Probability

1. List and count:

Example 1: A fair six-sided die is rolled once, what are the chance that the number appearing on its face is greater than two?

Step 1: list all possible outcomes.

A fair six-sided die is rolled once, there are 6 possible outcomes: 1, 2, 3, 4, 5, 6.

Step 2: count how many outcomes satisfies the requirement.

Requirement in this example: greater than two.

So there are 4 out of 6 outcomes satisfies the requirement

Probability is : $\frac{4}{6} = \frac{2}{3}$.

Why this works? We assume every possible outcome has exactly the same chance of happening.

Example 2: A six-sided die is rolled twice. What are the

Chance that the sum of the numbers appear on its face is greater than 10?

List outcomes first:

first roll \ second roll	1	2	3	4	5	6
1	(1,1) 2	(1,2) 3	(1,3) 4	(1,4) 5	(1,5) 6	(1,6) 7
2	(2,1) 3	(2,2) 4	(2,3) 5	(2,4) 6	(2,5) 7	(2,6) 8
3	(3,1) 4	(3,2) 5	(3,3) 6	(3,4) 7	(3,5) 8	(3,6) 9
4	(4,1) 5	(4,2) 6	(4,3) 7	(4,4) 8	(4,5) 9	(4,6) 10
5	(5,1) 6	(5,2) 7	(5,3) 8	(5,4) 9	(5,5) 10	(5,6) 11
6	(6,1) 7	(6,2) 8	(6,3) 9	(6,4) 10	(6,5) 11	(6,6) 12

requirement: sum greater than 10.

represent the outcomes satisfy the requirement.

$P = \frac{3}{36} = \frac{1}{12}.$

Example 3: A fair coin is tossed 3 times. What are the chance that all tosses are heads?

List of possible outcomes: H: head T: tail.

HHH HHT HTH HTT

THH THT TTH TTT.

requirement: all three tossess appear head.

represent the outcome satisfies the requirement.

$p = \frac{1}{8}$.

Another way of solving question of this type is by binomial formula.

Suppose in an experiment, the probability of success is p. This same experiment is performed n times. the chance of seeing m success $(m \leq n)$ is: $\binom{n}{m} p^m (1-p)^{n-m}$

$\binom{n}{m}$: n choose m, use calculator to compute.

In this example, experiment is toss a fair coin. It is repeated 3 times, so $n=3$. The success is showing head, so $p=\frac{1}{2}$. We want the probability of seen 3 success. so probability is:

$$\binom{3}{3}\left(\frac{1}{2}\right)^3\left(1-\frac{1}{2}\right)^{3-3}=\binom{3}{3}\left(\frac{1}{2}\right)^3\cdot\left(\frac{1}{2}\right)^0=1\times\left(\frac{1}{2}\right)^3\times 1=\frac{1}{8}$$

Example 4: The chance that a hiker sees at least one banana slug during a hike is 0.4. Over three independent hikes, what are the chances that exactly one hike is banana free?

Experiment: hiker go for a hike

Success: sees at least one banana slug. $P=0.4$

We want the probability of seen 2 success.

$$P=\binom{3}{1}0.4^2(1-0.4)^{3-2}=\binom{3}{2}0.4^2 0.6^1=3\times 0.4^2\times 0.6=0.288$$