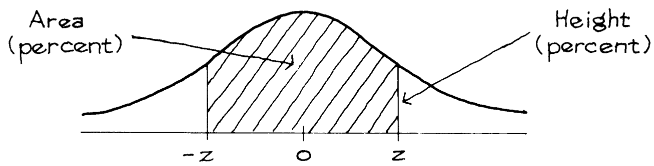
Reading Normal Tables.

Tables

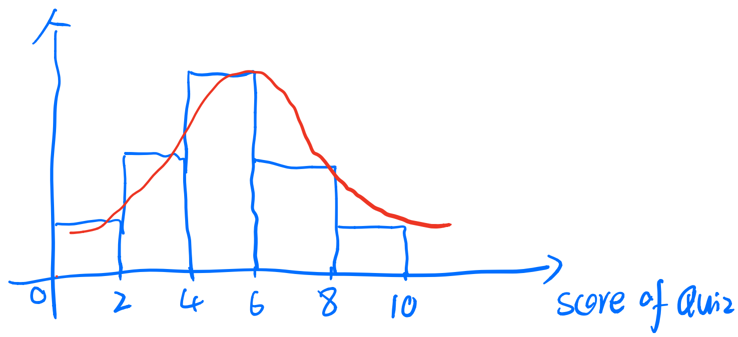


A NORMAL TABLE

z	*Height*	*Area*	*z*	*Height*	*Area*	*z*	*Height*	*Area*
0.00	39.89	0	1.50	12.95	86.64	3.00	0.443	99.730
0.05	39.84	3.99	1.55	12.00	87.89	3.05	0.381	99.771
0.10	39.69	7.97	1.60	11.09	89.04	3.10	0.327	99.806
0.15	39.45	11.92	1.65	10.23	90.11	3.15	0.279	99.837
0.20	39.10	15.85	1.70	9.40	91.09	3.20	0.238	99.863
0.25	38.67	19.74	1.75	8.63	91.99	3.25	0.203	99.885
0.30	38.14	23.58	1.80	7.90	92.81	3.30	0.172	99.903
0.35	37.52	27.37	1.85	7.21	93.57	3.35	0.146	99.919
0.40	36.83	31.08	1.90	6.56	94.26	3.40	0.123	99.933
0.45	36.05	34.73	1.95	5.96	94.88	3.45	0.104	99.944
0.50	35.21	38.29	2.00	5.40	95.45	3.50	0.087	99.953
0.55	34.29	41.77	2.05	4.88	95.96	3.55	0.073	99.961
0.60	33.32	45.15	2.10	4.40	96.43	3.60	0.061	99.968
0.65	32.30	48.43	2.15	3.96	96.84	3.65	0.051	99.974
0.70	31.23	51.61	2.20	3.55	97.22	3.70	0.042	99.978
0.75	30.11	54.67	2.25	3.17	97.56	3.75	0.035	99.982
0.80	28.97	57.63	2.30	2.83	97.86	3.80	0.029	99.986
0.85	27.80	60.47	2.35	2.52	98.12	3.85	0.024	99.988
0.90	26.61	63.19	2.40	2.24	98.36	3.90	0.020	99.990
0.95	25.41	65.79	2.45	1.98	98.57	3.95	0.016	99.992
1.00	24.20	68.27	2.50	1.75	98.76	4.00	0.013	99.9937
1.05	22.99	70.63	2.55	1.54	98.92	4.05	0.011	99.9949
1.10	21.79	72.87	2.60	1.36	99.07	4.10	0.009	99.9959
1.15	20.59	74.99	2.65	1.19	99.20	4.15	0.007	99.9967
1.20	19.42	76.99	2.70	1.04	99.31	4.20	0.006	99.9973
1.25	18.26	78.87	2.75	0.91	99.40	4.25	0.005	99.9979
1.30	17.14	80.64	2.80	0.79	99.49	4.30	0.004	99.9983
1.35	16.04	82.30	2.85	9.69	99.56	4.35	0.003	99.9986
1.40	14.97	83.85	2.90	0.60	99.63	4.40	0.002	99.9989
1.45	13.94	85.29	2.95	0.51	99.68	4.45	0.002	99.9991

1. What is a normal curve?

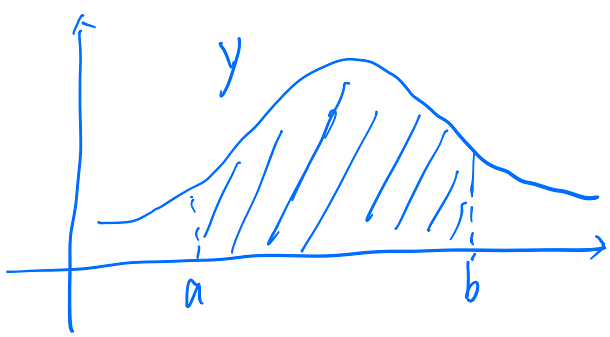
When we draw (density) histogram of many random variables in real life, the plot will have "bell shape".



Properties: 1. Symmetric around the center.

2. Determined by two parameters: mean and SD.

Given mean and SD, the corresponding normal curve is unique.



Suppose the density of Y has a bell shape. We often interested in the shadow area which is $Pr(a \le Y \le b)$.

There are many normal curves (infinite many). How to calculate those area of interests in a simple way?

Define some standard normal curve, use this as a baseline, then for any other normal curve, we find a way to transfer it into the standard normal curve, then use the standard normal result to calculate.

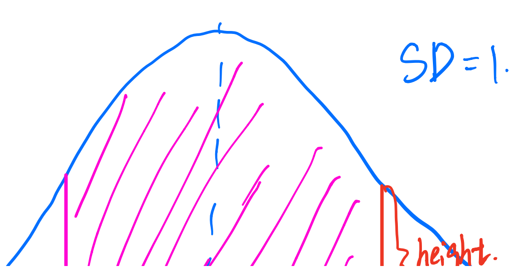
What is standard normal curve?

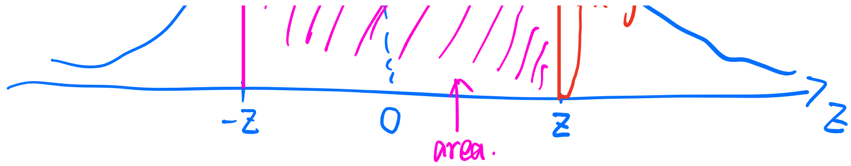
mean and SD define a normal curve.

mean: center of data. we use: 0

SD: spread of data we use: 1.

Therefore, we define the normal curve determined by mean 0 and SD 1 as the standard normal curve. The random variable, whose density has this standard normal curve shape is called Z.



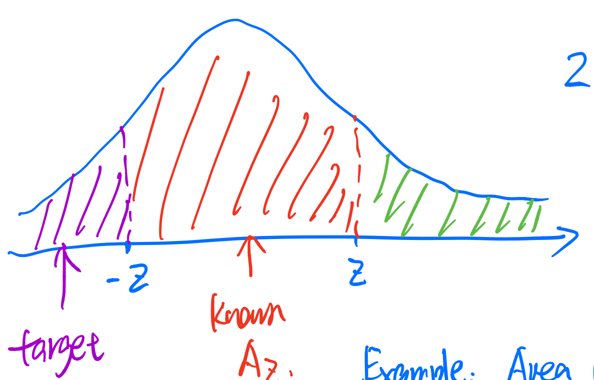


Z score table: $z^{(z \geq 0)}$: height. area.

Typical Questions Related to Z-table:

z	*Height* (H_z)	*Area* (A_z)
0.00	39.89	0
0.05	39.84	3.99
0.10	39.69	7.97
0.15	39.45	11.92
0.20	39.10	15.85

1. Area under the normal curve between $-z$ and z? A_z.
2. Area under the normal curve to the left of $-z$?



$2 \times \text{purple} + A_z = 100$

$\Rightarrow \text{purple} = \frac{100 - A_z}{2}$.

Example: Area under normal curve to the left of -0.20?

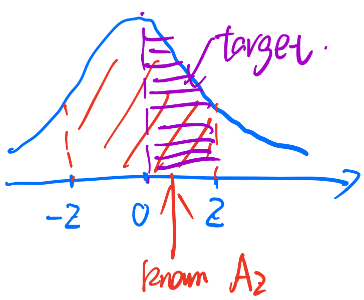
$z = 0.20$ $\quad A_{0.20} = 15.85$ $\quad \frac{100 - 15.85}{2} = 42.075\%$

$\approx 42.08\%$.

3. Area under normal curve to the right of z?

green = purple. $\frac{100-A_z}{2}$

4. Area under normal curve between 0 and z?

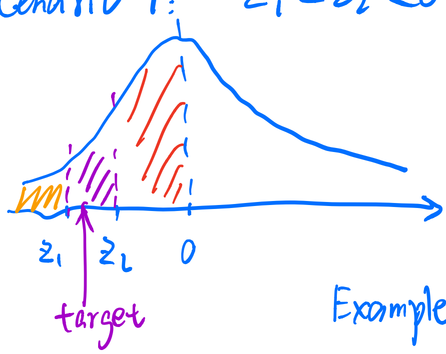


Purple $= \frac{A_z}{2}$ by symmetric.

Example: Area under normal curve between 0 and 0.15? $\frac{11.92}{2} = 5.96\%$

5. Area under normal curve between z_1 and z_2?

Scenario 1: $z_1 < z_2 < 0$.



Purple = 50 − red − orange

$= 50 - \frac{A_{-z_2}}{2} - \frac{100-A_{-z_1}}{2}$

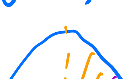
Example: area under normal curve between -0.15 and -0.05?

$50 - \frac{A_{0.05}}{2} - \frac{100-A_{0.15}}{2}$

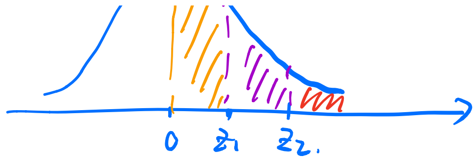
$= 50 - \frac{3.99}{2} - \frac{100-11.92}{2}$

$= 3.965\% = 3.97\%$.

Scenario 2: $0 < z_1 < z_2$.



Purple = 50 − red − orange

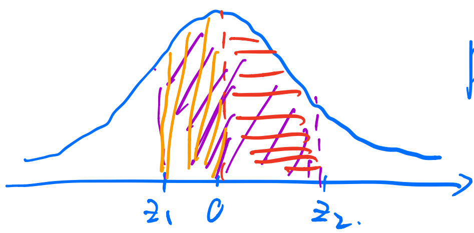


$$= 50 - \frac{100 - A_{z_2}}{2} - \frac{A_{z_1}}{2}$$

Example: area under curve between 0.05 and 0.20?

$$50 - \frac{100 - A_{0.20}}{2} - \frac{A_{0.05}}{2} = 50 - \frac{100 - 15.85}{2} - \frac{3.99}{2} = 5.93\%$$

Scenario 3: $z_1 < 0 < z_2$.



Purple = red + orange

$$= \frac{A_{z_2}}{2} + \frac{A_{-z_1}}{2}$$

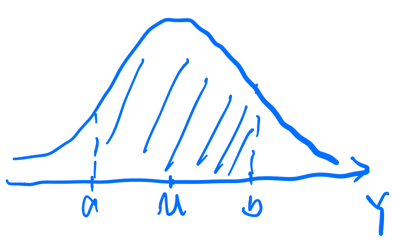
Example: area under normal curve between −0.05 to 0.2?

$$\frac{A_{0.2}}{2} + \frac{A_{0.05}}{2} = \frac{15.85}{2} + \frac{3.99}{2} = 9.92\%$$

Transfer a non-standard normal curve to a standard normal curve.

Y has a normal curve density with mean $\neq 0$ and SD $\neq 1$.

$\Pr(a \leq Y \leq b)$



a, b are in Y unit. we need to transform them to z unit.

Formula for transform number in Y unit to number in Z unit?

$$\frac{y - mean_Y}{SD_Y}$$

a in Z unit: $\frac{a - mean_Y}{SD_Y} = Z_a$

b in Z unit: $\frac{b - mean_Y}{SD_Y} = Z_b$

then the question becomes: area under normal curve between Z_a and Z_b, use the previous method!

Example: Y has mean 10 and SD 2, what is the probability that Y is from 9.8 to 10.4?

9.8 in Y unit is $\frac{9.8-10}{2} = \frac{-0.2}{2} = -0.1$ in Z unit.

10.4 in Y unit is $\frac{10.4-10}{2} = \frac{0.4}{2} = 0.2$ in Z unit.

So the question becomes the area under normal curve between -0.1 and 0.2, it is $\frac{A_{-z_1}}{2} + \frac{A_{z_2}}{2} = \frac{A_{0.1}}{2} + \frac{A_{z_2}}{2} = \frac{7.97}{2} + \frac{15.85}{2}$

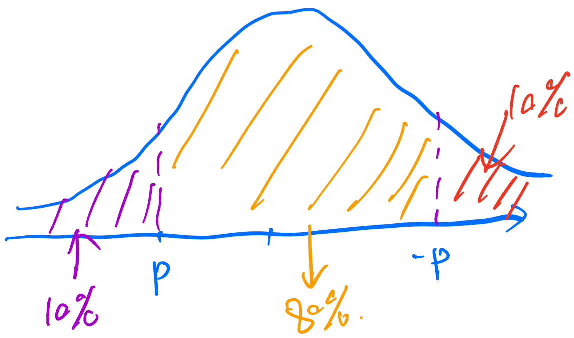
$= 11.91\%$.

Another type of questions related to normal table is percentile.

What is the ith percentile of Z? (i given).

Suppose the ith percentile of Z is p, it means area under normal curve to the left of p is i.

Example: What is the 10th percentile of Z?



Find the number in normal table such that A_z is approximately 80%.

$Z = 1.3$. so the percentile is -1.3.

The question can also given in Y unit.

Example: Suppose Y has mean 10 and SD 2, what is the 10th percentile of Y?

$$\frac{Y\text{ unit} - \text{mean}}{SD} = Z\text{ unit} \Rightarrow (Z\text{ unit} \times SD) + \text{mean} = Y\text{ unit}.$$

Since 10th percentile of Z is -1.3, $(-1.3 \times 2) + 10 = 7.4$ is the 10th percentile of Y.