

Review: Probability & Box Model.

Example 1: Two fair six-sided dice are rolled and the sum of the two faces is observed. What is the preferred box model for this scenario?

Possible outcomes? 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.

How many of each?

1: 2, 12	4: 5, 9
2: 3, 11	5: 6, 8
3: 4, 10	6: 7.

How many draws? 1.

2, 3, 3, 4, 4, 4, 5, 5, 5, 5, 6, 6, 6, 6, 6, 7, 7, 7
7, 7, 7, 8, 8, 8, 8, 8, 9, 9, 9, 9, 10, 10, 10, 11, 11, 12.

Example 2: A coin, which is known to be biased toward

heads, is repeatedly and independently flipped n times. If there are more heads than tails on the n trials, you win \$500. You get to select the number of n . Which n should you prefer?

Use the box model:

Numbers in the box: 500, 0

How many of each? a of 500 and b of 0.

a should be greater than b , because the coin is biased towards head.

How many draws? n (You need to determine this).

$$\begin{aligned}\text{Expected return} &= (\text{number of draws}) \times (\text{Avg. of each box}) \\ &= n \times \frac{500a}{a+b}\end{aligned}$$

We want Expected return large, so n also need to be large. Therefore, we will pick n as large as possible.

Example 3: A fair coin is independently tossed 100 times. What is the mean and standard error of the number the number of heads obtained?

Box:

1	0
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 100 draws.

$$\begin{aligned}\text{mean} &= (\text{number of draws}) \times (\text{Avg. of each box}) \\ &= 100 \times \frac{1+0}{2} = 50.\end{aligned}$$

$$\begin{aligned}\text{SE} &= (\sqrt{\text{number of draws}}) \times (\text{SD. of each box}) \\ &= \sqrt{100} \times \sqrt{\frac{(1-\frac{1}{2})^2 + (0-\frac{1}{2})^2}{2}} \\ &= 10 \times \frac{1}{2} = 5.\end{aligned}$$

Example 4: A fair coin is flipped 100 times. Approximate the chance that between 48 and 52 heads are observed.

From the last problem, we know mean = 50 and SE = 5.

so $\frac{52.5 - 50}{5} = 0.5$ go to Z table $\rightarrow 38.29\% \approx 38\%$

Here you need to plus 0.5.

Example 5: What are the exact chances of getting exactly 4 rainy days in a week of seven independent days if the daily chance of rain is $\frac{1}{2}$?

Binomial formula: $\binom{7}{4} \left(\frac{1}{2}\right)^4 \left(1 - \frac{1}{2}\right)^3 = \frac{7 \times 6 \times 5 \times 4}{4 \times 3 \times 2 \times 1} \cdot \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^3 = \frac{35}{128}$

We can not use box model because we are asked to calculate the exact probability, and the number of draws (7) in this example is not large enough.