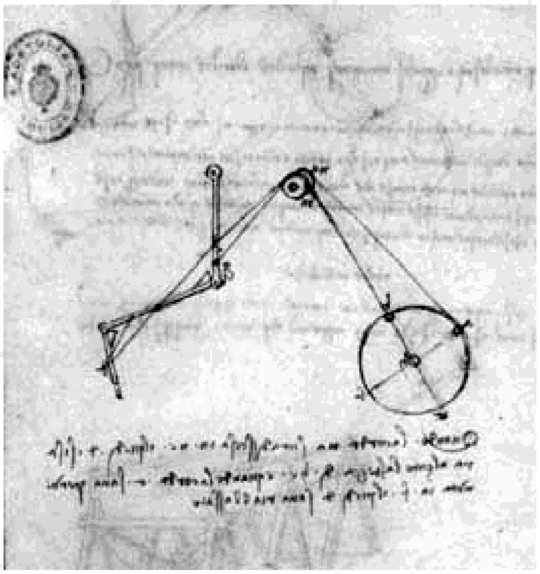
Chapter 6

Methods II: Arcs & Circles

A circle is the set of all points equidistant from a point. That point, the circle center, and that set of points, the circle's rim, account for the circle's geometric power. From the circle's hub and its rim visual forces wheel outward, steered by lines and angles, into those spaces under the designer's authority. The circle puts in place the visual mechanics upon which the designer may build motion and continuity.



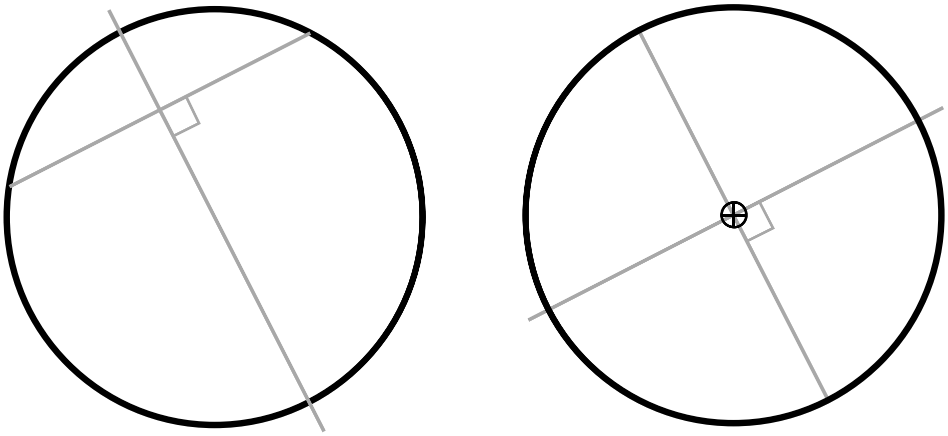
Leonardo Da Vinci, a study for motion of a robot leg. Among Da Vinci's many un-built inventions was a mechanical replica of a human whose motions were to be directed by internal mechanisms. This sketch shows the inventor working out a method for translating a single rotary motion into the multiple movements of a leg.

Chord Constructions

The most important construction on a circle or arc is to locate its center. With the center in place it is a simple matter to draw a radius; a radius, in turn, makes it possible to draw a tangent to the circle. Both of the constructions below establish the circle center by using the perpendicular bisectors of chords.

13)   Locate the center point of a given circle.

Draw any chord on the circle and then construct its perpendicular bisector. Extend this line to form the diameter of the circle:

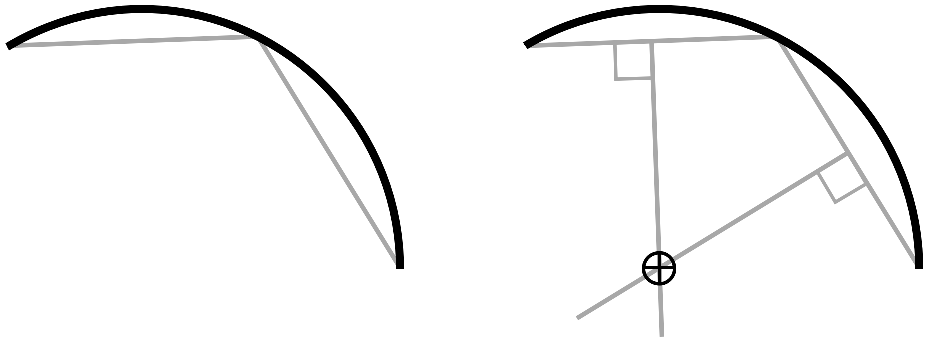


On this diameter construct the perpendicular bisector to draw a second diameter. Since both diameters, by definition, pass through the circle center, their intersection marks that center. The principles at work here are that the perpendicular bisector of a chord is always a radius of the circle, and that by definition the radial line passes through the circle center.

14)   Locate the center point of a given arc.

Unlike fully closed circles arcs may not possess a diameter, and thus their centers cannot be determined by bisecting a diameter. An arc, however, always has chords on hand to determine its center. This method uses two intersecting radii to establish the arc center.

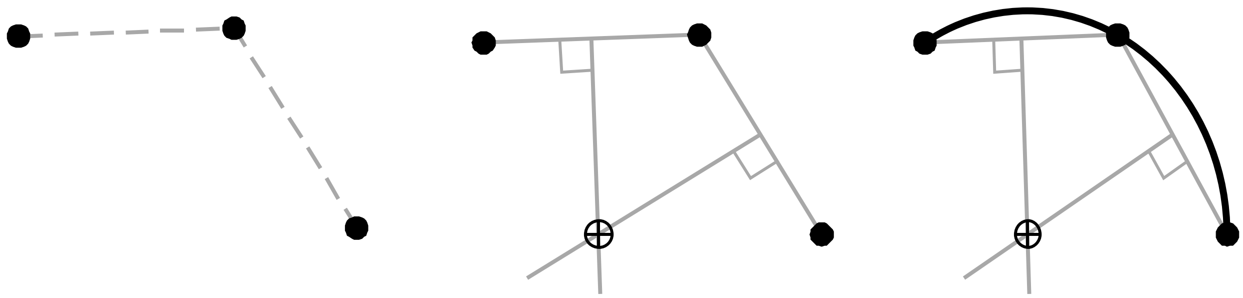
Draw any two chords on an arc and then construct their perpendicular bisectors. As radii these bisectors will both pass through the arc center with their intersection designating that point.



Draw any two chords on an arc and then construct their perpendicular bisectors. As radii these bisectors will both pass through the arc center with their intersection designating that point.

15) Find the arc or circle of three given points.

In the previous diagram the two chords were drawn adjacent and co-terminal, that is, they share a common endpoint. Consequently a designer needs only three points in order to define the two lines. Any set of three points (with the exception of co-linear points) will form two adjacent chords of an arc. That arc and its corresponding circle will be unique; one and only one circle will connect the points. The construction below will determine that arc:



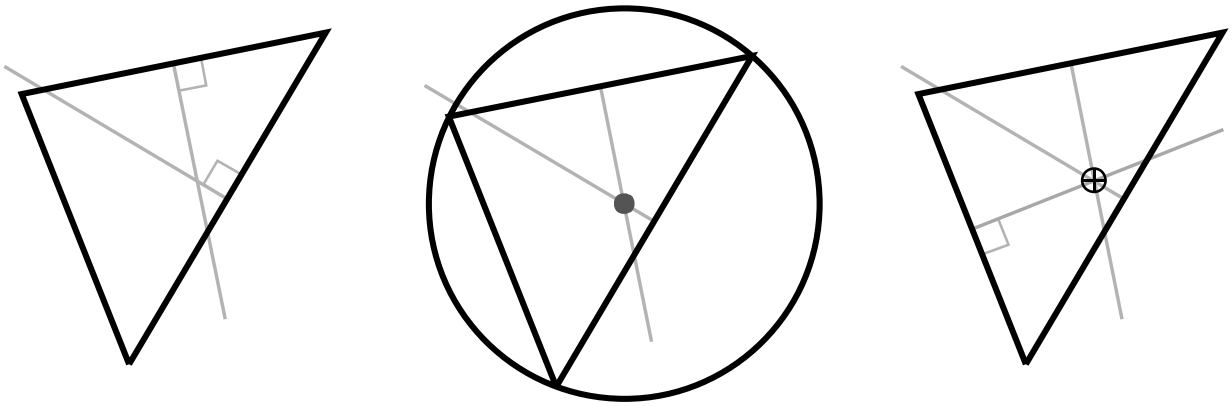
Draw two lines to connect the three points. These lines are two chords of the desired arc. Draw their perpendicular bisectors and from the point of intersection of the bisectors draw the arc or circle through the points.

Specifying the location of three points on a circle is one of the three most common methods designers use to denote a unique circle. The other two utilize a center location plus a radius length or the locations of the endpoints of the diameter line.

16) Circumscribe a given triangle.

Since the three corners, or vertices, of a triangle are points, any triangle can have its vertices joined by a circle through a procedure is known as circumscribing. Once circumscribed each side of the triangle becomes a chord of the circumscribing circle.

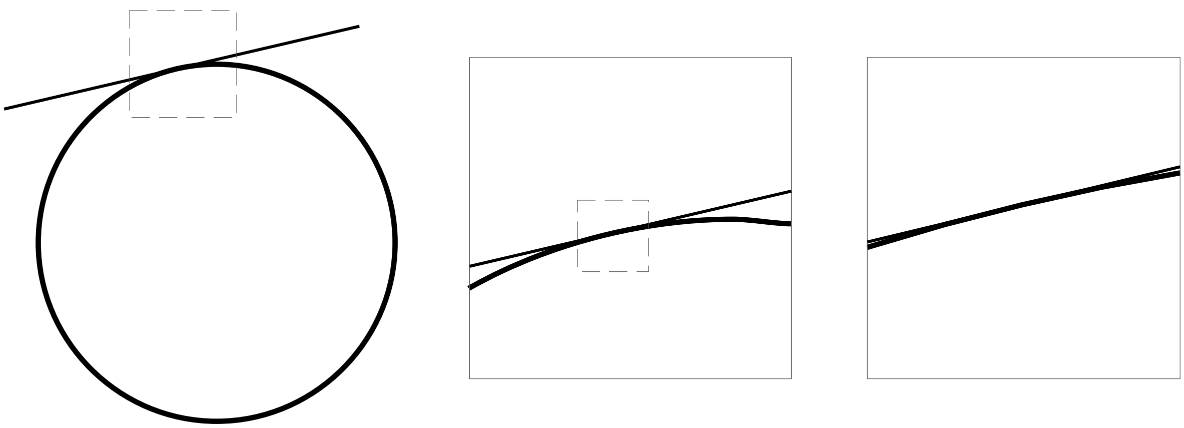
The construction for circumscribing a triangle is in principle the same as drawing a three-point circle:



Draw perpendicular bisectors from two sides of the given triangle and from the point of intersection of the bisectors draw the circle through the vertices. The bisector of the third side of the triangle will also meet at the center of the circumscribing circle.

Tangent Constructions

A tangent line intersects an arc at one and only one point, but that is a difficult point to visually determine with any precision whatsoever. The tangent line and arc merge gradually until they appear to join well before the point of tangency is reached.

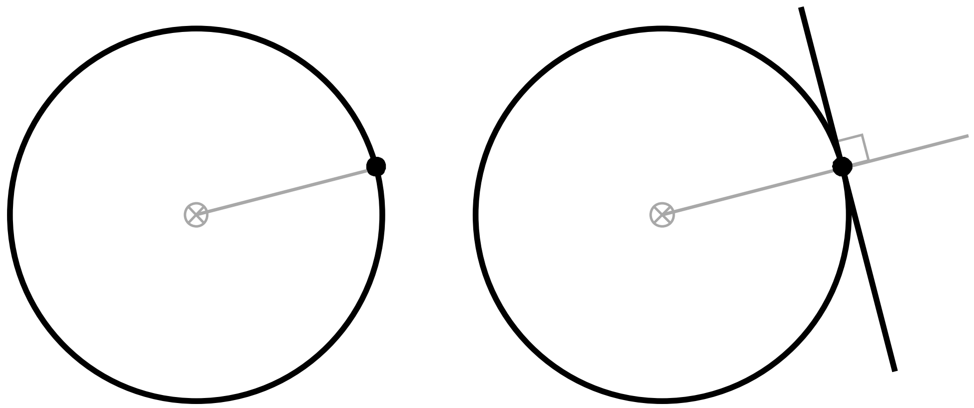


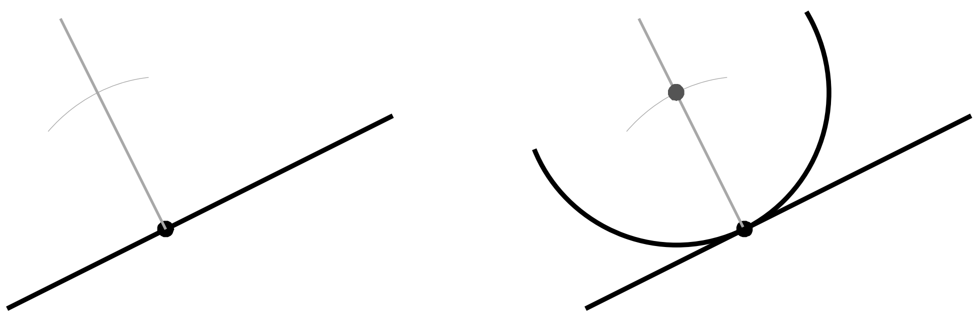
Trying to “eyeball” a tangent line is doomed to geometric failure. Zooming in on a tangent point does no real good except to highlight this difficulty.

On the other hand placing the point of a compass on an intersection or on an endpoint an easy and necessary aspect of construction drawing. To locate points of tangency construction methods like those below rely on determining points by means of intersections between lines and arcs.

17) Draw a tangent from a given point on a circle.

Tangents are perpendicular to the radius of a circle at the point of tangency, so it is necessary to find the radius that passes through the given point prior to constructing the tangent. To do this use construction 13 or 14 to locate the center point and then draw a line through this center and the given point. From the given point draw a perpendicular to this radius. The radius and the tangent both intersect the circle at the same unique point.



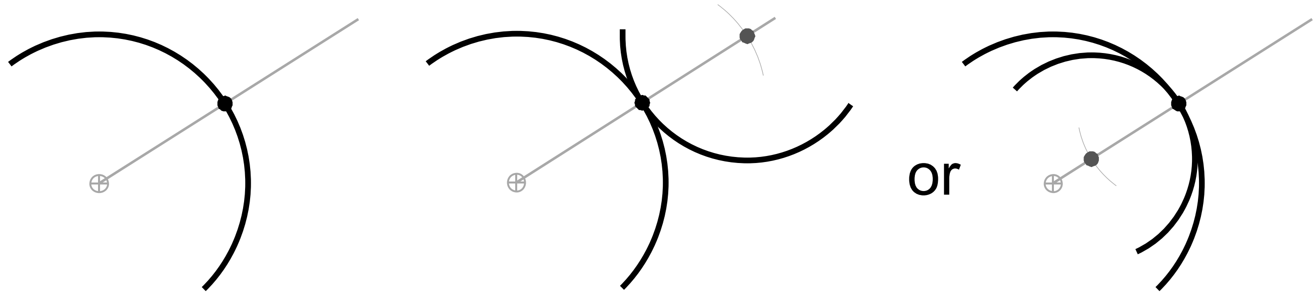


18) Draw an arc of given radius tangent to a point on a line.

Construct a perpendicular from a point on a line. Open the compass to the given radius and, from the same point, draw an the perpendicular. Without changing its opening set the compass on this intersect and draw the tangent arc through the given point.

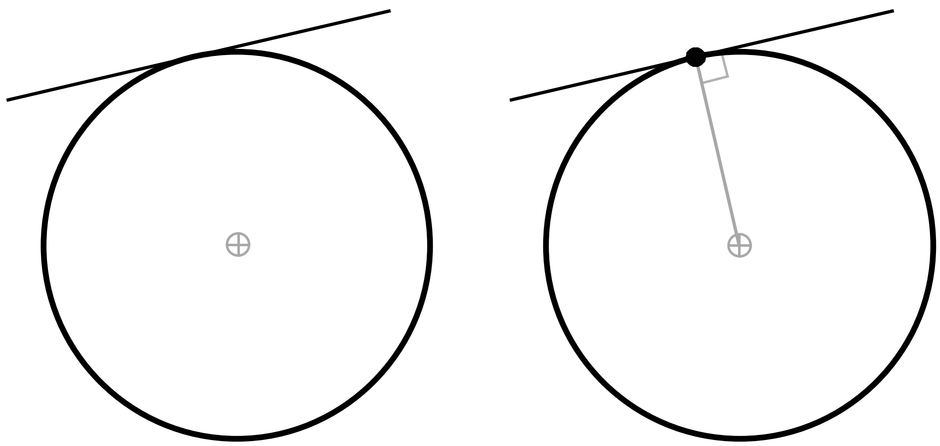
t19) Draw an arc of given radius tangent to a point on an arc.

A related construction can give an arc tangent to a point on another arc. Draw a radius through the point on the arc and extend it beyond the curve. This extended radius functions similar to the perpendicular in the previous construction, so continue by marking the center of the tangent arc as on the perpendicular. Note that there are two options for tangency, internal or external, to the given arc.



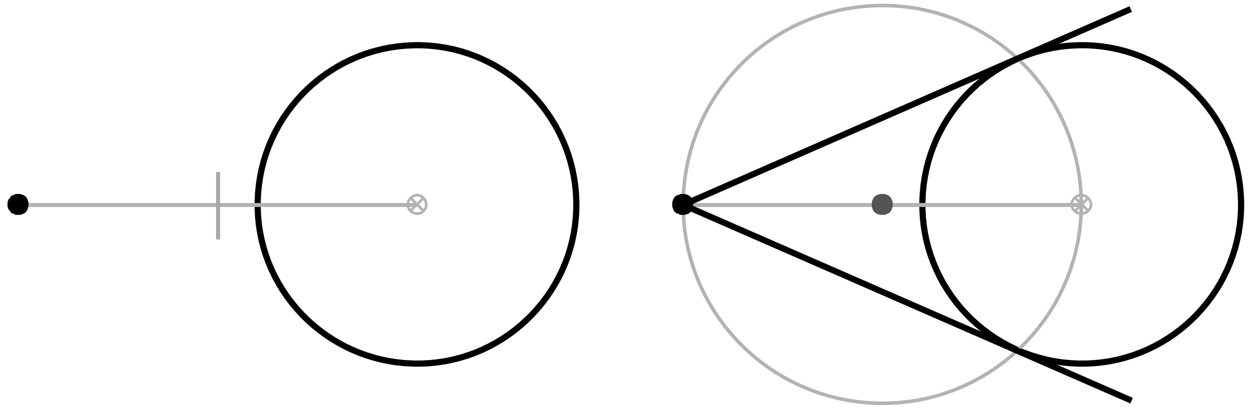
20) Locate the point of tangency of a line to a circle.

Finding the radius that passes through it can solve the visual problems of spotting the tangent point. By the same principle governing constructions 17 and 18 this radius will be the perpendicular from the circle center to the tangent line. Locate the center, and then draw a perpendicular from the center to the tangent line:

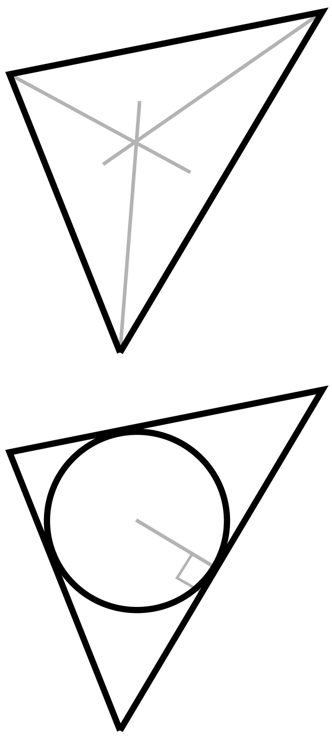


21)   Draw a tangent from a given point to a circle.

Draw a line between the given point and the center of the given circle. From the mid-point of this line draw a circle with this line as its diameter. Connect the given point with the intersections of the constructed circle and the given circle:



22)   Inscribe a circle into a triangle.



A circle tangent to all sides of a triangle is defined as inscribing the triangle. It is possible to derive the construction of such a circle from the previous construction.

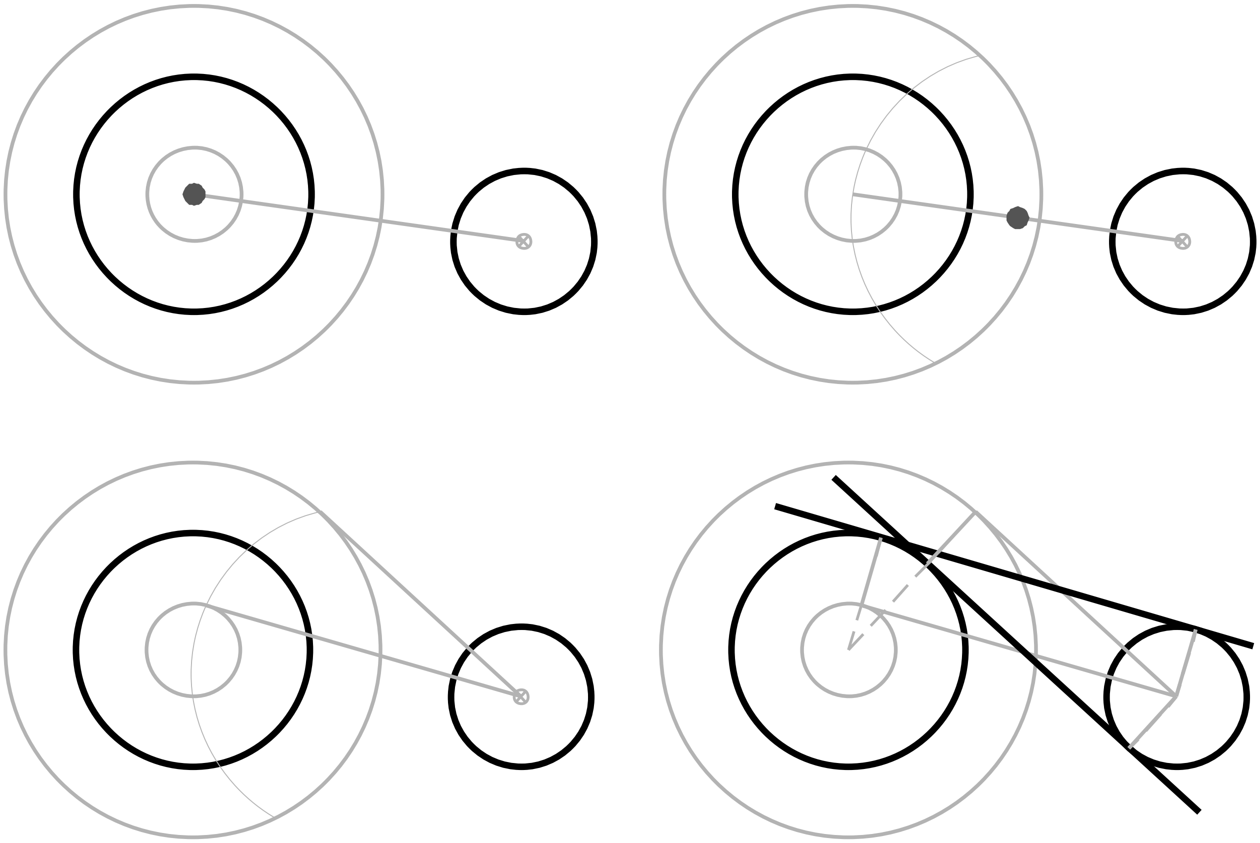
Note how the line between the given point and the center of the given circle bisects the angle of the two tangent lines. The vertex of two adjacent sides of a triangle is equivalent to the given point and the adjacent sides are similarly equivalent to the constructed tangents. By this it is known that the center of the inscribed circle must lie on the bisector of any two adjacent sides of its host triangle. In order to be tangent to all three sides of the triangle it is necessary for the center of the inscribed circle to lie on the bisectors of all three vertex angles.

Construct bisectors to at least two angles of the triangle. The bisectors will intersect at the center of the inscribing circle. A perpendicular dropped from that point onto any side of the triangle determines the radius of the circle and its point of tangency.

23)   Draw the tangents common to two circles.

This construction is similar to construction 12 in the previous chapter. It requires a helper construction akin to construction 21.

Choose the larger of the two circles and draw two concentric circles whose radii are, respectively, larger and smaller than that of the host circle by a difference equal to the radius of the smaller circle. From the center of the smaller circle draw tangents to each

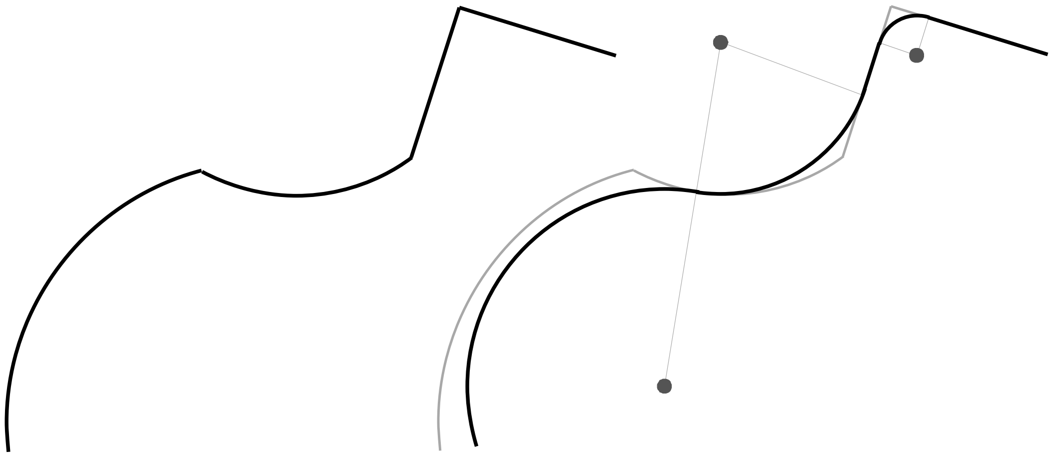


Continuity

Continuity provides smooth flow from one element to another. Any sense of stoppage, such as at the vertex of an angle, will break continuity.

A good way to picture continuity is to imagine a series of connected lines and curves as if they described a road, and then imagine driving along that road. Where the road angles into a sharp corner the car must come to a halt or at least slow to a crawl; round off the corner into a curve and the car can easily move on with hardly slowing at all. Similarly a viewer's eye will stop and start, stop and start repeatedly when following a multi-angled line, but will change direction smoothly if the angles become curves.

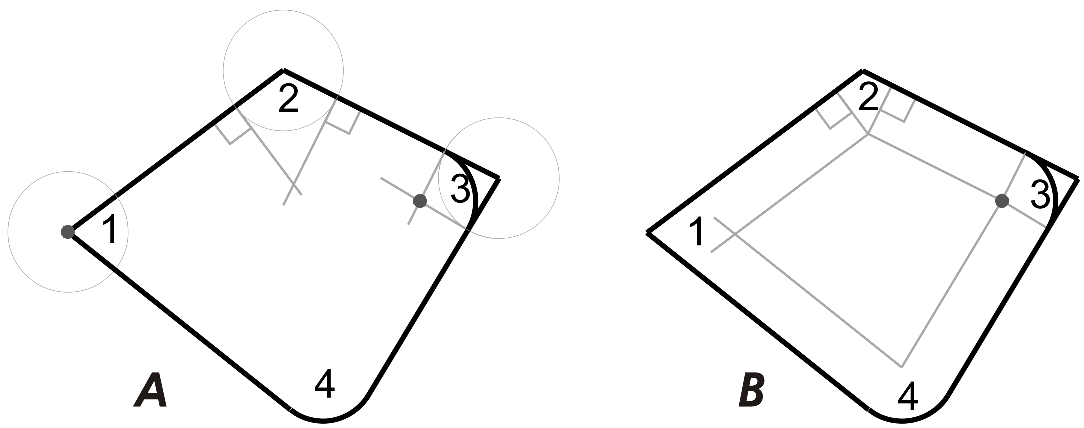
n constructive geometry continuity results when arcs and lines are tangent at their points of juncture. This is called tangent continuity and is the only form of continuity found in traditional geometry. In the first example below two arcs and two lines meet to form a discontinuous path. The second example reinterprets that path by ensuring tangent continuity between each of the three meeting points. An intermediary arc, called a fillet, provides continuity where the two lines meet and tangency is otherwise impossible.



One common design application of tangent continuity is the filleting of angles, especially the vertices of polygons. A second, and particularly elegant, application occurs in the design of type faces.

24) Fillet the vertices of a polygon.

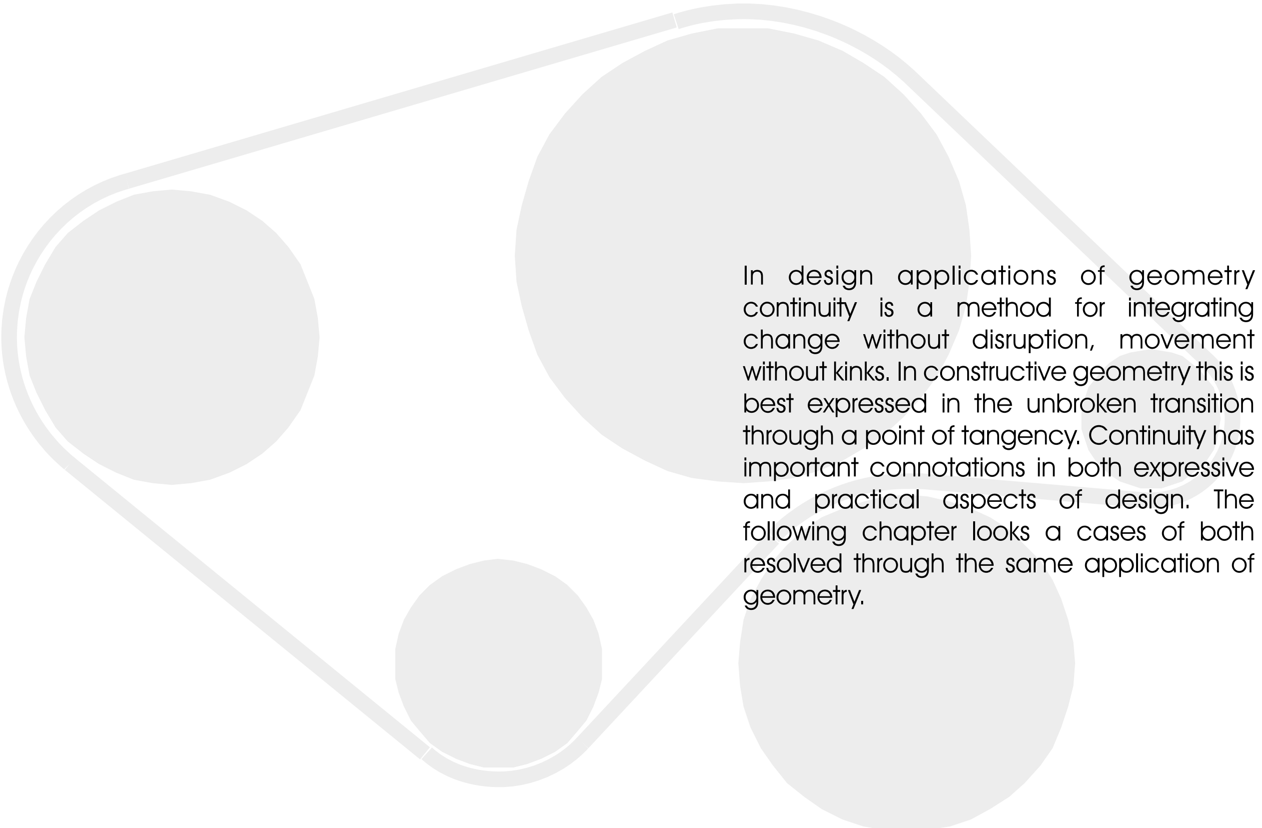
To fillet (pronounced FILL-et) is to round off the corners of a linear figures with a circular arc. Smooth, round transitions soften the sharp edges of the vertices. There are two methods to fillet corners: method A below uses tangent points setback equal distances on both sides of each vertex, while method B uses tangent arcs of



25) *Fillet the vertices of a polygon.*

A. To create a fillet by the setback method draw a circle or arc whose radius is equal to the desired distance of the point of tangency from the vertex point. From both intersections of the arc with the vertex sides draw perpendiculars to the inside. Set the compass at the intersection of the perpendiculars and draw an arc tangent to the vertex sides connecting the base points of the perpendiculars.

B. To fillet by the equal arc method first draw interior lines parallel to the sides of the polygon. From the points where these parallels meet draw perpendiculars to the sides of the polygons and, from the same points, draw arcs tangent to the sides of the polygon. In practice only one perpendicular needs to be drawn, as it establishes the radius common to all of the filleting arcs.



In design applications of geometry continuity is a method for integrating change without disruption, movement without kinks. In constructive geometry this is best expressed in the unbroken transition through a point of tangency. Continuity has important connotations in both expressive and practical aspects of design. The following chapter looks a cases of both resolved through the same application of geometry.