Chapter 8
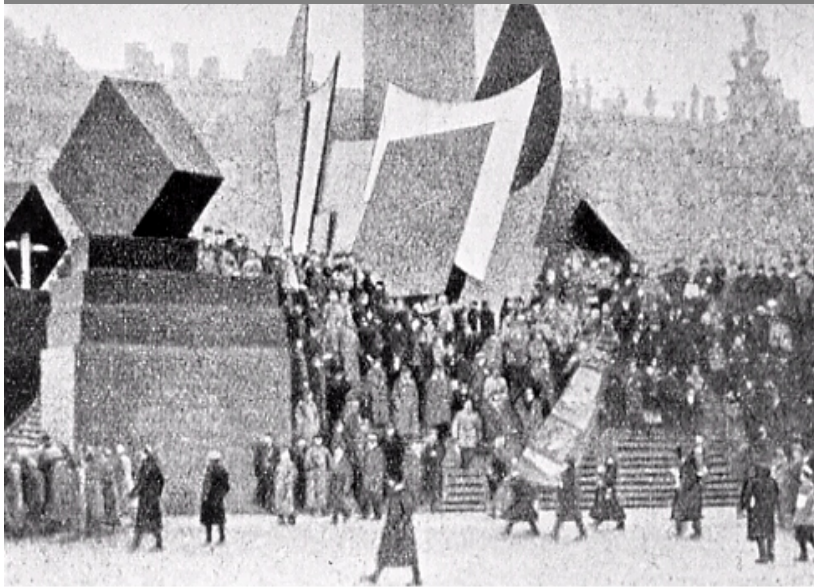
Shapes
And Figures

Shape

In geometry plane figures are closed two-dimensional forms bounded by lines and curves. In the language of visual design they are simply shapes. Visual design typically places shapes into two basic categories: organic shapes and geometric shapes.

Organic shapes, also called biomorphic shapes, comprise free-form curves with no angles or straight edges to break the flowing character of their edges. The primary source of these shapes are living plants and creatures found in nature including the human body. An examination of either a seashell or the human body will turn up no straight edges or flat surfaces.

Geometric shapes by contrast flourish in the architecture and machinery of human culture. Sharp, straight edges and flat planar surfaces are the rule. Curves tend to be circular, as with the rim of wheels or the edge of a compact disk.

Designers often create composite shapes with both organic and geometric characteristics (many typefaces, for example, display characteristics of both categories), but these two categories are usually regarded as dichotomous, at two opposite ends of a continuum. This view sees the contrast between the organic and geometric as indicative of an analogous contrast between nature and culture: the organic is a product of life's evolutionary engineering, while geometry springs from the minds and hands of people.

The dichotomy is one of visual meanings and not necessarily one of actual structure. As seen in the previous chapter traditional construction can, through sequences of tangent arcs, nicely approximate freeform curves. A much more precise depiction of organic curves becomes possible with the use of spline geometry. This geometry, which derives from the analysis of stress on physical material, is the basis for creating highly naturalistic computer models of living creatures.
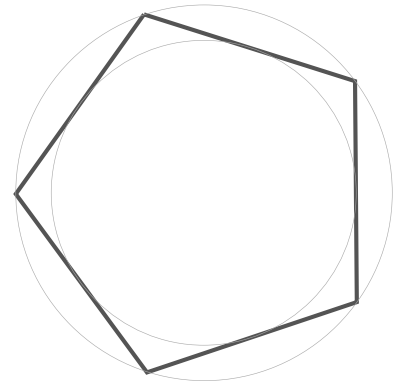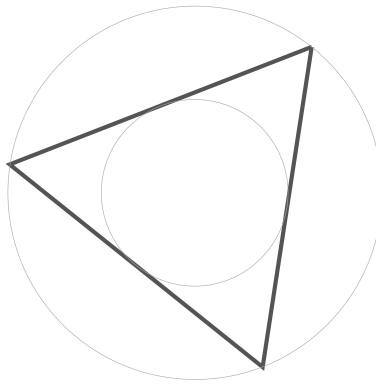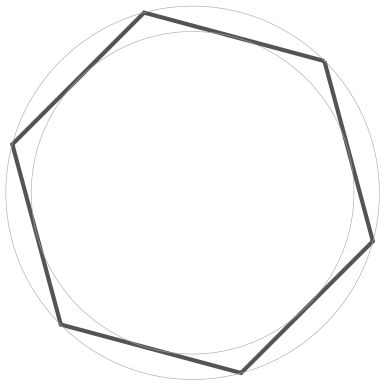
Polygons

Polygons are plane figures bounded by straight lines. Broken down, polygon is a Greek word meaning "many (poly-) sides (-gon)". A polygon has two important features: the line segments forming its sides and the vertex points where sides meet.
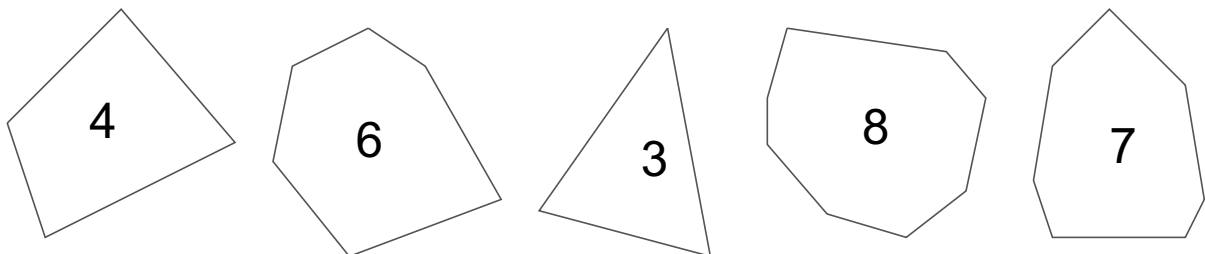
Polygons can group according to their number of sides. A pentagon, for instance, has five sides, while a nonagon has nine sides. This nomenclature pairs the Greek word denoting the number of sides with the suffix -gon. The exceptions are triangles and quadrilaterals (four-sided figures).

A polygon may be either regular or irregular. A regular polygon has all of its sides and all of its vertex angles equal. As a result all sides and vertices of a regular polygon will lay equidistant from a center point. This point will also mark the centers of the inscribed and circumscribed circles of the polygon. All regular polygons will fit perfectly into a circle. Any polygon not meeting these criteria will be irregular.

Polygons also divide between convex and concave. The exterior angles of a convex polygon are all greater than 180°, but one or more exterior angles of less than 180° qualifies the polygon as concave.
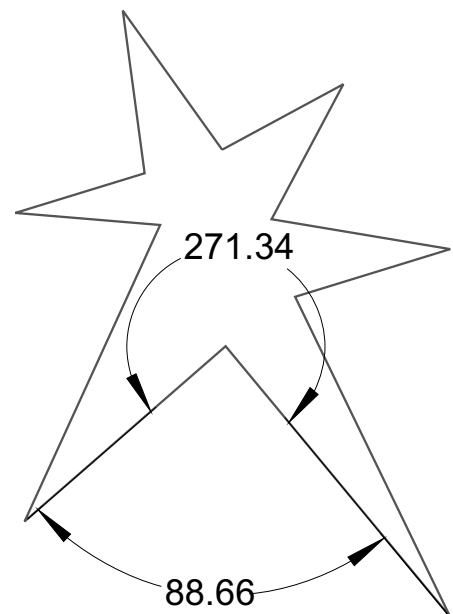
Triangles

The triangle is the most basic, the most used and, therefore, the most important of all the plane figures. It represents the minimum number of sides necessary to enclose an area of surface. From a structural and engineering point of view the triangle is the only arrangement of linear parts that will remain stable under force. Triangles are used for all manner of sophisticated layout where fixing points in space is a necessity. GPS (geographical positioning system) operations use triangulations compiling spatial and temporal data to fix a user's position on the earth to within a few feet. With all this going for it, it's no wonder that geometers identify several varieties of this simple figure.
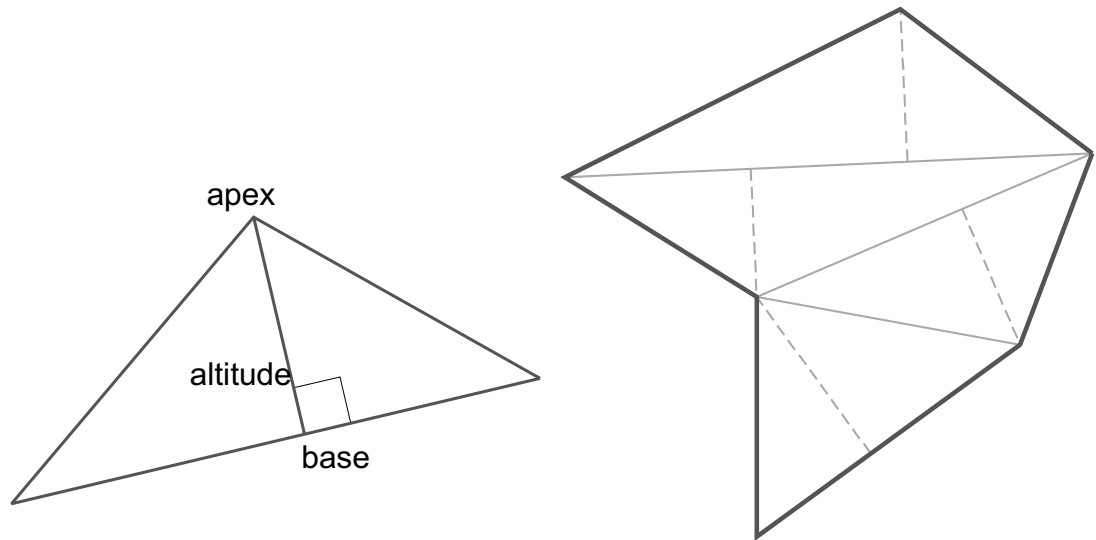
right triangles

Due to its function in determining slope and its role in trigonometry the most common triangle is the right triangle. Of these the most famous is the 3-4-5, or Pythagorean triangle (see chapter 1). If all of the angles in a triangle are under 90° it is termed an acute triangle; if above 90° it becomes an obtuse triangle.

However, any acute or obtuse triangle may be divided into two right triangles along a perpendicular line, or altitude, connecting the base of a triangle to its apex. As a result a designer can specify any triangle by means of its base and altitude. Moreover, since any polygon may be broken down into component triangles by drawing its diagonals, that polygon also divide into right triangles.

The practical virtue of these relationships is that they allow computational methods for calculating geometric relationships. Both the Pythagorean Theorem and fundamental trigonometry (see chapter 14) exploit relationships inherent in the right triangle.

271.34

88.66

apex

altitude

base

isosceles triangles

When two of its sides are equal a triangle becomes symmetrical along one axis and is known as an isosceles triangle. On each side of the axis of symmetry lay two mirrored right triangles. The regular triangle, i.e., all sides and angles equal, is the equilateral triangle and is a special case of this triangle. Like all regular polygons, the equilateral triangle, is symmetrical on all sides.

A
A   S

S
S
S   S

S
A   S

A
A
S
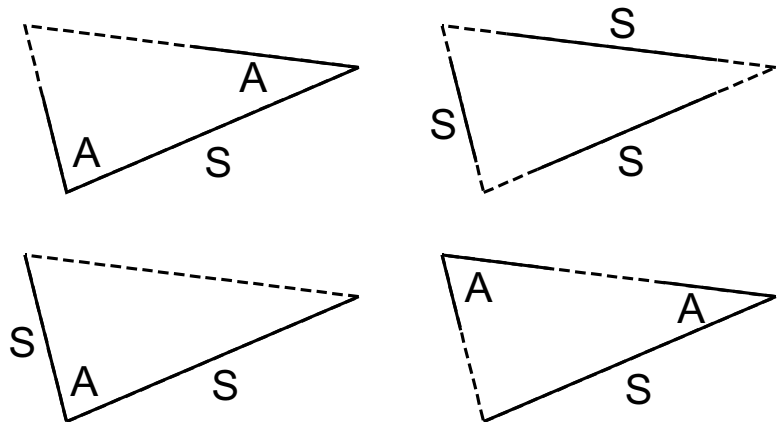
congruent and similar triangles

Two triangles are congruent if all three angles and all three sides of one triangle are equal to the corresponding sides and angles of the other. Some important uses of triangles in design stem from knowing the minimum amount of information needed to replicate a triangle congruently. For example, knowing the length of one

side of a triangle and the measure of the two angles adjacent on either end is sufficient to duplicate that triangle. In geometry this relationship is customarily referred to as angleside-angle, or ASA. The use of the dioptra to measure the distance across a river in chapter 5 is an application of ASA.

Quadrilaterals

Close on the heels of the triangle in its importance to design is the quadrilateral. Far and away the most important of these are the rectangle, a figure comprised of four right angles, and its regular version, the square. A useful property of this class of figures is that both diagonals are equal. To "check for square", i.e., assure perpendicularity, when building a rectangular object like a paint stretcher or doorway, a craftsperson simply compares the diagonals.
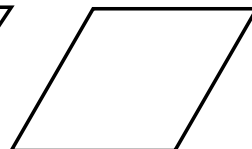
There are quite a few types of quadrilaterals, named according to their degree of regularity. Though distant contenders, the parallelogram and trapezoid follow rectangles in the frequency that they show up in design. The parallelogram is a skewed rectangle: it retains opposite sides that are the same size and parallel, but the corner angles shift from right to acute and obtuse.
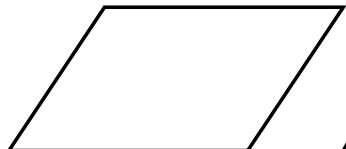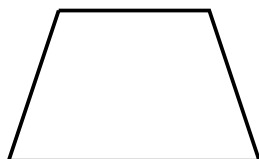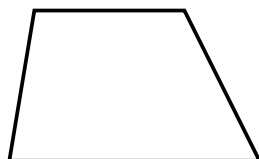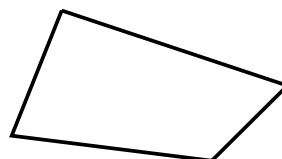
rectangle   parallelogram   rhombus

Trapezoids maintain two parallel sides but the lengths vary, as do the angles: it does, however, remain symmetric. Trapeziums are less regular kin to the trapezoids in that two sides keep parallel, but the figure overall is asymmetric. All other quadrilaterals are categorized as irregular.

trapezoid   trapezium   irregular

A quadrilateral is cyclic if it a circle can pass through all of its vertices. This is true all triangles, since any three points define a unique circle, but the quadrilateral has a fourth vertex that may or may not lie on the circle. All rectangles are cyclic, i.e., able to be inscribed in a circle, as are all trapezoids. No parallelograms or trapeziums are cyclic.
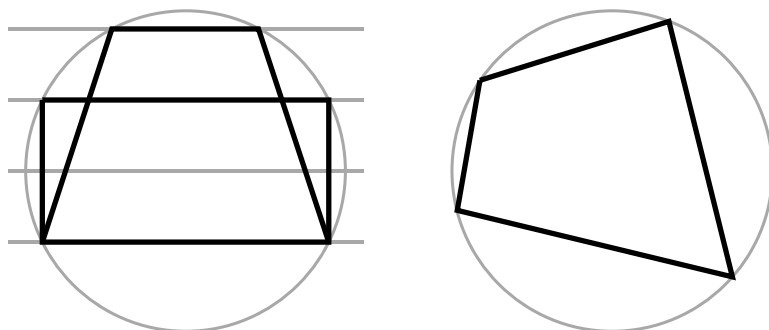
Constructing Regular Polygons

All regular polygons inscribe in a circle. So the most common methods for drawing a regular polygon is to construct them in a circle. Since they are simpler shapes, though, equilateral triangles and squares can be constructed more directly.
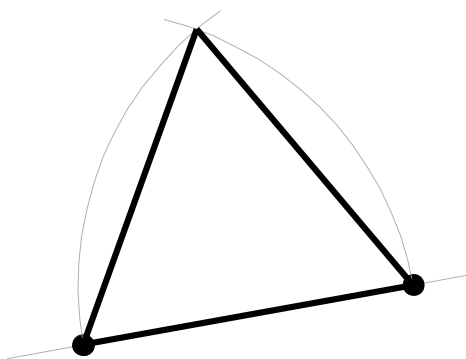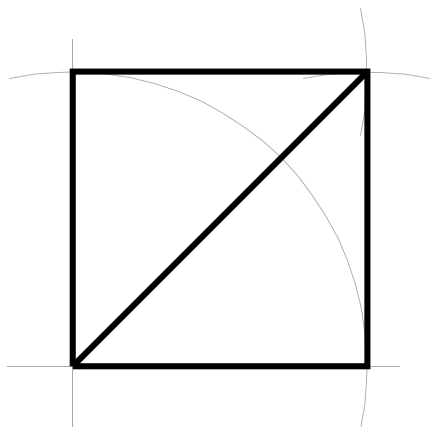
27)   Draw an equilateral triangle.

Mark two points on a line at a distance equal to the side of the desired triangle. Open the compass to a radius equal to this distance and from these points strike arcs to intersect above the line. Connect the points and the arc intersection to complete the triangle.

28)   Draw a square.

Draw two lines perpendicular and from their intersection strike two arcs equidistant along each line. Keeping the same arc radius strike two arcs, one from each equidistant point to intersect in one quadrant formed by the perpendiculars, Connect all arc intersections to complete the square.

This same method will yield a 45° angle by drawing the diagonal of the square.

29)   Inscribe a regular hexagon and an equilateral triangle in a circle.

This construction appeared in chapter three as an example of the mathematical rigor of constructive geometry.
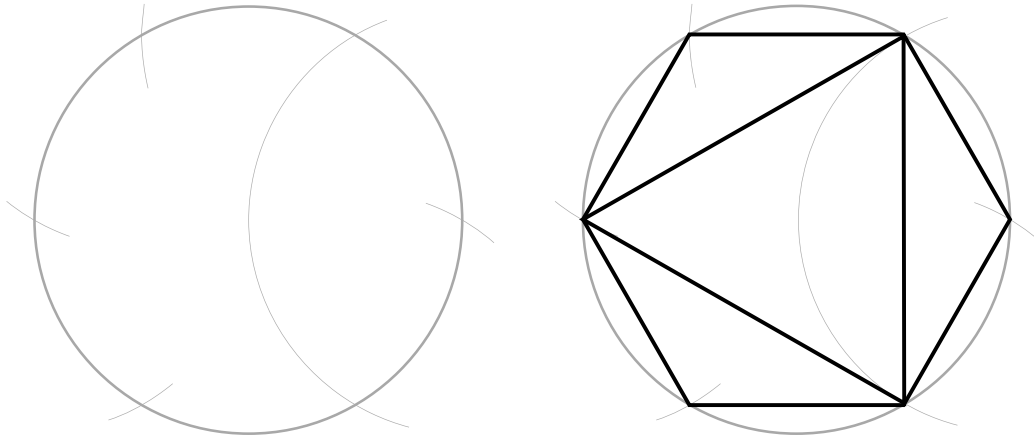
Set the compass to the radius of the circle and divide its circumference into six equal arcs. Connect the dividing points to create the hexagon and join every other dividing point to draw the triangle.
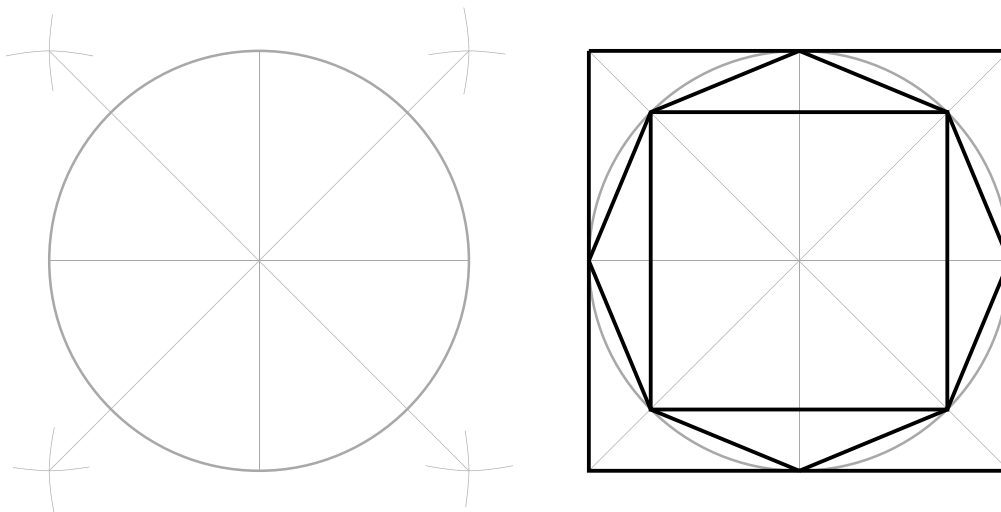
30)   Inscribe a regular octagon and a square in a circle.

Draw two perpendicular diameters of the circle and, keeping the radius of the circle, strike arcs from each end point of the diameters to intersect in all four quadrants draw two more 45° diameters by connecting the opposite arc intersections.

This will divide the circle into eight equal segments. Connect the points where the four diameters split the circumference to construct the octagon; connect every other division point to inscribe the square. As an added bonus connect the four arc intersections to circumscribe a square on the circle.

Non-Mathematical Constructions

The only additional inscriptions of regular polygons that are mathematically possible are the pentagon, the heptakaidecagon and doubles of the equilateral triangle or the square.

The pentagon is an unusual construction, both in its complexity and in the unique relationships it contains. This construction will be featured in the upcoming chapter on sacred geometry.

The heptakaidecagon with seventeen sides is an interesting oddity, but as far as design is concerned is just another polygon with a lot of sides. The very vast majority of regular polygons used in design have less than twelve sides. Beyond that number a regular polygon appears more circular than polygonal: the distinct separation of visual elements that allows the part-to-part relationships defining a polygon's visual strength perceptually merges into a rough curve. In fact early computer graphics programs from the 1970's used a 10-sided figure to represent circles in order to conserve memory.

Albrecht Durer's Melancholia *ranks as one of the most studied images in art.*

Filled with geometric and alchemical objects it marks an age passing from magical and symbolic thought to the scientific concepts that mark the modern world. Durer represents the obsession and intellectual sadness of melancholy under the weight of new and changing knowledge opening up to querying minds like his own. As a result of the invention of movable type and thus the quick printing of books for international distribution the world experienced its first rapid spread of new ideas.

As a printer and publisher Durer played a key role in distributing these ideas and some of the ensuing melancholy.

(The date for this work, 1514, is hidden the middle two cells in the bottom row of the magic square located in the top right of this print. A magic square is a figure in which each horizontal and vertical row and both diagonal rows add up to the same number, in this case, 34.)

The hexagon, octagon and the decagon (ten sides) construct from bisecting the sides of the equilateral triangle, the square and the pentagon respectively. This leaves artists and designers with three potentially useful polygons that cannot be validly constructed: the heptagon with seven sides, the nonagon with nine sides and the hendecagon with eleven sides.

Durer's constructions

Through the ages people engaged in practical design have devised methods for constructing these three figures to an acceptably accurate approximation. The constructions below are taken from Albrecht Durer's "Underweysung der Messung mit dem Zirckel und Richtscheyt", usually known in English as "The Art of Measurement," published in 1525.

Historically, Durer has achieved equal respect as an artist and as a mathematician, and "Der Messung" reflects both honors, in that Durer usually distinguishes between those constructions which are provable and those which are useful approximations for craftspeople. For example, he gives one construction of a pentagon that is approximate and another by the ancient Greco-Roman astronomer Ptolemy that is accurate. As a craftsman (Durer ran a print shop in Nuremberg, Germany in addition to his artistic endeavors) he recognized the need for practical approximation and as a mathematician (Durer traveled to Italy to confer with the most noted mathematicians of the 16th century) he respected geometric validity. Durer may or may not have developed the constructions below, but he was the first to present them in print.

31)   Draw a regular heptagon.

This is a particularly elegant construction. It uses a length one-half that of the side of an inscribed equilateral triangle to determine the length of one side of a heptagon in that same circle.

From a point on the circle draw an arc passing through the circle center and intersecting the circle at each end of the arc. Connect the two intersections with a chord line. Note that this chord is one side of an equilateral triangle inscribed into the circle. Bisect the chord and use this distance to divide the circle.

This division will go into the circle seven times, falling short of closure by 0.78°. For most visual applications this error of closure is acceptable.

32)   Draw a regular nonagon.

Durer's method for drawing this nine-sided figure is less direct, but produces a similarly precise approximation.

0.78
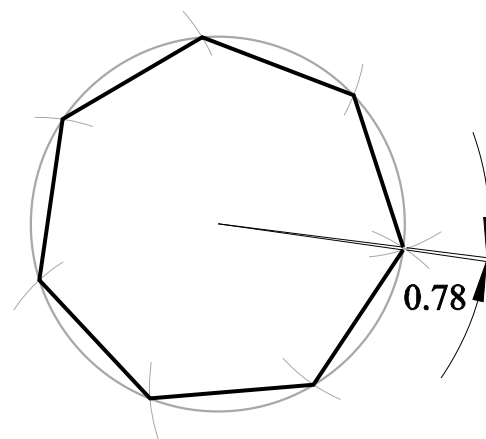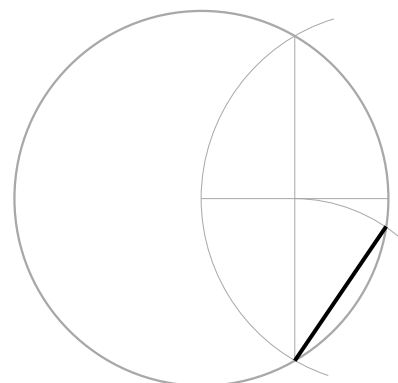
Divide a circle into six equal arcs as in construction #29. From every second dividing point strike arcs of the circle's radius to pass through the circle center and circumference creating a three-lobed figure. In one lobe of the figure draw a radius and divide it into three equal segments by using construction #9.

Using the division point nearest the center, draw a concentric circle one-third of the radius of the original circle. The chord lines drawn within each lobe to connect the intersections of the inner circle with the lobe arcs approximate one side of a nonagon inscribed in that circle. Bisect the remaining three arcs to distribute the approximation error and ensure closure of the figure.
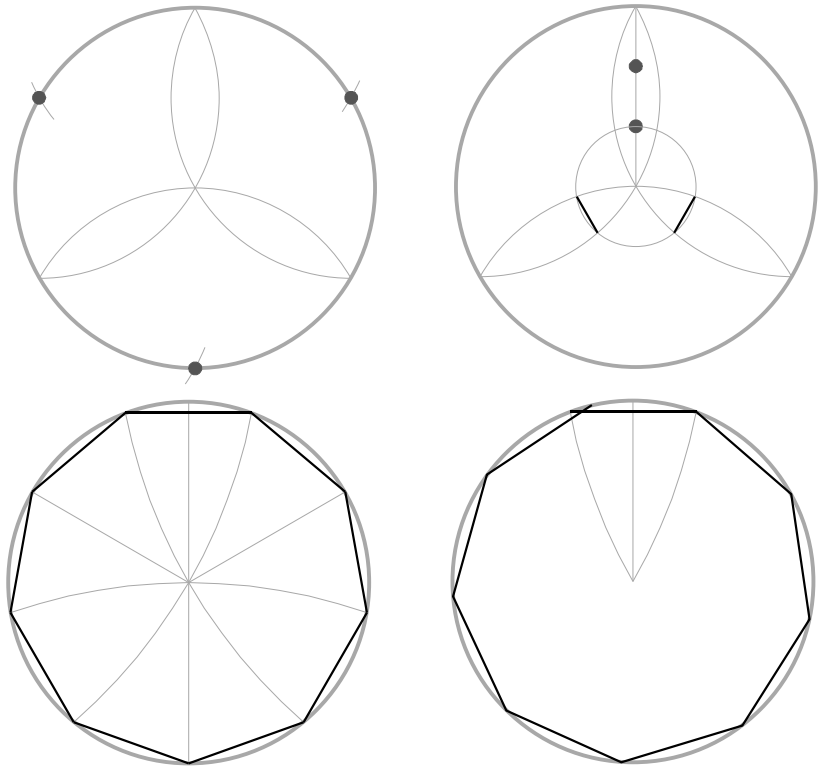

*33)   Draw a regular hendecagon.*

Durer's method for an 11-sided figure is straightforward: he asks only that one set the compass to a length one-eighth longer than half the radius. As the very small error of closure demonstrates, this is an exceptionally accurate construction.

Rednaldini's rule

A little over century after Durer published these constructions a mathematician and Italian aristocrat Carlo Renaldini garnered a small place in history by developing a construction method for approximating regular polygons of any number of sides*.*

1°
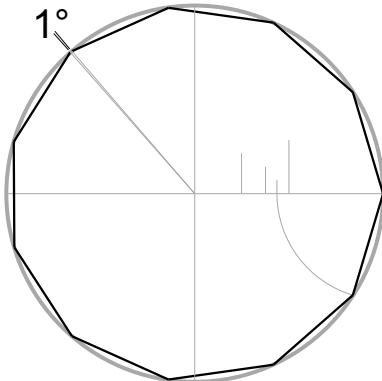
Rednaldini's method, while consistent is not all that accurate attaining a 4% error before leaving the teens and eventually surpassing 8%. His construction for the nonagon yields an error of closure of about 1%, comparable to Durer's construction. However, Durer's method distributes the error in order to make it visually acceptable.


3°
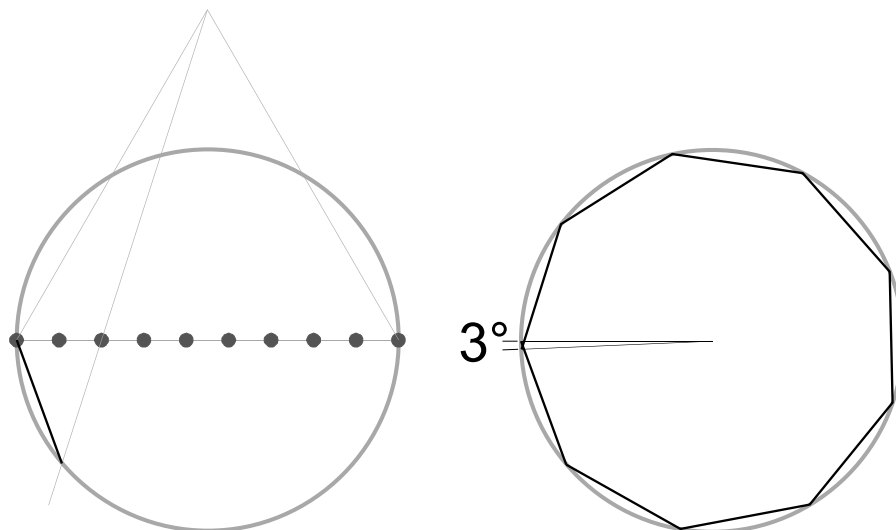
35)   Draw a regular **n**-gon.

Divide the diameter of a circle into equal segments of the number of sides of the desired polygon. Using this same diameter as a base construct an equilateral triangle using construction #27. From the apex of this triangle draw a line through the second point of division and continue this line to intersect with the circle. The distance from the nearest end of the diameter to the point of intersection approximates the side of the desired polygon.

Though its mathematics are minor – more coincidental than significant – Rednaldini's method is one of a long line of attempts to link clusters of relationships under a single principle. This tendency is essential to scientific theory and equally essential to building belief systems in philosophy and religion and to higher expression in the arts. When such an attempt is successful and hits upon truth deeper than happenstance, it generates an emotional power to rival any in human experience. This translates into a faith in knowledge that drives the life and efforts of the scientist, the artist and the priest.

The next chapter looks at examples of geometry working to bolster religious belief and at some of the significant discoveries that the sacred applications of geometry brought to the development of mathematics.
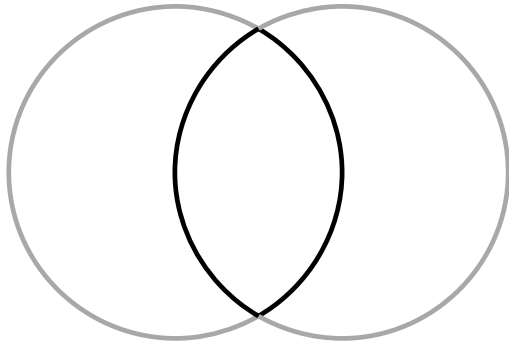
One of the most studied figures of ancient times was the *vesca picsis*. *This elegant Latin translates as "fish bladder" and describes the football-like shape formed by two circles passing through one another's centers. The fish bladder intrigued both practical and religious geometers.*
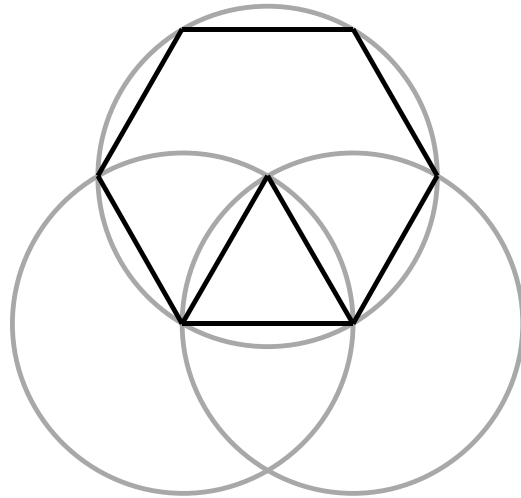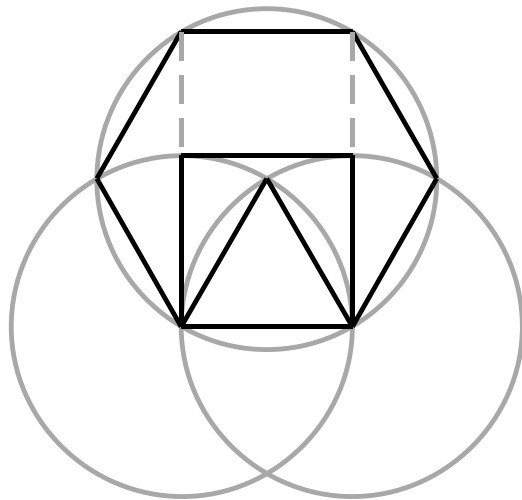
On the practical side the vesca pisces served as the basis for "rusty compass" construction of the equilateral triangle, the square, the pentagon and the hexagon. Rusty refers to the fact that the compass remains fixed during the entire construction and this, as a rule, ensures the craftsperson greater precision during layout. It also ensures speed. The Islamic architect used intricate layouts of equal interlocking circles as guides to quickly design tiling patterns for mosque walls.

On the religious side this simple form contained the makings for deriving most of the important polygons and proportions used in religious symbolism.