Chapter 17

CAD Geometry

Vector Geometry

Early graphic programs were initially developed for industrial applications. Used by draftsmen, who drew architectural and mechanical plans, these computer-aided drafting (CAD) programs imitated the traditional constructive drawing principles of that field.

To do so the CAD program must translate traditional constructive processes in computable, numerical form, that is, as mathematical functions performed in a numerically described space. These programs employ special algebraic geometry to carry out such operations. With it the program can describe constructive relationships to a high degree of accuracy, about 5 - 7 decimal places. For all practical purposes the CAD user designs as if using traditional design geometry. The calculations take place deep in the program, so that the user need not be aware of the mathematics at work.

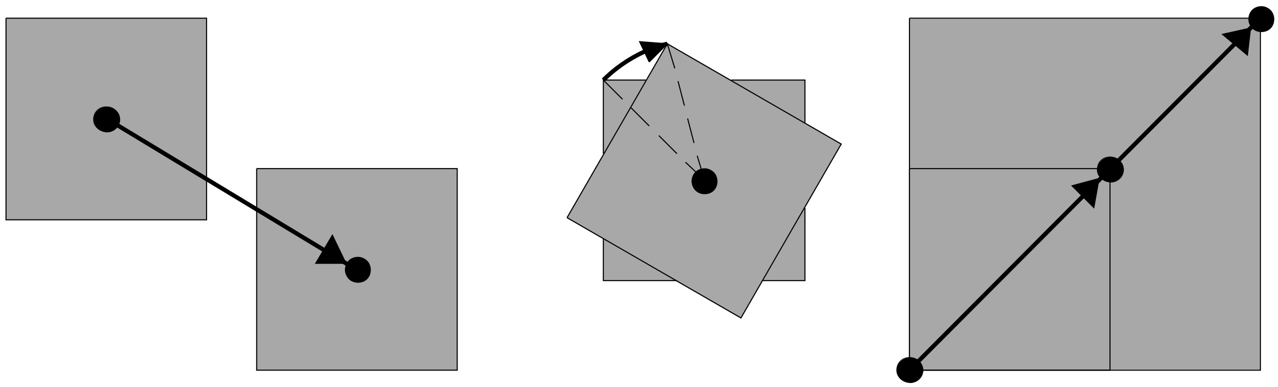
CAD programs are said to represent or *express* design geometry, especially traditional constructive geometry. Computers cannot draw or visualize geometry; they can only compute numerical values and relationships and configure this information to the monitor screen or printer. In order to enable computation vector geometry is a computational geometry that employs trigonometric functions, matrices and a coordinate grid to describe geometry.

While trigonometric functions and matrices are beyond the scope of a book for non-mathematicians, it is possible to develop an intuition for the principles underlying vector representation. This chapter takes a look at what constitutes a vector, how and why the computer uses vectors in this representation and what this knowledge offers the contemporary designer,

Vector Transformations

A vector is a ray segment that has a specific length, direction and position in space. Change the direction of the segment and it becomes a new vector. As a ray, the vector is also assigned a heading , or angular direction.

Changing the length or position or direction will transform the ray segment into a new vector. An operation that changes the size, direction or position of a vector does not only change an attribute of a vector, but creates an entirely a new vector. The three basic transformations are scale, rotation and translation. Scale is a change of size and is expressed by a change in length of vectors; rotation is a change of direction of vectors; and translation is a shift in position, also called move. These are the same transformations in principle as those governing symmetry relationships.



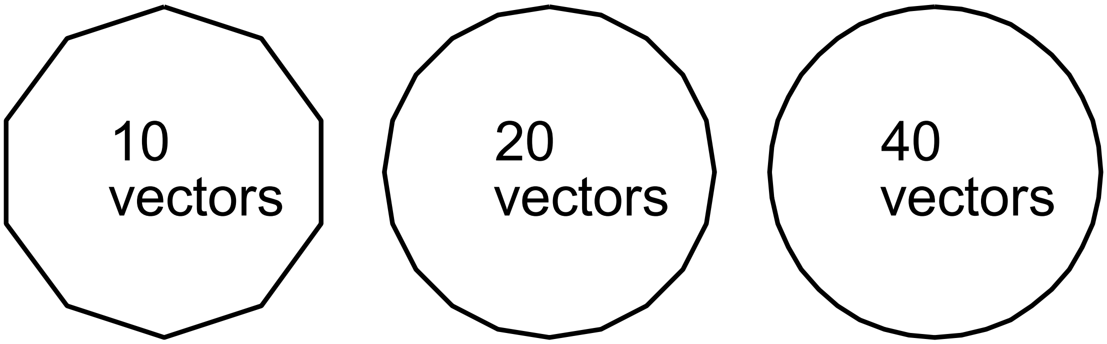
The vector transformations, left to right: 1) translation – the program translates the change from one coordinate position to another.
2) rotation – the program assigns a new direction stipulated by new angular value.
3) scale – the vector dimension is stretched out by extending the endpoint to a new coordinate.

Vector Representation

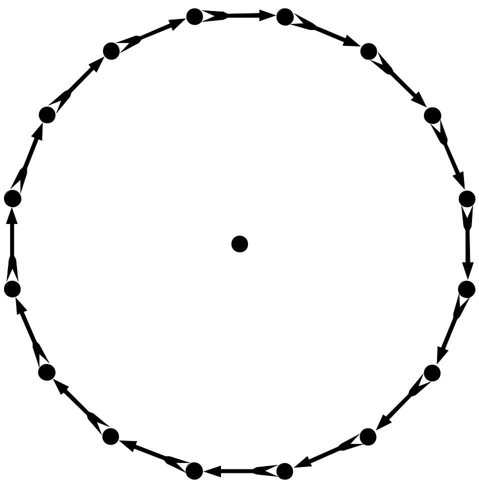
Vector geometry represents geometric figures as compounds of vectors. A square, for instance, comprises four vectors connected end to end and directed on paths 90 to one another. In vector geometry just the appearance of a square does not necessarily constitute a square. The vector program must *group* the four vectors as one closed figure rather than four independent vectors.

If we draw four lines into a square on a piece of paper it, by definition, is a square. In the computer, though, the vector program may not recognize a square, but four aligned vectors instead. A little complicated perhaps, but such distinctions greatly enhance the computer's ability to manage geometry with ease and flexibility.

Curves are expressed as a series of vectors set end to end with gradual changes in direction from one vector to the next. The vectors appear in such small increments relative to the arc of the curve that the visual impression is one of smooth continuity. A series of equal-length vectors, for example, that each veer to the same side by the same number of degrees will arc into a circle. In practice a circle is represented as a series of vectors connecting a set of points equidistant from a center location. The more points being connected the smoother the perceived curve.



The vectors in this representation of a circle constitute, in effect, the chords of an inscribed regular polygon with many, many sides. This, though, is only the graphic representation of the circle. The program can assign this representation the mathematical properties of either a circle or of a polygon. In the former case the polygonal representation becomes a *virtual* circle and is treated as such in all future computations. The program will then perform operations involving tangents, diameters, and other relationships of a circle. In the latter case the program will not perform these operations, but will treat the vectors as sides of a polygon.



In many vector programs users can "explode" the circle into a polygon, thus removing its curve properties. The polygon can further "explode" into a series of independent vectors. When designing in a CAD program the principles of constructive geometry are essential, but a deeper knowledge of the underlying vector representation will greatly increase the sophistication of the design process.

Vector Graphics

Vectors are efficient for the computer to manage. To describe a vector all the computer need do is specify the two endpoints of a line segment. For many operations the computer must also assign which endpoint is the start and which is the stop of the vector. In these cases the vector is a truncated ray. Once these points have been established the program draws a line on the screen to connect the two points. This takes far less memory than specifying the line pixel by pixel.

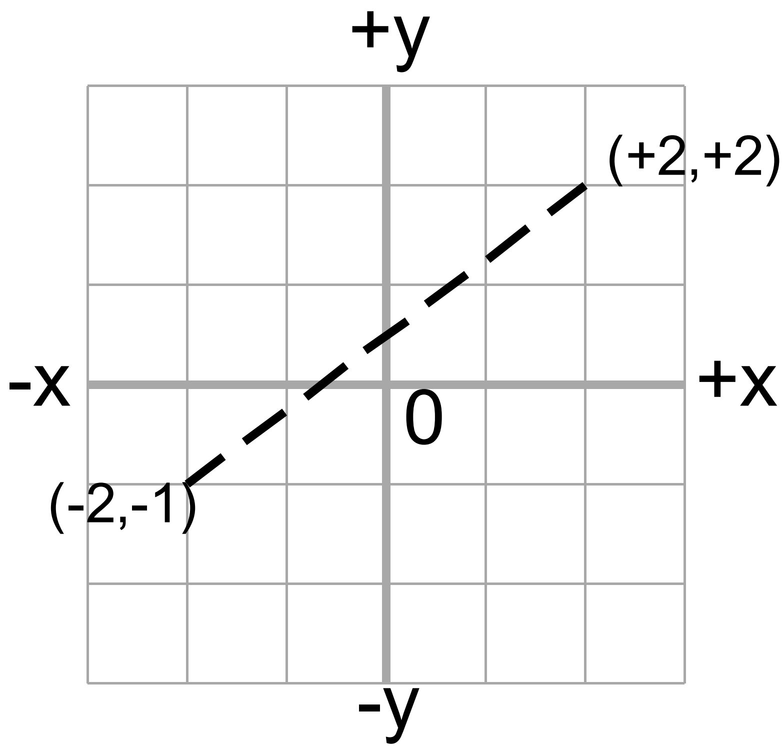
On the whole vector graphics used in drawing and modeling programs taxes the computer far less than does raster graphics used in photo and paint programs. Vector graphics stores the graphic information needed to create an image as vectors organized into geometric patterns; raster graphics stores the graphic information as pixel properties and position. Usually this means that less information is needed for vector graphics to stipulate image details, but in highly complex images, where the vector count rises into the many thousands, the vector information can easily exceed the pixel information. This is especially true if the graphic contains information on color and line weight (thickness) or has been run through a filter.

Nevertheless, vector graphics will still hold one advantage. Since its information is held geometrically, it can increase in scale indefinitely with no increase in information. By contrast the raster information increases by the square of the scale increase.

The Coordinate System

In order to specify the endpoints of a vector the computer requires a mathematical system to pinpoint these locations in space with numbers. This system is called the coordinate system and, in the case of two-dimensional space (2-space), it uses two coordinates, ***x,y***, to specify the horizontal and vertical distance from a set reference point called the origin.

Invented in the early 17th century by French mathematician and philosopher Rene Descartes, these coordinates are sometimes labeled Cartesian coordinates. The coordinate system is essentially a technological improvement on the origin and baseline introduced earlier. The ***x,y*** coordinate system improves on this method by integrating it with a grid. The baseline becomes, in effect, the horizontal or ***x***-axis with the vertical or ***y***-axis contributing the perpendicular direction. These two axes intersect at the origin. Paralleling each of these axes is a series of lines spaced one unit apart creating a square grid over the entire 2-space.



A point can now be specified numerically. Two numbers are used: the first states the unit distance to move from the origin along the ***x***-axis and the second denotes the unit distance to move vertically, following a line parallel to the ***y***-axis. A move to the left of the origin is expressed as a negative ***x*** value, while a move to the right is positive. A positive ***y*** value denotes an upward move, while a negative value defines a downward move.

Descartes conceived of the coordinate grid as a method for numerically analyzing geometric features and to manipulate them computationally. The new field of mathematics he founded was dubbed analytic geometry.

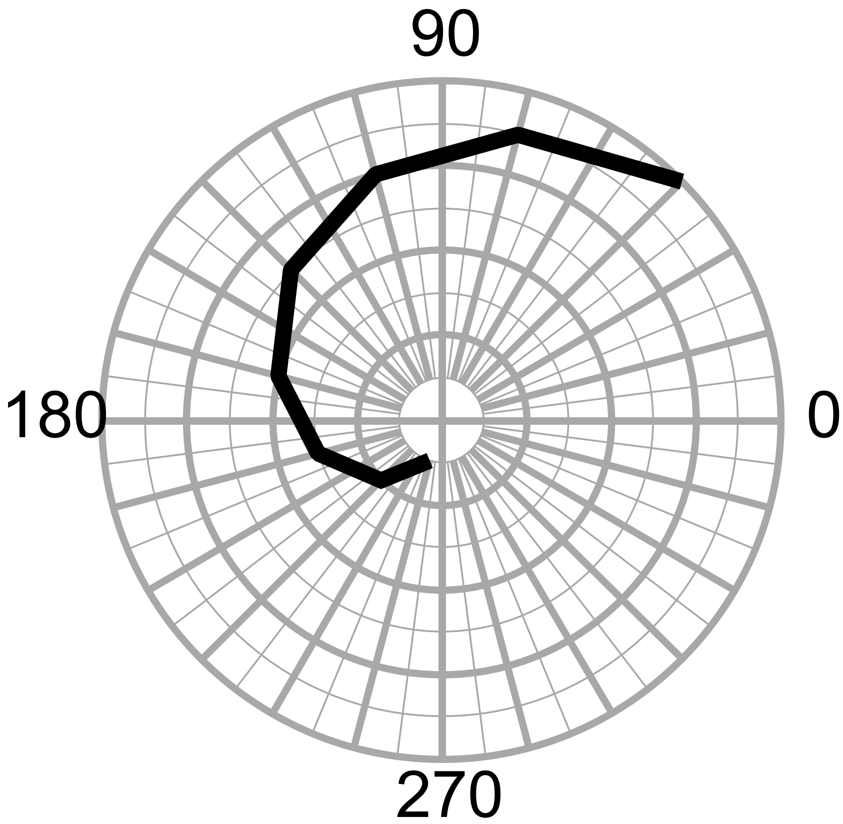
Vector geometry employs aspects of analytic geometry to position, direct and size its elements. Unlike human thought computing does not work with spatial perceptions, but rather with computable numerical descriptions.

Polar Coordinates

Because the standard ***x,y*** coordinate system relies on perpendicular relationships it is also referred to as the orthogonal grid or the orthogonal coordinate system. It is not the only coordinate system available to computer graphics. An alternative is the polar coordinate system.

Polar coordinate system also uses two coordinates to place a point in space relative to its origin. The first coordinate specifies the angular direction (also called bearing or azimuth) in degrees from the origin and the second coordinate specifies the unit distance from the origin. In this system a line headed horizontally to the right is assigned an angular value of 0. The angular value increases as the line rotates counter-clockwise around the origin: straight up is 90°, to the left is 180° and straight down is 270°. As the line swings through all 360 degrees, it returns to 0. On a polar grid radial lines are typically set every 5 or 10 degrees. The distance out from the origin is represented by a series of concentric circles whose radii increase in increments of one unit.

Recall how ancient builders used slope and distance to spot a point on a building site and note the similarity to polar coordinates. By integrating a grid system with this earlier method geometric features can, as with the orthogonal grid, be represented numerically.

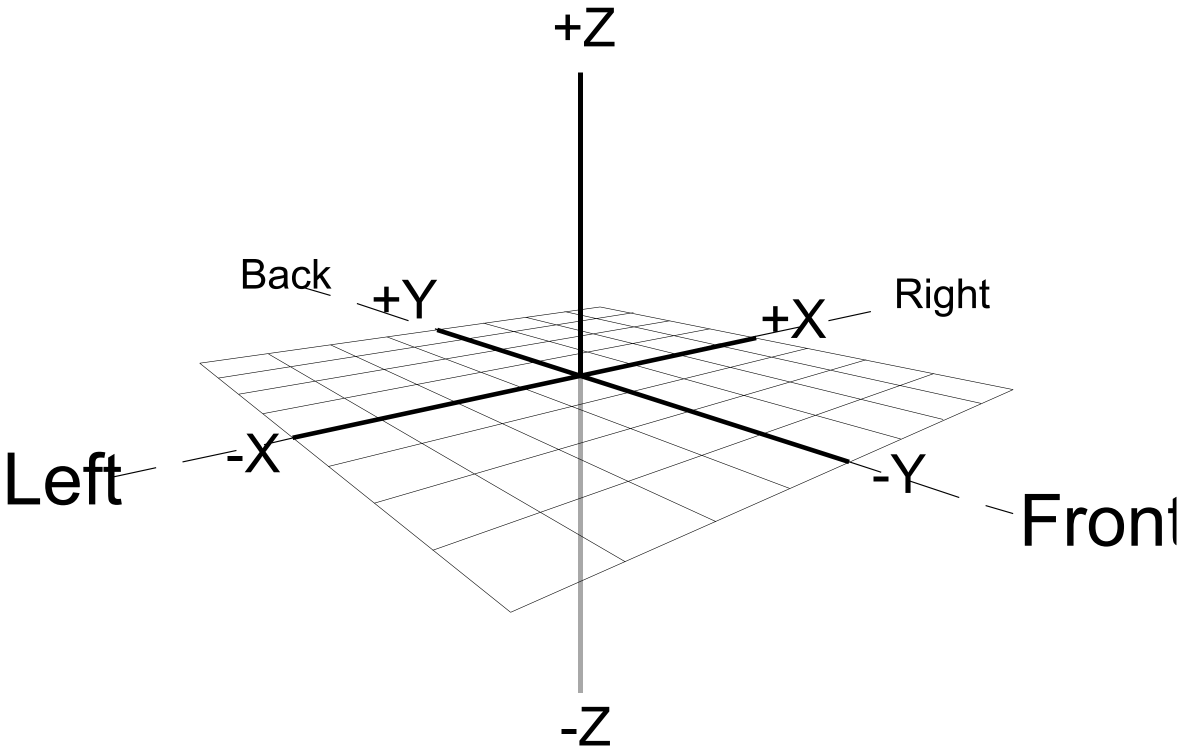


Polar coordinates simplify the computing and drawing of a large variety of curves, especially spirals. CAD programs offer both types of coordinate systems. Some features of a CAD drawing may be produced using a polar grid, while other features of the same drawing utilize the orthogonal grid.

3-Space

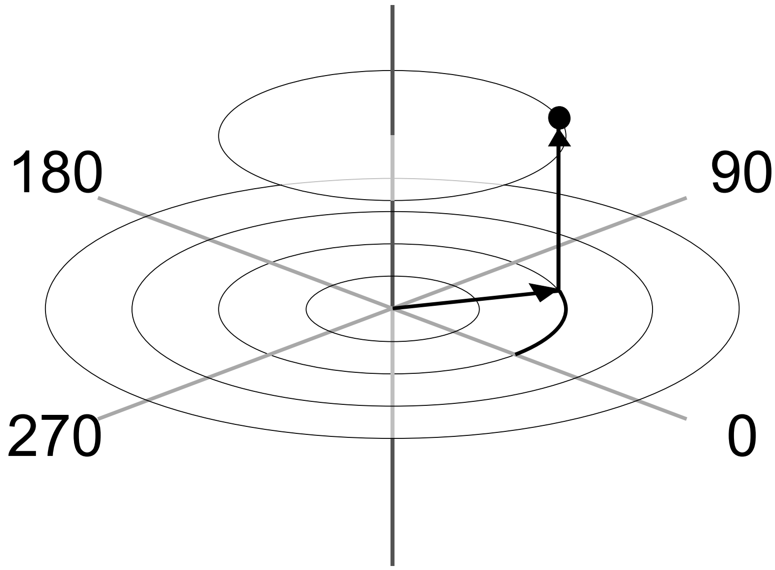
To model fully dimensional objects we require a third dimension of space. Adding a dimension means adding an axis and a third coordinate to the system. This new axis is the ***z***-axis. It runs perpendicular to the plane demarcated by the ***x***- and ***y***-axes and passes through the origin. With the addition of this new axis the ***x,y*** plane defaults to the horizontal plane and the ***z*** value determines the vertical position of a coordinate point ***x,y,z***. In conjunction with the ***x***- and ***y***-axes the ***z***-axis determines two new *planes of space* the ***x,z*** plane and the ***y,z*** plane.

When presenting the workspace for 3D procedures modeling programs provide up to six orthographic views. The most used are Top, Front and Right views. By convention Top is the orthographic view of the ***x,y*** plane with +x values on the right; Front is the orthographic view of the ***x,z*** plane with the +x values to the right; Right is the orthographic view of the ***y,z*** plane with the +y values to the right. Note that these are the favored views because they always keep the positive coordinates to the top and to the right. This consistency helps to navigate 3-space, which can be quite challenging on a 2-space monitor screen.

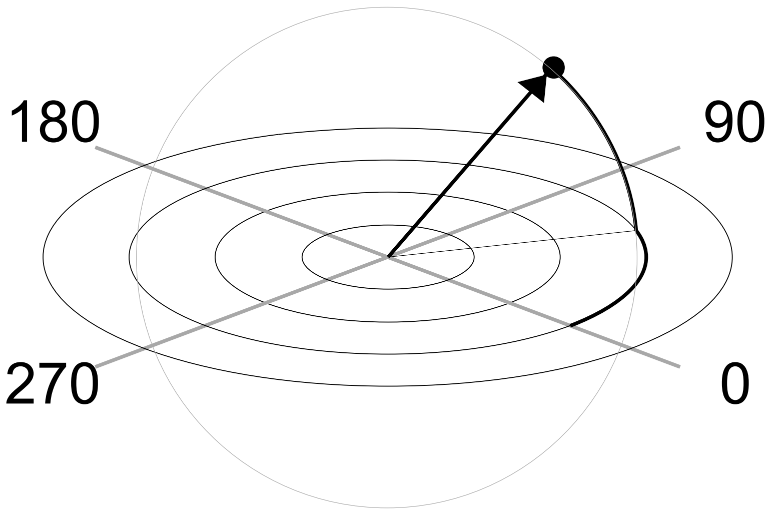


Very few programs offer 3-space polar grids on-screen, but the underlying code may use these coordinates. There are two types of polar grids used in 3-space: the cylindrical grid and the spherical grid.

The cylindrical grid defaults the 2-space grid to the horizontal plane and inserts a vertical axis through the origin. The three coordinates denote, in order, the horizontal angle bearing, the distance on the bearing line and the vertical distance (angle, distance out, distance up/down). This grid may be used for architectural applications where a circular layout is most effective.



The spherical grid also defaults the 2-space polar grid to the horizontal plane, but does not add a third axis. Instead the vertical dimension is defined by a vertical angle. The three coordinates specify, in order, the horizontal angle, the vertical angle and the distance out. All points in spherical coordinates are pictured as lying on a radius of the sphere. The first two coordinate values determine that radius, while the third value provides the distance from the origin.



This is the system used in astronomy and navigation. In these fields the horizontal angle is called the azimuth and the vertical angle is called the declination. The position of the sun on a certain date and time can be specified by azimuth and declination. Since the horizon curves in a circle around us, and the sky arches overhead like a dome (or hemisphere), spherical coordinates are an apt choice.

Geometric Spaces

Mathematicians often use the term space to apply to an abstract realm of relationships that constitute their own closed environment defined by systems controlling those relationships. This space may relate to actual space or it may be a logical, conceptual realm of relationships. Only objects that conform to the tenets of that space can exist in that space. A cube, for instance cannot, exist in 2-space, because that space does not allow for a third dimension. Its shadow or its picture can occupy that space, but not the cube itself.

Euclidean space is the conception of space governed by the axioms of traditional geometry. It was the only notion of space governing people's perception until the 19th century. It successfully explained the operation of tools, machines, maps, and building construction, and it still does. In regard to some aspects of space, especially the very large and the very small, Euclid's assumptions and axioms do not work. Astronomers, for example, struggled for centuries to create a conceptual model of the universe, while limited by the nature of their Euclidean perceptions. The same applied to the study of the atom.

Einstein found that he needed to adopt another form of geometry in order to improve the study of space and physics on an astronomical scale. To do so he borrowed from one the new geometries developed in the 19th century. Mathematicians discovered these by assuming one or more of Euclid's initial axioms were false, most notably his axiom that parallel lines never meet. If parallel lines do meet, they thought, what form would space itself take? The conclusion adopted by Einstein was that space must ultimately curve.

It is almost impossible to imagine space as curving, rather than extending indefinitely in all directions. This defies common sense experience. Yet it explains a host of principles of astrophysics. The workings of space at the atomic level defy common sense even more. Our most basic assumptions about space, such as position, direction, motion and distance, fly out the window. Our brains and perceptions adapted to survive in the spaces in which we live. The astronomic and the atomic exist beyond that limit.

Scientists have had to grudgingly develop a purely mathematical language to deal with space at these extremes and simply accept the truth of the mathematics even though their senses rebelled.

Most mathematical spaces are numerical constructions, which serve to solve certain key problems. Vector space, laid out above, is a numerical space. It comprises that set of geometric relationships made possible by truncated rays in a numerically ordered system of dimensions. In computer graphics the constructive geometry of the engineer and designer is expressed in a manner to fit that space.

Vector space can also be used to represent forms not treated in Euclidean geometry, algebraic curves and differential surfaces among them. The key word here is "represent": the vectors in CAD programs do not really create curves so much as build acceptable images of the curves and approximate their mathematical behavior to a precision within the tolerances needed for design.

Vector geometry permits representation of curves that cannot be drawn with a compass or straight edge, but which can be plotted on a grid. These include curves described by quadratic and cubic equations. The formulas for such curves include variables raised to powers of 2 (quadratic) and 3 (cubic).

Strings and Ropes

Unlike humans computers do not perceive space – they are built only to manipulate numbers. It is up to the computer graphics specialists to devise the numerical structures and programming that can adapt machine computation to conform so elegantly to human perceptions. In this these technicians are the modern descendants of the ancient rope pullers, who devised the first simple tools to express the logic of space into humanly expressive form. Strings of code may have replaced ropes, but the spirit remains the same.