Review: Probability & Box Model.

Example 1: Two fair six-sided dice are rolled and the sum of the two faces is observed. What is the preferred box model for this scenario?

Notes: another correct solution: 2 draws from the box | 1 2 3 4 5 6 |

Possible outcomes? 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.

How many of each? 1: 2, 12 4: 5, 9

2: 3, 11 5: 6, 8

3: 4, 10 6: 7.

How many draws? 1.

2, 3, 3, 4, 4, 4, 5, 5, 5, 5, 6, 6, 6, 6, 6, 7, 7, 7
7, 7, 7, 8, 8, 8, 8, 8, 9, 9, 9, 9, 10, 10, 10, 11, 11, 12.

Example 2: A coin, which is known to be baised toward

heads, is repeatedly and independently flipped n times. If there are more heads than tails on the n trails, you win $500, You get to select the number of n. which n should you prefer?

Use the box model:

Numbers in the box: 500, 0

How many of each? a of 500 and b of 0.

a should be greater than b, because the coin is biased towards head.

How many draws? n (You need to determine this).

Expected return = (number of draws) x (Avg. of each box)

$$= n \times \frac{500a}{a+b}$$

We want Expected return large. so n also need to be large. Therefore, we will pick n as large as possible.

Example 3: A fair coin is independently tossed 100 times. What is the mean and standard error of the number the number of heads obtained?

Box: | 1 | 0 | 100 draws.

mean = (number of draws) × (Avg. of each box)

$= 100 \times \frac{1+0}{2} = 50.$

SE = $(\sqrt{\text{number of draws}}) \times (\text{SD. of each box})$

$= \sqrt{100} \times \sqrt{\frac{(1-\frac{1}{2})^2+(0-\frac{1}{2})^2}{2}}$

$= 10 \times \frac{1}{2} = 5.$

Example 4: A fair coin is flipped 100 times. Approximate the chance that between 48 and 52 heads are observed.

From the last problem, we know mean = 50 and SE = 5.

so $\frac{52.5-50}{5} = 0.5$ → go to Z table → 38.29% ≈ 38%

Here you need to plus 0.5.

Example 5: What are the exact chances of getting exactly 4 rainy days in a week of seven independent days if the daily chance of rain is $\frac{1}{2}$?

Binomial formula: $\binom{7}{4}\left(\frac{1}{2}\right)^4\left(1-\frac{1}{2}\right)^3 = \frac{7\times6\times5\times4}{4\times3\times2\times1}\cdot\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^3 = \frac{35}{128}$

We can not use box model because we are asked to calculate the exact probability, and the number of draws (7) in this example is not large enough.