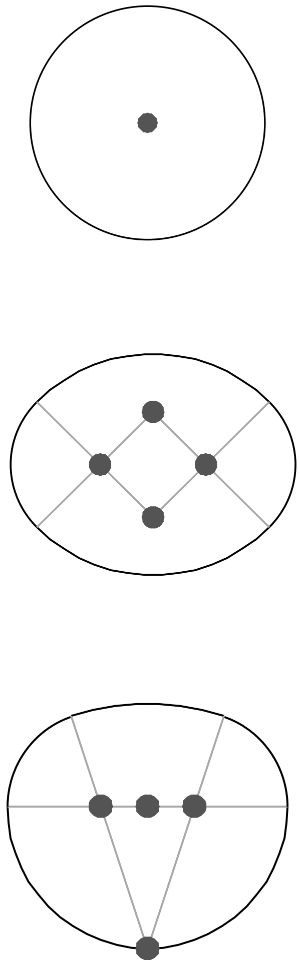
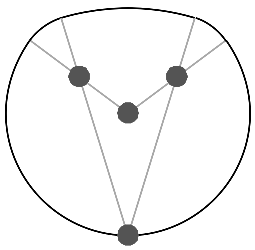
Chapter 1

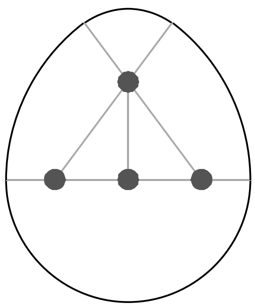
Introduction to Design Geometry

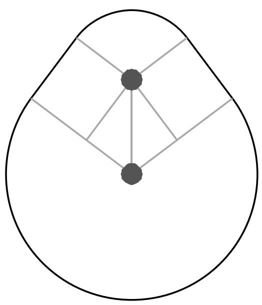


The most common geometry used in art and design traces its beginnings to the first great builders, to the origins of architecture in Neolithic cultures. These builders developed a basic geometry to ensure accurate layouts and precise angles for the process of construction. This geometry progressed in sophistication through the building of the great cities of ancient Babylon, Egypt and Greece.

The earliest tools for crafting geometry were simple. Of these the simplest and most important were taut cords and stakes. The stakes provided fixed points on the earth and taut cords connected these points into significant figures such as the footprint of a building or a ceremonial emblem. A stretched cord created a straight line; pivoted from a center stake it could define arcs and circles. A string line pulled by a suspended weight became the vertical reference, or plumb line, and defined a perpendicular with the earth. The third tool in the ancient geometers repertoire, then, was gravity.







Previous page: The Ring of Brodgar is a majestic stone circle that sweeps around its coastal Scotland site. Such ceremonial rings, built in the 3rd and 4th millennium BCE, dot the countryside of the United Kingdom and Western Europe. Not all are simple circles, but manifest five more variations of elongated and flattened circles (previous page and right) with sophisticated geometric construction. Engineer/archaeologist Alexander Thom discerned this geometry after thousands of field measurements. He also introduced the controversial notion that two standardized units of measure, the Megalithic "yard" and its half, the Megalithic "cubit", governed the construction of these circles on the British Isles and the western coast of Europe.

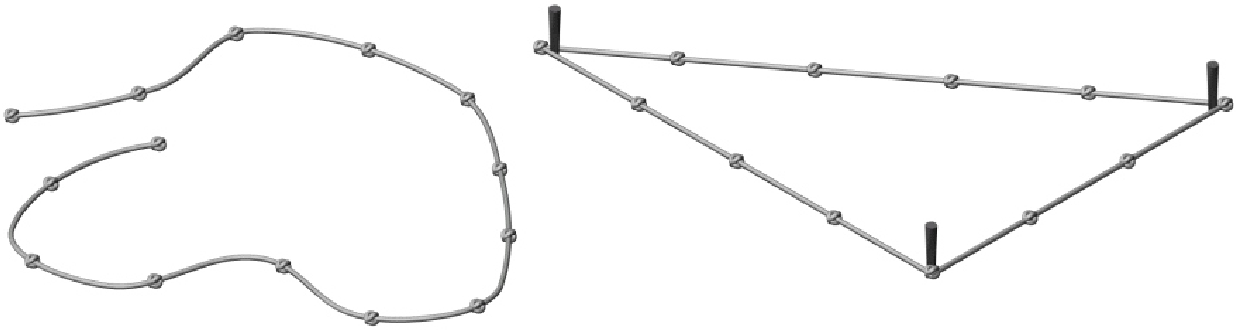
One reason for the distortion may have been to create a directional axis aligned with solstice sunrises and sunsets.

These pre-historic people also observed the heavens in hopes of discerning the behavior of their gods. They soon discovered that the same geometry that created useful and symbolic forms on the earth could serve to describe and even predict celestial activity. Geometry entered the realm of the sacred and the scientific.

Pulling Ropes

Unlike the stone tools for which the Neolithic age is named, its geometry exists only in the patterns inhering in the building projects whose layouts it guided. Fashioned from fiber and wood, the layout tools of these ancient engineers have long ago decayed. The most important of these was the rope, actually a heavy cord, used to demarcate borders and extend alignments.

The earliest written evidence for the applications of rope geometry appears in later religious texts, most especially the Sulbasutras, which were written as appendices to early Hindu religious tracts. This set of prescribed rules for laying out ceremonial structures dates back to at least 800BCE and offers the most complete evidence for early Indian geometry. Archaeological evidence points to a knowledge of rope geometry in India well before 2000BCE when the Harappans, a pre-Hindu culture, were known to employ it.



The Megalithic builders studied by Thom utilized lengths of rope, probably calibrated with equally spaced knots, as both compass and ruler. Stakes driven into the earth fixed the points determined with the rope. Three stakes and a 12 segment stretch of rope can form a 3-4-5 triangle that served for staking a 90 corner. This same proportion is still found in the standard 18 inch by 24 inch carpenter's square sold today.

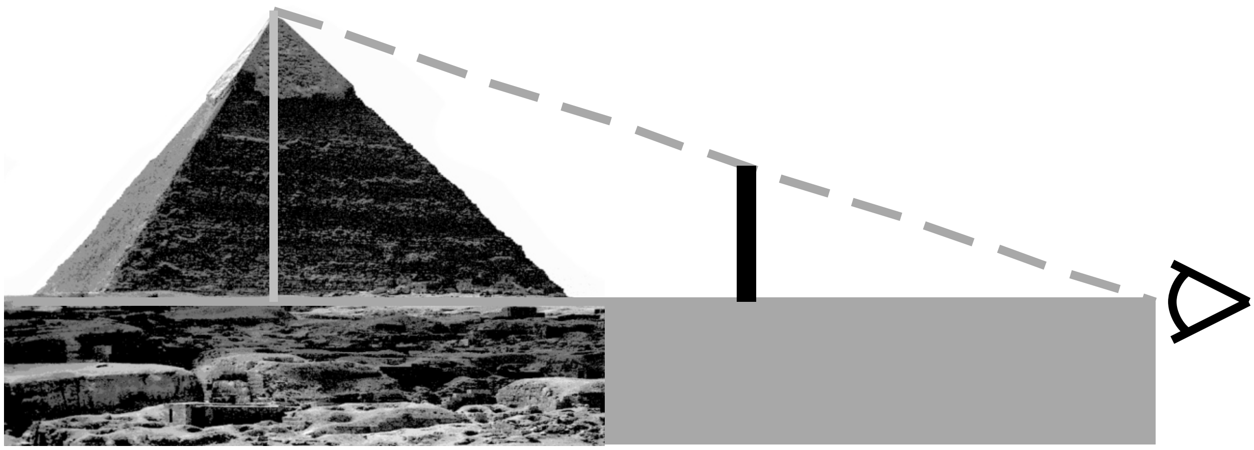
Classic geometry of the West had more secular roots in the leisure ruminations of ancient Egyptian surveyors. These surveyors -- or rope pullers, as their Egyptian name translates -- spent a sort of busman's holiday solving practical and hypothetical problems during the long season of the Nile's annual flooding. Until the river's waters ebbed from the inundated fields the rope pullers put pen to papyrus and stylus to clay in the professional development of their geometric methods.

Hieroglyphic depictions and a paragraph written in 440BCE by the early Greek historian Herodotus provide a sketch of the profession of the rope puller. Notable in the hieroglyphic illustrations is a distinctive clasp worn by the rope pullers as a mark of their caste. The ostensible purpose of the clasp was to hold the coiled rope, but it symbolized the special mathematical knowledge and attendant social status to which the pullers were privy. Mathematics could not be studied by just anyone and was usually a priestly reserve. Consequently, the rope puller belonged to a quasi-priestly social caste.

Not quite priests, because the rope pullers were professional tax agents of the pharaoh. Herodotus sets the beginnings of that status at about 1400BCE when the pharaoh Sesostris assigned regular plots of land to individual farmers and had these recorded in order to systematize the process of taxation. The rope pullers performed an initial survey to layout the lots and record ownership, but, due to the annual floods, were kept busy in perpetuity updating that survey. The Nile's flooding tended to wreak havoc on the orderly assignments, scouring land from the upstream banks and adding it onto downstream banks or covering field markings with mud. This necessitated annual surveys to re-establish boundaries for accurate tax assessments.

In the late 6th century BCE Thales travelled from his native Miletus on the western shores of Turkey to study in Egypt. During his stay, he was storied to have devised a method for measuring the height of a pyramid by sighting with a pole. Fixing the pole upright, Thales backed up until his line of sight matched the tip of the pole to the apex of the pyramid. He then calculated the proportion between the height of the pole above his eye and the distance he had backed. Next he strode to the base of the pyramid and added half the length of one side to find the total distance to the pyramid's center. This distance multiplied by the proportion gave the height of the pyramid.

The ancient Greek philosopher Thales returned from Egypt around 585BCE bearing notes on the rope pullers learning and the early Greek mathematicians soon generalized the rope-and-stake applications of the rope pullers into a system of points, lines and arcs linked by logic. At the same time they took geometry from the fields to the page by restricting geometric description to two drawing tools, the straightedge to describe straight lines and a pivoting tool, the compass, for executing arcs. The Greeks honored their Egyptian predecessors by dubbing their paper explorations geometry for "earth measure".



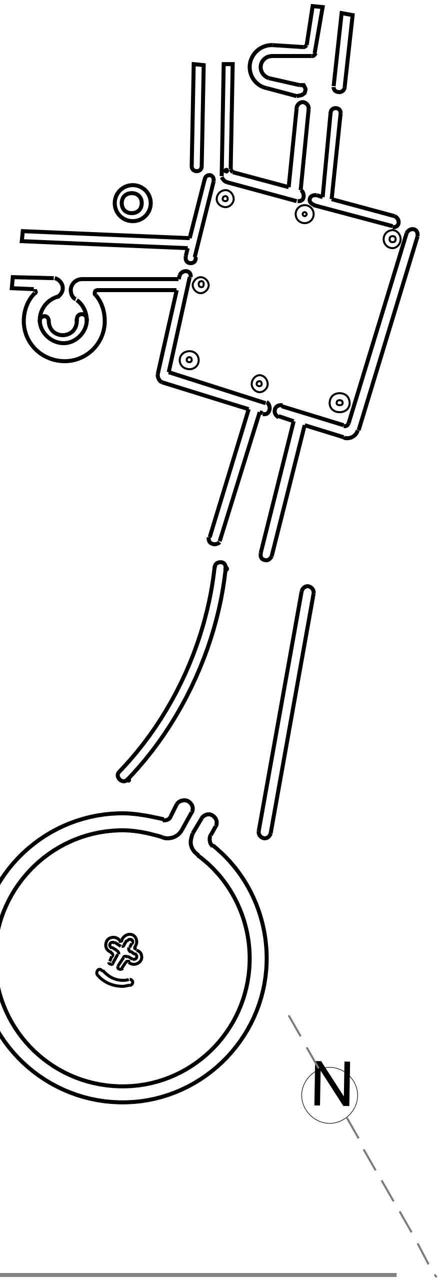
Egyptian rope pullers refined methods that had been in place for at least three millennia earlier. Babylonian tablets from as early as 4000BCE contain records of land ownership bearing the seal of the surveyor to verify the accuracy of the lands size and location. But rope-pulling geometry most likely dates back another millennium, in the practical and ritualistic functions of Neolithic tomb building (chapter 3). All indications are that the early European geometer served as both priest and architect.

Measure

Knotting is most often cited as the likely method for marking off the units. To compensate for the inevitable errors in knotting, beads may have been threaded onto the thin rope or cord and abutted to the knot. Thin notches on the beads would then more accurately calibrate the distances on the rope. Ropes were also pre-stretched and then saturated with wax or pitch in anticipation of the rope lengthening with the fatiguing of continued use or in damp weather.

This is less precise than it sounds, since ropes will still vary in length depending on the force of tension, or fatigue through repeated use or the dampness of the day. Accuracy fell well short of modern standards. Early errors of 1% were typical. This is evident in the layouts of the huge Hopewell ceremonial sites, which dot the Ohio valley. Built between 100BCE and 500AD, these sites paired circular enclosures and square enclosures, each surrounded by a perimeter mound between 900 and 1000 feet across. The circle and square related in a number of ways, all geometrically based. The diameter of the circle would in some cases equal the side of the square or in other cases its diameter.

At the Newark Fairgrounds Earthworks the perimeters of the two enclosures are nearly equal. At 3736.6 feet for the circle and 3712.0 feet for the square their measures fall within 1% of one another. This is close enough to signal the designer's intent to create symbolic equivalence between circle and square, the most common pairing in sacred geometry around the world (see chapter 9). Errors notwithstanding, this demonstrates a remarkably sophisticated and consistent approach to layout and well served its ceremonial function.



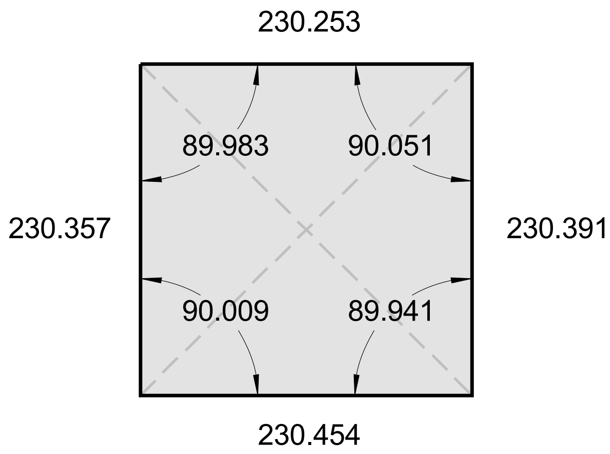
Newark Fairgrounds Earthworks.

Newark, Ohio is home to one of the most intriguing arrays of earthworks in an area known for the number and variety of these ancient structures. The photograph to the left is of the circle depicted above in a diagram adapted from an 1848 survey by Ephraim Squier and Edwin Davis for the Smithsonian Institution.

Photo courtesy of William F. Romain

The ceremonial squares also are off by about 1% on their corners angles. The angular and the distance errors of the Hopewell geometers would be about right for an early rope puller. Stretched ropes can yield errors of +/- 2%. Since the error will vary between too long and too short for each pull of the rope, this inaccuracy will partially average out over the long run. This plus the skill of the rope pullers narrowed the range of error. On simpler layouts their accuracy increased to within 0.25%

Egyptian rope pullers fared even better, cutting that error to about 0.05%. The development of writing accounted for the bulk of that improvement. With writing came computational methods, and a way to check the errors based on field data. The Egyptians could, for example, accurately calculate fractions. Their value for *pi* at 22/7 remained the most accurate until the 17th century.



Dimensions of the base of the Great Pyramid at Giza.

In 1925 the Egyptian government hired J. H. Cole to perform an official definitive survey of the Pyramid of Cephren at the Giza pyramid field near Cairo. Cole's was the first survey to locate the actual cornerstones of the pyramid. Distances are in meters.

The square base of the Great Pyramid at Giza is a particularly good example of ancient Egyptian accuracy. The north side is 8 inches smaller than the south. Two of the angles are off by .06 %; the other two are almost perfect 90° angles. Though by modern standards the 8 inches of error is relatively poor (today's average construction engineer with a tape measure from the neighborhood hardware store would fare better), in its day this was state of the art.

As documented in extant papyri, Egyptian mathematics held very strong rules for calculating measure and rigorous checks were standard practice. One common check for squareness is to measure the diagonals: if these are equal then the square should have true right angles. However, equal diagonals produce a true square only if all of the sides are equal. In the case of the Great Pyramid the diagonals are almost perfectly equal, so a check would not reveal the discrepancy. If anything the equivalence of the diagonals could indicate a significant effort on the part of the rope pullers to true up the layout. The culprit for error was likely the rope. The Egyptians performed excellently with the data supplied by their tools.

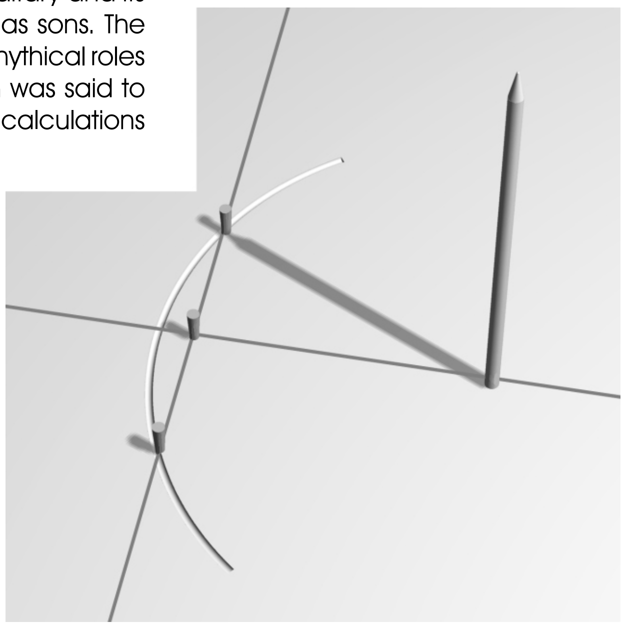
Ritual

There is no direct evidence of knotted ropes in field surveying and knotting makes for poor calibration. Evidence cited for the knotted rope as the standard surveying tool of ancient times actually points to the ceremonial use of knotted ropes, where symbolism counted more than accuracy. In ancient Egypt priests employed a length of rope knotted thirteen times to form twelve intervals. The intervals were sufficient to layout a 3,4,5 triangle and thus a right angle. Known today as the Pythagorean triangle, it has through history also borne the label of Egyptian triangle.

To kick off the construction of an Egyptian temple the pharaoh accompanied by a priestess performed a "stretching of the cord" foundation ritual in which pegs driven by golden hammers marked the alignment of the temple. Regarded as a microcosm of the universe on earth, the temple geometry was believed to be that of the cosmos itself. The priestess personified Seshat, the goddess of writing and measure, or simply computation.

From Seshat, through the agency of the priestess and her rope, the geometry governing the cosmos was thought to emerge. In images of this goddess a leaf, looking remarkably like hemp whose fibers make a high quality rope, grows from her head encircled by the hieroglyph for the number ten. Like many of the signs pertaining to quantity and measure this glyph is in the image of a rope.

Geometry in ancient Egypt was not a males-only field and for a time a female dominated field. Priesthood was hereditary and its status and learning passed on to daughters as well as sons. The roles of the pharaoh and the priestess paralleled the mythical roles of Thoth, the father, and Seshat, the daughter. Thoth was said to obtain the measure while Seshat performed the calculations ascertaining accuracy.



The first step in laying out a temple was to determine its orientation. Most commonly this orientation had a cosmological importance and so used astronomical factors such as the sun, the stars and the moon. Like many modern buildings many ancient temples orient according to the four cardinal directions.
The oldest method to determine these direction was to use the shadow of a vertical pole. During late morning the rope puller marked the end of the pole's shadow and then drew an arc using the base of the pole and that mark as the radius. In early afternoon the rope puller marked the point where the shadow again touched the arc. These two points are symmetric across north and south and define an east-west line.

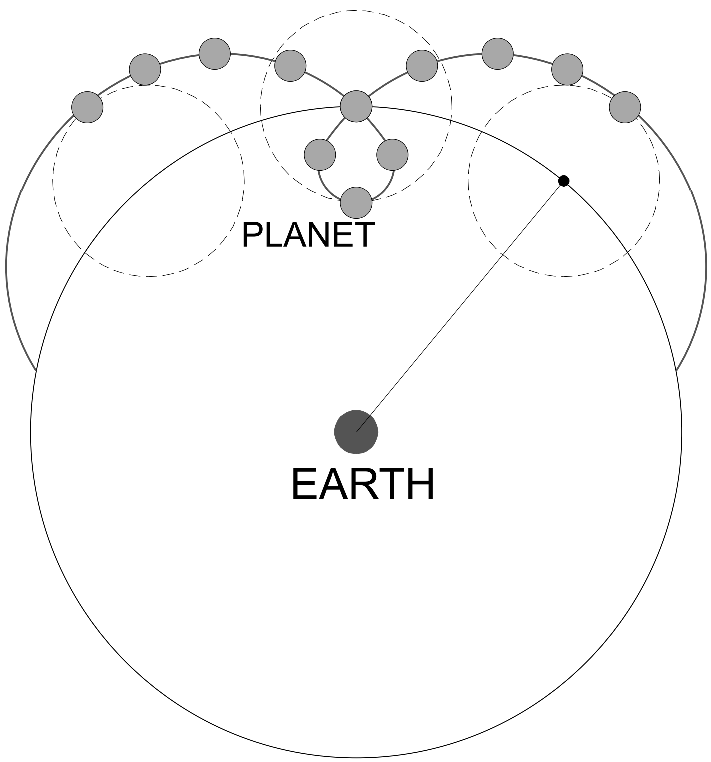
Euclidean Geometry

Euclid, in 320BCE, organized the known elements of practical and sacred constructive geometry into a logical, mathematical system. Geometry became the purview of abstract thought and deductive reasoning. The stakes of the Neolithic builder were reduced to points in space; their taut cords were transformed into lines.

Euclidean geometry, as the Greek system came to be called, was extremely successful. Its value is such that it is still taught today in high school. It remains a necessary predecessor to trigonometry and calculus.

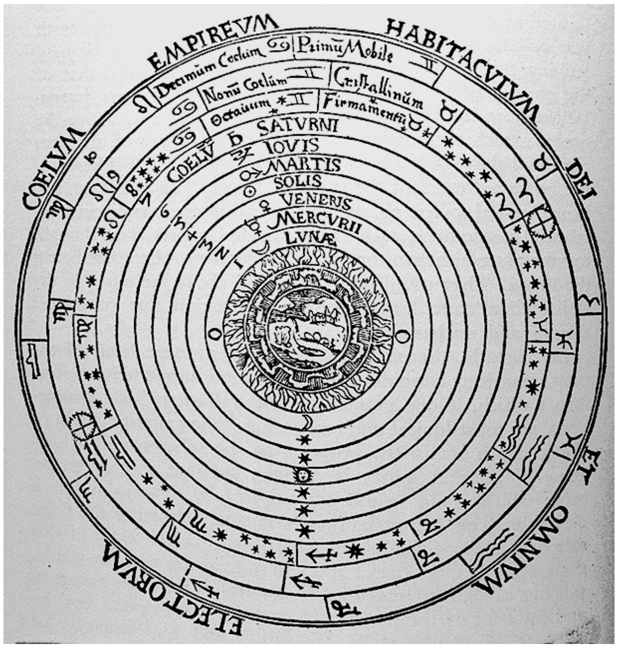
During the so-called Dark Ages of early Medieval Europe, Arab scholars kept the geometry of the Greeks alive and growing. The city of Baghdad became the meeting ground for this geometry and the more advanced computational methods of Indian mathematics. In addition to adding algebra and trigonometry to the field of mathematics, this union led to some of the most splendid visual patterns ever devised. The related geometries of cartography and perspective painting that marked the European Renaissance developed directly from Arabic studies of trigonometry, optical geometry and astronomy.

Epicycles in planetary motion. Early religions saw geometric perfection in the universe as manifest in its precise circular motion. This universe, however, did not quite fith the subsequent detailed data gathered by Babylonian, Greek and Egyptian astronomers. The planets did not follow smooth paths like the sun, stars and moon, but seemed to wander and backtrack through the heavens (planet is from the Greek for wanderer). Almost five centuries after Aristotle the Egypto-Roman astronomer Claudius Ptolemy published his classic Almagest in 150 AD. In this text Ptolemy presents his theory of epicycles to account for the backtracking paths of the planets. The planets circled a point that in turned circled the earth. Ptolemy's model thus restored the geometric perfection of circular motion about the earth, while accounting for the apparent retrogression of the planets.



Euclid's ordering of space held on as the basis of astronomy for over 2000 years until Isaac Newton and Liebnitz developed calculus in the late 17^{th} century and Newton applied this new computational method in his explanation of gravity's relationship to planetary motion. Though Newton formulated a physical rather than a geometric model for astronomical events, he still conceived of space as ultimately ordered by Euclidean geometry.

It wasn't until the 19^{th} century that any other forms of geometry were imagined. Non-Euclidean geometry, in fact, was the major tool by which Albert Einstein developed his theory of relativity. In Einstein's universe space itself curves and parallel lines meet a notion that to most people defies common sense.

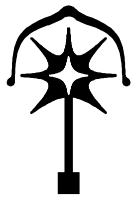


Top: Peter Apian, Cosmographia. In 1524 Apian published this schemata of Aristotle's conception of the universe to a receptive audience. Aristotle, like other philosophers of Classical Greece, believed that the universe is formed on the perfect geometry of the sphere. The planets and stars, mounted on revolving and nesting spheres, orbit the earth and not the sun. These spheres gyrated under the supervision of a Prime Mover, that existed outside of the outermost sphere. For Medieval Christians and Muslims this motivating entity was obviously the Deity. Also, any doubt concerning the sphericity of the universe had been eradicated by Ptolemy. Consequently, Aristotle's universe became a religious fact.

Practically speaking the ongoing success of Euclid's efforts to mold geometric constructions into a logical system of relationships and its ability to persist through history was due to its ongoing usefulness in building, planning, surveying, engineering, navigation or any design discipline where the practical structuring of space is a necessity. More importantly this system of geometry conforms to our day-to-day observations our “common sense” perceptions of space.

Such was the perceived power of this geometry to govern space that its study took on moral, even sacred imperatives. Just as the ancient Egyptians deified mathematics, so did Medieval and Renaissance thinkers read the mark of God in many intriguing

Depictions of Seshat. The link between divinity and geometry may be one of the oldest religious concepts. In ancient Egypt geometry and number received its own deity, Seshat. The only depictions of a female scribe from ancient Egypt is that of Seshat. The emblem of her divinity is variously a leaf, flower and/or star canopied by the hieroglyph for the number 10, a symbol of power. In the actual hieroglyph the image is ambiguous and appears to be either a flower or a star, underscoring this deity's role in transferring sacred geometry from the heavens to the earth. The hieroglyph for 10 evinces similar dual readings. Like may hieroglyphs alluding to quantity it is derived from a rope. At the same time it traces the arc of the sun, with the rising, setting and midday positions of the sun emphasized.



geometric relationships. The rigorous study of constructive geometry was thought to contribute to moral character and required of all gentlemen's education. Rectitude and rectangles went hand-in-hand.

Even today most design applications of geometry are constructive in nature. This is especially true of computer graphics. The constructive geometry of Euclid remains as important to building structures in virtual space as it has to building structures in real space. Most, if not all, of the tools featured in computer-aided design (CAD) programs assume that constructive geometry is the designer's geometry of choice. By necessity the underlying manipulation of geometry is computational and not constructive, but the programs are carefully designed to present the geometry on screen as constructive.

Purpose

This book draws examples of classical constructive geometry as a tool for design from its inception 7500 years ago to its current day replication in computer environments. The book is written for the non-mathematician. Equations and formulas are practically non-existent except in the chapter on trigonometry, which is inserted to demonstrate the role of computation in geometry.

The upcoming chapters are divided between practical methods and illustrations of its application through history. From the basic relationships of points and lines, to the drawing of geometric figures, to the creation of symmetry and proportion this book provides the classic geometric tools with which to structure visual space. As a tool for design this geometry is also a tool for meaning and for the imagination. To stir some of that the imagination this book offers a host of examples from the ancient architecture of Western Europe, to the high pampas of the Pre-Columbian Andes, to most of the great cultures of the Mid and Far East.