} + \frac{\tau_2}{\gamma_2}\right]\phi_R(\infty) \\ &= 1 - \frac{T_{\mathrm{se}}}{1 - T_{\mathrm{se}}}\phi_R(\infty) \\ &= 1 - \frac{T_{\mathrm{se}}}{1 - T_{\mathrm{se}}}\left[\theta(\infty) - \frac{(1-\rho)e^{-\xi(\infty)}\psi'(\theta(\infty))}{\langle K \rangle}\right]. \end{aligned}$	$None$
28	$\begin{aligned} \xi(\infty) &= \frac{\beta}{\gamma_1}\left[1 - (1-\rho)e^{-\xi(\infty)}\psi(\theta(\infty))\right] \\ \theta(\infty) &= 1 - T_{\mathrm{se}} + T_{\mathrm{se}}\frac{(1-\rho)e^{-\xi(\infty)}\psi'(\theta(\infty))}{\langle K \rangle}. \end{aligned}$	$\begin{aligned} \xi(\infty) &= \frac{\beta}{\gamma_1}\left[1 - (1-\rho)e^{-\xi(\infty)}\psi(\theta(\infty))\right] \\ \theta(\infty) &= 1 - T_{\mathrm{se}} + T_{\mathrm{se}}\frac{(1-\rho)e^{-\xi(\infty)}\psi'(\theta(\infty))}{\langle K \rangle}. \end{aligned}$
29	$\xi(\infty) = \frac{\beta}{\gamma_1}\left[1 - (1-\rho)e^{-\xi(\infty)}\psi(\theta(\infty))\right]$	$\xi(\infty) = \frac{\beta}{\gamma_1}\left[1 - (1-\rho)e^{-\xi(\infty)}\psi(\theta(\infty))\right]$
30	$\begin{aligned} \dot{V}_S &= B - \beta_1 I_1 V_S - \delta V_S \\ \dot{V}_I &= \beta_1 I_1 V_S - \delta V_I. \end{aligned}$	$\begin{aligned} \dot{V}_S &= B - \beta_1 I_1 V_S - \delta V_S \\ \dot{V}_I &= \beta_1 I_1 V_S - \delta V_I. \end{aligned}$
31		$\begin{aligned} S &= (1-\rho)e^{-\xi}\psi(\theta) \\ \dot{I}_1 &= 1 - S - I_2 - R \\ \dot{I}_2 &= \gamma_1 I_1 - \gamma_2 I_2 \\ \dot{I}_2 &= \gamma_2 I_1 - \gamma_2 I_2 \end{aligned}$

	$\begin{aligned} &S=(1-\rho)e^{-\xi}\psi(\theta) \\ &I_1=1-S-I_2-R \\ &\dot{I}_2=\gamma_1 I_1-\gamma_2 I_2 \\ &\dot{R}=\gamma_2 I_2 \\ &\dot{V}_S=V-\beta_1 I_1 V_S-\delta V_S \\ &\dot{V}_I=\beta_1 I_1 V_S-\delta V_I \\ &\dot{\xi}=\beta_2 V_I \\ &\dot{\theta}=-\tau_1\phi_{I,1}-\tau_2\phi_{I,2} \\ &\phi_S=(1-\rho)e^{-\xi}\frac{\psi'(\theta)}{\langle K\rangle} \\ &\phi_{I,1}=\theta-\phi_S-\phi_{I,2}-\phi_R \\ &\dot{\phi}_{I,2}=\gamma_1\phi_{I,1}-(\gamma_2+\tau_2)\phi_{I,2} \\ &\dot{\phi}_R=\gamma_2\phi_{I,2}. \end{aligned}$	
32	$\begin{pmatrix} R_{\mathrm{mo}} & R_{\mathrm{mo}} \\ R_{\mathrm{se}\|\mathrm{mo}} & R_{\mathrm{se}\|\mathrm{se}} \end{pmatrix}$	$\begin{pmatrix} R_{\mathrm{mo}} & R_{\mathrm{mo}} \\ R_{\mathrm{se}\|\mathrm{mo}} & R_{\mathrm{se}\|\mathrm{se}} \end{pmatrix}$
33		$\begin{aligned} &S=(1-\rho)\psi(\theta_{\mathrm{se}},\theta_{\mathrm{so}}) \\ &\dot{I}_1=1-S-I_2-R \\ &\dot{I}_2=\gamma_1 I_1-\gamma_2 I_2 \\ &\dot{R}=\gamma_2 I_2 \end{aligned}$

	$S = (1-\rho)\psi(\theta_{\mathrm{se}}, \theta_{\mathrm{so}})$ $I_1 = 1 - S - I_2 - R$ $\dot{I}_2 = \gamma_1 I_1 - \gamma_2 I_2$ $\dot{R} = \gamma_2 I_2$ $\dot{\theta}_{\mathrm{se}} = -\tau_1 \phi_{I,1}^{\mathrm{se}} - \tau_2 \phi_{I,2}^{\mathrm{se}}$ $\phi_{I,1}^{\mathrm{se}} = \theta - (1-\rho)\frac{\psi_x(\theta_{\mathrm{se}}, \theta_{\mathrm{so}})}{\langle K_{\mathrm{se}} \rangle} - \phi_{I,2}^{\mathrm{se}} - \phi_R^{\mathrm{se}}$ $\dot{\phi}_{I,2}^{\mathrm{se}} = \gamma_1 \phi_{I,1}^{\mathrm{se}} - (\gamma_2 + \tau_2)\phi_{I,2}^{\mathrm{se}}$ $\dot{\phi}_R^{\mathrm{se}} = \gamma_2 \phi_{I,2}^{\mathrm{se}}$ $\dot{\theta}_{\mathrm{so}} = -\beta \phi_{I,1}^{\mathrm{so}}$ $\phi_{I,1}^{\mathrm{so}} = \theta - (1-\rho)\frac{\psi_y(\theta_{\mathrm{se}}, \theta_{\mathrm{so}})}{\langle K_{\mathrm{so}} \rangle} - \phi_R^{\mathrm{so}}$ $\dot{\phi}_R^{\mathrm{so}} = \gamma_1 \phi_{I,1}^{\mathrm{so}}.$	
34	$R_{\mathrm{se}\vert\mathrm{se}} = T_{\mathrm{se}}\frac{\psi_{xx}(1,1)}{\langle K_{\mathrm{se}} \rangle}$ $R_{\mathrm{se}\vert\mathrm{so}} = T_{\mathrm{se}}\frac{\psi_{xy}(1,1)}{\langle K_{\mathrm{so}} \rangle}$ $R_{\mathrm{so}\vert\mathrm{se}} = T_{\mathrm{so}}\frac{\psi_{xy}(1,1)}{\langle K_{\mathrm{se}} \rangle}$ $R_{\mathrm{so}\vert\mathrm{so}} = T_{\mathrm{so}}\frac{\psi_{yy}(1,1)}{\langle K_{\mathrm{so}} \rangle}.$	$None$
35	$\begin{pmatrix} N_{\mathrm{se}}(g+1) \\ N_{\mathrm{so}}(g+1) \end{pmatrix} = \begin{pmatrix} R_{\mathrm{se}\vert\mathrm{se}} & R_{\mathrm{se}\vert\mathrm{so}} \\ R_{\mathrm{so}\vert\mathrm{se}} & R_{\mathrm{so}\vert\mathrm{so}} \end{pmatrix} \begin{pmatrix} N_{\mathrm{se}}(g) \\ N_{\mathrm{so}}(g) \end{pmatrix}.$	$\begin{pmatrix} N_{\mathrm{se}}(g+1) \\ N_{\mathrm{so}}(g+1) \end{pmatrix} = \begin{pmatrix} R_{\mathrm{se}\vert\mathrm{se}} & R_{\mathrm{se}\vert\mathrm{so}} \\ R_{\mathrm{so}\vert\mathrm{se}} & R_{\mathrm{so}\vert\mathrm{so}} \end{pmatrix} \begin{pmatrix} N_{\mathrm{se}}(g) \\ N_{\mathrm{so}}(g) \end{pmatrix}.$
36	$\theta_{\mathrm{se}}(\infty) = 1 - T_{\mathrm{se}} + T_{\mathrm{se}}(1-\rho)\frac{\psi_x(\theta_{\mathrm{se}}(\infty), \theta_{\mathrm{so}}(\infty))}{\langle K_{\mathrm{se}} \rangle}$ $\theta_{\mathrm{so}}(\infty) = 1 - T_{\mathrm{so}} + T_{\mathrm{so}}(1-\rho)\frac{\psi_y(\theta_{\mathrm{se}}(\infty), \theta_{\mathrm{so}}(\infty))}{\langle K_{\mathrm{so}} \rangle}.$	$None$
37	$\phi_S = (1-\rho)e^{-\xi}\zeta \sum_{k_v} \frac{k_v P(k_v)}{\langle K \rangle}\theta^{k_v - 1} = (1-\rho)e^{-\xi}\zeta \frac{\psi'(\theta)}{\langle K \rangle}.$	$\phi_S = (1-\rho)e^{-\xi}\zeta \sum_{k_v} \frac{k_v P(k_v)}{\langle K \rangle}\theta^{k_v - 1} = (1-\rho)e^{-\xi}\zeta \frac{\psi'(\theta)}{\langle K \rangle}.$

38	$\dot{\phi}_S = -\dot{\xi}\phi_S + \dot{\zeta}(1-\rho)e^{-\xi}\frac{\psi'(\theta)}{\langle K\rangle} + \dot{\theta}(1-\rho)e^{-\xi}\zeta\frac{\psi''(\theta)}{\langle K\rangle}$ $= -\dot{\xi}\phi_S + \eta\theta\pi_S - \eta\phi_S - (\tau_1\phi_{I,1} + \tau_2\phi_{I,2})\phi_S\frac{\psi''(\theta)}{\psi'(\theta)}.$	$None$
39	$S = (1-\rho)e^{-\xi}\psi(\theta)$ $I_1 = 1 - S - I_2 - R$ $\dot{I}_2 = \gamma_1 I_1 - \gamma_2 I_2$ $\dot{R} = \gamma_2 I_2$ $\dot{\xi} = \beta I_1$ $\dot{\theta} = -\tau_1\phi_{I,1} - \tau_2\phi_{I,2}$ $\dot{\phi}_S = -\beta I_1\phi_S + \eta\theta\pi_S - \eta\phi_S - (\tau_1\phi_{I,1} + \tau_2\phi_{I,2})\phi_S\frac{\psi''(\theta)}{\psi'(\theta)}$ $\phi_{I,1} = \theta - \phi_S - \phi_{I,2} - \phi_R$ $\dot{\phi}_{I,2} = \gamma_1\phi_{I,1} + \eta\theta\pi_{I,2} - (\eta + \gamma_2 + \tau_2)\phi_{I,2}$ $\dot{\phi}_R = \eta\theta\pi_R + \gamma_2\phi_{I,2} - \eta\phi_R$ $\pi_S = (1-\rho)\frac{\theta e^{-\xi}\psi'(\theta)}{\langle K\rangle}$ $\pi_{I,1} = 1 - \pi_S - \pi_{I,2} - \pi_R$ $\dot{\pi}_{I,2} = \gamma_1\pi_{I,1} - \gamma_2\pi_{I,2}$ $\dot{\pi}_R = \gamma_2\pi_{I,2}.$	$S = (1-\rho)e^{-\xi}\psi(\theta)$ $\dot{I}_1 = 1 - S - I_2 - R$ $\dot{R} = \gamma_1 I_1 - \gamma_2 I_2$ $\dot{R} = \gamma_2 I_2$
40	$I_2(0) = R(0) = 0$ $\theta(0) = 1$ $\phi_S(0) = (1-\rho)$ $\phi_{I,2}(0) = \phi_R(0) = 0$ $\pi_{I,2} = \pi_R = 0$ $\xi(0) = 0.$	$I_2(0) = R(0) = 0$ $\theta(0) = 1$ $\phi_S(0) = (1-\rho)$ $\phi_{I,2}(0) = \phi_R(0) = 0$
41	$N(g+1) = \frac{\beta}{\gamma_1}N(g) + \left(\frac{\tau_1}{\gamma_1} + \frac{\tau_2}{\gamma_2}\right)y(g)$	$N(g+1) = \frac{\beta}{\gamma_1}N(g) + \left(\frac{\tau_1}{\gamma_1} + \frac{\tau_2}{\gamma_2}\right)y(g)$
42	$y(g+1) = \frac{\beta}{\gamma_1}\langle K\rangle N(g) + \left(\frac{\tau_1}{\gamma_1} + \frac{\tau_2}{\gamma_2}\right)\frac{\langle K^2\rangle}{\langle K\rangle}y(g).$	$y(g+1) = \frac{\beta}{\gamma_1}\langle K\rangle N(g) + \left(\frac{\tau_1}{\gamma_1} + \frac{\tau_2}{\gamma_2}\right)\frac{\langle K^2\rangle}{\langle K\rangle}y(g).$
43	$\begin{pmatrix} \frac{\beta}{\gamma_1} & \frac{\tau_1}{\gamma_1} + \frac{\tau_2}{\gamma_2} \\ \frac{\beta}{\gamma_1}\langle K\rangle & \left(\frac{\tau_1}{\gamma_1} + \frac{\tau_2}{\gamma_2}\right)\frac{\langle K^2\rangle}{\langle K\rangle} \end{pmatrix}$	$\begin{pmatrix} \frac{\beta}{\gamma_1} & \frac{\tau_1}{\gamma_1} + \frac{\tau_2}{\gamma_2} \\ \frac{\beta}{\gamma_1}\langle K\rangle & \left(\frac{\tau_1}{\gamma_1} + \frac{\tau_2}{\gamma_2}\right)\frac{\langle K^2\rangle}{\langle K\rangle} \end{pmatrix}$

44	$S=(1-\rho)e^{-\xi}\sum_k P(k)\theta_k^k$ $I_1=1-S-I_2-R$ $\dot{I}_2=\gamma_1 I_1-\gamma_2 I_2$ $\dot{R}=\gamma_2 I_2$ $\dot{\theta}_k=-\tau_1\phi_{I,1\mid k}-\tau_2\phi_{I,2\mid k}$ $\phi_{S\mid k}=(1-\rho)e^{-\xi}\sum_{\widehat{k}} P_n\left(\widehat{k}\middle\vert k\right)\theta_{\widehat{k}}^{\widehat{k}-1}$ $\phi_{I,1\mid k}=\theta_k-\phi_{S\mid k}-\phi_{I,2\mid k}-\phi_{R\mid k}$ $\dot{\phi}_{I,2\mid k}=\gamma_1\phi_{I,1\mid k}-(\tau_2+\gamma_2)\phi_{I,2\mid k}$ $\dot{\phi}_{R\mid k}=\gamma_2\phi_{I,2\mid k}$ $\dot{\xi}=\beta I_1.$	$S=(1-\rho)e^{-\xi}\sum_k P(k)\theta_k^k$ $\dot{I}_1=1-S-I_2-R$ $\dot{I}_2=\gamma_1 I_1-\gamma_2 I_2$
45	$I_2(0)=R(0)=0$ $\theta_k(0)=1$ $\phi_{I,2\mid k}=\phi_{R\mid k}=0$ $\xi(0)=0.$	$I_2(0)=R(0)=0$ $\theta_k(0)=1$ $\phi_{I,2\mid k}=\phi_{R\mid k}=0.$
46	$\begin{array}{ccc} P_n(1\mid 1)=1/2 & P_n(1\mid 5)=3/40 & P_n(1\mid 10)=1/80 \\ P_n(5\mid 1)=3/8 & P_n(5\mid 5)=1/2 & P_n(5\mid 10)=17/80 \\ P_n(10\mid 1)=1/8 & P_n(10\mid 5)=17/40 & P_n(10\mid 10)=62/80 \end{array}$	$None$
47	$N_{\widehat{k}}(g+1)=\sum_k\left(T_{\mathrm{sekP}}\left(\widehat{k}\middle\vert k\right)+P\left(\widehat{k}\right)\frac{\beta}{\gamma_1}\right)N_k(g).$	$N_{\widehat{k}}(g+1)=\sum_k\left(T_{\mathrm{sekP}}\left(\widehat{k}\middle\vert k\right)+P\left(\widehat{k}\right)\frac{\beta}{\gamma_1}\right)N_k(g).$
48	$\begin{pmatrix} N_0(g+1) \\ N_1(g+1) \\ N_2(g+1) \\ N_3(g+1) \\ \vdots \end{pmatrix}=\left[T_{\mathrm{se}}\begin{pmatrix} 0 & 0 & 0 & 0 & \dots \\ 0 & P(1\mid 1) & 2P(1\mid 2) & 3P(1\mid 3) & \dots \\ 0 & P(2\mid 1) & 2P(2\mid 2) & 3P(2\mid 3) & \dots \\ 0 & P(3\mid 1) & 2P(3\mid 2) & 3P(3\mid 3) & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}+\frac{\beta}{\gamma_1}\begin{pmatrix} P(0) & P(0) & P(0) & P(0) & \dots \\ P(1) & P(1) & P(1) & P(1) & \dots \\ P(2) & P(2) & P(2) & P(2) & \dots \\ P(3) & P(3) & P(3) & P(3) & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}\right]\begin{pmatrix} N_0(g) \\ N_1(g) \\ N_2(g) \\ N_3(g) \\ \vdots \end{pmatrix}.$	$None$
49	$\theta_k(\infty)=(1-T_{\mathrm{se}})+T_{\mathrm{se}}\phi_{S\mid k}(\infty)=(1-T_{\mathrm{se}})+T_{\mathrm{se}}(1-\rho)e^{-\xi(\infty)}\sum_{\widehat{k}}P_n\left(\widehat{k}\middle\vert k\right)\theta_{\widehat{k}}^{\widehat{k}-1}$	$\theta_k(\infty)=(1-T_{\mathrm{se}})+T_{\mathrm{se}}\phi_{S\mid k}(\infty)=(1-T_{\mathrm{se}})+T_{\mathrm{se}}(1-\rho)e^{-\xi(\infty)}\sum_{\widehat{k}}P_n\left(\widehat{k}\middle\vert k\right)\theta_{\widehat{k}}^{\widehat{k}-1}$

50	$\xi(\infty)=\frac{\beta}{\gamma_1}[1-S(\infty)]=\frac{\beta}{\gamma_1}\left[1-(1-\rho)e^{-\xi(\infty)}\sum_k P(k)\theta_k^k(\infty)\right].$	$\xi(\infty)=\frac{\beta}{\gamma_1}[1-S(\infty)]=\frac{\beta}{\gamma_1}\left[1-(1-\rho)e^{-\xi(\infty)}\sum_k P(k)\theta_k^k(\infty)\right].$
51	$R(\infty)=1-(1-\rho)e^{-\xi(\infty)}\sum_k P(k)\theta_k^k(\infty).$	$R(\infty)=1-(1-\rho)e^{-\xi(\infty)}\sum_k P(k)\theta_k^k(\infty).$

Wikipedia (cosmos-detected Equations)

Eqn Num	Original Image	Decoded Image
0	$\frac{dS}{dt} = -\frac{\beta IS}{N}$	$\frac{dS}{dt} = -\frac{\beta IS}{N}$
1	$\frac{dI}{dt} = \frac{\beta IS}{N} - \gamma I$	$\frac{dI}{dt} = \frac{\beta IS}{N} - \gamma I$
2	$\frac{dR}{dt} = \gamma I \qquad (3)$	$\frac{dR}{dt} = \gamma I \qquad (3)$
3	$\frac{dS}{dt} + \frac{dI}{dt} + \frac{dR}{dt} = 0$	$\frac{dS}{dt} + \frac{dI}{dt} + \frac{dR}{dt} = 0$
4	$S(t) + I(t) + R(t) = \text{Constant} = N$	$S(t) + I(t) + R(t) = \text{Constant} = N$
5	$R_0 = \frac{\beta}{\gamma}$	$R_0 = \frac{\beta}{\gamma}$
6	$S(t) = S(0)e^{-R_0(R(t)-R(0))/N}$	$S(t) = S(0)e^{-R_0(R(t)-R(0))/N}$
7	$R_\infty = N - S(0)e^{-R_0(R_\infty - R(0))/N}$	$R_\infty = N - S(0)e^{-R_0(R_\infty - R(0))/N}$
8	$\frac{dI}{dt} = (R_0 S/N - 1)\gamma I$	$\frac{dI}{dt} = (R_0 S/N - 1)\gamma I$

9	$R_0 > \frac{N}{S(0)},$	$R_0 > \frac{N}{S(0)},$
10	$\frac{dI}{dt}(0) > 0,$	$\frac{dI}{dt}(0) > 0,$
11	$R_0 < \frac{N}{S(0)},$	$R_0 < \frac{N}{S(0)},$

1927 (Manually-detected Equations)

Eqn Num	Original Image	Decoded Image
0	$v_{0,0} = v_0 + y_0.$	$\overline{\left\| \frac{\overline{m}}{\overline{\mathrm{dit}}\,\overline{\frac{\mathrm{d}}{\mathrm{min}}\,\overline{\mathrm{thin}}}\,\|\overline{\mathrm{th}}\;\overline{\mathrm{dit}}}\ \overline{\mathrm{dit}}_{\overline{\mathrm{dit}}} \right.}$
1	$v_{t,\theta} = v_{t-1,\theta-1}(1-\psi(\theta-1)) = v_{t-2,\theta-2}(1-\psi(\theta-1)(1-\psi(\theta-2)) = v_{t-\theta,0}\,\mathrm{B}_\theta,$	$None$
2		$None$

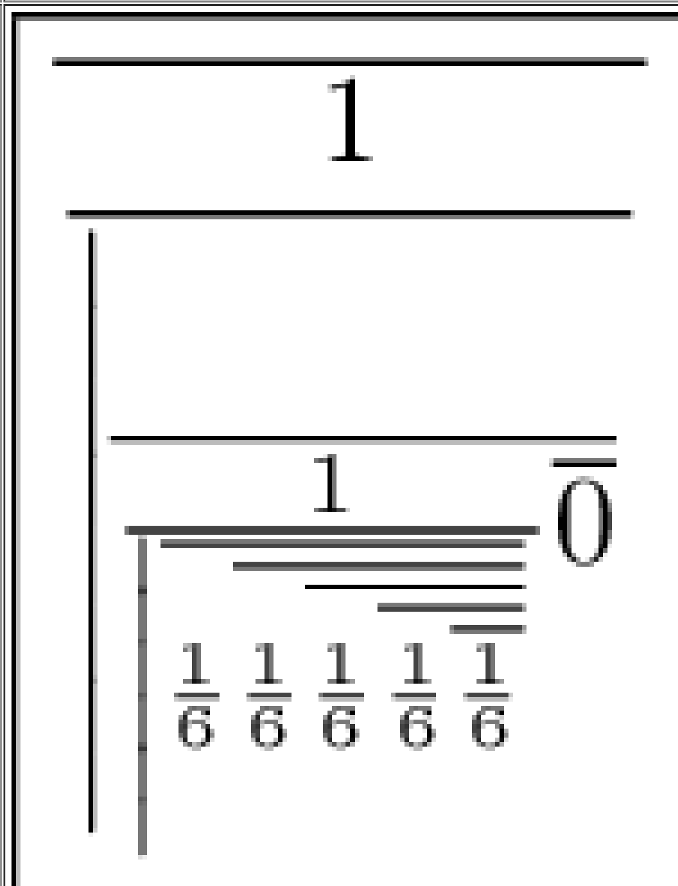
	$$x_t = \mathrm{N} - \sum_0^t v_{t,0}$$ $$= \mathrm{N} - \sum_0^t v_t - y_0,$$	
3	$$x_t + y_t + z_t = \mathrm{N}.$$	None
4	$$v_t = x_t \sum_1^t \phi_\theta v_{t,\theta} = x_t \sum_1^t \phi_\theta \mathrm{B}_\theta v_{t-\theta,0} \quad \text{(by 2)}$$ $$= x_t\left(\sum_1^t \mathrm{A}_\theta v_{t-\theta} + \mathrm{A}_t y_0\right) \quad \text{(by 1)},$$	None
5	$$y_t = \sum_0^t v_{t,\theta} = \sum_0^t \mathrm{B}_\theta v_{t-\theta} + \mathrm{B}_t y_0.$$	None
6	$$-v_t = x_{t+1} - x_t,$$	None
7	$$x_t - x_{t+1} = x_t\left(\sum_1^t \mathrm{A}_\theta v_{t-\theta} + \mathrm{A}_t y_0\right).$$	$$\frac{1}{\frac{}{\frac{d}{d\frac{1}{d}\overline{\frac{d}{dt}\,\frac{d}{d\frac{d}{d\frac{d}{d\frac{dt}{d\frac{1}{dit}\frac{1}{6}}\overline{th}}}}}}}}$$
8		

	$z_{t+1} - z_t = \sum_1^t \mathrm{C}_\theta v_{t-\theta} + \mathrm{C}_t y_0.$	None
9	$y_{t+1} - y_t = x_t [\sum_1^t \mathrm{A}_\theta v_{t-\theta} + \mathrm{A}_t y_0] - [\sum_1^t \mathrm{C}_\theta v_{t-\theta} + \mathrm{C}_t y_0].$	None
10	$x_t + y_t + z_t = \mathrm{N}.$ $v_t = -\frac{dx_t}{dt},$ $\frac{dx_t}{dt} = -x_t\left[\int_0^t \mathrm{A}_\theta v_{t-\theta}\, d\theta + \mathrm{A}_t y_0\right],$ $\frac{dz_t}{dt} = \int_0^t \mathrm{C}_\theta v_{t-\theta}\, d\theta + \mathrm{C}_t y_0,$ $y_t = \int_0^t \mathrm{B}_\theta v_{t-\theta}\, d\theta + \mathrm{B}_t y_0,$	$\boxed{\boxed{\boxed{\boxed{\frac{1}{\sqrt{\frac{1}{\frac{1}{\sqrt{\frac{1}{6}\,\overline{\frac{n}{n}}}}}}}}}}}$
11	$\mathrm{B}_\theta = e^{-\int_0^\theta \psi(a)\, da}, \qquad \mathrm{A}_\theta = \phi_\theta \mathrm{B}_\theta, \qquad \text{and} \qquad \mathrm{C}_\theta = \psi_\theta \mathrm{B}_\theta.$	None
12	$\begin{aligned} \frac{dx}{dt} &= -x\left[\int_0^t \mathrm{A}_\theta v_{t-\theta}\, d\theta + \mathrm{A}_t y_0\right], \\ &= -x\left[\int_0^t \mathrm{A}_{t-\theta} v_\theta\, d\theta + \mathrm{A}_t y_0,\right] \\ &= x\left[\int_0^t \mathrm{A}_{t-\theta} \frac{dx_\theta}{d\theta}\, d\theta - \mathrm{A}_t y_0\right], \end{aligned}$	None
13	$\begin{aligned} \frac{d \log x}{dt} &= \mathrm{A}_{t-\theta} x_\theta \Big\vert_0^t - \int_0^t x_\theta \frac{d\,\mathrm{A}_{t-\theta}}{d\theta}\, d\theta - \mathrm{A}_t y_0, \\ &= \mathrm{A}_0 x_t - \mathrm{A}_t x_0 + \int_0^t x_\theta \mathrm{A}'_{t-\theta}\, d\theta - \mathrm{A}_t y_0, \end{aligned}$	None

14	$A'_{t-\theta} = \frac{d\,A_{t-\theta}}{d\,(t-\theta)} = -\frac{dA_{t-\theta}}{d\theta}.$	$\frac{1}{\lvert\frac{1}{\lvert}\frac{1}{\lvert}\frac{1}{\lvert\overline{m}\rvert\frac{1}{\lvert t\rvert}\frac{1}{\lvert m\rvert}\overline{in}}\overline{0}}$
15	$\left.\begin{aligned}\frac{d\log x}{dt} &= -A_t(x_0+y_0)+\int_0^t x_\theta A'_{t-\theta}\,d\theta,\\ &= -A_tN+\int_0^t A'_\theta x_{t-\theta}\,d\theta.\end{aligned}\right\}$	*None*
16	$f(t) = \phi(t) + \int_0^t N(t,\theta)\,\phi(\theta)\,d\theta,$	*None*
17	$\frac{d\log x}{dt} = A_t + \lambda\int_0^t N(t,\theta)\,x(\theta)\,d\theta,$	*None*
18	$x = f_0(t) + \lambda f_1(t) + \lambda^2 f_2(t) + \text{etc.}$	*None*
19	$\frac{dx}{dt} = x\left[A_t + \lambda\int_0^t N(t,\theta)\,x(\theta)\,d\theta\right],$	*None*
20	$\begin{aligned}\frac{d}{dt}f_n(t) &= f_n(t)A_t + f_{n-1}(t)\int_0^t N(t,\theta)f_0(\theta)\,d\theta + f_{n-2}(t)\int_0^t N(t,\theta)f_1(\theta)\,d\theta\\ &\quad + \dots + f_0(t)\int_0^t N(t,\theta)f_{n-1}(\theta)\,d\theta\\ &= L_{n-1}(t)\quad\text{say.}\end{aligned}$	*None*
21	$f_n(t)\,e^{-\int_0^t A_t dt} = \int_0^t L_{n-1}(t)\,e^{-\int_0^t A_t dt}\,dt + \text{constant},$	*None*

22	$\dfrac{df_0(t)}{dt} = f_0(t)\,\mathrm{A}_t,$	None
23	$f_0(t) = f_0(0)\,e^{\int_0^t \mathrm{A}_s dt},$	None
24	$x = x_0 \mathrm{E}_t + \sum_{n=1}^{\infty} \lambda^n \mathrm{E}_t \int_0^t \dfrac{\mathrm{L}_{n-1}(t)}{\mathrm{E}_t}\,dt,$ $= \mathrm{E}_t \left[x_0 + \sum_1^{\infty} \lambda^n \int_0^t \dfrac{\mathrm{L}_{n-1}(t)}{\mathrm{E}_t}\,dt \right],$	None
25	$x = \mathrm{E}_t \left[x_0 + \sum_1^{\infty} \int_0^t \dfrac{\mathrm{L}_{n-1}(t)}{\mathrm{E}_t}\,dt \right].$	None
26	$\dfrac{d \log x}{dt} = \mathrm{A}_t + \int_0^t \mathrm{Q}_{t-\theta} x_\theta d\theta.$	None
27	$\int_0^{\infty} e^{-zt} \dfrac{d \log x}{dt}\,dt = \int_0^{\infty} e^{-zt} \mathrm{A}_t dt + \int_0^{\infty} e^{-zt} \int_0^t \mathrm{Q}_{t-\theta} x_\theta\, d\theta\, dt,$	None
28	$-\log x_0 + \int_0^{\infty} z e^{-zt} \log x\, dt = \mathrm{F}(z) + \int_0^{\infty} e^{-z\theta} \mathrm{Q}_\theta\, d\theta \int_0^{\infty} e^{-zt} x_t\, dt,$ $= \mathrm{F}(z) + \mathrm{F}_1(z) \int_0^{\infty} e^{-zt} x_t\, dt,$	None
29	$\int_0^{\infty} e^{-zt} (z \log x - \mathrm{F}_1(z)\, x)\, dt = \mathrm{F}(z) + \log x_0.$	None
30	$\int_0^{\infty} \phi(x, z)\, \psi(z, t)\, dt = \chi(z),$	

		$\frac{1}{\sqrt{\frac{1}{\sqrt{d\frac{1}{6}\frac{1}{6}\frac{1}{6}\frac{1}{6}\frac{1}{6}}}\frac{1}{\lvert R\rvert\frac{1}{\lvert R\rvert}}}}$
31	$-\int_0^\infty \frac{d\log x}{dt}\,dt = \int_0^\infty\int_0^t A_\theta v_{t-\theta}\,d\theta\,dt + y_0\int_0^\infty A_t\,dt,$	$None$
32	$\log\frac{x_0}{x_\infty} = \int_0^\infty A_\theta\,d\theta\int_0^\infty v_t\,dt + y_0\int_0^\infty A_t\,dt.$	$None$
33	$\log\frac{x_0}{x_\infty} = A(x_0 - x_\infty) + Ay_0 = A(N - x_\infty).$	$None$
34	$-\log\frac{1-p}{1-\frac{y_0}{N}} = ANp.$	$None$
35	$\int_0^\infty y_t\,dt = N\,p\int_0^\infty B_\theta\,d\theta.$	$None$
36		

	$$-\int_0^\infty e^{-zt}\frac{d\log x}{dt}\,dt = \int_0^\infty e^{-zt}\int_0^t A_\theta v_{t-\theta}\,d\theta\,dt + y_0\int_0^\infty e^{-zt}A_t\,dt,$$ $$= \int_0^\infty e^{-z\theta}A_\theta\,d\theta\int_0^\infty e^{-zt}v_t\,dt + y_0\int_0^\infty e^{-zt}A_t\,dt,$$	None
37	$$\int_0^\infty e^{-zt}A_t\,dt = \frac{-\int_0^\infty e^{-zt}\frac{d\log x}{dt}\,dt}{y_0+\int_0^\infty e^{-zt}v_t\,dt},$$	None
38	$$A_\theta = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} e^{zt}F_2(z)\,dz.$$	
39	$$\int_0^\infty e^{-zt}y_t\,dt = \int_0^\infty e^{-zt}\int_0^t B_\theta v_{t-\theta}\,d\theta\,dt + y_0\int_0^\infty e^{-zt}B_t\,dt,$$	None
40	$$\int_0^\infty e^{-zt}B_t\,dt = \frac{\int_0^\infty e^{-zt}y_t\,dt}{y_0+\int_0^\infty e^{-zt}v_t\,dt},$$	None
41		

	$$B_\theta = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} e^{zt}F_3(z)\,dt.$$	$$\frac{1}{\frac{1}{\vert}\frac{1}{\vert m\vert\frac{1}{\vert m\vert}\frac{1}{\vert m\vert}n}i}$$
42	$$-\frac{dx}{dt} = v_t = N\left[\int_0^\infty A_\theta v_{t-\theta}\,d\theta + A_t y_0\right],$$	*None*
43	$$\int_0^\infty e^{-zt}v_t\,dt = \frac{Ny_0\int_0^\infty e^{-zt}A_t\,dt}{1-N\int_0^\infty e^{-zt}A_t\,dt}$$	*None*
44	$$v_t = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} e^{zt}F_4(z)\,dz.$$	*None*
45	$$\begin{aligned}\int_0^\infty e^{-zt}y_t\,dt &= \int_0^\infty e^{-zt}\int_0^t B_\theta v_{t-\theta}\,d\theta\,dt + y_0\int_0^\infty e^{-zt}B_t\,dt,\\ &= \int_0^\infty e^{-zt}v_t\,dt\int_0^\infty e^{-z\theta}B_\theta\,d\theta + y_0\int_0^\infty e^{-zt}B_t\,dt,\\ &= \frac{Ny_0\int_0^\infty e^{-zt}A_t\,dt\int_0^\infty e^{-zt}B_t\,dt}{1-N\int_0^\infty e^{-zt}A_t\,dt} + y_0\int_0^\infty e^{-zt}B_t\,dt,\\ &= \frac{y_0\int_0^\infty e^{-zt}B_t\,dt}{1-N\int_0^\infty e^{-zt}A_t\,dt}\end{aligned}$$	*None*
46	$$y_t = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} e^{zt}F_5(z)\,dz.$$	*None*
47		

	$y_t = \int_0^t B_{t-\theta} v_\theta \, d\theta + B_t y_0,$ $= N \int_0^t B_{t-\theta} \left(\int_0^\theta A_{\theta-z} v_z \, dz + A_\theta y_0 \right) d\theta + B_t y_0,$ $= N \int_0^t B_{t-\theta} \int_0^\theta A_{\theta-z} v_z \, dz \, d\theta + N y_0 \int_0^t B_{t-\theta} A_\theta \, d\theta + B_t y_0,$ $= N \int_0^t A_{t-\theta} \int_0^\theta B_{\theta-z} v_z \, dz \, d\theta + N y_0 \int_0^t A_{t-\theta} B_\theta \, d\theta + B_t y_0,$ $= N \int_0^t A_{t-\theta} (y_\theta - B_\theta y_0 + B_\theta y_0) \, d\theta + B_t y_0,$ $= N \int_0^t A_{t-\theta} y_\theta \, d\theta + B_t y_0.$	$\boxed{\boxed{\boxed{\frac{1}{\boxed{\boxed{\frac{1}{\sqrt{\frac{1}{\frac{1}{6}}\frac{\overline{n}}{\overline{n}}}}}}}}}}$
48	$v_{t,0} = \int_0^t A_\theta v_{t-\theta,0} \, d\theta,$	$None$
49	$v_{t,0} = \frac{1}{2\pi i} \int_{a-i\infty}^{a+i\infty} e^{zt} \frac{N_0}{1 - \int_0^\infty e^{-z\theta} A_\theta \, d\theta} \, . dz.$	$None$
50	$v_{t,0} = v_{t,0} - v_{\epsilon,0} + v_{\epsilon,0},$ $= \int_\epsilon^t A_{t-\theta} v_{\theta,0} \, d\theta + \int_0^\epsilon A_{t-\theta} v_{\theta,0} \, d\theta,$ $= \int_0^t A_{t-\theta} v_\theta \, d\theta + A_{t-\epsilon'} \int_0^\epsilon v_{\theta,0} \, d\theta,$ where $0 < \epsilon' < \epsilon,$ $= \int_0^t A_{t-\theta} v_\theta \, d\theta + A_t y_0.$	

		$\frac{1}{\frac{1}{\sqrt{\frac{1}{\sqrt{\frac{1}{6}\frac{\overline{n}}{n}\bar{i}}}}}}$
51	$v_{t,0} = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} e^{zt} F(z)\,dz,$	$None$
52	$F(z) = \frac{y_0}{1-\int_0^\infty e^{-z\theta}A_\theta\,d\theta}$: let us denote this by $\frac{y_0}{1-A}$.	$None$
53	$F_4(z) = -y_0 + \frac{y_0}{1-A} = \frac{Ay_0}{1-A},$	$None$
54	$A_\theta = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} e^{zt}\frac{\int_0^\infty e^{-zt}v_t\,dt}{Ny_0+N\int_0^\infty e^{-zt}v_t\,dt}\,dz,$	$None$
55	$B_\theta = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} e^{zt}\frac{\int_0^\infty e^{-zt}y_t\,dt}{y_0+\int_0^\infty e^{-zt}v_t\,dt}\,dz.$	$None$
56		

	$$\int_0^\infty e^{-zt} e^{\alpha t} t^c \, dt = \frac{c\,!}{(z-\alpha)^{c+1}},$$	None
57	$$\int_0^\infty e^{-zt} \phi(t)\, dt = \Sigma\Sigma \frac{A_{r,s}}{(z-\alpha_r)^s}$$	None
58	$\phi(t) = \Sigma\Sigma \frac{A_{r,s}}{(s-1)\,!} t^{s-1} e^{\alpha_r t}$: see Fock (*loc. cit.*).	None
59	$$\left.\begin{aligned} \frac{dx}{dt} &= -\kappa xy \\ \frac{dy}{dt} &= \kappa xy - ly \\ \frac{dz}{dt} &= ly \end{aligned}\right\}$$	None
60	$$\frac{dz}{dt} = l(\mathrm{N} - x - z),$$	None
61	$$\frac{dz}{dt} = l\left(\mathrm{N} - x_0 e^{-\frac{\kappa}{l} z} - z\right).$$	None
62	$$\frac{dz}{dt} = l\left\{\mathrm{N} - x_0 + \left(\frac{\kappa}{l} x_0 - 1\right) z - \frac{x_0 \kappa^2 z^2}{2l^2}\right\}.$$	

		None
63	$z = \frac{l^2}{\kappa^2 x_0}\left\{\frac{\kappa}{l}x_0 - 1 + \sqrt{-q}\tanh\left(\frac{\sqrt{-q}}{2}lt - \phi\right)\right\}$	None
64	$\phi = \tanh^{-1}\frac{\frac{\kappa}{l}x_0 - 1}{\sqrt{-q}},$	None
65	$\sqrt{-q} = \left\{\left(\frac{\kappa}{l}x_0 - 1\right)^2 + 2x_0y_0\frac{\kappa^2}{l^2}\right\}^{\frac{1}{2}}.$	None
66	$\frac{dz}{dt} = \frac{l^3}{2x_0\kappa^2}\sqrt{-q}\,\mathrm{sech}^2\left(\frac{\sqrt{-q}}{2}lt - \phi\right).$	None
67	$\frac{dz}{dt} = 890\,\mathrm{sech}^2\,(0\cdot 2t - 3\cdot 4).$	None
68	$z = \frac{2l}{\kappa x_0}\left(x_0 - \frac{l}{\kappa}\right)$	None
69	$2\frac{l}{\kappa}\frac{n}{N} \quad \text{or} \quad 2n - \frac{2n^2}{N}.$	None
70		

	$$-\log\frac{1-p}{1-\frac{y_0}{\mathrm{N}}} = \mathrm{A}p\mathrm{N},$$	None
71	$$\mathrm{A} = \int_0^\infty \mathrm{A}_\theta\, d\theta = \int_0^\infty \phi_\theta e^{-\int_0^\theta \psi_a\, da}\, d\theta.$$	None
72	$$p + \frac{p^2}{2} + \frac{p^3}{3} + \dots = \mathrm{A}p\mathrm{N} = p\left(1+\frac{n}{\mathrm{N}_0}\right),$$	None
73	$$\frac{p}{2} + \frac{p^2}{3} + \dots = \frac{n}{\mathrm{N}_0},$$	None
74	$$p\mathrm{N} = 2n\frac{\mathrm{N}}{\mathrm{N}_0} = 2n\left(1+\frac{n}{\mathrm{N}_0}\right) = 2n,$$	None
75	$$\mathrm{A} = \int_0^\infty \kappa e^{-\int_0^\theta l\, da}\, d\theta = \kappa\int_0^\infty e^{-l\theta}\, d\theta = \frac{\kappa}{l}.$$	None
76	$$\left.\begin{aligned} \frac{d\log x}{dt} &= \int_0^t \mathrm{A}'_\theta v'_{t-\theta}\, d\theta + \mathrm{A}'_t y'_0 \\ \frac{d\log x'}{dt} &= \int_0^t \mathrm{A}_\theta v_{t-\theta}\, d\theta + \mathrm{A}_t y_0 \end{aligned}\right\},$$	None
77		None

	$\left.\begin{aligned} -\log \frac{1-p}{1-\frac{y_0}{N}} &= A'p'N' \\ -\log \frac{1-p'}{1-\frac{y_0'}{N'}} &= ApN \end{aligned}\right\}.$	
78	$p\left(1+\frac{p}{2}\right)p'\left(1+\frac{p'}{2}\right) = AA'pp'NN',$	None
79	$\frac{p}{2}+\frac{p'}{2} = AA'NN' - 1.$	None
80	$p = \frac{2n}{N_0}\frac{A'N_0'}{1+A'N_0'}.$	None
81	$N_0'pN_0 + N_0p'N_0' = 2N_0N_0'(AA'NN' - 1).$	None
82	$N_0N_0' = 1/AA' = \pi_0,$	None
83	$\begin{aligned} N_0'pN_0 + N_0p'N_0' &= 2(NN' - N_0N_0') \\ &= 2(\pi - \pi_0), \end{aligned}$	None
84	$\begin{aligned} \pi - \bar{\pi} &= NN' - (N - \Delta N)(N' - \Delta N'), \\ &= N\Delta N' + N'\Delta N - \Delta N\Delta N', \\ &= Np'N' + N'pN - pNp'N', \\ &= NN'(p + p' - pp'), \\ &= N_0N_0'(p + p' - pp') + (NN' - N_0N_0')(p + p' - pp'). \end{aligned}$	None

85	$\pi - \bar{\pi} = \mathrm{N}_0\mathrm{N}_0{}'\,(p + p') = 2\,(\pi - \pi_0).$	None

1979 (cosmos + Manually-detected Equations)

Eqn Num	Original Image	Decoded Image
0	$\mathrm{d}X/\mathrm{d}t = A - bX - \beta XY + \gamma Z$ $\mathrm{d}Y/\mathrm{d}t = \beta XY - (b + \alpha + v)Y$ $\mathrm{d}Z/\mathrm{d}t = vY - (\gamma + b)Z$	None
1	$\mathrm{d}N/\mathrm{d}t = A - bN - \alpha Y$	$\mathbb{H}\mathbb{M}_\ell^T \# \underline{t} \equiv \mathcal{A} - \#\mathbb{N}^{\mathbb{T}} = \# \mathbb{F}^{p'p}$
2	$A/b > (\alpha + b + v)/\beta$	$\mathbb{R}\#\mathbb{H}\ \#\ \mathbb{E}\mathbb{H}\mathbb{H}\mathbb{B}\ \mathbb{B}\ \mathbb{B}\ \#\ \mathbb{R}\ \mathbb{R}\mathbb{B}\mathbb{B}\ \mathbb{B}\mathbb{B}\ \mathbb{B}\ \mathbb{B}\ \mathbb{B}\ \mathbb{B}\ \mathbb{B}$
3	$N^* = \dfrac{A + D(\alpha + b + v)/\beta}{b + D}$	$\mathbb{R}^\# \equiv \underline{\underline{d}} \pm \underline{\mathbb{H}}\|\underline{d}\pi + \mathbb{H} + \mathbb{H}\|\overline{p}\|\mathbb{R}$ $\mathbb{R}^\# \equiv \qquad \frac{\hbar}{\hbar} + \underline{E}$
4	$D = \alpha/[1 + v/(b + \gamma)]$	$\mathbb{B} = \mathbb{H}\mathbb{H}[\mathbb{H} \pm \mathbb{E}\mathbb{H}[\mathbb{H} \pm \mathfrak{p}p]]\underline{1}\|\underline{1}\|\underline{h}\|\underline{h}\|\underline{h}\|\underline{1}\|\underline{1}\|\underline{1}\|\underline{1}\|\underline{1}\|\underline{1}\|\underline{1}\|\underline{1}\|\underline{1}\|\underline{1}\|\underline{1}$
5	$\mathrm{d}X/\mathrm{d}t = a(X + Y + Z) - bX - \beta XY + \gamma Z$ $\mathrm{d}Y/\mathrm{d}t = \beta XY - (\alpha + b + v)Y$ $\mathrm{d}Z/\mathrm{d}t = vY - (b + \gamma)Z$	None
6	$\mathrm{d}N/\mathrm{d}t = (a - b)N - \alpha Y \qquad (11)$	$\mathtt{d\ N\ d\ t} \ = (\boldsymbol{a} - \boldsymbol{b})\boldsymbol{N} - \boldsymbol{a}Y \qquad (11)$
7	$\mathrm{d}N/\mathrm{d}t = (r - \alpha y)N \qquad (12)$	$\mathtt{d\ N\ d\ t} \ = (\boldsymbol{r} - \boldsymbol{ay})\boldsymbol{N} \qquad (\ 1\ 2\ )$
8	$\alpha > r\left[1 + \dfrac{v}{b + \gamma}\right]$	$\Phi \ \approx r\left[\left[\pm\dfrac{\mathbb{E}}{\mathbb{H} + \gamma}\right]\right]$
9		$\mathcal{R}^\mp = \dfrac{\Phi[\mathbf{a} + \sharp\#\mathbf{w}]}{\mathbb{B}[\mho_{\boldsymbol{a}} - r\#\# + p^\#\mathbb{H} + *\mathbb{H})]}$

	$N^* = \frac{\alpha(\alpha+b+v)}{\beta[\alpha - r(1+v/\{b+\gamma\})]}$	
10	$y^* = r/\alpha$	$\mathbb{F}^{\pm} = r_{\ell}^{\#} \mathrm{d}\pi$
11	$\rho = [B^2 - (b+\gamma)(\alpha - r) + rv]^{1/2} - B$	$None$
12	$y \equiv Y/N \to (r-\rho)/\alpha \quad (17)$	$\mathbf{y} \equiv \boldsymbol{Y}/\boldsymbol{N} \to (\boldsymbol{r} - \boldsymbol{\rho})/\boldsymbol{\alpha} \quad (17)$
13	$N_T = (\alpha + b + v)/\beta \quad (18)$	$\mathbf{N_T} = \{\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{v})/\boldsymbol{\beta} \quad (18)$

2000 (Manually-detected Equations)

Eqn Num	Original Image	Decoded Image
0	$R_0 \geq \sigma \geq R,$	$R_0 \ \geq \ \sigma \ \geq \ R,$
1	$\begin{array}{ll} dS/dt = -\beta IS/N, & S(0) = S_o \geq 0, \\ dI/dt = \beta IS/N - \gamma I, & I(0) = I_o \geq 0, \\ dR/dt = \gamma I, & R(0) = R_o \geq 0, \end{array}$	$None$
2	$\begin{array}{ll} ds/dt = -\beta is, & s(0) = s_o \geq 0, \\ di/dt = \beta is - \gamma i, & i(0) = i_o \geq 0, \end{array}$	$None$
3	$T = \{(s,i)\,\|s \geq 0, i \geq 0, s+i \leq 1\}$	$T = \left\{ \left(s, \frac{i}{i} \right) \|s \ \geq \ 0, i \ \geq \ (0, s + i^* \ \leq \ 1 \right\}$
4	$i_o + s_o - s_\infty + \ln(s_\infty/s_o)/\sigma = 0.$	$i_\phi + 8_\phi - 8_\infty + \ln(\ell_\infty/k_\phi)\Big/\!/\sigma = 0.$
5	$i(t) + s(t) - [\ln s(t)]/\sigma = i_o + s_o - [\ln s_o]/\sigma$	$\begin{array}{l} i\,(\underline{t}) + \ \mathrm{s}\ (\ \mathrm{t}\ ) - \left[\ln \ \delta(\underline{t})\right] \Big/ \\ \pi = i_\phi^* + \ 8_\varrho - [\mathrm{1.5mmIn}\ s\ _{\mathrm{lambda}}] \Big\| f\, d \end{array}$

6	$\begin{aligned} dS/dt &= \mu N - \mu S - \beta IS/N, & S(0) &= S_o \geq 0, \\ dI/dt &= \beta IS/N - \gamma I - \mu I, & I(0) &= I_o \geq 0, \\ dR/dt &= \gamma I - \mu R, & R(0) &= R_o \geq 0, \end{aligned}$	$None$
7	$\begin{aligned} ds/dt &= -\beta is + \mu - \mu s, & s(0) &= s_o \geq 0, \\ di/dt &= \beta is - (\gamma + \mu)i, & i(0) &= i_o \geq 0, \end{aligned}$	$None$
8	$\sigma \approx \dfrac{\ln(s_o/s_\infty)}{s_o - s_\infty}$	$\sigma \approx \dfrac{\ln(s_o/s_\infty)}{s_e - s_\infty}$
9	$\begin{aligned} dM/dt &= b(N-S) - (\delta + d)M, \\ dS/dt &= bS + \delta M - \beta SI/N - dS, \\ dE/dt &= \beta SI/N - (\varepsilon + d)E, \\ dI/dt &= \varepsilon E - (\gamma + d)I, \\ dR/dt &= \gamma I - dR, \\ dN/dt &= (b-d)N. \end{aligned}$	$None$
10	$\begin{aligned} dm/dt &= (d+q)(e+i+r) - \delta m, \\ de/dt &= \lambda(1-m-e-i-r) - (\varepsilon+d+q)e \\ &\text{with } \lambda = \beta i, \\ di/dt &= \varepsilon e - (\gamma+d+q)i, \\ dr/dt &= \gamma i - (d+q)r. \end{aligned}$	$None$
11	$\mathfrak{D} = \{(m,e,i,r) : m \geq 0, e \geq 0, i \geq 0, r \geq 0, m+e+i+r \leq 1\}.$	$\mathcal{T} = \{(\mathcal{H}_h, \notin i, r^*) : T\mathcal{H} \geq (l, \in (l, i \geq (l, r^* \geq (l, {}_{Tl} + ' + i + r^* \leq 1\}_. \}$
12	$R_0 = \sigma = \dfrac{\beta\varepsilon}{(\gamma+d+q)(\varepsilon+d+q)}.$	$None$
13		$None$

	$m_e = \frac{d+q}{\delta+d+q}\left(1-\frac{1}{R_0}\right),$ $e_e = \frac{\delta(d+q)}{(\delta+d+q)(\varepsilon+d+q)}\left(1-\frac{1}{R_0}\right),$ $i_e = \frac{\varepsilon\delta(d+q)}{(\varepsilon+d+q)(\delta+d+q)(\gamma+d+q)}\left(1-\frac{1}{R_0}\right),$ $r_e = \frac{\varepsilon\delta\gamma}{(\varepsilon+d+q)(\delta+d+q)(\gamma+d+q)}\left(1-\frac{1}{R_0}\right),$	
14	$\lambda = \delta(d+q)(R_0-1)/(\delta+d+q),$	$\lambda = \delta\big(dl+q\big)\big(R_{0_\iota}-1\big)\Big/\big(\delta+d+q\big)\big],$
15	$\frac{\partial U}{\partial a} + \frac{\partial U}{\partial t} = -d(a)U,$	$\frac{\breve{Q}U}{\partial t} + \frac{\bar{\partial}\mathbb{U}}{\partial t} = -d[d)\vert\mathbb{U},$
16	$B(t) = U(0,t) = \int_0^\infty f(a)U(a,t)da.$	$B(t) \ = U[\![0,\ t\,\vert\ = \ \int_0^\infty \ f(d\,)U\langle\!\langle d_\gamma, t)ddL_\star$
17	$B(t) = U(0,t) = \int_0^t f(a)B(t-a)e^{-\int_0^a d(v)dv}da + \int_t^\infty f(a)U_0(a)e^{-\int_{a-t}^a d(v)dv}da.$	$B(t) = U[(0,t) = \int_{\mathbb{I}_0}^t f(d)B(t-d)e^{-\int_0^a d(v)dv}dd + \int_t^\infty f(a)]U_0(d)e^{-\int_{a-t}^a d(\psi)dv}dt_d.$
18	$1 = \int_0^\infty f(a)\exp[-D(a)-qa]da.$	$1 = \int_0^\infty f\ (a)\exp[-D(a)-qa]dd.$
19	$R_{pop} = \int_0^\infty f(a)\exp[-D(a)]da$	$R_{pop} = \int_0^\infty f\ (d)\exp[-\bar{D}(d)]ddd$
20	$U(a,t) = \rho e^{qt}e^{-D(a)-qa}$ with $\rho = 1\Big/\int_0^\infty e^{-D(a)-qa}da$.	$U\big(d,t\big) = \rho e^{qt}e^{-D(a)-qa}$ with $\rho = 1\ \int\int_0^\infty\int_0^\infty e^{-D(a)-qq}dd$.
21	$N_i(t) = \int_{a_{i-1}}^{a_i} U(a,t)da = e^{qt}\int_{a_{i-1}}^{a_i} A(a)da = e^{qt}P_i,$	$N_i(t) = \int_{d_{i-1}}^{a_i} U\big(d,t\big)da = e^{qt}\int_{d_{i-1}}^{a_i} A(a)dd = e^{qt}P_i,$
22	$A(a) = A(a_{i-1})\exp[-(d_i+q)(a-a_{i-1})].$	$A\,(a) = A(a_{i-1})\exp[-(d_i+q)](a-a_{i-1})]\,.$
23	$P_i = A(a_{i-1})\{1-\exp[-(d_i+q)(a_i-a_{i-1})]\}/(d_i+q).$	$P_i = A\big(d_{i-1}\big)\,\{1-\exp[-\big(d_i+q\big)\big(d_i-a_{i-1}\big)]\}\,/\big(d_i+q\big).$
24	$c_i = \frac{A(a_i)}{P_i} = \frac{d_i+q}{\exp[(d_i+q)(a_i-a_{i-1})]-1}.$	$c_i = \frac{A\left(d_i\right)}{P_i} = \frac{d_i+q}{\exp\left[(d_i+q)\left[d_i-d_{i-1}\right)\right]-1}.$
25	$dN_1/dt = \sum_{j=1}^n f_jN_j - (c_1+d_1)N_1,$ $dN_i/dt = c_{i-1}N_{i-1} - (c_i+d_i)N_i, \qquad 2 \le i \le n-1,$ $dN_n/dt = c_{n-1}N_{n-1} - d_nN_n.$	$dN_1/dt = \sum_{j=1}^n f_jN_j - \big(c_1+d_1\big)N_1,$ $dN_i/dt = c_{i-1}N_{i-1} - (C_1+d_1)N_i,$ $dN_i/dt = c_{n-1}N_{n-1} - d_nN_n.$
26		

	$P_i = \frac{c_{i-1} \cdots c_1 P_1}{(c_i + d_i + q) \cdots (c_2 + d_2 + q)}.$	$P_i = \frac{c_{i-1} \cdots c_1 P_1}{(C_i + d_i + q) \cdots (C_2 + d_2 + q]}.$
27	$1 = \frac{f_1 + f_2 \frac{c_1}{(c_2 + d_2 + q)} + \cdots + f_n \frac{c_{n-1} \cdots c_1}{(c_n + d_n + q) \cdots (c_2 + d_2 + q)}}{(c_1 + d_1 + q)}.$	$1 = \frac{f_1 + f_2 \frac{c_1}{(c_2+d_2+q]} + \cdots + f_n \frac{c_{n-1} \cdots c_1}{(c_n+d_n+q)} \cdots (c_2 + d_2 + q_1)}{(c_1 + d_1 + q)}.$
28	$R_{pop} = f_1 \frac{1}{(c_1 + d_1)} + f_2 \frac{c_1}{(c_2 + d_2)(c_1 + d_1)} + \cdots + f_n \frac{c_{n-1} \cdots c_1}{(c_n + d_n + q) \cdots (c_1 + d_1)}.$	$R_{\mathrm{pop}} = f_1 \frac{1}{(c_1 + d_1)} + f_2 \frac{c_1}{(c_2 + d_2)\,(c_1 + d_1)} + \cdots + f_n \frac{C_{n-1} \cdots c_1}{(c_n + d_{\eta_h} + q] \cdots (c_1 + d_1)}.$
29	$\begin{aligned} &\partial M/\partial a + \partial M/\partial t = -(\delta + d(a))M, \\ &\partial S/\partial a + \partial S/\partial t = \delta M - (\lambda(a,t) + d(a))S \\ &\quad \text{with } \lambda(a,t) = \int_0^\infty b(a)\tilde{b}(\tilde{a})I(\tilde{a},t)d\tilde{a} \Big/ \int_0^\infty U(\tilde{a},t)d\tilde{a}\ , \\ &\partial E/\partial a + \partial E/\partial t = \lambda(a,t)S - (\varepsilon + d(a))E, \\ &\partial I/\partial a + \partial I/\partial t = \varepsilon E - (\gamma + d(a))I, \\ &\partial R/\partial a + \partial R/\partial t = \gamma I - d(a)R. \end{aligned}$	None
30	$\begin{aligned} M(0,t) &= \int_0^\infty f(a)[M + E + I + R]da, \\ S(0,t) &= \int_0^\infty f(a)Sda, \end{aligned}$	$\begin{aligned} M(0,\, t) &= \int_0^\infty f(a)\,[M + E + I + R\vert dd, \\ S(0,\, t) &= \int_0^\infty f(a)\,Sdd, \end{aligned}$
31	$\begin{aligned} &\partial m/\partial a + \partial m/\partial t = -\delta m, \\ &\partial s/\partial a + \partial s/\partial t = \delta m - \lambda(a,t)s \\ &\quad \text{with } \lambda(a,t) = b(a) \int_0^\infty \tilde{b}(\tilde{a})i(\tilde{a},t)\rho e^{-D(\tilde{a})-q\tilde{a}}d\tilde{a}, \\ &\partial e/\partial a + \partial e/\partial t = \lambda(a,t)s - \varepsilon e, \\ &\partial i/\partial a + \partial i/\partial t = \varepsilon e - \gamma i, \\ &\partial r/\partial a + \partial r/\partial t = \gamma i, \end{aligned}$	None
32	$\begin{aligned} m(0,t) &= \int_0^\infty f(a)[1 - s(a,t)]e^{-D(a)-qa}da, \\ s(0,t) &= \int_0^\infty f(a)s(a,t)e^{-D(a)-qa}da, \end{aligned}$	None
33	$\begin{aligned} m(a) &= (1 - s_0)e^{-\delta a}, \\ s(a) &= e^{-\Lambda(a)} \left[s_0 + \delta(1 - s_0) \int_0^a e^{-\delta x + \Lambda(x)}dx \right], \\ e(a) &= e^{-\varepsilon a} \int_0^a \lambda(y)e^{\varepsilon y - \Lambda(y)} \left[s_0 + \delta(1 - s_0) \int_0^y e^{-\delta x + \Lambda(x)}dx \right] dy, \\ i(a) &= e^{-\gamma a} \int_0^a \varepsilon e^{(\gamma - \varepsilon)z} \int_0^z \lambda(y)e^{\varepsilon y - \Lambda(y)} \left[s_0 + \delta(1 - s_0) \int_0^y e^{-\delta x + \Lambda(x)}dx \right] dydz, \end{aligned}$	None
34	$\begin{aligned} \lambda(a) = b(a) &\int_0^\infty \tilde{b}(\tilde{a})\rho e^{-D(\tilde{a})-q\tilde{a}-\gamma\tilde{a}} \int_0^{\tilde{a}} \varepsilon e^{(\gamma-\varepsilon)z} \\ &\times \int_0^z \lambda(y)e^{\varepsilon y - \Lambda(y)} \left[s_0 + \delta(1 - s_0) \int_0^y e^{-\delta x + \Lambda(x)}dx \right] dydzd\tilde{a}. \end{aligned}$	None
35		$8_0 = s_0 F_\lambda \ + \delta(1 - s_0) F_\pm,$

	$s_0 = s_0 F_\lambda + \delta(1 - s_0) F_*,$	
36	$F_* = \int_0^\infty f(a) e^{-\Lambda(a) - D(a) - qa} \int_0^a e^{-\delta x + \Lambda(x)} dx da.$	$F_+^v = \int_0^\infty f\left(d\right) e^{-\Lambda(a) - D(a) - qa} \int_0^d e^{-\delta x + \Lambda(x)} dlxdt.$
37	$1 = \int_0^\infty \tilde{b}(\tilde{a}) \rho e^{-D(\tilde{a}) - q\tilde{a} - \gamma\tilde{a}} \int_0^{\tilde{a}} \varepsilon e^{(\gamma - \varepsilon) z} \int_0^z b(y) e^{\varepsilon y}$ $\times \left[\delta F_* e^{-k \int_0^y b(\alpha) d\alpha} + \delta(1 - F_\lambda) \int_0^y e^{-\delta x - k \int_x^y b(\alpha) d\alpha} dx\right] \Big/ (\delta F_* + 1 - F_\lambda) dy dz d\tilde{a}.$	$None$
38	$R_0 = \int_0^\infty \tilde{b}(\tilde{a}) \rho e^{-D(\tilde{a}) - q\tilde{a} - \gamma\tilde{a}} \int_0^{\tilde{a}} \varepsilon e^{(\gamma - \varepsilon) z} \int_0^z b(y) e^{\varepsilon y} dy dz d\tilde{a}.$	$R_{\mathrm{t}0} = \int_0^\infty \vec{b}(\underline{\tilde{d}}_l) \rho \mathbb{R}^- \overline{L}(\underline{\vec{a}}) - \underline{q}\bar{\vec{a}} - \gamma \bar{\mathrm{d}} \int_0^{\vec{d}} \varepsilon \in {}^{(\gamma - \mathbb{E})\mathbb{Z}} \int_0^{\overline{x}} b(\underline{y}$
39	$V = \int_0^\infty [\alpha(a) e(a, t) + \beta(a) i(a, t)] da,$	$V = \int_0^\infty \Big[c(d) e\big(d_l, \bar{t}\big) + \Big|\beta(q)_i i(d_l\,, t)\big)\Big] dl_l\,,$
40	$\dot{V} = \int_0^\infty \{\alpha(a)[\lambda s - \varepsilon e - \partial e/\partial a] + \beta(a)[\varepsilon e - \gamma i - \partial i/\partial a]\} da$ $= \int_0^\infty \{\lambda s \alpha(a) + e[\alpha'(a) - \varepsilon\alpha(a) + \varepsilon\beta(a)] + [\beta'(a) - \gamma\beta(a)] i\} da.$	$\dot{V} = \int_0^\infty \Big\{ \mathbb{G}\big(d\big) \Big[\lambda_8 - \varepsilon \in - \bar{\varrho} \notin \bar{G} d \underline{d}\Big] + \breve{j}\!/\!\big(\quad \big(\underline{d}\big)\Big]$ $= \int_0^\infty \{ \lambda$
41	$\dot{V} = \int_0^\infty s b(a) \varepsilon e^{\varepsilon a} \int_a^\infty e^{-\varepsilon z} \beta(z) dz da \int_0^\infty \tilde{b}(\tilde{a}) i \rho e^{-D(\tilde{a}) - q\tilde{a}} d\tilde{a} + \int_0^\infty [\beta' - \gamma\beta] i da.$	

42	$\dot{V} = \left[\int_0^\infty s b(a)\varepsilon e^{\varepsilon a} \int_a^\infty e^{(\gamma-\varepsilon)z} \int_z^\infty \tilde{b}(x)\rho e^{-D(x)-qx-\gamma x} dx dz da - 1\right] \times \int_0^\infty \tilde{b}(\tilde{a}) i(\tilde{a},t)\rho e^{-D(\tilde{a})-q\tilde{a}} d\tilde{a}.$	$None$

43	$\dot V \le (R_0-1)\int_0^\infty \tilde b(\tilde a)i(\tilde a,t)\rho e^{-D(\tilde a)-q\tilde a}d\tilde a \le 0 \quad \text{if } R_0 \le 1.$	$None$
44	$1 = \int_0^\infty \tilde b(\tilde a)\rho e^{-D(\tilde a)-q\tilde a-\gamma\tilde a}\int_0^{\tilde a} b(y)e^{\gamma y-k\int_0^y b(\alpha)d\alpha}dyd\tilde a$	$1 = \int_0^\infty \vec b(\widetilde d)\rho e^{-L(\vec{\vec\mu}) - q\vec{\vec q} - \gamma\,\overline{\mathrm d}}\int_0^{\vec a} b(y)e^{\gamma\,\underline{y}-\hbar}\int_0^y b(\mathrm{d}r)\,dd\mathrm{e}$
45	$R_0 = \int_0^\infty \tilde b(\tilde a)\rho e^{-D(\tilde a)-q\tilde a-\gamma\tilde a}\int_0^{\tilde a} b(y)e^{\gamma y}dyd\tilde a.$	
46	$A = E[a] = \frac{\int_0^\infty a\lambda(a)e^{-D(a)}[\delta F_* e^{-\Lambda(a)} + \delta(1-F_\lambda)\int_0^a e^{-\delta x-\Lambda(a)+\Lambda(x)}dx]da}{\int_0^\infty \lambda(a)e^{-D(a)}[\delta F_* e^{-\Lambda(a)} + \delta(1-F_\lambda)\int_0^a e^{-\delta x-\Lambda(a)+\Lambda(x)}dx]da}.$	$None$
47	$A = E[a] = \frac{\int_0^\infty a\lambda(a)e^{-\Lambda(a)-D(a)}da}{\int_0^\infty \lambda(a)e^{-\Lambda(a)-D(a)}da}.$	$A = E^*[q] = \frac{\int_0^\infty d\lambda}{\int_0^\infty \lambda(d)e^{-\frac{\Lambda}{A}(a)} - L(q)}\frac{dl}{dlq}.$
48	$R_0 = \beta c/[(\gamma \mid d \mid q)(c \mid d \mid q)],$	$None$
49	$1 = \frac{(d+q)R_0}{\lambda+d+q}\left[s_0 + \frac{\delta(1-s_0)}{\delta+d+q}\right].$	$1 = \frac{(dl+q)\,R_{l0}}{\lambda+dl+q}\left[80 + \frac{\delta(1-8_0)}{\delta+\ell-\ell}\right].$
50	$s_0 = \frac{\delta-\lambda s_0}{\delta-\lambda}F_\lambda - \frac{\delta(1-s_0)}{\delta-\lambda}F_\delta,$	$None$
51	$s_0 = \delta(F_\lambda - F_\delta)/[\delta(1-F_\delta) - \lambda(1-F_\lambda)].$	$8_0 = \delta\,(F^*_\lambda - F^*_\delta)\int\left[\delta\left(1-F^*_6\right) - \lambda\,(1-F^*_\lambda)\right].$
52	$1 = \frac{R_0(d+q)\delta\left[F_\lambda - F_\delta + \frac{(\delta-\lambda)(1-F_\lambda)}{\delta+d+q}\right]}{(\lambda+d+q)[\delta(1-F_\delta)-\lambda(1-F_\lambda)]},$	$1 = \frac{R_{t_0}(d\,l+q)\,\delta\,\Big[F^v_\lambda - F_\delta + \frac{(\delta-\lambda)(1-F_\lambda)}{\delta+F^+_\lambda}\Big)}{\delta + \frac{d}{F}\lambda\Big]},$
53	$A = E[a] = \frac{\lambda d\int_0^\infty a[c_1e^{-(\lambda+d)a} + c_2e^{-(\delta+d)a}]da}{\lambda d\int_0^\infty [c_1e^{-(\lambda+d)a} + c_2e^{-(\delta+d)a}]da} = \frac{\frac{\delta-\lambda s_0}{(\lambda+d)^2} - \frac{\delta(1-s_0)}{(\delta+d)^2}}{\frac{\delta-\lambda s_0}{(\lambda+d)} - \frac{\delta(1-s_0)}{(\delta+d)}}.$	$None$
54		$R_0 = \frac{[q+1/(A-p)](1+pq)}{(q+1/L)\Big\|(1-p/L)}.$

	$R_0 = \frac{[q + 1/(A - p)](1 + pq)}{(q + 1/L)(1 - p/L)}.$	
55	$1 = \frac{R_0(d+q)}{\lambda + d + q}\left[s_0 + \frac{\delta(1 - s_0)}{\delta + d + q} - g[c_1 e^{-(\lambda+d+q)A_v} + c_2 e^{-(\delta+d+q)A_v}]\right],$	$1 = \frac{R_0(\|d+q)}{\lambda + d + q}\left[s_0 + \frac{\delta(1 - s_0)}{\delta + d + q} - g\left[e_1 e^{-\left(\lambda+d+q\right)A_v} + \epsilon_2 e^{=\left(\delta+d+q\right) 4 v_\|}\right]\right],$
56	$s_0 = c_1 F_\lambda + c_2 F_\delta - g\left[c_1 + c_2 e^{(\lambda-\delta)A_v}\right] F_{A_v},$	$8_0 = \epsilon_1 F_\lambda^\dagger + c_2 F_\delta^* - g\left[c_1 + c_2^\dagger \varepsilon^{(\lambda-\delta)\ A_v}\right]\ F_4$, F_4
57	$1 = \frac{R_0(d+q)}{\lambda + d + q}\left[1 - ge^{-(\lambda+d+q)A_v}\right].$	$1 = \frac{R_0\left(dl + q\right)}{\lambda + d + q}\left[1 - ge^{-\left(\lambda+\#+q\right)A\ v_w}\right].$
58	$ge^{-(d+q)A_v} \geq 1 - 1/R_0,$	$None$
59	$A = \frac{1}{\lambda + d} - \frac{gA_v[c_1 e^{-(\lambda+d)A_v} + c_2 e^{-(\delta+d)A_v}] + c_2 \frac{\delta - \lambda}{(\delta+d)^2}}{c_1[1 - ge^{-(\lambda+d)A_v}] + c_2[\frac{\lambda+d}{\delta+d} - ge^{-(\delta+d)A_v}]}.$	$A = \frac{1}{\lambda + 4} - \frac{gA_\psi\left[c_1 e^{-(\lambda+d)} A_v\ +\ c_2 e^{-(\delta+d)A_v}\right] + c_2 \frac{\delta-\lambda}{(\delta+d)^2}}{c_1\left[1 - ge^{-(\lambda+d)A_v}\right] + c_2 \frac{\|\lambda+d\|}{\delta+d} - ge^{-(\delta+d)/4}}$
60	$R_0 = \frac{\beta}{\gamma}\left[1 + \frac{\gamma}{\varepsilon - \gamma}\frac{1 - e^{-\varepsilon L}}{\varepsilon L} - \frac{\varepsilon}{(\varepsilon - \gamma)}\frac{1 - e^{-\gamma L}}{\gamma L}\right].$	$R_\mathrm{t} = \frac{\beta}{\gamma}\left[1 + \frac{\gamma}{\varepsilon - \gamma}\frac{1 - e^{-\varepsilon L}}{\varepsilon L} - \frac{\varepsilon}{(\varepsilon - \gamma)}\frac{1 - e^{-\gamma\, L}}{\gamma L}\right].$
61	$1 = \beta\varepsilon\left[\frac{1 - e^{-\lambda L}}{(\gamma - \lambda)(\varepsilon - \lambda)\lambda L} + \frac{1 - e^{-\varepsilon L}}{(\varepsilon - \lambda)(\varepsilon - \gamma)\varepsilon L} - \frac{1 - e^{-\gamma L}}{(\gamma - \lambda)(\varepsilon - \gamma)\gamma L}\right].$	$1 = \beta\varepsilon\left[\frac{1 - e^{-\lambda\, L}}{(\gamma - \lambda)(\varepsilon - \lambda)\lambda L} + \frac{1 - e^{-\varepsilon}}{(\varepsilon - \lambda)(\varepsilon - \gamma)\varepsilon L} - \frac{1 - e^{-\gamma\, L}}{(\gamma - \lambda)\,\|\varepsilon - \gamma\|\,\gamma L}\right].$
62	$R_0 = \frac{\beta}{\gamma}\left[1 - \frac{1 - e^{-\gamma L}}{\gamma L}\right],$	$R_0 = \frac{\beta}{\gamma}\left[1 - \frac{1 - e^{-\gamma L}}{\gamma L}\right]\ ,$
63	$1 = \frac{\beta}{\gamma - \lambda}\left[\frac{1 - e^{-\lambda L}}{\lambda L} - \frac{1 - e^{-\gamma L}}{\gamma L}\right].$	$1 = \frac{\beta}{\gamma - \lambda}\left[\frac{1 - e^{-\lambda L}}{\lambda L} - \frac{1 - e^{-\gamma L}}{\gamma L}\right].$
64	$A = \frac{1}{\lambda} - \frac{Le^{-\lambda L}}{1 - e^{-\lambda L}}.$	$A = \frac{1}{\lambda} - \frac{Le^{-\lambda L}}{1 - e^{-\lambda L}}.$
65	$\bar{s} = \int_0^L \frac{e^{-\lambda a}}{L} da = \frac{1 - e^{-\lambda L}}{\lambda L}.$	$\overline{s} = \int_0^L \frac{e^{-\lambda a}}{L} dd = \frac{1 - e^{-\lambda L}}{\lambda L}.$
66		

	$\begin{aligned} &\partial S/\partial a + \partial S/\partial t = -\lambda(a,t)S - d(a)S, \\ &\qquad \lambda(a,t) = \int_0^\infty b(a)\tilde{b}(\tilde{a})I(\tilde{a},t)d\tilde{a} \bigg/ \int_0^\infty U(\tilde{a},t)d\tilde{a}\ , \\ &\partial E/\partial a + \partial E/\partial t = \lambda(a,t)S - \varepsilon I - d(a)E, \\ &\partial I/\partial a + \partial I/\partial t = \varepsilon I - \gamma I - d(a)I, \\ &\partial R/\partial a + \partial R/\partial t = \gamma I - d(a)R. \end{aligned}$	$None$
67	$\begin{aligned} &ds_1/dt = (c_1 + d_1 + q)P_1 - [\lambda_1 + c_1 + d_1 + q]s_1, \\ &ds_i/dt = c_{i-1}s_{i-1} - [\lambda_i + c_i + d_i + q]s_i, \qquad i \geq 2, \\ &\qquad \lambda_i = b_i \sum_{j=1}^{n} \tilde{b}_j i_j, \\ &de_1/dt = \lambda_1 s_1 - [\varepsilon_1 + c_1 + d_1 + q]e_1, \\ &de_i/dt = \lambda_i s_i + c_{i-1}e_{i-1} - [\varepsilon_i + c_i + d_i + q]e_i, \quad i \geq 2, \\ &di_1/dt = \varepsilon_1 e_1 - [\gamma_1 + c_1 + d_1 + q]i_1, \\ &di_i/dt = \varepsilon_i e_i + c_{i-1}i_{i-1} - [\gamma_i + c_i + d_i + q]i_i, \quad i \geq 2, \\ &dr_1/dt = \gamma_1 i_1 - [c_1 + d_1 + q]r_1, \\ &dr_i/dt = \gamma_i i_i + c_{i-1}r_{i-1} - [c_i + d_i + q]r_i, \qquad i \geq 2. \end{aligned}$	$None$
68	$\begin{aligned} &s_1 = \hat{c}_1 P_1/\hat{\lambda}_1,\ s_i = c_{i-1}s_{i-1}/\hat{\lambda}_i && \text{for } i \geq 2, \\ &e_1 = \lambda_1 s_1/\hat{\varepsilon}_1,\ e_i = (\lambda_i s_i + c_{i-1}e_{i-1})/\hat{\varepsilon}_i && \text{for } i \geq 2, \\ &i_1 = \varepsilon_1 e_1/\hat{\gamma}_1,\ \ i_i = (\varepsilon_i e_i + c_{i-1}i_{i-1})/\hat{\gamma}_i && \text{for } i \geq 2, \end{aligned}$	$None$
69	$e_i = \frac{\lambda_i C_{i-1}}{\hat{\varepsilon}_i \hat{\lambda}_i \cdots \hat{\lambda}_1} + \frac{\lambda_{i-1} C_{i-1}}{\hat{\varepsilon}_i \hat{\varepsilon}_{i-1} \hat{\lambda}_{i-1} \cdots \hat{\lambda}_1} + \frac{\lambda_{i-2} C_{i-1}}{\hat{\varepsilon}_i \hat{\varepsilon}_{i-1} \hat{\varepsilon}_{i-2} \hat{\lambda}_{i-2} \cdots \hat{\lambda}_1} + \cdots + \frac{\lambda_1 C_{i-1}}{\hat{\varepsilon}_i \cdots \hat{\varepsilon}_1 \hat{\lambda}_1}$	$None$
70	$\begin{aligned} \frac{i_i}{C_{i-1}} = &\frac{\varepsilon_i}{\hat{\gamma}_i}\left(\frac{\lambda_i}{\hat{\varepsilon}_i \hat{\lambda}_i \cdots \hat{\lambda}_1} + \frac{\lambda_{i-1}}{\hat{\varepsilon}_i \hat{\varepsilon}_{i-1} \hat{\lambda}_{i-1} \cdots \hat{\lambda}_1} + \cdots + \frac{\lambda_1}{\hat{\varepsilon}_i \cdots \hat{\varepsilon}_1 \hat{\lambda}_1}\right) \\ &+ \frac{\varepsilon_{i-1}}{\hat{\gamma}_i \hat{\gamma}_{i-1}}\left(\frac{\lambda_{i-1}}{\hat{\varepsilon}_{i-1} \hat{\lambda}_{i-1} \cdots \hat{\lambda}_1} + \frac{\lambda_{i-2}}{\hat{\varepsilon}_{i-1} \hat{\varepsilon}_{i-2} \hat{\lambda}_{i-2} \cdots \hat{\lambda}_1} + \cdots + \frac{\lambda_1}{\hat{\varepsilon}_{i-1} \cdots \hat{\varepsilon}_1 \hat{\lambda}_1}\right) \\ &+ \cdots + \frac{\varepsilon_2}{\hat{\gamma}_i \cdots \hat{\gamma}_2}\left(\frac{\lambda_2}{\hat{\varepsilon}_2 \hat{\lambda}_2 \hat{\lambda}_1} + \frac{\lambda_1}{\hat{\varepsilon}_2 \hat{\varepsilon}_1 \hat{\lambda}_1}\right) + \frac{\varepsilon_1}{\hat{\gamma}_i \cdots \hat{\gamma}_1}\left(\frac{\lambda_1}{\hat{\varepsilon}_1 \hat{\lambda}_1}\right). \end{aligned}$	$None$
71	$\begin{aligned} 1 = \sum_{j=1}^{n} \tilde{b}_j C_{j-1} \Bigg[&\frac{\varepsilon_j}{\hat{\gamma}_j}\left(\frac{b_j}{\hat{\varepsilon}_j \hat{b}_j \cdots \hat{b}_1} + \frac{b_{j-1}}{\hat{\varepsilon}_j \hat{\varepsilon}_{j-1} \hat{b}_{j-1} \cdots \hat{b}_1} + \cdots + \frac{b_1}{\hat{\varepsilon}_j \cdots \hat{\varepsilon}_1 \hat{b}_1}\right) \\ &+ \frac{\varepsilon_{j-1}}{\hat{\gamma}_j \hat{\gamma}_{j-1}}\left(\frac{b_{j-1}}{\hat{\varepsilon}_{j-1} \hat{b}_{j-1} \cdots \hat{b}_1} + \frac{b_{j-2}}{\hat{\varepsilon}_{j-1} \hat{\varepsilon}_{j-2} \hat{b}_{j-2} \cdots \hat{b}_1} + \cdots + \frac{b_1}{\hat{\varepsilon}_{j-1} \cdots \hat{\varepsilon}_1 \hat{b}_1}\right) \\ &+ \cdots + \frac{\varepsilon_2}{\hat{\gamma}_j \cdots \hat{\gamma}_2}\left(\frac{b_2}{\hat{\varepsilon}_2 \hat{b}_2 \hat{b}_1} + \frac{b_1}{\hat{\varepsilon}_2 \hat{\varepsilon}_1 \hat{b}_1}\right) + \frac{\varepsilon_1}{\hat{\gamma}_j \cdots \hat{\gamma}_1}\left(\frac{b_1}{\hat{\varepsilon}_1 \hat{b}_1}\right)\Bigg], \end{aligned}$	$None$
72	$\begin{aligned} R_0 = \sum_{j=1}^{n} \tilde{b}_j C_{j-1} \Bigg[&\frac{\varepsilon_j}{\hat{\gamma}_j}\left(\frac{b_j}{\hat{\varepsilon}_j \hat{c}_j \cdots \hat{c}_1} + \frac{b_{j-1}}{\hat{\varepsilon}_j \hat{\varepsilon}_{j-1} \hat{c}_{j-1} \cdots \hat{c}_1} + \cdots + \frac{b_1}{\hat{\varepsilon}_j \cdots \hat{\varepsilon}_1 \hat{c}_1}\right) \\ &+ \frac{\varepsilon_{j-1}}{\hat{\gamma}_j \hat{\gamma}_{j-1}}\left(\frac{b_{j-1}}{\hat{\varepsilon}_{j-1} \hat{c}_{j-1} \cdots \hat{c}_1} + \frac{b_{j-2}}{\hat{\varepsilon}_{j-1} \hat{\varepsilon}_{j-2} \hat{c}_{j-2} \cdots \hat{c}_1} + \cdots + \frac{b_1}{\hat{\varepsilon}_{j-1} \cdots \hat{\varepsilon}_1 \hat{c}_1}\right) \\ &+ \cdots + \frac{\varepsilon_2}{\hat{\gamma}_j \cdots \hat{\gamma}_2}\left(\frac{b_2}{\hat{\varepsilon}_2 \hat{c}_2 \hat{c}_1} + \frac{b_1}{\hat{\varepsilon}_2 \hat{\varepsilon}_1 \hat{c}_1}\right) + \frac{\varepsilon_1}{\hat{\gamma}_j \cdots \hat{\gamma}_1}\left(\frac{b_1}{\hat{\varepsilon}_1 \hat{c}_1}\right)\Bigg], \end{aligned}$	$None$
73	$\dot{V} = \sum \alpha_i b_i s_i \sum \tilde{b}_j i_j - (\beta_1 \hat{\gamma}_1 - \beta_2 c_1)i_1 - \cdots - (\beta_{n-1}\hat{\gamma}_{n-1} - \beta_n c_{n-1})i_{n-1} - \beta_n \hat{\gamma}_n i_n.$	$\dot{V} = \sum_i c_{ti} b_{i\bar{s}_i} \sum_j \tilde{b}_j \tilde{i}_j - (\beta_1 \hat{\gamma}_1 - \beta_2 \epsilon_1)\, i_1 - \cdots - (\beta_{n-1}\hat{\gamma}_{n-1} - \beta_n$

74	$1=\sum_{j=1}^{n}\tilde{b}_jC_{j-1}\left[\frac{b_j}{\hat{\gamma}_j\hat{b}_j\cdots\hat{b}_1}+\frac{b_{j-1}}{\hat{\gamma}_j\hat{\gamma}_{j-1}\hat{b}_{j-1}\cdots\hat{b}_1}+\cdots+\frac{b_2}{\hat{\gamma}_j\cdots\hat{\gamma}_2\hat{b}_2\hat{b}_1}+\frac{b_1}{\hat{\gamma}_j\cdots\hat{\gamma}_1\hat{b}_1}\right],$	$1=\sum_{j=1}^{n}\tilde{b}_jC_{j-1}\left[\frac{b_j}{\hat{\gamma}_j\hat{b}_j\cdots\hat{b}_1}+\frac{b_{j-1}}{\hat{\gamma}_j\hat{\gamma}_{j-1}\hat{b}_{j-1}\cdots\hat{b}_1}+\cdots+\frac{b_2}{\hat{\gamma}_2\cdots\hat{\gamma}_2\hat{b}_2}\right.$
75	$R_0=\sum_{j=1}^{n}\tilde{b}_jC_{j-1}\left[\frac{b_j}{\hat{\gamma}_j\hat{c}_j\cdots\hat{c}_1}+\frac{b_{j-1}}{\hat{\gamma}_j\hat{\gamma}_{j-1}\hat{c}_{j-1}\cdots\hat{c}_1}+\cdots+\frac{b_2}{\hat{\gamma}_j\cdots\hat{\gamma}_2\hat{c}_2\hat{c}_1}+\frac{b_1}{\hat{\gamma}_j\cdots\hat{\gamma}_1\hat{c}_1}\right].$	*None*
76	$A=E[a]=\frac{\int_0^\infty a\lambda(a)s(a)e^{-D(a)}da}{\int_0^\infty\lambda(a)s(a)e^{-D(a)}da}=\frac{\sum_{i=1}^{n}\int_{a_{i-1}}^{a_i}a\lambda(a)s(a)e^{-D(a)}da}{\sum_{i=1}^{n}\int_{a_{i-1}}^{a_i}\lambda(a)s(a)e^{-D(a)}da}.$	$A=E[a]=\frac{\int_0^\infty d\lambda(d)s(d)e^{-D(a)}{}_{dd}}{\int_0^\infty\lambda(a)s(a)e^{-D(a)}dd}=\frac{\sum_{i=1}^{n}\int_{a_{i-1}}^{a_i}d\lambda(d)\,s(d)e^{-D(a)}\,dd}{\sum_{i=1}^{n}}$
77	$A=\frac{\sum_{i=1}^{n}b_is_i\pi_{i-1}[1+d_ia_{i-1}-(1+d_ia_i)e^{-d_i\Delta_i}]/d_i^2}{\sum_{i=1}^{n}b_is_i\pi_{i-1}[1-e^{-d_i\Delta_i}]/d_i}.$	*None*
78	$R\cong\frac{\sum_{j=1}^{16}\lambda_js_jP_j\left(\frac{1}{\gamma+d_j+q}\right)\left(\frac{\varepsilon}{\varepsilon+d_j+q}\right)}{\sum_{j=1}^{16}i_jP_j}.$	*None*
79	$R_0=\sigma\cong\frac{\sum_{j=1}^{16}\lambda_jP_j\left(\frac{1}{\gamma+d_j+q}\right)\left(\frac{\varepsilon}{\varepsilon+d_j+q}\right)}{\sum_{j=1}^{16}i_jP_j}.$	*None*
80	$A\cong\frac{\sum_{j=1}^{16}[\frac{a_{j-1}+a_j}{2}]\lambda_js_jP_j}{\sum_{j=1}^{16}\lambda_js_jP_j}.$	$A\triangleq\frac{\sum_{j=1}^{16}\left[\frac{\mathrm{d}_{j-1}+a_j}{2}\right]\lambda_j\,s_jP_j}{\sum_{j=1}^{16}\lambda_j8_j\,P_j}.$
81	$R_0=[1+\lambda/(d+q)][1+(d+q)/\delta],$	$R_{0_i}=\left[1+\lambda_f^j\left[dl+q_i\right)\right.\left|\right|1+\left(d\,+\,q\right)/f\left|,\right|,$
82	$R\cong\frac{\sum_{j=1}^{32}\lambda_j(s_j+r_{1j}+r_{2j})P_j/(\gamma+d_j)}{\sum_{j=1}^{32}(i_j+i_{mj}+i_{wj})P_j},$	*None*
83	$\sigma\cong\frac{\sum_{j=1}^{32}\lambda_jP_j/(\gamma+d_j)}{\sum_{j=1}^{32}(i_j+i_{mj}+i_{wj})P_j},$	$\sigma\cong\frac{\sum_{j=1}^{32}}{\sum_{j=1}^{32}\left(\frac{i}{l_j}+i_{mij}+\frac{i}{t_{wj}j}\right)P_j^2},$
84	$\sigma=R_0[I+\rho_mI_m+\rho_wI_w]/[I+I_m+I_w]<R_0,$	$\mathcal{F}=R_0[I+\rho_mI_{m_i}+\rho_{iw}I_w]/[I+I_{p_i}+I_w]<R_0,$

2016 (cosmos-detected Equations)

Eqn Num	Original Image	Decoded Image
0	$\dot{S} = -\beta IS$ (1a) $\dot{I} = \beta IS - \gamma I$ (1b)	$\dot{\mathcal{S}} = -\beta \mathrm{IS}$ (la) $\dot{I} = \beta \mathrm{IS} - \gamma \mathsf{l}$ (la)
1	$\dot{R} = \gamma I$	$\dot{\mathsf{R}} = \gamma I$
2	$S(0) = 1 - \rho$ $I(0) = \rho$ $R(0) = 0.$	$S(\mathbf{0}) = 1 - \rho$ $\mathcal{R}(\mathbf{0}) = \mathbf{0}$
3	$\dot{\theta} = -\tau \phi_I$	$\dot{\theta} = -\tau \phi_l$
4	$\phi_I = \theta - (1 - \rho)\dfrac{\psi'(\theta)}{\langle K \rangle} - \dfrac{\gamma}{\tau}(1 - \theta)$	$\phi_I = \theta - (1 - \rho)\dfrac{V'(\theta)}{\langle \mathbb{K} \rangle} - \dfrac{\gamma}{\tau}(1 - \theta)$
5	$S = (1 - \rho)\psi(\theta)$	$\boldsymbol{S} = (\mathbf{1} - \rho)\mathbb{V}(\theta)$
6	$I = 1 - S - R$ (2d) $\dot{R} = \gamma I$ (2e)	$l = 1 - \mathsf{S} - \mathsf{R}$ (2d) $\dot{\mathsf{R}} = \gamma I$ (2d)
7		

	$\theta(0) = 1$ $R(0) = 0$	$\theta(\mathbf{0}) = \mathbf{1}$ $\mathrm{R}(\mathbf{0}) = \mathbf{0}$
8	$\phi_S = (1-\rho)\psi'(\theta)/\langle K\rangle$	$\phi_{\mathrm{S}} = (1-\rho)\psi'(\theta)/\langle K\rangle$
9	$\phi_R = \gamma(1-\theta)/\tau.$	$\phi_{\mathbb{R}} = \gamma(1-\theta)/\tau.$
10	$\phi_I = \theta - \phi_S - \phi_R$ $= \theta - (1-\rho)\frac{\psi'(\theta)}{\langle K\rangle} - \frac{\gamma}{\tau}(1-\theta).$	$\phi_l = \theta - \phi_{\mathrm{S}} - \phi_{\mathrm{R}}$ $= \theta - (1-\rho)\frac{\psi'(\theta)}{\langle K\rangle} - \frac{\gamma}{\tau}(1-\theta).$
11	$\dot{S} + \beta IS = 0$	$\dot{S} + \beta lS = \emptyset$
12	$\frac{\mathrm{d}}{\mathrm{d}t}\left(Se^{\beta\int_0^t I\,\mathrm{d}\hat{t}}\right) = 0\ .$	$\frac{\mathbf{d}}{\mathbf{dt}}\left(\quad \overset{t}{\beta} \quad \widehat{\mathbf{dt}} = \mathbf{0}\ .\right.$
13	$S(t) = S(0)e^{-\xi(t)} = (1-\rho)e^{-\xi(t)}$	$\boldsymbol{S}(\boldsymbol{t}) = \boldsymbol{S}(\mathbf{0})\boldsymbol{e}^{-\hat{\mathbf{z}}(\mathbf{t})} = (\mathbf{1}-\rho)\boldsymbol{e}^{-\bar{\xi}(t)}$
14	$S = (1-\rho)e^{-\xi}$	$\boldsymbol{S} = (\mathbf{1}-\boldsymbol{\rho})\boldsymbol{e}^{-\frac{\mathbf{z}}{5}}\boldsymbol{e}^{-\frac{\mathbf{z}}{5}}\boldsymbol{e} = (\mathbf{1}-\boldsymbol{\rho})\boldsymbol{e}^{-\frac{\mathbf{z}}{5}}\boldsymbol{e}^{-\frac{\mathbf{z}}{5}}\boldsymbol{e} = (\mathbf{1}-\boldsymbol{\rho})\boldsymbol{e}^{-\frac{\mathbf{z}}{5}}\boldsymbol{e}^{-\mathbf{z}}$

15	$I = 1 - S - R = 1 - (1-\rho)e^{-\xi} - \frac{\gamma\xi}{\beta}$	$\mathrm{I} = 1 - \mathrm{S} - \mathrm{R} = 1 - (1-\rho)\mathrm{e}^{-\mathfrak{z}} - \frac{\gamma\xi}{\mathrm{B}}$
16	$R = \frac{\gamma\xi}{\beta}$	$\mathrm{R} = \frac{\gamma\xi}{\beta}$
17	$\dot{\xi} = \beta I = \beta\left[1 - (1-\rho)e^{-\xi} - \frac{\gamma\xi}{\beta}\right]$	$\dot{\xi} = \boldsymbol{\beta} l = \boldsymbol{\beta}\left[1 - (\mathbf{1} - \boldsymbol{\rho})\boldsymbol{e}^{-\dot{\varsigma}} - \frac{\gamma\boldsymbol{\xi}}{\boldsymbol{\beta}}\right]$
18	$\xi(0) = 0$.	$\xi(\mathbf{0}) = \mathbf{0}$.
19	$\mathscr{R}_0 = \beta/\gamma$.	$\partial_0 = \beta/\gamma$
20	$\mathscr{R}_0 = \sum_k \frac{kP(k)}{\langle K\rangle}(k-1)\frac{\tau}{\tau+\gamma}$	$\beta_0 = \sum_k \frac{\mathsf{kp}(k)}{\langle K\rangle}(k-1)\frac{\tau}{\tau+\gamma}$
21	$= \frac{\tau\langle K^2 - K\rangle}{(\tau+\gamma)\langle K\rangle}$.	$= \frac{\tau\langle \mathcal{K}^2 - K\rangle}{(\tau+\gamma)\langle K\rangle}$.
22	$R(\infty) = 1 - (1-\rho)e^{-\mathscr{R}_0 R(\infty)}$	$\mathrm{R}(\mathbf{w}) = \mathbf{1} - (\mathbf{1} - \mathbf{g})\boldsymbol{e}^{-\beta\mathfrak{d}_0\mathcal{R}(\infty)}$
23	$\theta(\infty) = (1-\rho)\frac{\psi'(\theta(\infty))}{\langle K\rangle} = \frac{\gamma(1-\theta(\infty))}{\tau}$.	$\theta(\infty) = (1-\rho)\frac{\psi(\theta(\infty))}{\langle K\rangle} = \frac{\gamma(1-\theta(\infty))}{\tau}$.

24	$S=(1-\rho)e^{-\xi}\psi(\theta)$	$\boldsymbol{S}=(\mathbf{1}-\rho)\boldsymbol{e}^{-?}\psi(\theta)$
25	$I_1=1-S-I_2-R$ (5b) $\dot{I}_2=\gamma_1 I_1-\gamma_2 I_2$ (5c) $\dot{R}=\gamma_2 I_2$ (5d) $\dot{\xi}=\beta I_1$ (5e) $\dot{\theta}=-\tau_1\phi_{I,1}-\tau_2\phi_{I,2}$ (5f)	$None$
26	$\phi_S=\frac{(1-\rho)e^{-\xi}\psi'(\theta)}{\langle K\rangle}$	$\phi_{\mathrm{S}}=\frac{(1-\rho)\boldsymbol{e}^{-5}\psi'(\theta)}{\langle\mathbb{K}\rangle}$
27	$\phi_{I,1}=\theta-\phi_S-\phi_{I,2}-\phi_R$	$\phi_{R1}=\theta-\phi_S-\phi_{I2}-\phi_R$
28	$\dot{\phi}_{I,2}=\gamma_1\phi_{I,1}-(\gamma_2+\tau_2)\phi_{I,2}$	$\dot{\phi}_{l2}=\gamma_1\phi_{l,1}-(\gamma_2+\tau_2)\phi_{l2},$
29	$\dot{\phi}_R=\gamma_2\phi_{I,2}\ \cdot$	$\dot{\phi}_{\mathrm{R}}=\gamma_2\theta_{I,2}\ \cdot$
30	$\begin{pmatrix} N_{\mathrm{ma}}(g+1) \\ N_{\mathrm{se}}(g+1)\end{pmatrix}=\begin{pmatrix} R_{\mathrm{ma}} & R_{\mathrm{ma}} \\ R_{\mathrm{se\|ma}} & R_{\mathrm{se\|se}}\end{pmatrix}\begin{pmatrix} N_{\mathrm{ma}}(g) \\ N_{\mathrm{se}}(g)\end{pmatrix}.$	$None$
31	The dominant eigenvalue is $\mathscr{R}_0=\frac{R_{\mathrm{ma}}+R_{\mathrm{se\|se}}+\sqrt{\left(R_{\mathrm{ma}}+R_{\mathrm{se\|se}}\right)^2-4\left(R_{\mathrm{ma}}R_{\mathrm{se\|se}}-R_{\mathrm{se\|ma}}R_{\mathrm{ma}}\right)}}{2}$ $=\frac{R_{\mathrm{ma}}+R_{\mathrm{se\|se}}+\sqrt{\left(R_{\mathrm{ma}}-R_{\mathrm{se\|se}}\right)^2+4R_{\mathrm{se\|ma}}R_{\mathrm{ma}}}}{2}.$	$None$
32	$\begin{pmatrix} N(g+1) \\ y(g+1)\end{pmatrix}=\begin{pmatrix} R_{\mathrm{ma}} & R_{\mathrm{se\|ma}}/\langle K\rangle \\ R_{\mathrm{ma}}\langle K\rangle & R_{\mathrm{se\|se}}\end{pmatrix}\begin{pmatrix} N(g) \\ y(g)\end{pmatrix}.$	$None$
33	$\mathscr{R}_0=\frac{R_{\mathrm{ma}}+R_{\mathrm{se\|se}}+\left\|R_{\mathrm{ma}}-R_{\mathrm{se\|se}}\right\|}{2}+\frac{R_{\mathrm{se\|ma}}R_{\mathrm{ma}}}{\left\|R_{\mathrm{ma}}-R_{\mathrm{se\|se}}\right\|}+\mathscr{O}\left(\frac{\left(R_{\mathrm{se\|ma}}R_{\mathrm{ma}}\right)^2}{\left\|R_{\mathrm{ma}}-R_{\mathrm{se\|se}}\right\|^3}\right).$	$\mathcal{R}_{\mathrm{ma}}+\mathcal{R}_{\mathrm{se\|se}}+\left\|\mathsf{R}_{\mathrm{ma}}-\mathsf{F}\right.$
34	$\mathscr{R}_0=\frac{2R_{\mathrm{se\|se}}+\varepsilon+\sqrt{\left(2R_{\mathrm{se\|se}}+\varepsilon\right)^2-4R_{\mathrm{se\|se}}^2-4\varepsilon R_{\mathrm{se\|se}}+4R_{\mathrm{se\|ma}}R_{\mathrm{ma}}}}{2}$ $=R_{\mathrm{se\|se}}+\frac{\varepsilon+\sqrt{4R_{\mathrm{se\|ma}}R_{\mathrm{ma}}+\varepsilon^2}}{2}$ $\approx R_{\mathrm{se\|se}}+\sqrt{R_{\mathrm{se\|ma}}R_{\mathrm{ma}}}\ .$	$None$

35	$S(\infty) = (1-\rho)e^{-\xi(\infty)}\psi(\theta(\infty))$ $\phi_S(\infty) = (1-\rho)e^{-\xi(\infty)}\frac{\psi^{'}(\theta(\infty))}{\langle K\rangle}$.	$\boldsymbol{S}(\infty) = (\mathbf{1}-\rho)\boldsymbol{e}^{-\boldsymbol{\xi}(\boldsymbol{\sigma})}\mathbb{V}(\theta(\infty))$ $\phi_{\boldsymbol{S}}(\infty) = (\mathbf{1}-\rho)\boldsymbol{e}^{-\xi(\infty)}\frac{\hat{V}(\theta(\circ))}{\langle K\rangle}$.
36	$\xi(\infty) = \frac{\beta}{\gamma_1}R(\infty) = \frac{\beta}{\gamma_1}[1-S(\infty)] = \frac{\beta}{\gamma_1}\left[1-(1-\rho)e^{-\xi(\infty)}\psi(\theta(\infty))\right]$.	$\xi(\infty) = \frac{\beta}{\gamma_1}\mathsf{R}(\infty) = \frac{\beta}{\gamma_1}[\mathbf{1}-\mathbf{S}(\infty)] = \frac{\beta}{\gamma_1}\left[\mathbf{1}-(\mathbf{1}-\rho)\boldsymbol{e}^{-\xi(\mathbf{w})}\psi(\theta(\infty))\right]$.
37	$\theta(\infty) = 1-\tau_1\int_0^\infty \phi_{I,1}\mathrm{d}t - \tau_2\int_0^\infty \phi_{I,2}\mathrm{d}t$.	$\theta(\infty) = 1-\tau_1\int_0^\infty \phi_{l,1}\mathsf{dt} - \tau_2\int_0^\infty \phi_{l,2}\mathsf{dt}$.
38	$\gamma_1\int_0^\infty \phi_{I,1}\mathrm{d}t = \phi_R(\infty) + \tau_2\int_0^\infty \phi_{I,2}\mathrm{d}t$.	$\gamma_1\int_0^\infty \phi_{l,1}\mathsf{d}t = \phi_{\mathbb{R}}(\infty) + \tau_2\int_0^\infty \phi_{l,2}\mathsf{dt}$.
39	$\theta(\infty) = 1-\frac{\tau_1}{\gamma_1}\phi_R(\infty) - \left(\frac{\tau_1\tau_2}{\gamma_1}+\tau_2\right)\int_0^\infty \phi_{I,2}\mathrm{d}t$.	$\theta(\infty) = 1-\frac{\tau_1}{\gamma_1}\phi_{\mathbb{R}}(\infty) - \left(\frac{\tau_1\tau_2}{\gamma_1}+\tau_2\right)\int_0^\infty \phi_{I,2}\mathsf{dt}$.
40	$\theta(\infty) = 1-\left[\frac{\tau_1}{\gamma_1}+\frac{\tau_1\tau_2}{\gamma_1\gamma_2}+\frac{\tau_2}{\gamma_2}\right]\phi_R(\infty)$ $= 1-\frac{T_{\mathrm{se}}}{1-T_{\mathrm{se}}}\phi_R(\infty)$ $= 1-\frac{T_{\mathrm{se}}}{1-T_{\mathrm{se}}}\left[\theta(\infty)-\frac{(1-\rho)e^{-\xi(\infty)}\psi^{'}(\theta(\infty))}{\langle K\rangle}\right]$.	$\theta(\infty) = \mathbf{1}-\left[\frac{\tau_1}{\gamma_1}+\frac{\tau_1\tau_2}{\gamma_1\gamma_2}+\frac{\tau_2}{\gamma_2}\right]\phi_{\mathbb{R}}(\circ)$ $= \mathbf{1}-\frac{\Gamma_{\mathrm{se}}}{1-T_{\mathrm{se}}}\phi_{\mathbb{R}}(\infty)$
41	$\xi(\infty) = \frac{\beta}{\gamma_1}\left[1-(1-\rho)e^{-\xi(\infty)}\psi(\theta(\infty))\right]$ $\theta(\infty) = 1-T_{\mathrm{se}}+T_{\mathrm{se}}\frac{(1-\rho)e^{-\xi(\infty)}\psi^{'}(\theta(\infty))}{\langle K\rangle}$.	$None$
42	$\xi(\infty) = \frac{\beta}{\gamma_1}\left[1-(1-\rho)e^{-\xi(\infty)}\psi(\theta(\infty))\right]$	$\xi(\infty) = \frac{\beta}{\gamma_1}\left[\mathbf{1}-(\mathbf{1}-\rho)\boldsymbol{e}^{-\boldsymbol{\xi}(\infty)}\psi(\theta(\infty))\right)$
43	$\dot{V}_S = B - \beta_1 I_1 V_S - \delta V_S$	$\dot{V}_S = \mathbb{B} - \beta_1 I_1 V_{\mathrm{S}} - \delta V$
44	$\dot{V}_I = \beta_1 I_1 V_S - \delta V_I$	$\dot{V}_l = \beta_1 I_1 V_{\mathrm{S}} - \delta V$
45	$S = (1-\rho)e^{-\xi}\psi(\theta)$	$\boldsymbol{S} = (\mathbf{1}-\rho)\boldsymbol{e}^{-?}\psi(\theta)$
46	$I_1 = 1 - S - I_2 - R$	$\mathrm{I}_1 = 1 - S - \mathrm{l}_2 - \mathrm{R} -$

47	$\dot{I}_2 = \gamma_1 I_1 - \gamma_2 I_2$	$\dot{\mathrm{I}}_2 = \gamma_1 I_1 - \gamma_2 l_2$
48	$\dot{R} = \gamma_2 I_2$	$\dot{\mathrm{R}} = \gamma_2 l_2$
49	$\dot{V}_S = B - \beta_1 I_1 V_S - \delta V_S$ $\dot{V}_I = \beta_1 I_1 V_S - \delta V_I$.	$\dot{V}_{\mathrm{s}} = \mathbb{B} - \beta_1 I_1 V_8 - \delta V$ $\dot{V}_l = \beta_1 I_1 V_{\mathrm{S}} - \delta V_l$.
50	$\dot{\xi} = \beta_2 V_I$	$\dot{\xi} = \beta_2 V$
51	$\dot{\theta} = -\tau_1 \phi_{I,1} - \tau_2 \phi_{I,2}$	$\dot{\theta} = -\tau_1 \phi_{l,1} - \tau_2 \phi_{l2}$
52	$\phi_S = (1-\rho) e^{-\xi \frac{\psi'(\theta)}{\langle K \rangle}}$	$\phi_{\mathrm{S}} = (1-\rho) e^{-\xi} \frac{\psi'(\theta)}{\langle K \rangle}$
53	$\phi_{I,1} = \theta - \phi_S - \phi_{I,2} - \phi_R$	$\phi_{R1} = \theta - \phi_S - \phi_{I2} - \phi_R$
54	$\dot{\phi}_{I,2} = \gamma_1 \phi_{I,1} - (\gamma_2 + \tau_2) \phi_{I,2}$	$\dot{\phi}_{l2} = \gamma_{12} \phi_{l1} - (\gamma_2 + \tau_2) \phi_{l22}$
55	$\dot{\phi}_R = \gamma_2 \phi_{I,2}$.	$\dot{\phi}_{\mathrm{R}} = \gamma_2 \theta_{\mathrm{I},2}$.
56	$\begin{pmatrix} R_{\mathrm{mo}} & R_{\mathrm{mo}} \\ R_{\mathrm{se}\|\mathrm{mo}} & R_{\mathrm{se}\|\mathrm{se}} \end{pmatrix}$	$\begin{pmatrix} \mathsf{R}_{\mathsf{mo}} & \mathsf{R}_{\mathsf{mo}} \\ \mathsf{R}_{\mathsf{se}\|\mathsf{mo}} & \mathsf{R}_{\mathsf{sese}} \end{pmatrix}$
57		

	$S = (1-\rho)\psi(\theta_{\mathrm{se}}, \theta_{\mathrm{so}})$	$\boldsymbol{S} = (\mathbf{1}-\rho)\mathbb{V}(\theta_{\mathrm{se}}, \theta_{\mathrm{s0}})$
58	$I_1 = 1 - S - I_2 - R$	$\mathrm{I}_1 = 1 - S - \mathrm{I}_2 - \mathrm{R} -$
59	$\dot{I}_2 = \gamma_1 I_1 - \gamma_2 I_2$	$\dot{\mathrm{I}}_2 = \gamma_1 I_1 - \gamma_2 l_2$
60	$\dot{R} = \gamma_2 I_2$ (7d)	$\dot{\mathrm{R}} = \gamma_2 I_2$
61	$\dot{\theta}_{\mathrm{se}} = -\tau_1 \phi_{I,1}^{\mathrm{se}} - \tau_2 \phi_{I,2}^{\mathrm{se}}$	$\dot{\theta}_{\mathrm{se}} = -\tau_1 \phi_{l,1}^{\mathrm{se}} - \tau_2 \phi_{l2}^{\mathrm{se}}$
62	$\phi_{I,1}^{\mathrm{se}} = \theta - (1-\rho)\frac{\psi_x(\theta_{\mathrm{se}}, \theta_{\mathrm{so}})}{\langle K_{\mathrm{se}}\rangle} - \phi_{I,2}^{\mathrm{se}} - \phi_R^{\mathrm{se}}$	$\phi_{l,1}^{\mathrm{se}} = \theta - (1-\rho)\frac{\psi_\chi(\theta_{\mathrm{se}}, \theta_{\mathrm{s0}})}{\langle K_{\mathrm{se}}\rangle} - \phi_{l,2}^{\mathrm{se}} - \phi_R^{\mathrm{se}}$
63	$\dot{\phi}_{I,2}^{\mathrm{se}} = \gamma_1 \phi_{I,1}^{\mathrm{se}} - (\gamma_2 + \tau_2)\phi_{I,2}^{\mathrm{se}}$	$\dot{\phi}_{l2}^{\mathrm{se}} = \gamma_1 \phi_{l1}^{\mathrm{se}} - (\gamma_2 + \tau_2)\phi_{l2}^{\mathrm{se}}$
64	$\dot{\phi}_R^{\mathrm{se}} = \gamma_2 \phi_{I,2}^{\mathrm{se}}$	$\dot{\phi}_{\mathrm{R}}^{\mathrm{se}} = \gamma_2 \phi_{l2}^{\mathrm{se}}$
65	$\dot{\theta}_{\mathrm{so}} = -\beta \phi_{I,1}^{\mathrm{so}}$	$\dot{\theta}_{\mathrm{so}} = -\beta \phi_{l_l}^{\mathrm{s}} l_l$
66	$\phi_{I,1}^{\mathrm{so}} = \theta - (1-\rho)\frac{\psi_y(\theta_{\mathrm{se}}, \theta_{\mathrm{so}})}{\langle K_{\mathrm{so}}\rangle} - \phi_R^{\mathrm{so}}$	$\phi_{l,1}^{\mathrm{so}} = \theta - (1-\rho)\frac{\psi_{\mathrm{y}}(\theta_{\mathrm{se}}, \theta_{\mathrm{so}})}{\langle K_{\mathrm{sq}}\rangle} - \phi_R^{\mathrm{ss}}$
67	$\dot{\phi}_R^{\mathrm{so}} = \gamma_1 \phi_{I,1}^{\mathrm{so}} \cdot$	$\dot{\phi}_{\mathrm{R}}^{\mathrm{so}} = \gamma_1 \phi_{l,1}^{\mathrm{so}} \cdot$
68		

	$$R_{\mathrm{se}\vert\mathrm{se}} = T_{\mathrm{se}}\frac{\psi_{xx}(1,1)}{\langle K_{\mathrm{se}}\rangle}$$ $$R_{\mathrm{se}\vert\mathrm{so}} = T_{\mathrm{se}}\frac{\psi_{xy}(1,1)}{\langle K_{\mathrm{so}}\rangle}$$ $$R_{\mathrm{so}\vert\mathrm{se}} = T_{\mathrm{so}}\frac{\psi_{xy}(1,1)}{\langle K_{\mathrm{se}}\rangle}$$	$$\mathsf{R}_{\mathsf{se}\vert\mathsf{se}} = \mathsf{T}_{\mathsf{se}}\frac{\mathbb{V}_{\mathcal{A}}(\mathsf{I},1)}{\langle \mathsf{K}_{\mathsf{se}}\rangle}$$ $$\mathsf{R}_{\mathsf{se}\vert\mathsf{se}} = T_{\mathsf{se}}\frac{\mathbb{V}_{\mathsf{se}}(\mathbf{1},\mathbf{1})}{\langle K_{\mathsf{se}}\rangle}$$
69	$$R_{\mathrm{so}\vert\mathrm{so}} = T_{\mathrm{so}}\frac{\psi_{yy}(1,1)}{\langle K_{\mathrm{so}}\rangle}\,.$$	$$\mathsf{R}_{\mathrm{so}\vert\mathrm{so}} = \mathsf{T}_{\mathsf{so}}\frac{\mathbb{V}_{\mathbf{yy}}(\mathsf{I},\mathsf{I})}{\langle \mathrm{K}_{\mathrm{so}}\rangle}\,.$$
70	$$\begin{pmatrix} N_{\mathrm{se}}(g+1) \\ N_{\mathrm{so}}(g+1) \end{pmatrix} = \begin{pmatrix} R_{\mathrm{se}\vert\mathrm{se}} & R_{\mathrm{se}\vert\mathrm{so}} \\ R_{\mathrm{so}\vert\mathrm{se}} & R_{\mathrm{so}\vert\mathrm{so}} \end{pmatrix}\begin{pmatrix} N_{\mathrm{se}}(g) \\ N_{\mathrm{so}}(g) \end{pmatrix}\,.$$	$$\begin{pmatrix} N_{\mathrm{se}}(\mathbf{g}+\mathbf{1}) \\ N_{\mathbf{so}}(\mathbf{g}+\mathbf{1}) \end{pmatrix} = \begin{pmatrix} \mathsf{R}_{\mathsf{selse}} & \mathsf{R}_{\mathsf{selso}} \\ \mathsf{R}_{\mathsf{solse}} & \mathsf{R}_{\mathsf{solso}} \end{pmatrix}(N_{\mathsf{se}}(\mathbf{g})$$
71	$$\theta_{\mathrm{se}}(\infty) = 1 - T_{\mathrm{se}} + T_{\mathrm{se}}(1-\rho)\frac{\psi_x(\theta_{\mathrm{se}}(\infty),\theta_{\mathrm{so}}(\infty))}{\langle K_{\mathrm{se}}\rangle}$$ $$\theta_{\mathrm{so}}(\infty) = 1 - T_{\mathrm{so}} + T_{\mathrm{so}}(1-\rho)\frac{\psi_y(\theta_{\mathrm{se}}(\infty),\theta_{\mathrm{so}}(\infty))}{\langle K_{\mathrm{so}}\rangle}\,.$$ We can solve this iteratively. From the solution, we have $S(\infty) = (1-\rho)\psi(\theta_{\mathrm{se}}(\infty),\theta_{\mathrm{so}}(\infty))$.	$None$
72	$$\phi_S = (1-\rho)e^{-\xi}\zeta\sum_{k_v}\frac{k_v P(k_v)}{\langle K\rangle}\theta^{k_v-1} = (1-\rho)e^{-\xi}\zeta\frac{\psi^{'}(\theta)}{\langle K\rangle}\,.$$	$$\phi_{\mathrm{S}} = (1-\rho)\boldsymbol{e}^{-\xi}\zeta\sum_{k_v}\frac{k_y\mathbf{P}(k_v)}{\langle K\rangle}\theta^{k_p-1} = (1-\rho)\boldsymbol{e}^{-}\zeta\frac{V(\theta)}{\langle\mathbf{K}\rangle}\,.$$
73	$$\dot\phi_S = -\dot\xi\phi_S + \dot\zeta(1-\rho)e^{-\xi}\frac{\psi^{'}(\theta)}{\langle K\rangle} + \dot\theta(1-\rho)e^{-\xi}\zeta\frac{\psi^{'\prime}(\theta)}{\langle K\rangle}$$ $$= -\dot\xi\phi_S + \eta\theta\pi_S - \eta\phi_S - (\tau_1\phi_{I,1} + \tau_2\phi_{I,2})\phi_S\frac{\psi^{'\prime}(\theta)}{\psi^{'}(\theta)}\,.$$	$None$
74		$None$

	$S = (1-\rho)e^{-\xi}\psi(\theta)$ $I_1 = 1 - S - I_2 - R$ $\dot{I}_2 = \gamma_1 I_1 - \gamma_2 I_2$ $\dot{R} = \gamma_2 I_2$ $\dot{\xi} = \beta I_1$ $\dot{\theta} = -\tau_1 \phi_{I,1} - \tau_2 \phi_{I,2}$ $\dot{\phi}_S = -\beta I_1 \phi_S + \eta\theta\pi_S - \eta\phi_S - (\tau_1\phi_{I,1} + \tau_2\phi_{I,2})\phi_S \frac{\psi''(\theta)}{\psi'(\theta)}$ $\phi_{I,1} = \theta - \phi_S - \phi_{I,2} - \phi_R$ $\dot{\phi}_{I,2} = \gamma_1\phi_{I,1} + \eta\theta\pi_{I,2} - (\eta + \gamma_2 + \tau_2)\phi_{I,2}$ $\dot{\phi}_R = \eta\theta\pi_R + \gamma_2\phi_{I,2} - \eta\phi_R$ $\pi_S = (1-\rho)\frac{\theta e^{-\xi}\psi'(\theta)}{\langle K \rangle}$ $\pi_{I,1} = 1 - \pi_S - \pi_{I,2} - \pi_R$ $\dot{\pi}_{I,2} = \gamma_1\pi_{I,1} - \gamma_2\pi_{I,2}$ $\dot{\pi}_R = \gamma_2\pi_{I,2}\ .$	
75	$I_2(0) = R(0) = 0$ $\theta(0) = 1$ $\phi_S(0) = (1-\rho)$ $\phi_{I,2}(0) = \phi_R(0) = 0$ $\pi_{I,2} = \pi_R = 0$ $\xi(0) = 0\ .$	$\boldsymbol{l_2(0) = \mathbb{R}(0) = 0}$ $\boldsymbol{\theta(0) = 1}$ $\boldsymbol{\phi_{l2}(0) = \phi_{\mathbb{R}}(0) = 0}$
76	$N(g+1) = \frac{\beta}{\gamma_1}N(g) + \left(\frac{\tau_1}{\gamma_1} + \frac{\tau_2}{\gamma_2}\right)y(g)$	$\mathrm{N}(\mathrm{g}+1) = \frac{\beta}{\gamma_1}\mathrm{N}(\mathrm{g}) + \left(\frac{\tau_1}{\gamma_1} + \frac{\tau_2}{\gamma_2}\right)y(\mathrm{g}$
77	$y(g+1) = \frac{\beta}{\gamma_1}\langle K \rangle N(g) + \left(\frac{\tau_1}{\gamma_1} + \frac{\tau_2}{\gamma_2}\right)\frac{\langle K^2 \rangle}{\langle K \rangle}y(g)\ .$	$\boldsymbol{y}(\boldsymbol{g}+\mathbf{1}) = \frac{\boldsymbol{\beta}}{\gamma_1}\langle K \rangle \mathrm{N}(\boldsymbol{g}) + \left(\frac{\tau_1}{\gamma_1} + \frac{\tau_2}{\gamma_2}\right)\frac{\langle K^2 \rangle}{\langle K \rangle}\boldsymbol{y}(\boldsymbol{g})\ .$
78		

	$\begin{pmatrix} \frac{\beta}{\gamma_1} & \frac{\tau_1}{\gamma_1}+\frac{\tau_2}{\gamma_2} \\ \frac{\beta}{\gamma_1}\langle K\rangle & \left(\frac{\tau_1}{\gamma_1}+\frac{\tau_2}{\gamma_2}\right)\frac{\langle K^2\rangle}{\langle K\rangle} \end{pmatrix}$ is an upper bound for $\mathscr{R}_0$.	$None$
79	$S=(1-\rho)e^{-\xi}\sum_k P(k)\theta_k^k$	$\boldsymbol{S}=(\mathbf{1}-\rho)\boldsymbol{e}^{-5}\sum_k \mathbf{p}(\boldsymbol{k})\theta_k^k$
80	$I_1=1-S-I_2-R$	$\mathrm{I}_1=\mathrm{I}-\mathrm{S}-\mathrm{I}_2-\mathrm{R}_2-\mathrm{R}_2-\mathrm{R}_2-\mathrm{R}_2-\mathrm{R}_2-\mathrm{R}_2-\mathrm{R}_2-\mathrm{R}_2-\mathrm{R}_2-\mathrm{R}_2-\mathrm{R}_2-\mathrm{R}_2-\mathrm{R}_2-\mathrm{R}$
81	$\dot{I}_2=\gamma_1 I_1-\gamma_2 I_2$	$\dot{\mathrm{I}}_2=\gamma_1 I_1-\gamma_2 I_2$
82	$\dot{R}=\gamma_2 I_2$	$\dot{\mathrm{R}}=\gamma_2 I_2$
83	$\dot{\theta}_k=-\tau_1\phi_{I,1\|k}-\tau_2\phi_{I,2\|k}$	$\dot{\theta}_k=-\tau_1\phi_{l1\|k}-\tau_2\phi_{l,2\|\|}$
84	$\phi_{S\|k}=(1-\rho)e^{-\xi}\sum_{\widehat{k}} P_n\left(\widehat{k}\middle\|k\right)\theta_{\widehat{k}}^{\widehat{k}-1}$	$\phi_{\mathrm{S\|k}}=(1-\rho)\boldsymbol{e}^{-5}\sum_{\widehat{k}} p_n\left(\widehat{k}\middle\|k\right)\theta_{\widehat{k}}^{\widehat{k}-1}$
85	$\phi_{I,1\|k}=\theta_k-\phi_{S\|k}-\phi_{I,2\|k}-\phi_{R\|k}$	$\phi_{I1\|k}=\theta_k-\phi_{S\|k}-\phi_{l2\|k}-\phi_{\mathbb{R}\|}$
86	$\dot{\phi}_{I,2\|k}=\gamma_1\phi_{I,1\|k}-(\tau_2+\gamma_2)\phi_{I,2\|k}$	$\dot{\phi}_{l2\|k}=\gamma_!\phi_{l,1\|k}-(\tau_2+\gamma_2)\phi_{I,2\|k}$
87	$\dot{\phi}_{R\|k}=\gamma_2\phi_{I,2\|k}$	$\dot{\phi}_{R\|k}=\gamma_2\phi_{l2\|k\|k}$
88		$\dot{\xi}=\beta l_l\,.$

	$\dot{\xi} = \beta I_1$.	
89	$I_2(0) = R(0) = 0$ $\theta_k(0) = 1$ $\phi_{I,2\|k} = \phi_{R\|k} = 0$ $\xi(0) = 0$.	$l_2(\mathbf{0}) = \mathbb{R}(\mathbf{0}) = \mathbf{0}$ $\theta_l(\mathbf{0}) = \boldsymbol{\phi}_{\mathbb{R}k} = \mathbf{0}$ $\xi(\mathbf{0}) = \mathbf{0}$.
90	$P_n(1\|1) = 1/2 \quad P_n(1\|5) = 3/40 \quad P_n(1\|10) = 1/80$ $P_n(5\|1) = 3/8 \quad P_n(5\|5) = 1/2 \quad P_n(5\|10) = 17/80$ $P_n(10\|1) = 1/8 \quad P_n(10\|5) = 17/40 \quad P_n(10\|10) = 62/80$	$None$
91	$N_{\widehat{k}}(g+1) = \sum_k \left(T_{\mathrm{sekP}} \left(\widehat{k} \middle\| k \right) + P\left(\widehat{k}\right) \frac{\beta}{\gamma_1} \right) N_k(g)$.	$\mathsf{N}_{\widehat{k}}(\mathbf{g}+\mathbf{1}) = \sum_k \left(\boldsymbol{T}_{\mathrm{sekp}} \left(\widehat{k} \middle\| k \right) + \mathbf{p}\left(\widehat{k}\right) \frac{\beta}{\gamma_1} \right) \mathsf{N}_k(\boldsymbol{g})$.
92	Writing this as a matrix equation, we have $\begin{pmatrix} N_0(g+1) \\ N_1(g+1) \\ N_2(g+1) \\ N_3(g+1) \\ \vdots \end{pmatrix} = \left[T_{se} \begin{pmatrix} 0 & 0 & 0 & 0 & \cdots \\ 0 & P(1\|1) & 2P(1\|2) & 3P(1\|3) & \cdots \\ 0 & P(2\|1) & 2P(2\|2) & 3P(2\|3) & \cdots \\ 0 & P(3\|1) & 2P(3\|2) & 3P(3\|3) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} + \frac{\beta}{\gamma_1} \begin{pmatrix} P(0) & P(0) & P(0) & P(0) & \cdots \\ P(1) & P(1) & P(1) & P(1) & \cdots \\ P(2) & P(2) & P(2) & P(2) & \cdots \\ P(3) & P(3) & P(3) & P(3) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \right] \begin{pmatrix} N_0(g) \\ N_1(g) \\ N_2(g) \\ N_3(g) \\ \vdots \end{pmatrix}$	$None$
93	$\theta_k(\infty) = (1 - T_{se}) + T_{se}\phi_{S\|k}(\infty) = (1 - T_{se}) + T_{se}(1-\rho)e^{-\xi(\infty)} \sum_{\widehat{k}} P_n\left(\widehat{k} \middle\| k\right) \theta_{\widehat{k}}^{\widehat{k}-1}$	$None$
94	$\xi(\infty) = \frac{\beta}{\gamma_1}[1 - S(\infty)] = \frac{\beta}{\gamma_1}\left[1 - (1-\rho)e^{-\xi(\infty)} \sum_k P(k)\theta_k^k(\infty)\right]$.	$\xi(\infty) = \frac{\beta}{\gamma_1}[1 - S(\infty)] = \frac{\beta}{\gamma_1}\left[1 - (1-\rho)e^{-\xi(\infty)} \sum_k \mathsf{P}(k)\theta_k^k(\infty)\right]$.
95	$R(\infty) = 1 - (1-\rho)e^{-\xi(\infty)} \sum_k P(k)\theta_k^k(\infty)$.	$\mathsf{R}(\infty) = \mathbf{1} - (\mathbf{1} - \rho)\boldsymbol{e}^{-\xi(\mathbf{w})} \sum_k \mathbf{p}(k)\theta_k^k(\infty)$.