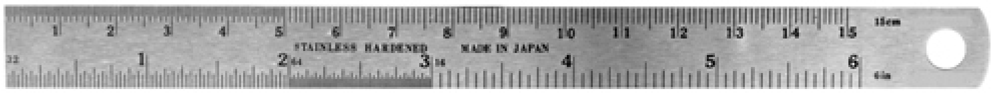
Chapter 4

Methods I: Line Interaction



Tools and Techniques

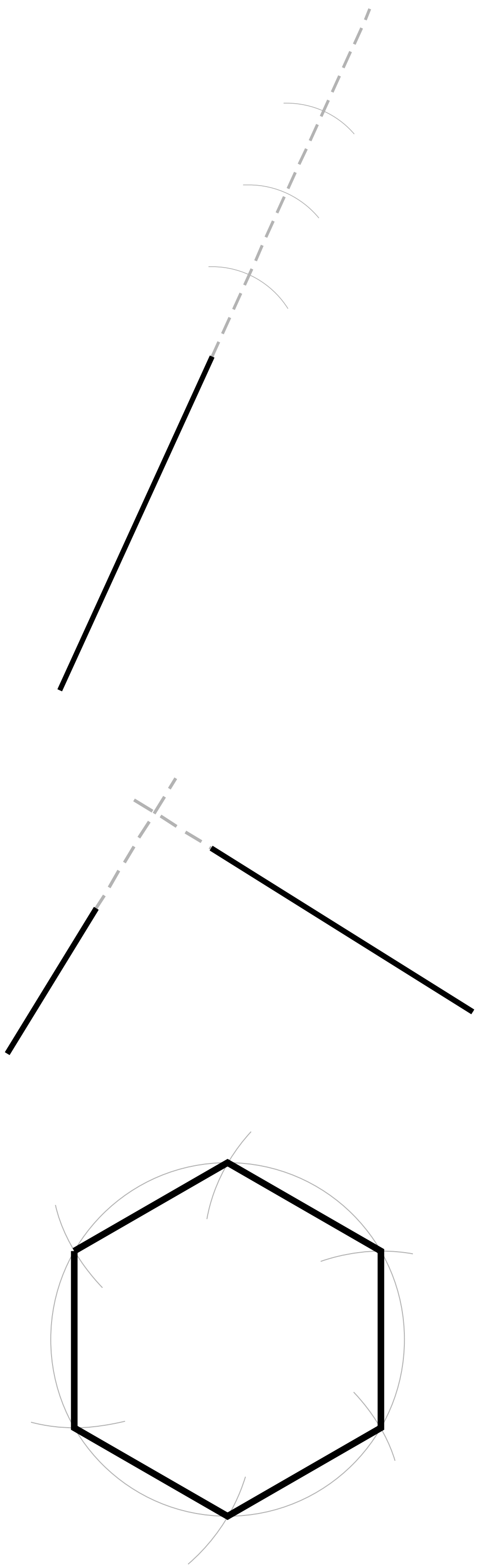
The layout techniques practiced by mapmakers, engineers, draftsmen, artists and architects through most of history can be boiled down to a relatively few essential constructions and their combinations. The prime tools are the compass and straight edge and pure construction uses these two exclusively.

Specialized tools quicken many of these constructions and to make them more practical out in the field or at the drawing board. The carpenter's square, for example, provides a template for quick, on-the-spot layout of perpendicular lines and sloped structures. Today digital technology has added to the list of tools, but, like the carpenter's square, these are just tools. Knowledge of constructions and their applications remains essential to the effective use of these tools.

The compass draws arcs and circles. Elegant in themselves, these elements gain even more importance, due to the fact that they each determine a set of points equidistant from a fixed point. The intersection of two equal arcs, then, can fix a point equidistant from two pre-existing points.

Another critical role of the compass is as a divider. In construction dividing is the process of marking off a segment of a line or arc to a pre-established length. To divide, the designer opens the compass to match a distance already determined in another point of the design, and then transfers this distance onto the arc or line segment to be divided. Dividing is especially useful for marking the line or arc into a sequence of equally spaced units of distance.

The major uses of the straight edge are to draw a straight line and to extend a previously drawn line. This last function enables the designer to locate the meeting point of two lines or to determine points and lines co-linear with an existing line. In conjunction with the dividing function of the compass, a designer may use the straight edge to determine the unit distance between two elements.



Top: Using a straight edge and compass divider to locate a point three units from and co-linear to a line segment. Middle: Locating a point co-linear to two line segments. Bottom: Dividing a circle by its radius to create a hexagon.

The major categories of construction include bisections, perpendiculars, congruencies, chords and tangencies. All of the geometric forms and figures used in classical design derive from these four operations. This chapter lays out some common methods for using the first three categories to build relationships and interactions among lines. The following chapter adds chord and tangency construction to create important relationships between arcs and lines.

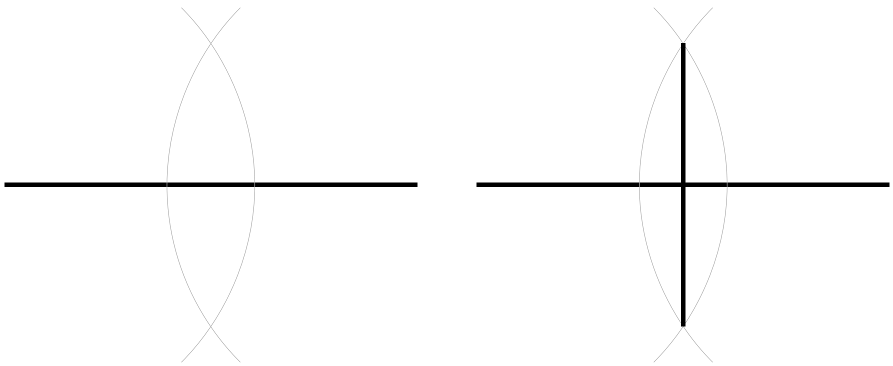
Note that in all of the illustrations below certain conventions are followed: 1) the given elements and the finished figures are presented in heavy black line, 2) the construction lines are light gray and do not appear in the final design, 3) dark gray dots stipulate the center of the arcs drawn at the stage of construction being illustrated.

In practicing these constructions a good designer attempts to keep all construction lines as light as possible. A general rule of layout is to only draw construction lines that will cleanly erase. In practice this means placing no pressure on the drawing lead. Instead let the weight of the pencil or compass do the work. Finally, always keep pencils sharp.

Bisection

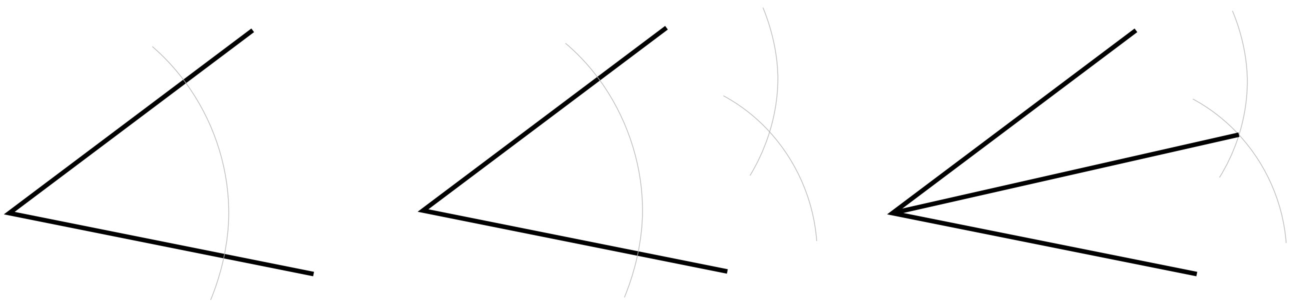
1) Bisect a line segment

Open the compass a bit larger than half of the length of the segment. (The distance is approximate, just as long as it is larger than half.) From each end point draw two arcs to intersect one another on both sides of the line segment. With the straight edge draw a line connecting the two points of intersection. This line bisects the original line and runs perpendicular to it.



2) Bisect an angle

Place the compass point on the vertex of the angle and draw an arc to intersect each leg of the angle. From these intersections draw two arcs of the same radius in the opening of the angle. A line drawn from the crossing of these arcs back to the vertex will bisect the angle.



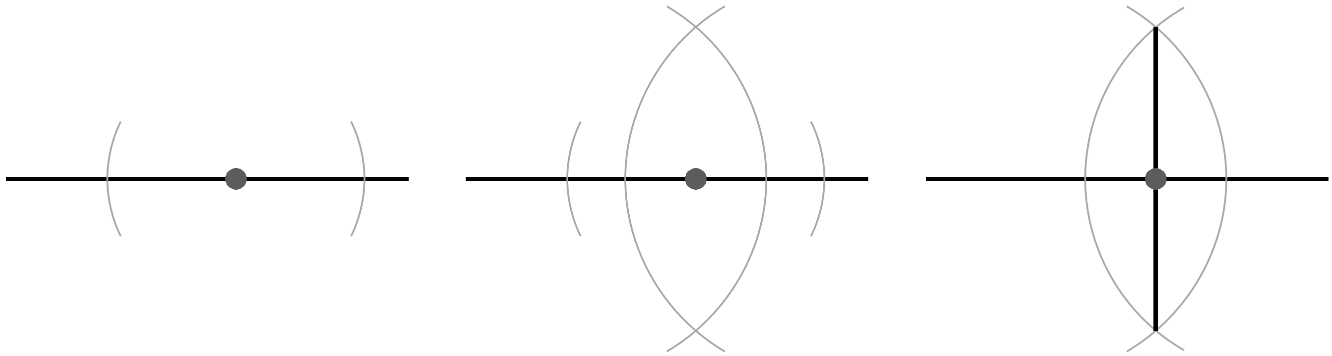
Perpendiculars

Gravity infuses the experience of space. The primary experience of the perpendicular is that of our own body to the surface of the earth. The verticality and balance of this sense of perpendicularity affects cultural and physical experience to the degree that our most basic spatial and social perceptions flow from it. We have named the angle between earth and body the right angle. People are upright if they are standing, and morally upright if they are behaving. Words like correct or rectify or rectangle derive from the Latin *rectus* for right. If something is askew, it is not right and this applies to angles as well as behavior.

Determining perpendiculars is perhaps the most essential construction used in solving design problems.

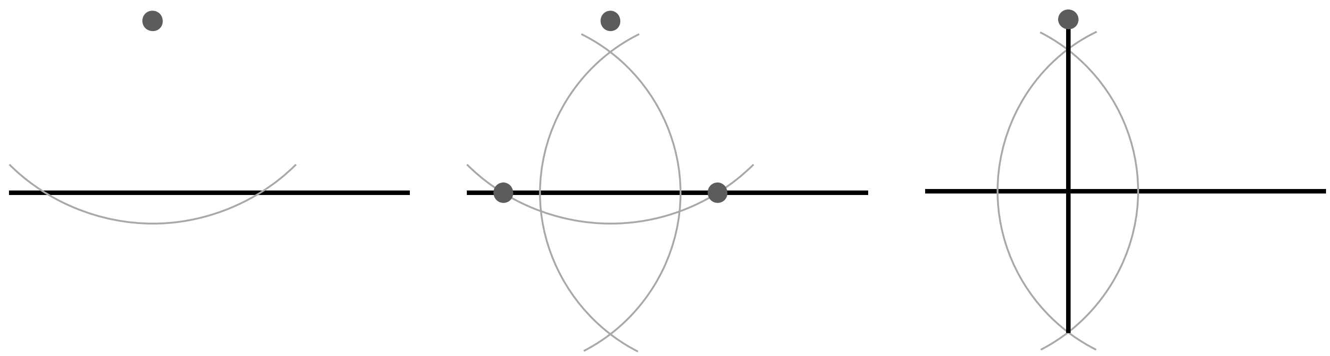
3) Draw the perpendicular to a given point on a line

This construction uses the same procedure employed to bisect the line segment, but with one preceding step: to place the point of the compass on the given point and draw two arcs of the same circle to intersect the line on each side of the given point. This in effect creates a line segment between the two arcs. It also ensures that the given point is at the center of this new segment. Now bisect the segment between the arcs to create the perpendicular at the given point.



4) Draw the perpendicular from a given point to a line

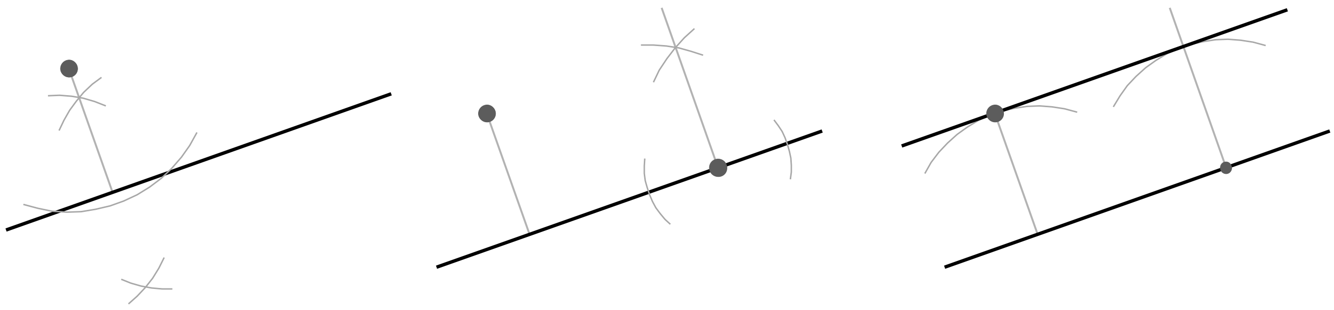
From the given point off the line draw an arc to intersect the given line at two points. A perpendicular bisector constructed on the line segment between these two points of intersection will pass through the given point. This construction will also yield the offset, i.e., the shortest distance, of the point from the line.



5) Draw a line parallel to a given line through a given point (I)

Both of the following constructions apply the perpendicular constructions to determine the parallel line.

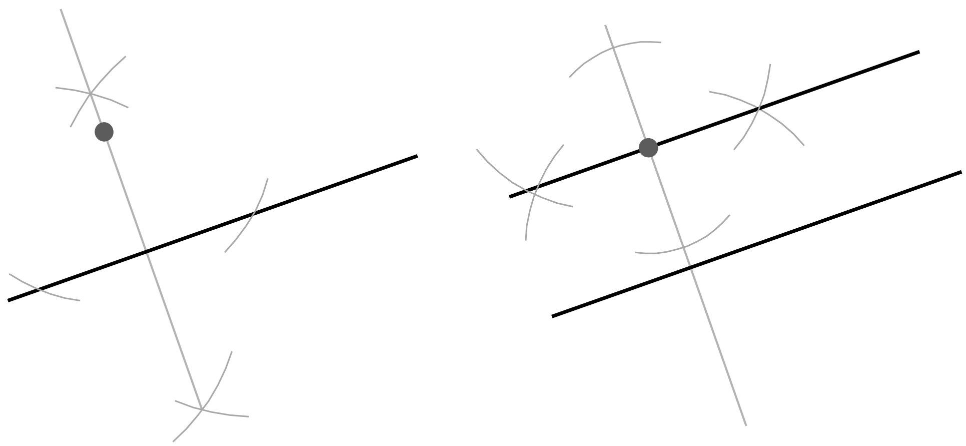
Two points will define a line and two points equidistant from a given line will define a parallel to that line. This construction is effective because it establishes a second point that is the same distance from the given line as the given point. A line drawn through these two points will be a parallel to the given line.



To determine the offset of the given point from the line, construct a perpendicular from the point to the line (construction # 4 above). Select a point on the given line a significant distance from the intersection of this perpendicular. Construct a perpendicular to the line from the selected point (construction #3 above). Set the compass radius to match the distance from the base of the first perpendicular to the given point. Strike an arc of this radius from the base of the second perpendicular. The line drawn through the given point and the intersection of this arc with the perpendicular will be parallel to the given line.

6) Draw a line parallel to a given line through a given point (II)

This construction also combines constructions #3 and #4 above. It uses the principle that two lines are parallel if they intersect a third line at the same angle.



The first step is to draw the third line mentioned in the above rule. To do this, use construction 4 to draw a line through the given point and perpendicular to the given line. Since this new line and the given line are perpendicular, any other line perpendicular to it will be parallel to the given line and vice versa. Using construction 3, draw a line through the given point perpendicular to the constructed line. This line will be parallel to the given line.

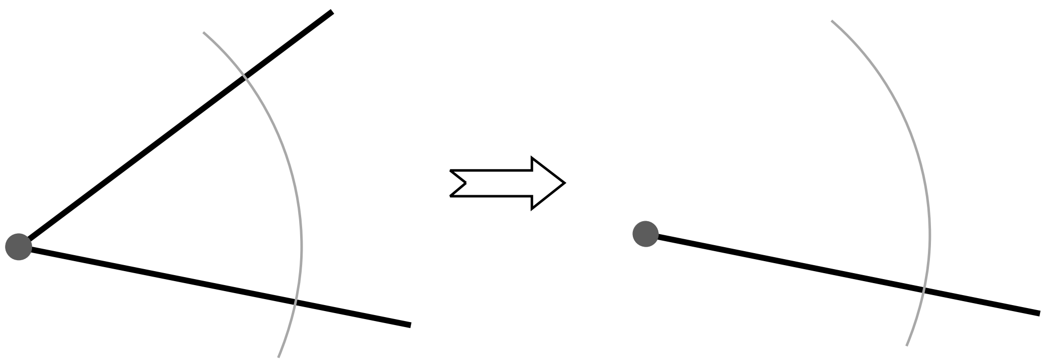
Congruency

Many of the most useful constructions are those based on congruency. Congruency means that one geometric figure can be fit perfectly on top of another by simply moving and rotating one of the figures. Both figures are perceived as perfect matches. A related concept is similarity. Two figures are similar if they are exactly alike except for their relative sizes. Their internal proportions, however, are exactly the same.

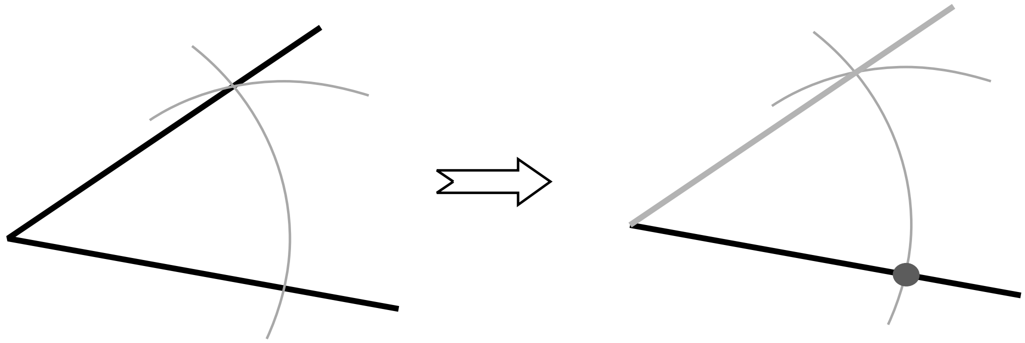
The most essential construction of congruency is the construction of an angle congruent to a given angle. It is the basis for all constructions using the principle of congruency.

7) Draw an angle congruent to a given angle

Draw an arc from the vertex of the given angle to intersect both legs of the angle. On another part of the page draw a line segment to serve as the first leg of the congruent angle. Without changing the setting of the compass, draw an arc from one end of this segment to intersect it.

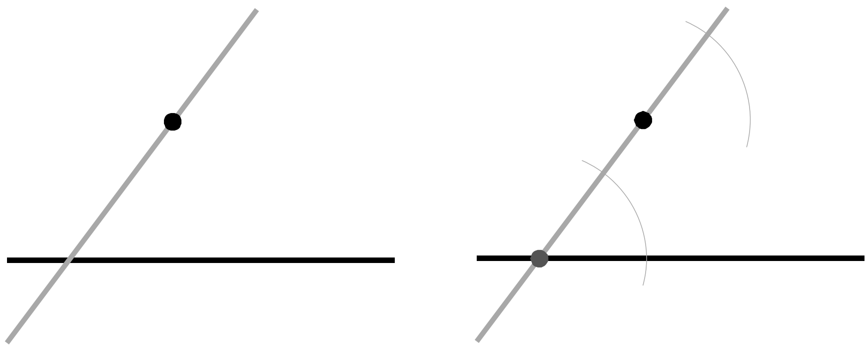


Return to the given angle and set the compass to the distance between the intersections of the arc. Transfer this distance onto the new construction to duplicate the original angle.

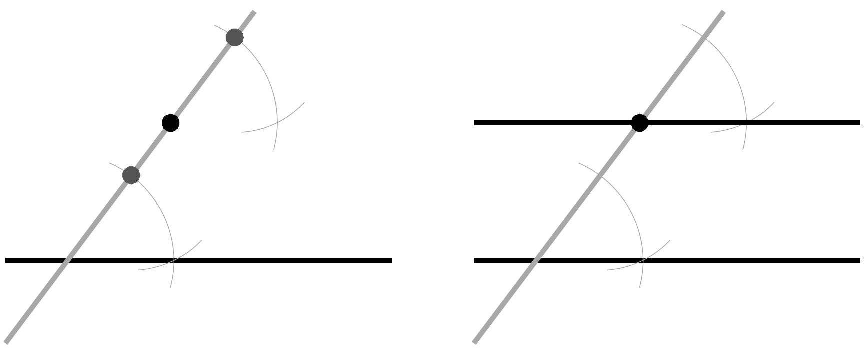


8) Draw a line parallel to a given line through a given point (III)

Like construction #6 this construction also uses the principle that all lines intersecting the same line at the same angle are parallel. It is quicker and more efficient than that previous construction.

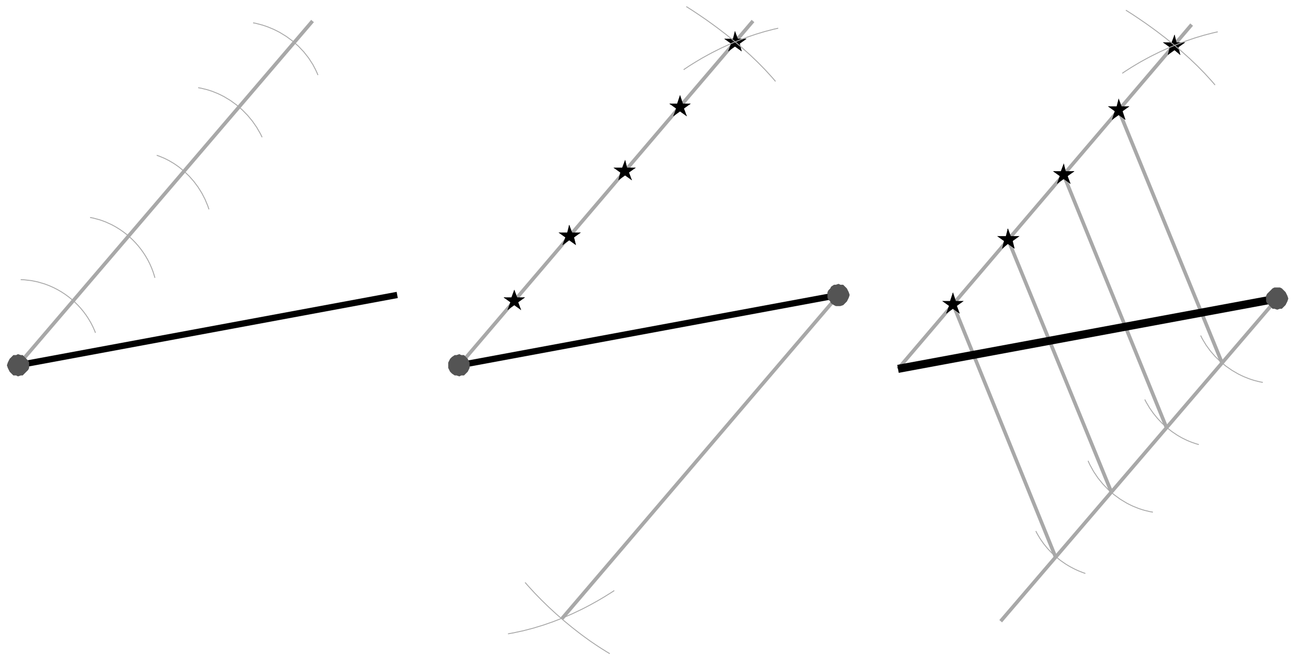


Draw any line through the given point and the given line. From the given point and the drawn line construct an angle congruent to that of the drawn line and the given line. The lower leg of this new angle will be parallel to the given line.



9) *Divide a line segment evenly into any number of equal smaller segments.*

This elegant construction relies on the principle of congruency to ensure accurate division of the line into a whole number of equal segments:



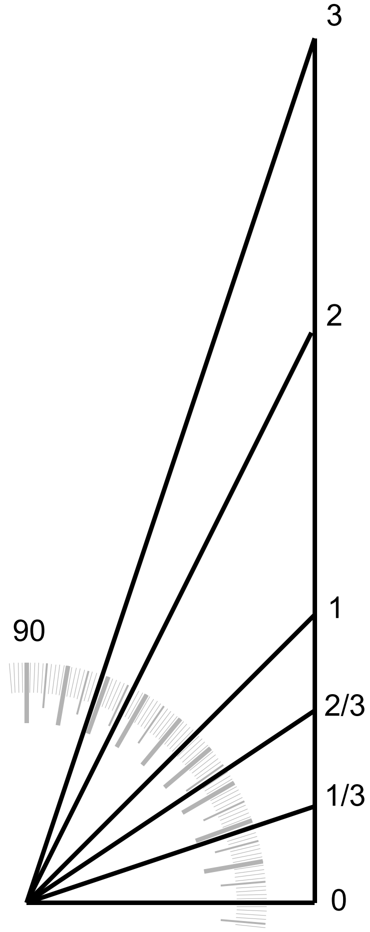
Arbitrarily draw a construction line from one end of the given line being divided. From the start of this new line use the compass to mark off a series of equal line segments of the same number of divisions to be performed on the given line. Like the angle of the construction line, the length of these segments is also arbitrary.

Next use construction 8 to draw a second construction line through the opposite endpoint of the given line and parallel to the first construction line. Segment this line just like the previous line. Connect the dividing points of each construction line to create a series of parallel lines passing through the original line.

In geometric jargon the given line is now a transverse to the set of parallel lines crossing it. The transverse through a set of equally spaced parallels will divide equally and the angles at each intersection will be congruent to those at all of the other intersections.

Slopes and Angles

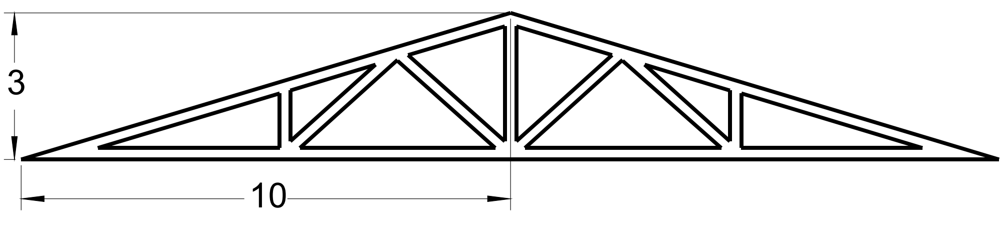
Slope and angle both measure the relative difference in direction of two lines. Any slope that can be determined rationally, that is, as the ratio between two whole numbers corresponding to the ratio of offset/run, can be determined constructively. In practice this means virtually all needed expressions of direction. The slope is, in effect, the direction of the hypotenuse of a right triangle to its base:



Angle, on the other hand, measures the increments of the rotation of a line around a point. The full rotation is divided into 360 even increments, or degrees. For finer measurements each degree divides into 60 minutes and each minute into 60 seconds. When held at arm's length the diameter of a dime intercepts an angle of about one degree in one's field of vision. One second equals 1/3600 (60 x 60) of that diameter, or less than .0001 inch.

This familiar system reached us from ancient Babylon due its eventual adoption by the ancient Greeks. The Babylonians based angle measures on the number 60, because their number system based on tens and sixties rather than on tens and hundreds. The high accuracy of this system reflects the Babylonians deep belief in astrology. A major task of the priest/astronomers was to keep minute records of planetary positions and study their correspondence to historical events in order to aid the king in his decision-making. When Alexander the Great headed east on his mission of world conquest he appropriated these records for Greek scholars.

Angle measure has the distinct advantage of breaking directional changes into units of measure. Its advantage over slope determination becomes clear as angles approach 90 and 0. The slope of an angle of 1 is .017455064 and the slope of an angle of 89 is 57.28996163. The former is very close to 4/229 (.01746) and the latter lies somewhere between slopes of 172/3 (57.33...) and 229/4 (57.25). Both are extremely cumbersome to construct.

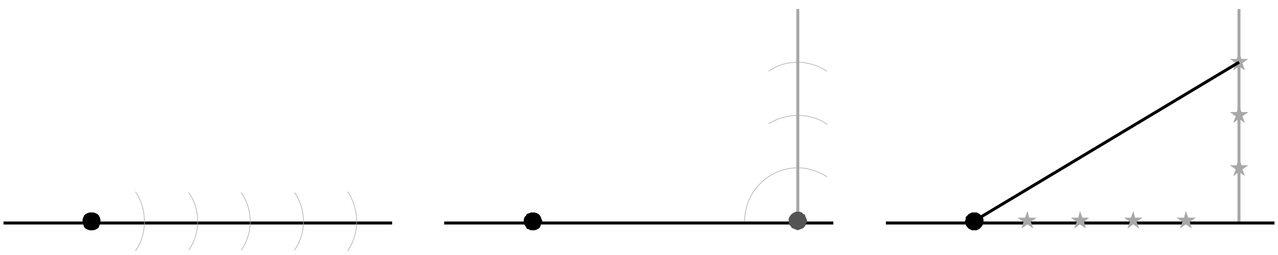


On the other hand most architectural angles are far more pronounced and slope determinations serve them well. A carpenter framing in a roof over a garage 20 feet wide would

Rather deal with a slope specification of 30% (3/10) than an angle specification of 16.7. The former instantly translates in to a center support strut of 3 feet.

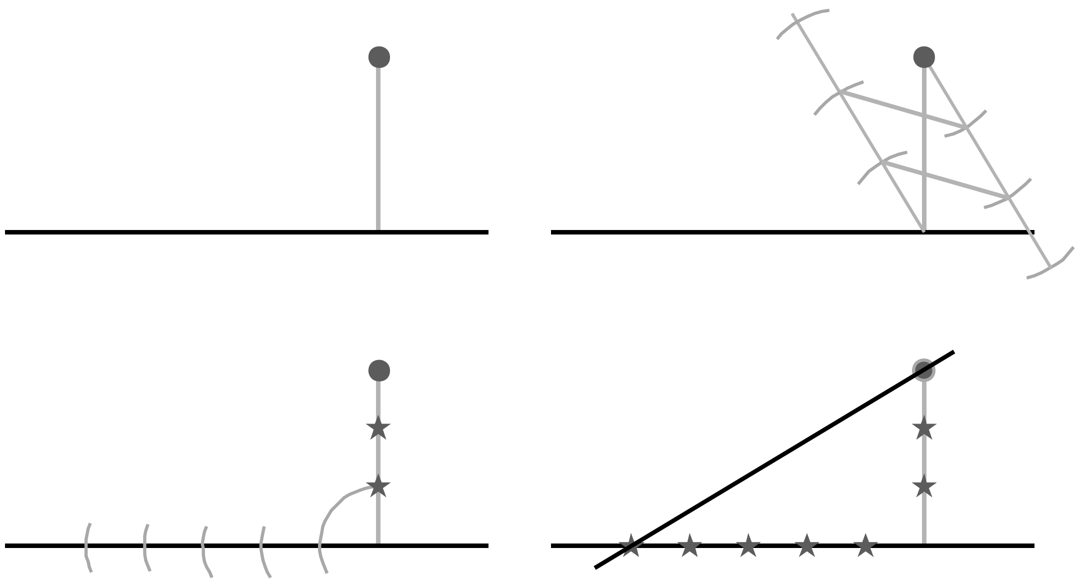
10) Draw a line of specified slope through a point on a line.

The example below demonstrates the construction of a slope specified at 3/5: three units of offset over five units of run. Set the compass for the chosen unit -- any unit of length will do -- and mark off the five units of run along the given line. At the fifth mark construct a perpendicular (construction 3) from the line and mark three units up this perpendicular. This will determine the offset

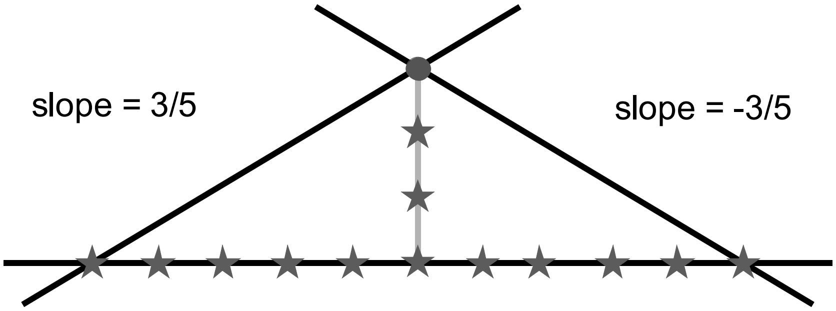


11) Draw a line of specific slope through a point to a line (l).

First determine the offset of the given point from the given line by drawing a perpendicular from the point to the line. Now divideand then divide the perpendicular into three equal units usinf construction 9. Copy this unit and, starting at the base of the perpendicular, divide five units on the line. Draw a line from the given point to the fifth dividing arc.

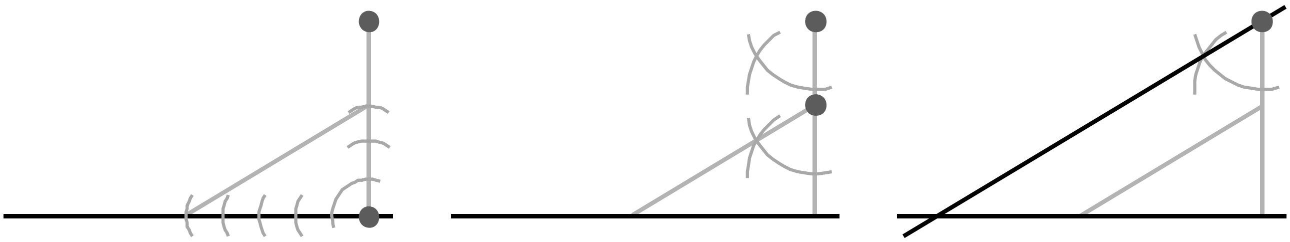


Note that by convention a positive slope rises upward from left to right and a negative slope falls as it moves from left to right. Thus the division in the example above moves to the left. Had the slope been specified as -3/5 the five units would run to the right.



12) Draw a line of specific slope through a point to a line (II).

This construction combines constructions 10 and 8 above. Though it entails two separate constructions, it is a bit quicker than construction 11:

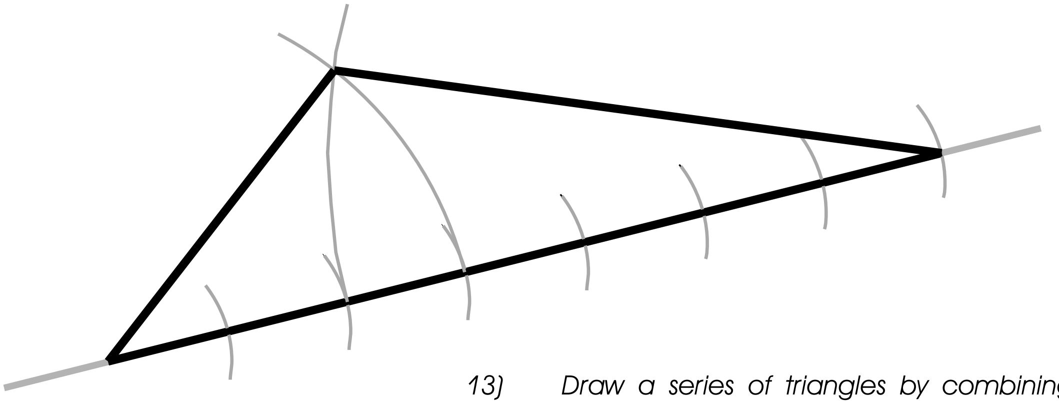


First draw a line of the specified slope as in construction 10, and then draw a line parallel to that line through the given point as in construction 8.

problem solving

Although construction 12 is fairly simple it nevertheless reveals two basic approaches used to solve more complex problems: breaking the problem down into a set of simpler problems and putting these into the order in which they need to be solved.

In this case the construction redefines the end goal into two such simpler goals: to draw a line of the required slope and to put that line through a point.



13) Draw a series of triangles by combining construction methods to fit the triangle.

The triangles on this page are offered for practice in problem solving. Each requires a different combination of dividing procedures and slope constructions:

- *A) Draw a triangle with sides of 3, 5 and seven units.*
- *B) Draw a triangle with sides of slopes 1 and -3/5.*
- *C) Draw a triangle with sides of slopes 4/5 and -3/4.*