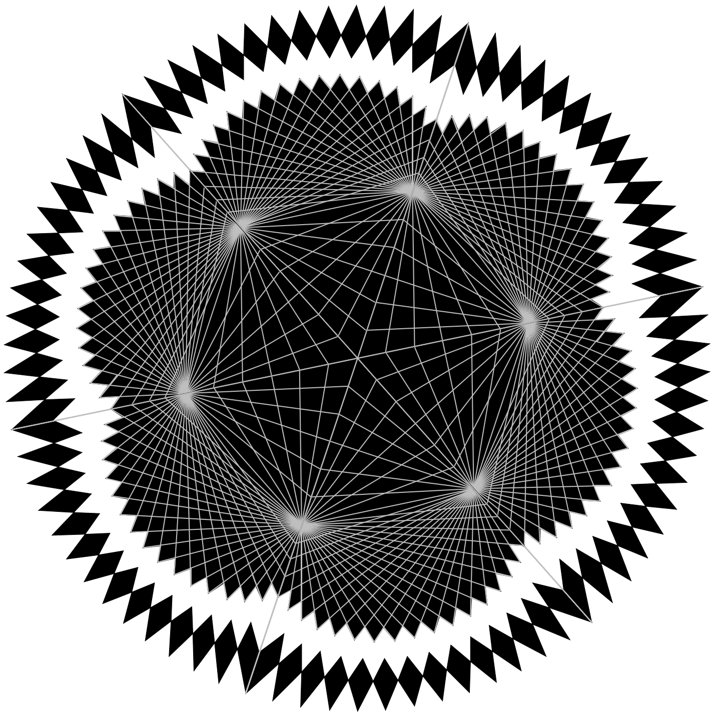
Chapter 2

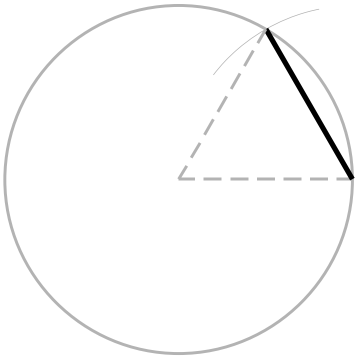
Elements of Construction



Above left: CAD construction of the pattern of a crop circle created near Wiltshire, England. Crop circles appear every summer as tromped down areas of grass crop in fields throughout the world, but most concentrated in England. Many of these circles are masterpieces of constructive design.

Theory of Constructions

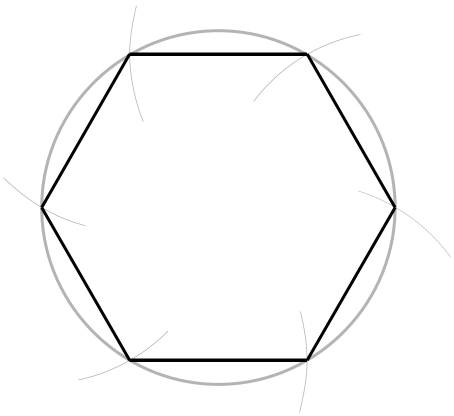
In geometry construction denotes the building of mathematically consistent geometric figures with string lines and stakes or their later surrogates. This means, for example, that the procedure for drawing a regular hexagon should logically and necessarily yield a figure with six equal sides and six equal corner angles. The appearance of equal sides and equal angles is not enough: the idea is that the construction procedure, discounting human error, provides the correct steps to a theoretically perfect hexagon.



Since a humanly executed process cannot be perfect, it is said that the construction is valid or consistent, that is, perfect in theory if not in execution. Constructive geometry embraces all of those figures and relationships that in theory can be realized using only strings and stakes.

Left: The crop circle on the previous page was designed by radiating lines from the six points of a hexagon. The hexagon itself was constructed by dividing the circle by its radius, which goes into its circumference six times. This is not some esoteric or magical relationship as some ancient priests may have wanted their followers to believe, but a necessary outcome of the relationships inherent in triangles and circles.

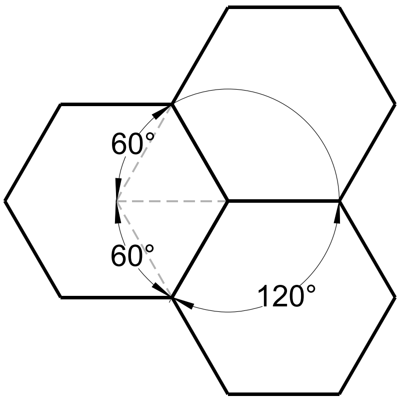
Marking a line on the circle equal to its radius defines an equilateral triangle with the two radii meeting the endpoints of the line. Since the sum of the angles of a triangle equals 180°, each angle of the equilateral triangle will be 60°. Six and exactly six such triangles will fit around the center of the circle whose circuit is 360°. The vertex angle of the hexagon is double that of the equilateral triangle (below left). Consequently, exactly three hexagons fit around a point.



The methods of classical construction replace the string line with a compass and straightedge in order to carry out the constructions accurately on paper. All of the geometric problems that concerned Greek, Roman, Arabic, and Renaissance geometers were investigated using only constructions derived with these tools. Until the development of CAD programs in the 1960's drafting and engineering drawing relied primarily on those tools.

- ***Measure and Number***

When executing constructions, a ruler is most often used as the straightedge, but not for measuring. Pure geometry uses proportion and not measure. The ideal straight edge possesses no markings.

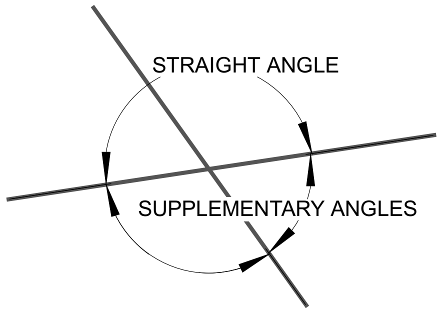


Number, however, does matter. Strictly speaking, geometric procedures apply number only when specifying units of proportion. The slope of a line, for example, may be denoted as 3/5. This means that for every 5 units a line moves forward it rises 3 units, or, put another way, for every unit it moves forward it rises 3/5 of a unit. Measure does not affect this slope: it remains the same no matter if the units are millimeters, inches or miles. What matters is that number denotes the proportion of the rise of the line to its forward run.

When crafting physical objects, however, measure becomes inmportant. In the physical world objects have absolute scale as well as relative scale. If a person is 5 feet - 8 inches tall, then that person may be short or tall relative to his or her friends, but absolute (or *world*) height remains fixed at 5 feet - 8 inches tall. Measure specifies this by assigning a precise distance in the world to each unit of proportion.

In the world of computer design the distinction between measure and proportion is especially important. On the one hand modeling programs concern themselves with the consistency of proportion among the elements of the model. In the computer, the relative scale among elements is purely numerical, i.e., proportional. On the other hand CAD programs assign measure to the figures they produce. This is because CAD programs have been developed for projects that are to be built in the physical world and to produce drawings to aid in that building. Within the computer, though, models remain in the virtual world of number.

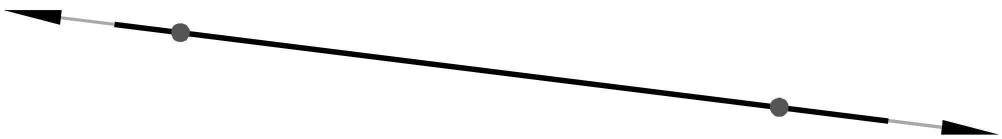
Elements



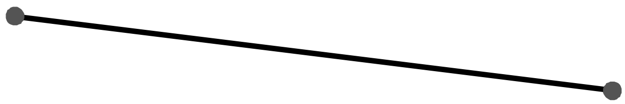
Constructions derive from a few basic elements: lines, arcs and points. These elements and their relationships account for the myriad of forms and ideas generated by the constructive process.

1) Lines and angles

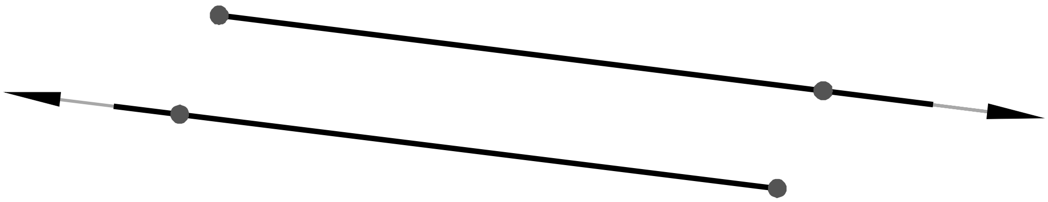
Strictly speaking, a line in constructive geometry is straight and it can be extended indefinitely. The position and direction of a line is determined by two points in space, but the line continues on track well past these points:

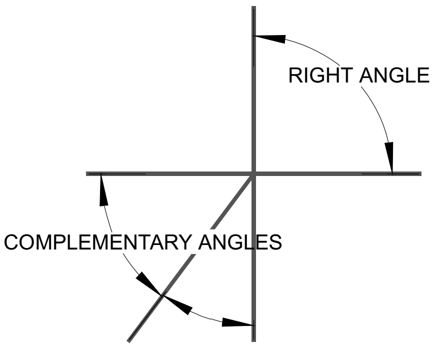


Should the actual drawn line end at these points, or any other points, it is referred to as a line segment:



A line with only one end point is termed a ray. This end point serves as the origin of the ray and the second point determines the direction of the ray. By definition a ray can extend indefinitely beyond this second point. The term ray segment is sometimes used to denote a directional line segment. In an angle, for example, the line segment originates at the point, or vertex, of the angle. Degree measurements or slope values determine its direction.

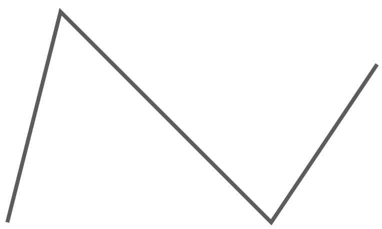




In construction two lines may or may not intersect. If they do not intersect they are, by definition, parallel. If they do intersect, then they will form angles. In constructive geometry certain angles are, practically speaking, more important than others. These are the straight angle (180 degrees) and the right angle (90 degrees), especially the right angle. When two lines meet at right angles, they are defined as perpendicular to one another.

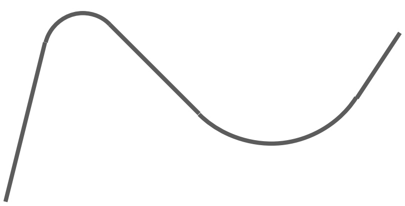
Constructing perpendicular lines is the most basic of all of the constructive drawing operations. Perpendicularity shows up again and again in constructive drawing.

Geometers note some important relationships in the intersection of two lines. One is that the opposite angles are equal, or, in the argot of geometry, congruent. Another is that adjacent angles are supplementary, i.e., they add up to 180 degrees, so that the size of one indicates the size of the other. A similar relationship exists between the two angles created by the division of a right angle. These are labeled complementary angles.



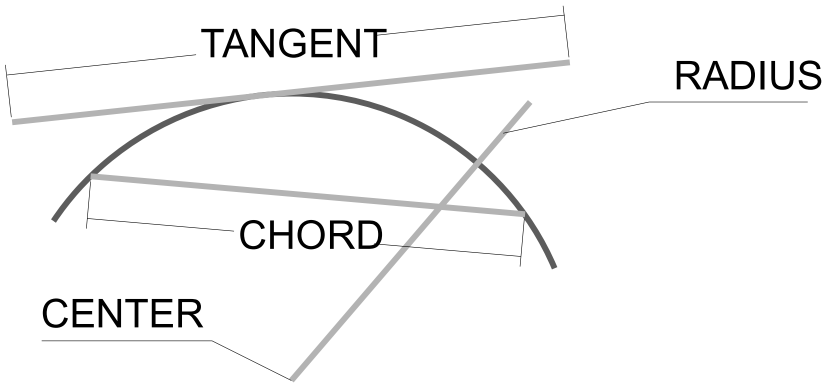
2) Arcs and circles

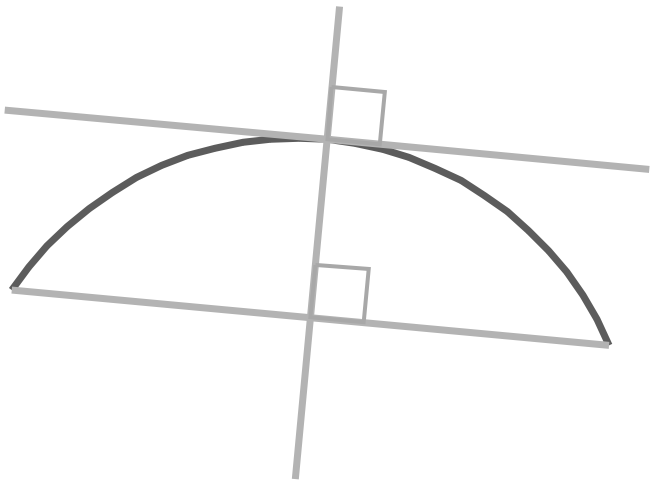
In constructive geometry the most important attribute of an arc is that it defines a set of points equidistant from a specified point, the arc center. Beyond that constructive drawing is most concerned with the relationship of lines to the arc. In terms of visual design, however, the arc provides a gentler contrast to those harsher layouts laden with only straight lines and corners. It substitutes smooth, continuous transitions for the abrupt shifts of direction of angular paths.



The arc and its mother, the circle, are the only true curves in constructive geometry, although constructive processes can plot a whole host of other curves. Plotting a curve means that the geometers cannot actually draw other curves, but that they can *represent* a non-circular curve by setting a series of separate points that lie on the desired curve.

The key lines related to the circle are tangents, chords and radii. A tangent line intersects an arc at only one point, while a chord line intersects (and usually terminates) on the arc at two points. A radius is a ray originating at the arc center:



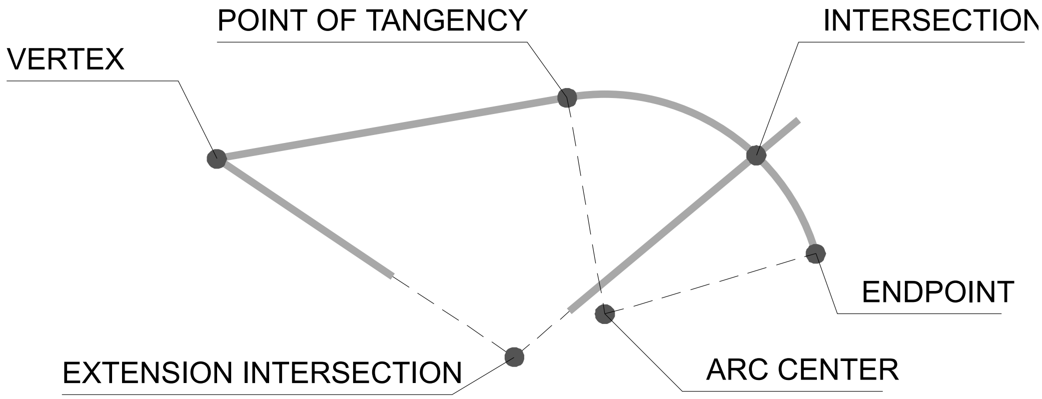


Some special relationships, important to construction, exist between all of these lines. The perpendicular bisector of the chord, for example, is a radial line, which by definition passes through the arc center. Also, a tangent line is always perpendicular to the radius intersecting the arc at the point of tangency.

3) Points

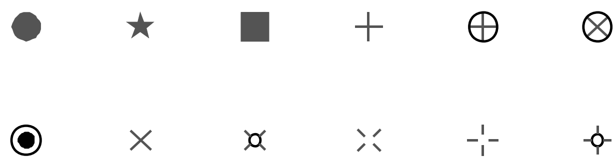
A point is a position in space. In principle it has no dimension and therefore no visual existence except through the interaction of lines and arcs. In fact, the main role of lines and arcs in construction is to determine points in space, i.e., to identify significant positions in the layout.

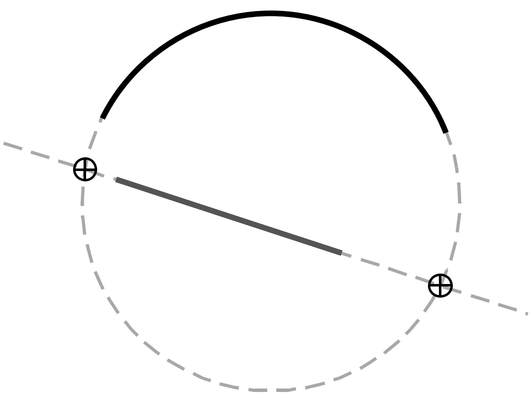
Points may exist as the vertex of angles, the ends of lines and arcs, centers of arcs, tangent points or intersections. In cases like an intersection or a vertex the point is easily seen, but in other cases like an arc center or a point of tangency it is necessary to mark the points with a point object. Though not a point in itself, a point object is understood to represent a point.



The best design for a point object is usually a small circle with two lines, one horizontal and one vertical, crossing at its center. It is both easily seen and precise. However, there may be hundreds of different point object designs depending on the drawing for which a construction is created. In such cases the point object also represents the feature, such as a fence post or electrical outlet, to be set at the location marked by the point object.

POINT OBJECTS:





Points may exist even though they do not appear in a design. Such points are called implied points. A common example is an arc center: it is a vital and effective element of the geometry without being visible.

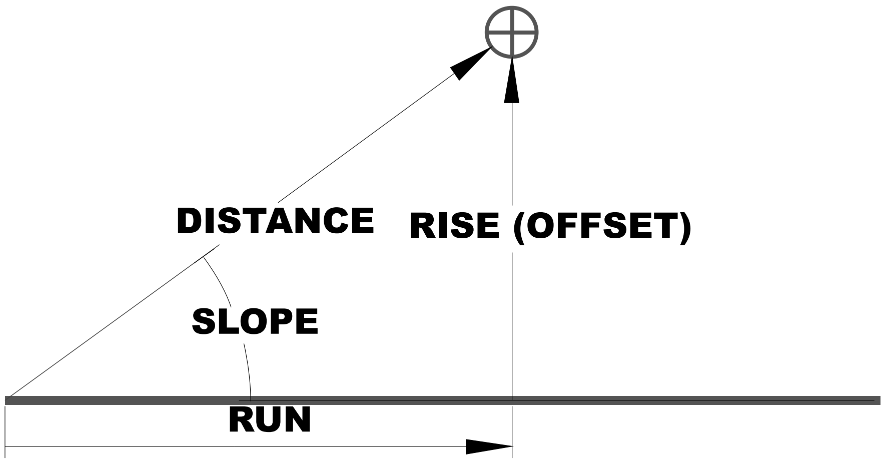
Another example is a point implied by the intersection of two extended line segments and/or arcs. In construction all line segments and arcs are regarded as extendable -- the line segment along its line and the arc along its circle. Where these extensions meet is often of key importance in a design layout.

Types of Constructions

Constructions divide into three categories of which the first two are covered in this chapter: 1) constructions from a baseline and 2) constructions on a circle and 3) plotted curves. Constructions from a baseline include methods for using perpendicular, parallel and angled lines. Constructions on a circle include applying these lines to a circle in the form of radii, diameters, chords and tangents. Plotting constructions comprise most methods for setting points on a non-circular curve.

- ***Construction from a Baseline***

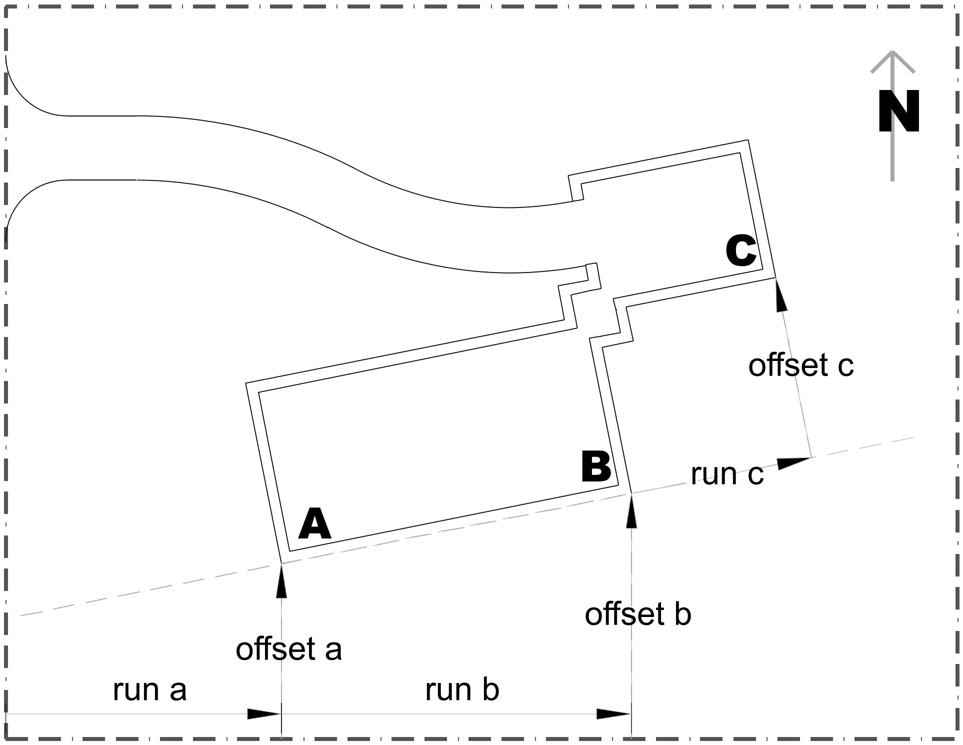
A baseline is a line on a 2D surface that functions as the primary reference from which angle and distances are taken. Typically a point on the baseline will be chosen as an origin, such that any point on the surface can be determined by a distance along the baseline from the origin and a distance perpendicular to the baseline, OR by the distance along a line drawn through the origin at a specified slope (or angle) to the baseline.



When builders stake out a construction site they will first stake the baseline, usually through the center of the building site or along one edge. Most commonly this is a north-south line. They will also select an origin on that line, typically a corner or center point. From these two references the baseline and the point of origin - any location on the site can be specified.

If the terrain of the site is rolling or sloping and the builder must consider vertical elevations, then a third reference is added called a benchmark. All elevations are determined as +/-, higher or lower, than the elevation of the benchmark.

Once the baseline and the origin are set, any point on a plane can be defined in one of two ways: 1) run and offset, or 2) slope and distance. Both methods locate the point by securing two pieces of data: the run/offset method first specifies the distance along the baseline and then specifies the perpendicular offset, while the slope/distance method specifies the slope (or angle) of a line running from the origin to the point and the distance along the sloped line. In the latter method the slope is determined with respect to the base line.

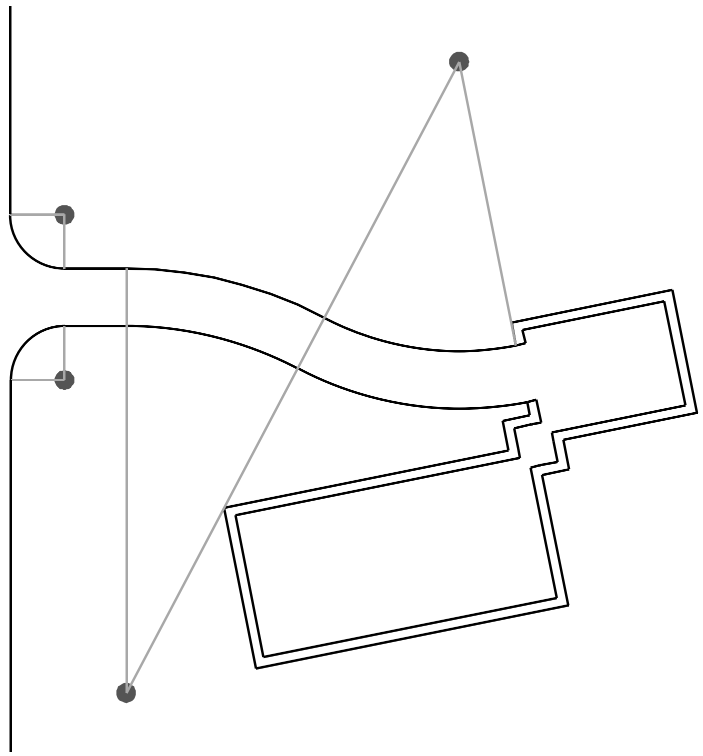


Layouts may begin with a primary origin and baseline, but may incorporate secondary, or relative, origins and baselines as well. The diagram above specifies the position and direction of a house and garage on a lot.

In this diagram the southwest corner of the lot furnishes the point of origin in the layout and the southern boundary defines the baseline. From the architect's specifications of runs and offsets the builder can stake the two corners of the house nearest the baseline. These staked points determine a secondary baseline. On this line a new origin can then be set as the rear corner of the house. From this point a run and distance will specify the rear corner of the garage. Any side of the house or garages can become a relative baseline in order to position light posts, trees and other lot features.

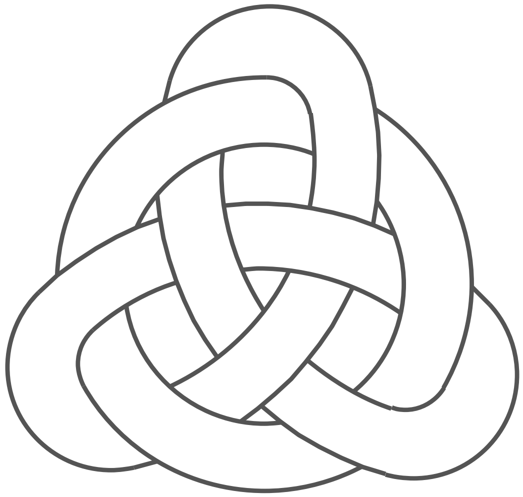
- ***Constructions on a Circle***

These constructions offer visual and practical counterpoint to the straight paths and abrupt turns of angular objects. The layout for the driveway in the above example uses two tangent arcs to provide a smooth and gracious transition between the street and the garage.



Another attribute of the circle is the centered, unified and radiating image of spaces it makes possible: a stark contrast to the tracked and divided spaces defined by lines and angles. As such the circle becomes a primary archetype on which to pattern conceptions of space, and the cropped space of lines and angles yet another. Often sacred geometries seek out symbolic patterns that merge these two primary archetypes into a unified whole expressive of fundamental spiritual ideas.

Continuous arcs describe the looping of a trefoil knot ensnaring the ring in the Celtic design to the right. No straight lines flatten the movement and no abrupt starts and stops jar the form in this design.



Ritual geometry serves symbolic purposes often seeking to acheive harmony among disparate elements. One example is the yantra, a Hindu canonical pattern dating back over 2000 years to ancient India. Translated as "conception tool" yantras may be used variously as meditation aids, talismans and templates for altar and temple design. The downward pointing triangle represents creative energy descending into the world. The circle radiates this energy as the petals of a lotus blossom. The outer architecture is the "earth-city" and signifies the gating of this energy into the concrete, physical world.