Example 1: A single random draw from a box has mean 10 and a standard deviation 3. What is the standard error of the average of nine draws from the box?

$$\text{SE of average} = \frac{\text{SE of sum}}{\text{number of draws}} = \frac{\sqrt{\text{number of draws}} \times \text{SD of box}}{\text{number of draws}}$$

$$= \frac{\text{SD of box}}{\sqrt{\text{number of draws}}} = \frac{3}{\sqrt{9}} = \frac{3}{3} = 1$$

Example 2: Consider a box containing the three tickets 1, 2, 3 and suppose that 9 draws are made at random from the box. To three decimal places, what is one standard error for the sample average of these draws?

SD of box: $\sqrt{\frac{(1-2)^2+(2-2)^2+(3-2)^2}{3}} = \sqrt{\frac{2}{3}}$.

SE of average: $\frac{\text{SD of box}}{\sqrt{\text{number of draws}}} = \frac{\sqrt{\frac{2}{3}}}{7} \approx 0.117$

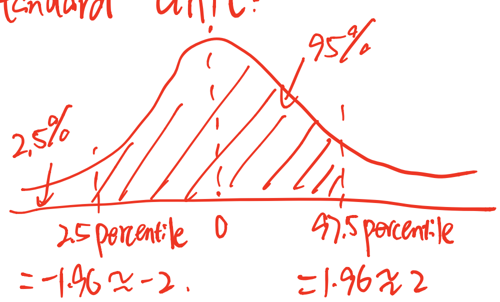
Example 3: A single random draw from a box has mean 10 and a standard deviation 3. What is the expected value of the average of nine draws from the box?

Expected value of average $= \dfrac{\text{Expected of sum}}{\text{number of draws}} = \dfrac{\text{number of draws} \times \text{avg. of box}}{\text{number of draws}}$

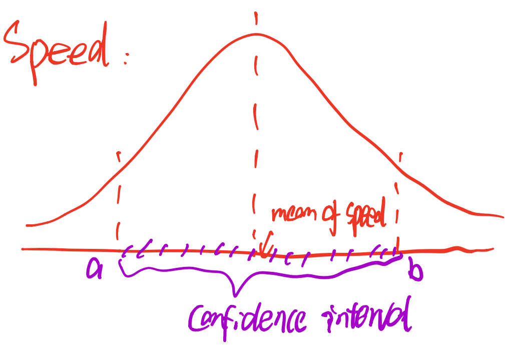
$= \text{avg. of box} = 10.$

Example 4: A state trooper clocks the speed of 100 randomly selected cars on Interstate 80 and obtains a sample mean of 70 mph and a standard deviation of 10 mph. Which of the following is an approximate 95% confidence interval for the true mean speed of cars at this location?

Standard unit:



Speed:



$$\frac{b - \text{mean of speed}}{\text{SD. of speed}} = 2 \Rightarrow b = \text{mean of speed} + 2 \times \text{SD. of speed}.$$

$$= 70 + 2 \times \frac{10}{\sqrt{100}} = 72$$

$$\frac{a - \text{mean of speed}}{\text{SD of speed}} = -2 \Rightarrow a = \text{mean of speed} - 2 \times \text{SD of speed}.$$

$$= 70 - 2 \times \frac{10}{\sqrt{100}} = 68.$$

why SD of speed $= \frac{10}{\sqrt{100}}$? We want SD of speed of a car.

but we are given SD of speed of 100 cars.

SD of speed of 100 cars $= \sqrt{100} \times$ SD of speed of a car

$\Rightarrow$ SD of speed of a car $= \frac{10}{\sqrt{100}} = 1$.

※: 95% confidence interval of any random variable: Approximately

mean of the random variable $\pm$ $\overset{1.96}{2}$ $\times$ SD. of the random variable.

$70 \pm 2 \times 1 = 70 \pm 2 = [68, 72]$

Example 5: Which of the following cannot be a 95% confidence interval for the true proportion of California voters that prefer Joe Biden to Donald Trump in the presidential election?

✓ [50, 70] • [0.5, 0.7] • [0.1, 0.9] • [0.45, 0.90]

proportion should be between 0 and 1.

Example 6: A coin is flipped 100 independent times in an effort to assess whether it is fair. Of these 100 flips, 40 result in heads. Which of the following is an approximate 95% confidence interval for the true heads proportion?

Box $\boxed{0 \quad 1}$

SD of box: $\sqrt{\frac{(0-\frac{1}{2})^2+(1-\frac{1}{2})^2}{2}} = \frac{1}{2}$.

SE of proportion: $\frac{\sqrt{\text{number of draws}} \times \text{SD of box}}{\text{number of draws}} = \frac{\text{SD of box}}{\sqrt{\text{number of draws}}}$

$= \frac{\frac{1}{2}}{\sqrt{100}} = \frac{1}{20}$.

95% C.I. mean $\pm$ $1.96 \times SD = \frac{40}{100} \pm 1.96 \times \frac{1}{20} = [0.302, 0.498]$

or if you write $\frac{40}{100} \pm 2 \times \frac{1}{20} = [0.3, 0.5]$ it should also be correct.