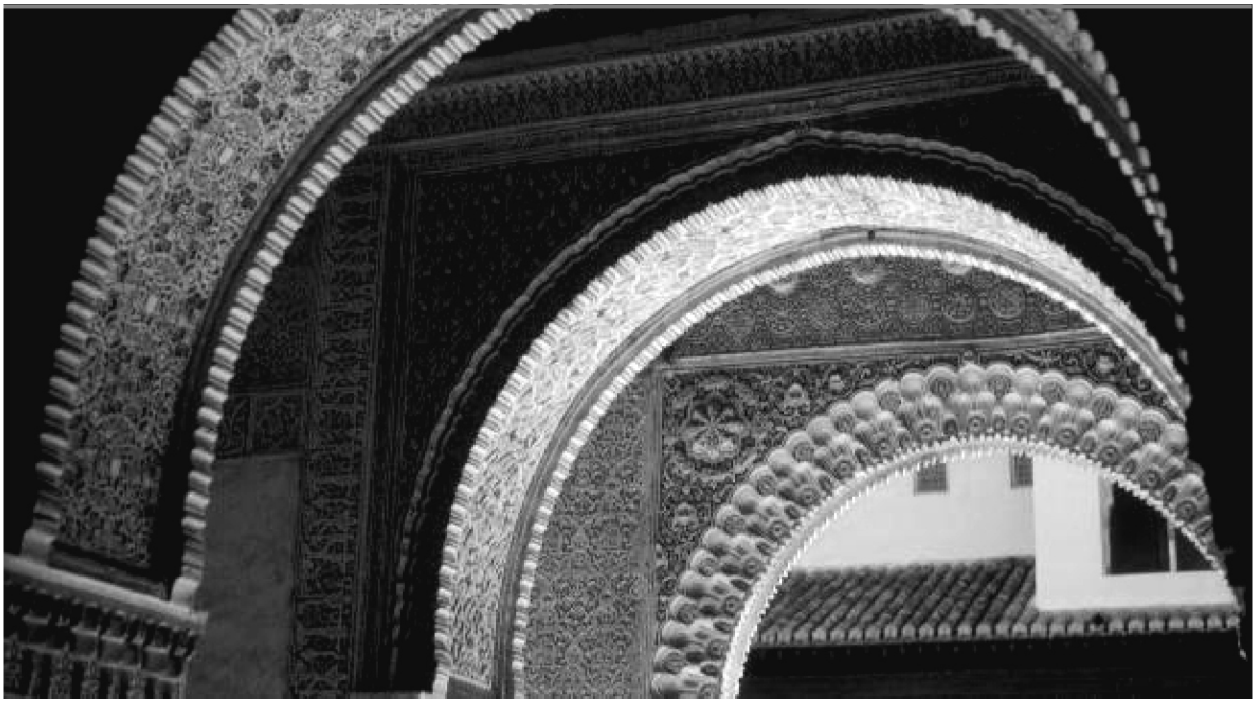
Chapter 8

Symmetry & Tessellation



Pattern on the Plane

Tiling geometry deals with the space filling characteristics of geometric figures. Also called tessellation, from the Latin word for tile, this area of geometry reveals much about the nature of symmetry. Most importantly tessellation highlights the relationship between pattern and symmetry, to what extent symmetry imposes order on arrangements of form.

In many ways this is the most engaging design application of geometry, as tessellation has through history achieved a very high order of elegance and sophistication. Most Non-Western cultures have accorded extensive visual research to patterned art. In contrast to the image making tradition of Western art, huge sectors of the world have engaged in pattern generation as their primary art form to express secular and religious values.

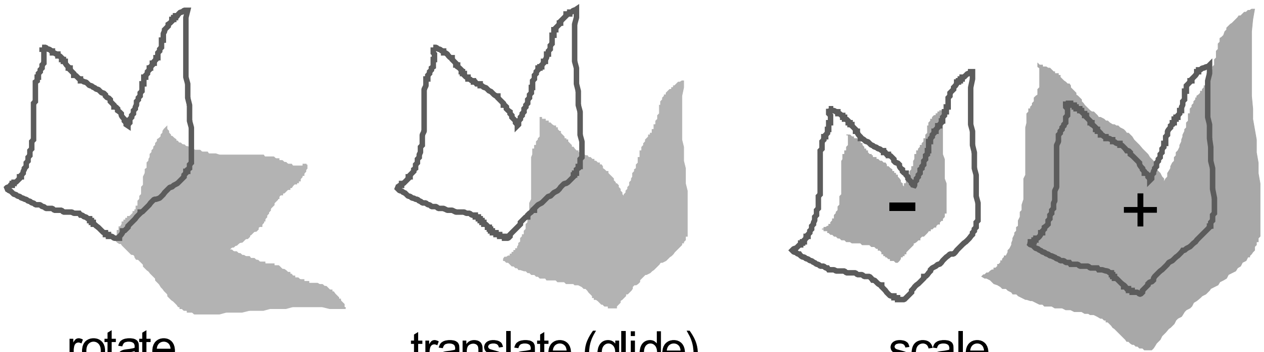
Symmetry is the basis for creating pattern and the masters of pattern were the artists of Islam,as attested to by this detail of arches found at the Alhambra in Granada, Spain. Bereft of imagery by the canons of that religion, Islamic art turned to geometric patterns to exalt their God. Mathematicians, too, joined the artists in their development of pattern. The result was that within a few centuries Islamic tilers had discovered all possible categories of symmetry patterns – a total of 17 – all found at the Alhambra palace.

Transformation and Symmetry

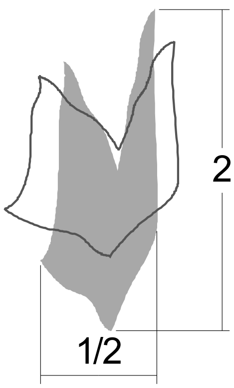
Symmetry is defined as the ability of a shape to undergo a transformation and still retain its appearance. Symmetry can function within a figure or between the figure and its copies in a space. In common speech symmetry refers to the mirroring of the left half of a figure onto its right half. Known to geometers as reflection symmetry and to designers as bi-lateral symmetry, this is but one of a number of potential symmetries effected by transformations on a plane.

transformations

A transformation, loosely defined, is a change in the spatial conditions of a geometric figure that keeps the fundamental shape properties of the figure intact. Rotation is an instance of a geometric transformation. A rotated figure will change directions in space without any change of shape.



rotate translate (glide) scale

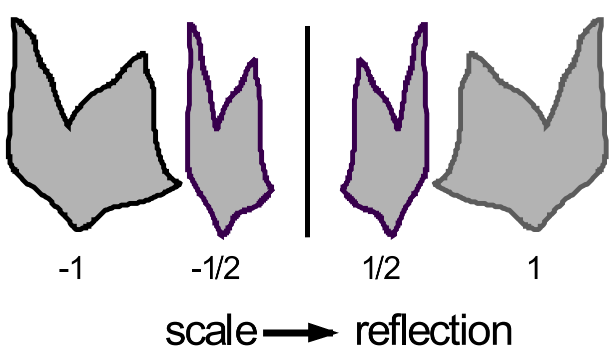


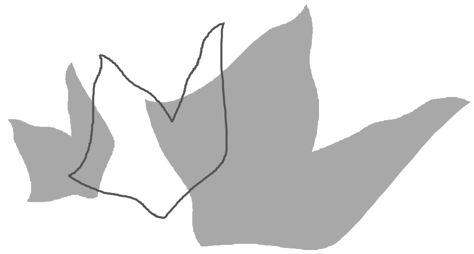
scale (variable)

Similarly, moving a figure changes its position in space, while retaining its shape. In tiling this transformation is known as translation or glide.

Enlarging or diminishing the size of a figure will change the area of space it occupies without modifying the relationship of its parts. This is a scale transformation. This is the one transformation that can produce a change in the shape of a figure, in that one dimension of a figure may be scaled independent of the other to create a change in the figure's internal proportions.

These are the three basic transformations. All others are variations or combinations of these. Reflection, for example, is actually a variation of the scale transformation. An affine transformation combines rotation, translation and scale into one operation.

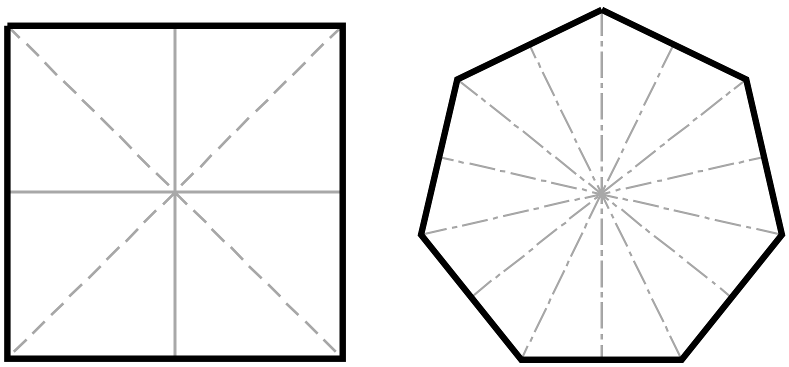




affine

symmetry

A square is said to have four-fold, rotational symmetry, because each 1/4 turn will return it to its original appearance. Similarly a regular pentagon has five-fold symmetry and a regular hexagon has six-fold symmetry. A figure has **n**-fold rotational symmetry if it can be rotated ***1/n*** times and fit back in on itself.

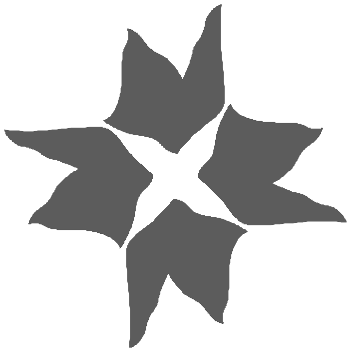


<u>Axes of symmetry</u>. Each line dividing the polygons above denotes an axis across which each shape can mirror itself.

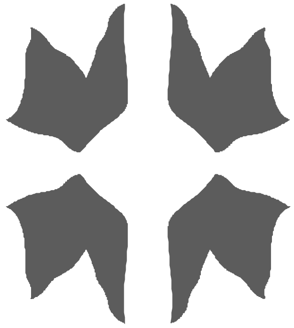
Regular polygons possess a number of axes of reflection symmetry equal to that of its sides. Each diagonal of a square and each line connecting the midpoints of its opposite sides is an axis of symmetry across which the square may reflect. The reflection axes of a regular polygon with an odd number of sides split the figure from vertex to midpoint of the opposite side.

A third symmetry that can appear in an object or patterned field is translation or glide symmetry. A pattern possesses this symmetry if a shape in the pattern can glide across the surface and fit to another copy of itself. Any copy of a shape that remains parallel to the original shape is related by translation.

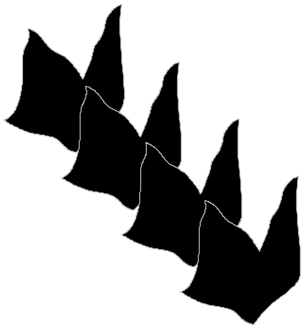
In the early 20th century the noted artist and mathematician M.C. Escher discovered a fourth surface symmetry while studying Arabic tessellations on the walls of the Alhambra, a medieval palace built in the city of Granada when Spain was part of the Islamic empire. This symmetry was labeled glide-reflection, and is in force when the original glides and then reflects to fit its copy.



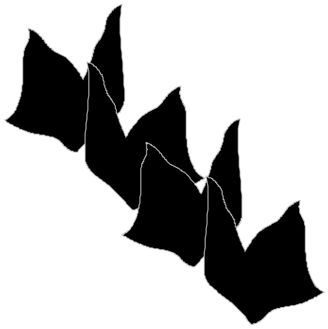
rotation



reflection



glide



glide-reflection

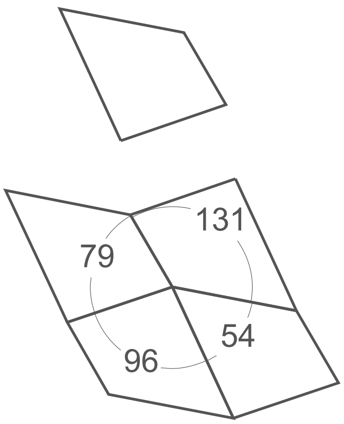
Space Filling

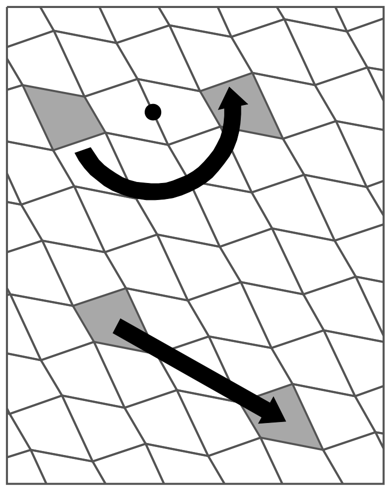
Space filling denotes the ability for a repeated figure to cover an entire surface without any gaps appearing. Clearly a square can do this, as it is the most common shape of floor tile sold. Rectangles, parallelograms, trapezoids, in fact any quadrilateral, will tile a surface without gapping. Since any triangle is half of a quadrilateral and two of the same triangles can fit together to form a quadrilateral, then any triangle can tile a surface.

***Tiling of irregular quadrilaterals*.**

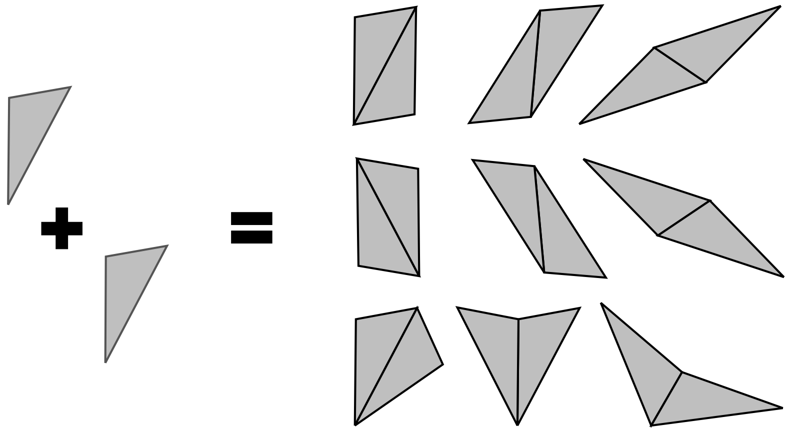
If an irregular quadrilateral and its copies are arranged such that each of the four angles meets around a point, then it will always tile a surface. This arrangement ensures that the angles around a point add up to 360°.

Tiles in a tessellation can relate through one of the four types of surface symmetry. The tessellation to the right incorporates glide and rotation symmetries.

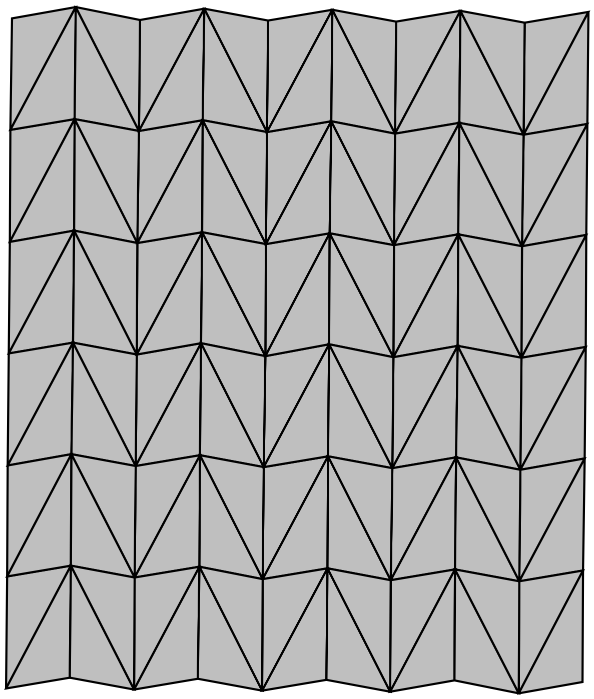


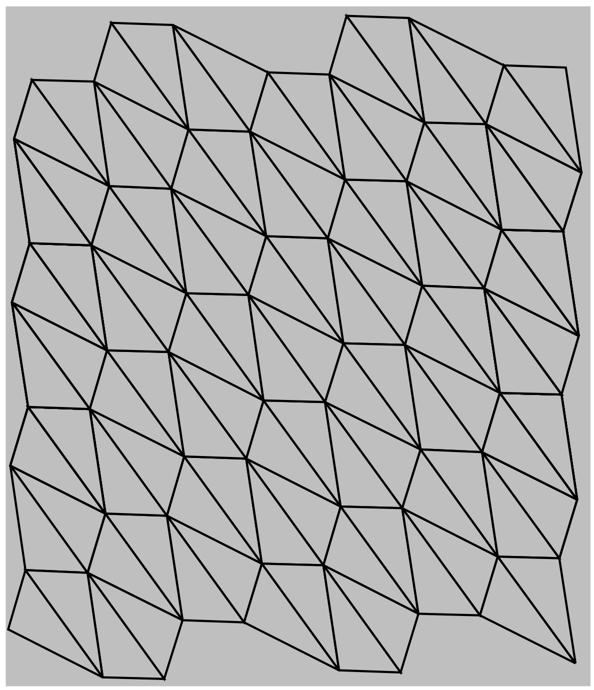


The property that allows quadrilaterals to tile a surface is the fact that their angles add up to 360°, the same as the number of degrees around a point. This means that four of the same quadrilateral can fit exactly at a corner. A regular hexagon can tile because each of its angles are 120°, so that three can fit evenly around a point.



Any two congruent triangles can join into nine different quadrilaterals: three parallelograms plus their mirrors and three "kites". These quadrilaterals will tile a surface.





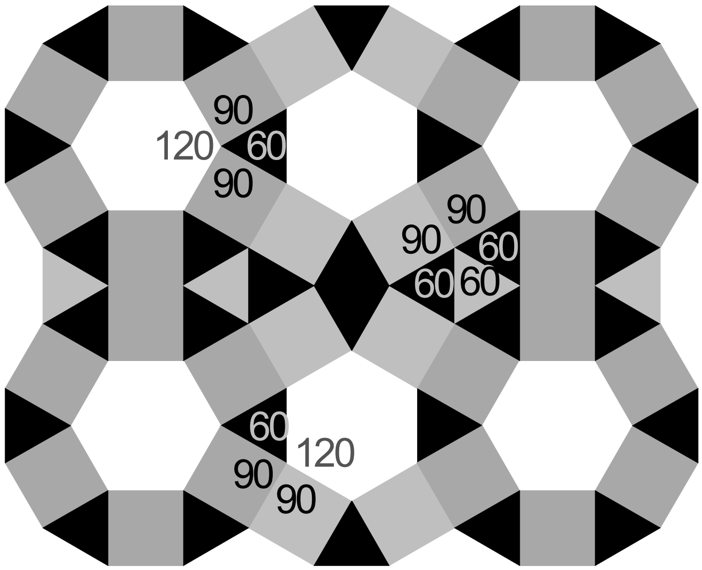
Generating Patterns

Patterns achieve beauty and complexity by preserving the ordering effects of symmetry while increasing the intricacy of form. Some of the most common methods are:

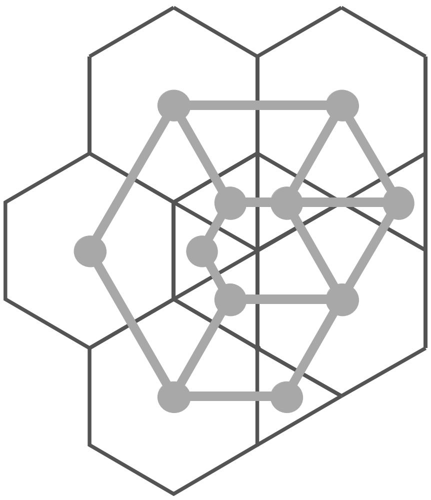
Semi-regular tiling

Many beautiful patterns combine more that one figure to achieve space filling. Combining two or more regular polygons around a point, for example, creates many space-filling patterns, as long as the combination of their angles adds up to 360°. This is known as semi-regular tiling. Regular tiling, which uses one regular polygon to tile the surface, is possible only with equilateral triangles, squares and hexagons. Such a tiling possesses all four types of symmetry.

Two squares and three equilateral triangles will fit around a point, since the angles, 90-90-60-60-60, add up to 360. Similarly, the combination of square, square, hexagon and equilateral triangle, 90-90-120-60, works, as does the combination of octagon, octagon, square, 135-135-90, or decagon, square, hexagon, 150-90-120.



Connecting centers

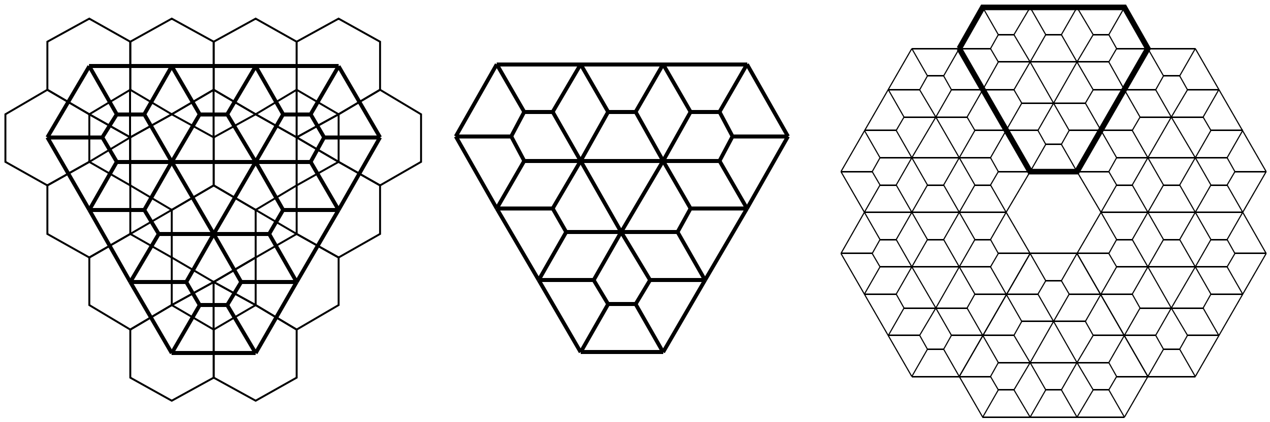


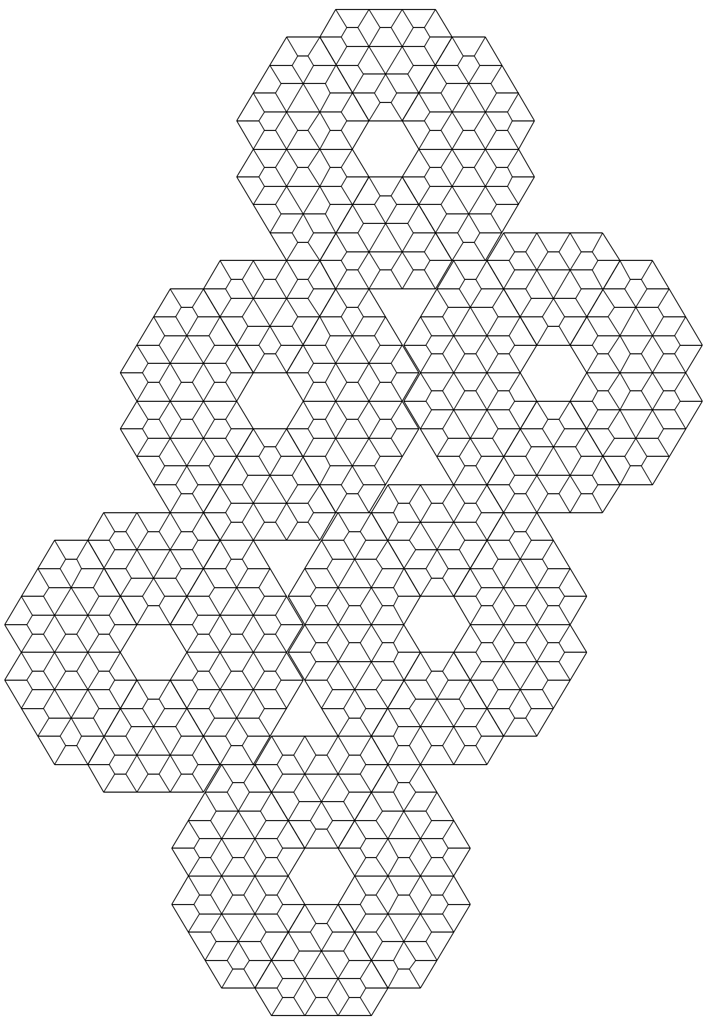
Connecting centers of the polygons can open more elegance and complexity.

This procedure works because it connects the center of symmetry of each tile. This center is the point around which a tile rotates and it lies on the axes of reflection of a tile. This ensures that the derived pattern is symmetrical as well.

Hierarchy of Structure

Smaller tiles can group into a pattern that may form a larger tile to fill the surface. This larger tile, in its turn, may become part of an even larger, more complex tile. This type of structure -- one that builds complexity by recombining previous, simpler levels of order -- is call hierarchical organization. Such an organization can also generate from subdividing a tile into smaller tiles, and then in turn subdividing those tiles. In the pattern below the triangular pattern from the previous figure has combined into a large complex hexagonal tile.

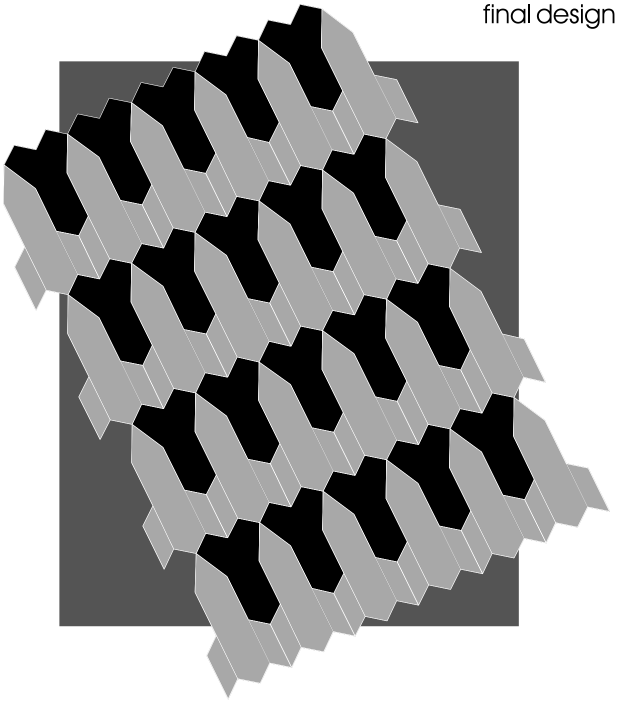


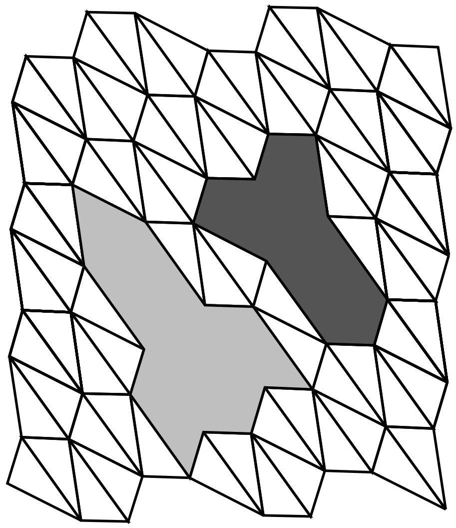


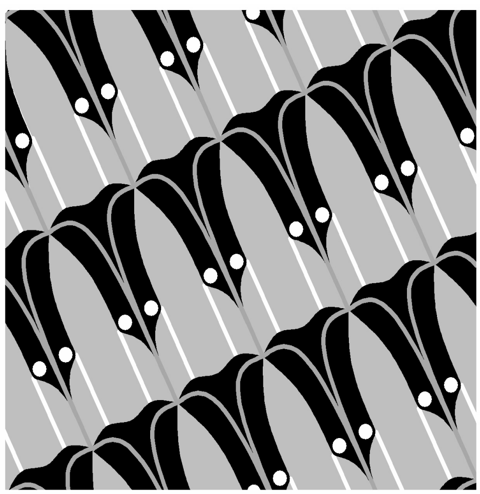
Joining tiles in a net

A net is similar to a tiling pattern with the emphasis placed on the pattern of lines. Nets are common in design as devices to systematically organize a composition. The most common net is the standard square grid used to aid layout in page design, architecture and other areas of design where a highly regularized organization is sought. However, nets can comprise almost any space filling system of simple shapes.

A net is generally comprised of the simplest shape underlying a pattern. The vast majority of nets used to generate tessellations are triangular or square depending on the overall symmetry of the final design.







<u>Using an irregular triangular net to build more complex tiles</u>. In this figure 14 triangles from a net of irregular triangles group to form a rocket shaped tile. Another 10 join into a smaller tile, also suggesting a spaceship. The two tiles interlock into a space-filling array. Smoothing the angular outlines of the tiles and adding image detail create a more elegant image and strengthens the smaller tiles to suggest an alien ship. As a result pattern and image merge into the representation of a densely fought and tightly regimented space battle.

No apparent indication of the original triangles remains in this final image. Nevertheless that simple early pattern greatly eased the development of this idea. The net assured that the symmetry necessary for tiling the plane was in place. It was up to the designer to provide the experimentation and imagination to unearth the imagery buried in that symmetry.