Chapter 9

Sacred Geometry

Figures and Magic

Virtually all of the world's religions, past and present, have shown a consistent eagerness to heap spiritual significance on numbers and geometric figures. Though they are the stuff of logic, the mental and visual tools of science and engineering, these mathematical objects have also aided culture after culture in crafting their supernatural and metaphysical realities.

In early cultures the boundaries between the physical and spiritual worlds were blurred -- if such boundaries even existed in early thought. Ancient believers wove geometry into the canonical rules, the sacred symbols and the mystic structure of their religions.

Geometry began as much in the design of ritual spaces and sacred objects as in the design of the practical spaces and objects. Often the specific geometric figures and their methods of application were prescribed in the sacred writings, or canons of a religion. Geometry's role in engineering the sacred has been just as crucial to its development as has its role in engineering the physical world.

Geometry imparted cohesion and order to both realms. The perceived properties of geometric figures can handily correspond to abstract relationships and thus serve as mnemonic devices to preserve, focus and connect such relationships in the minds of the faithful. In effect, geometric figures have served as excellent symbols for the intangible entities of religion.

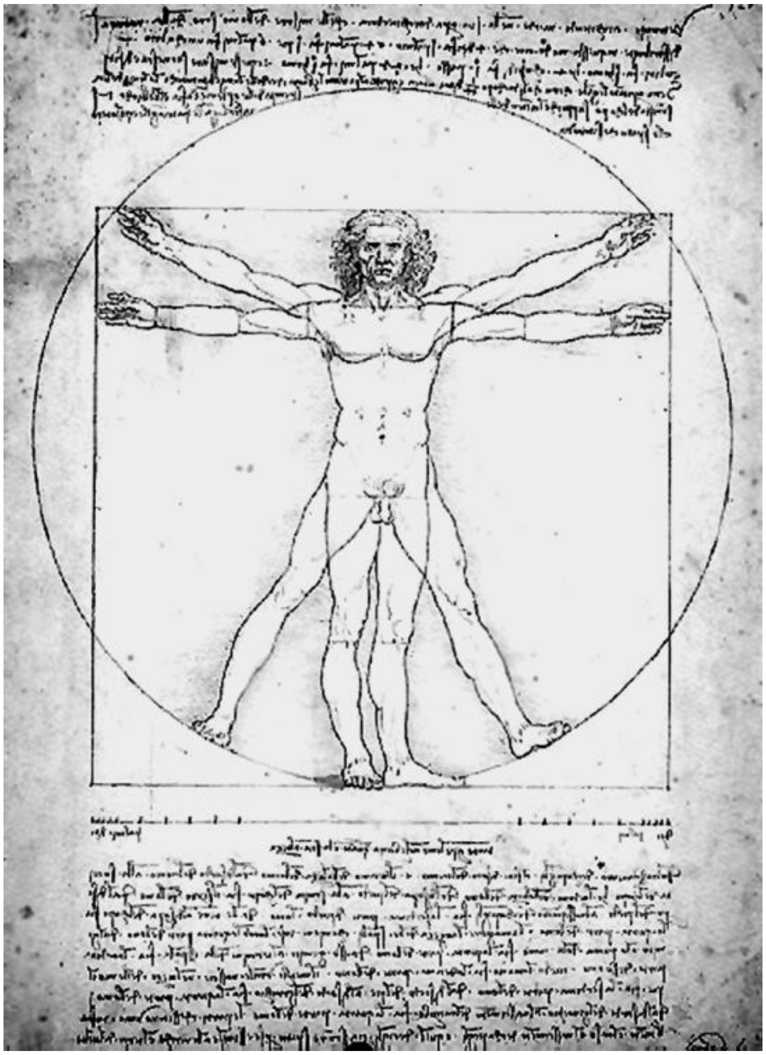
This symbolic role can intensify in the minds of believers to the degree that the geometric figure does not just represent the supernatural, but is, in itself, a manifestation of the supernatural. Knowledge of the properties of geometric forms was regarded as knowledge of the properties of the mystic realm and could lead to the ability to manipulate this realm.

Anthropologists denote belief systems that conflate supernatural entities with their symbolic surrogates as magic. Used in this way the term is not pejorative – most of the world's religions are magical to some extent. One example is belief in the efficacy of sacraments. This belief assumes that ritualistic acts, that is, symbolic behavior, can cause changes in spiritual structures. On a cruder level superstitious beliefs in curses, hexes, voodoo, and the like, assume further that symbolic behavior can have physical repercussions via supernatural channels.

After numerous earlier attempts to square the circle, in 1507 at age 55 Leonardo Da Vinci began a focused effort to seek a solution. Three fruitless years later he abandoned this quest, along his hopes of becoming the major geometer of his time, and began his famous research into detailing the human anatomy.

250 years later the German mathematician Johann Lambert provided the first rigorous proof that pi ***and, therefore, the area of a circle, are irrational and impossible to construct.***

In this 1492 sketch Da Vinci applied the "perfect" human proportions as prescribed by the ancient Roman architectural philosopher Vitruvius and interposed these into a conjunction of the square and the circle. This is by no means a scientific drawing, but a visual attempt to harmonize the "measure of man" with a mystic order represented through geometry.

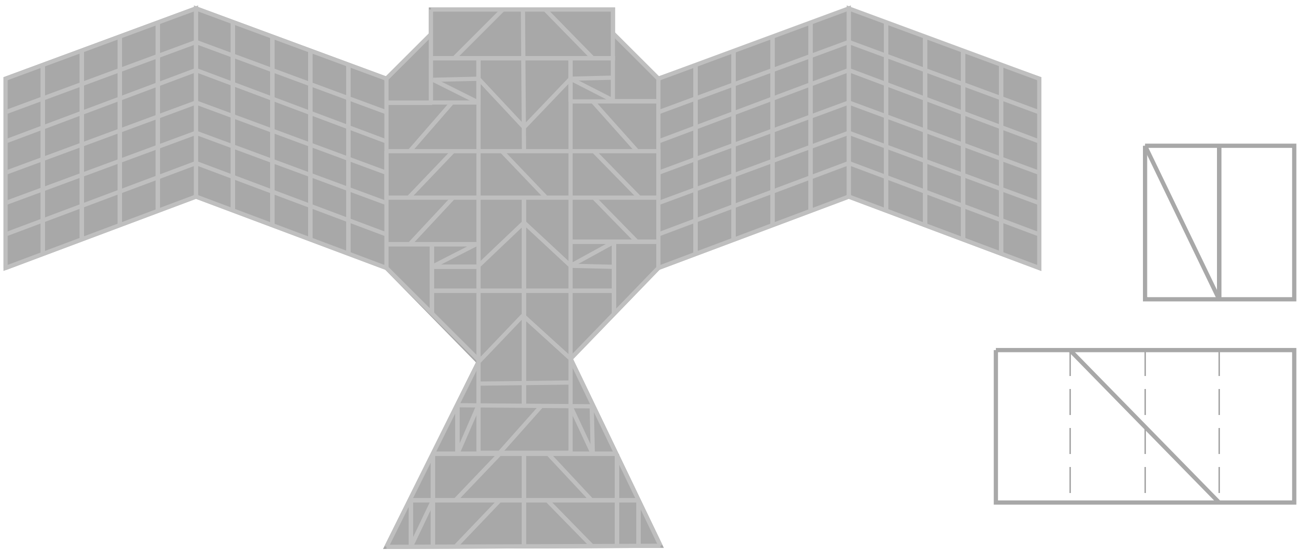


The Circle and the Square… and the Pentagon

One of the oldest problems tackled by a variety of ancient and medieval geometers had its origins in religious canons. The challenge was to draw a circle of the exact same area as a given square using only compass and straight edge construction. In the 17th century geometers found that there is no constructible solution to this problem, dubbed the squaring or quadrature of the circle, yet over the previous centuries the world's greatest minds exhausted countless hours in its pursuit.

Examining the relationship between the circle and square in sacred geometry, however, reveals much about the relationship between geometry and religion.

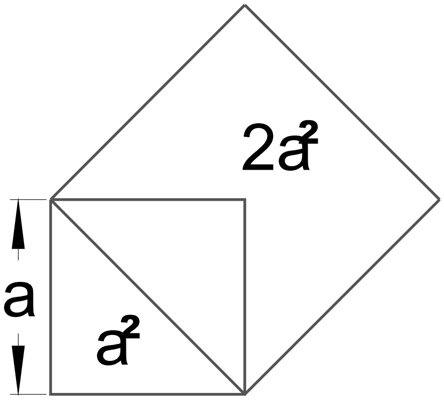
The diagram below illustrates the layout of a course of bricks from a Vedic altar in the form of a bird. Known as the falcon it is built from a mosaic of shapes derived from rectangles and their division into right triangles.



canonical geometry

The earliest record of the quadrature of the circle appeared around 800 BCE in an Indian religious text, the Sulbasultras. This text served as a manual of geometric constructions for the building of early Hindu, or Vedic, altars and appeared as an appendix to a large group of religious texts called the Vedangas. Sulbasutras translates as "codes of the measuring rope" after the chief measuring tool of ancient times.

The Sulbasutras set down precise rules for every element of the Vedic altar from the allowable shapes of bricks to its overall proportions. Proportion was especially important: the Vedangas, for example, required that any new altar added to a site be twice as large in area as the altar already there. Among the topics related to this canon were how to draw a square exactly twice the

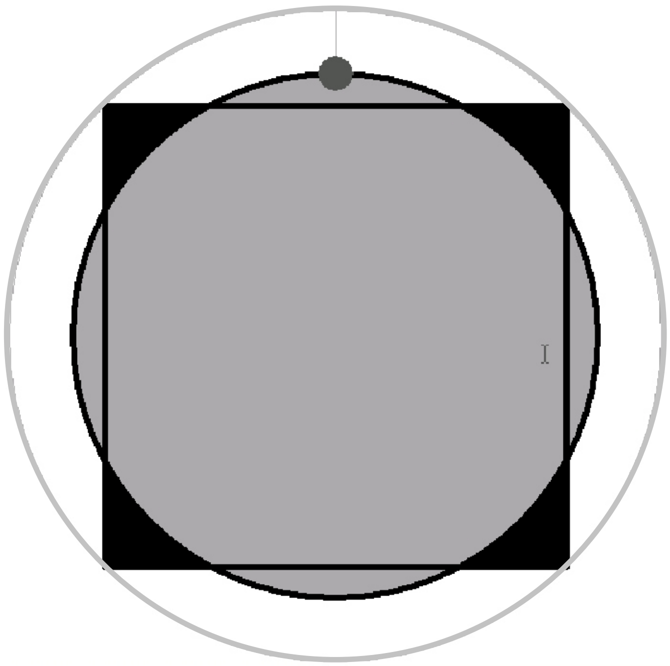


area of a given square and how to draw a square of exactly the same area as a given rectangle. Since all of the eight permissible brick shapes were rectangles, right triangles or their combinations, these two constructions, when used together, could double the area of any of the bricks.

The Vedangas included the quadrature of the circle as a method of proportion that could equalize and therefore strike a balance between the circle and square. Like most of the world's religions Veda saw the circle and square as closely related complements and set them up to represent harmonious opposites such as heaven and earth, space and form. Inscribing the circle in the square gave the square dominance, while inscribing the square in the circle gave the circle dominance. Neither of these options offered harmony. The answer was to square the circle.

The construction prescribed in the Sulbasutras offered a crude approximation deemed sufficient by the Vedic priests. This circle has an area about 2% greater than the square and gives a value for *pi* of 3.08.

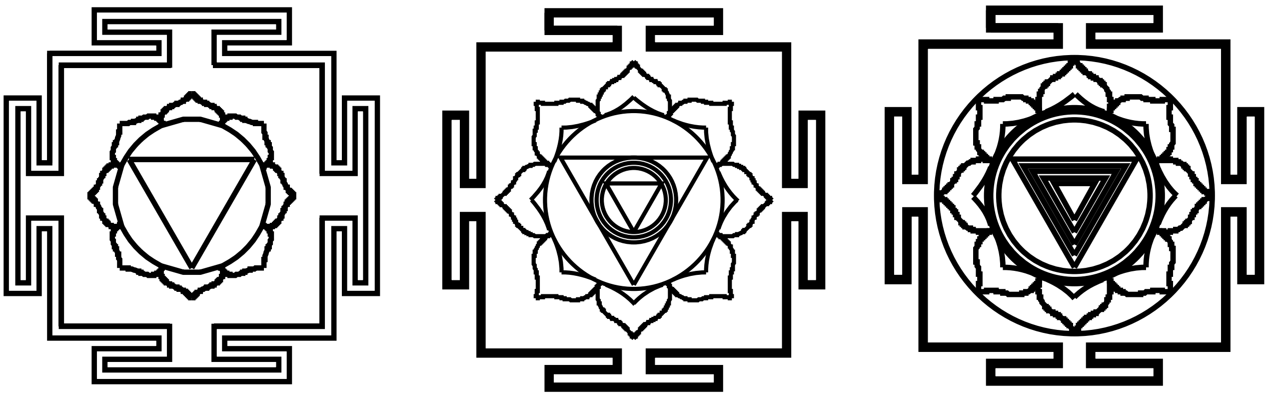
Draw a circle through the four corners of a square, and draw a perpendicular bisector from one side of the square to the circumscribed circle. Divide the perpendicular into thirds and draw a circle from the center of the square to the first third of this line.



symbolic geometry

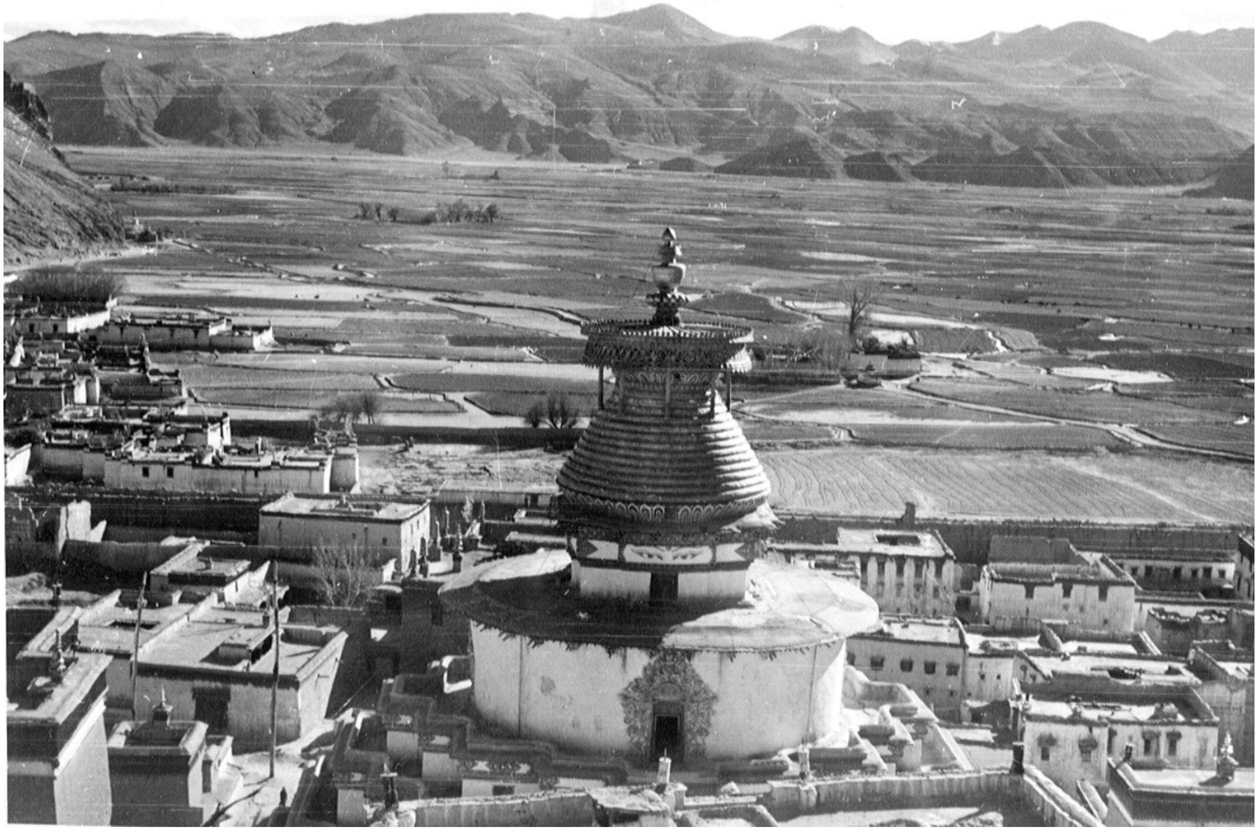
Since prehistory religions have assigned literally hundreds of meanings to circular and/or square emblems. Many of these meanings were cosmographical, that is, they sought to succinctly portray the shape of the universe. Broadly speaking the circle defined the form of the heavens: the dome of the sky, the rim of the horizon, the orbs of the sun and moon, the spinning of the stars at night. For its part the square suggested the four directions, the cells of architecture, the basic balance of horizontal and vertical, the stability of the right angle.

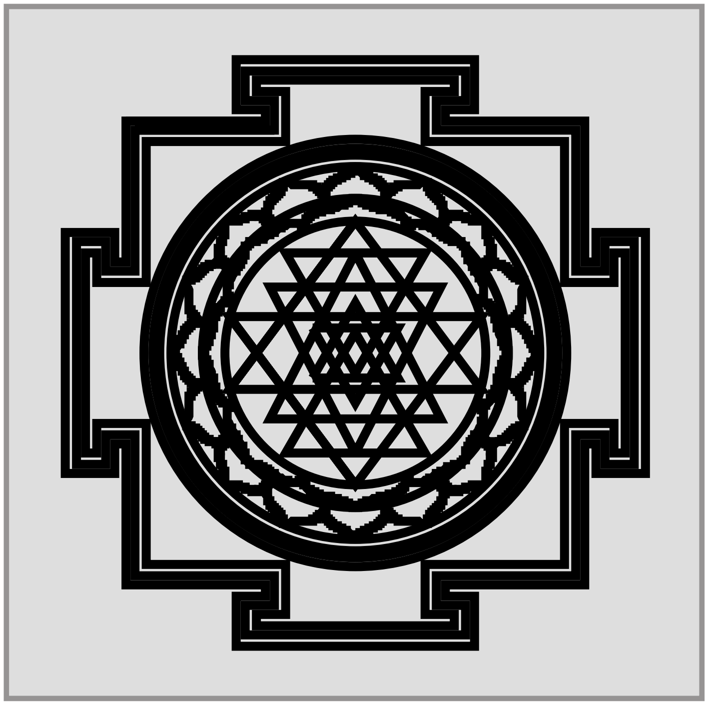
The Neolithic cultures that built circles of stones for their rituals to the sun and moon, organized their towns on a square grid similar to the Romans. They formed their sacred spaces to reflect the heavenly dome in which dwelt their gods, but built their own dwellings on the practical stability of the square.



Many other meanings derive from the shape properties of these two primary forms by using these properties as analogs for metaphysical ideas. A square, for example, partitions into corners and sides, while the circle is a single closed line. As a consequence the circle has often become a symbol of wholeness, unity and continuity. This is one of its symbolic uses in Hinduism. In other usages, however, that same religion regards the circle as an emblem of the void, representing absolute nothingness and the purity and perfection that implies. This is the source of zero being written as a circle.

Due to its partite construction the square lacks the wholeness and continuity of the circle. It does, though, frame and crop space into ordered segments. In contrast to the floating ethereality of the circle, it has often represented the earthly and the man-made with intimations rectitude, order and security.





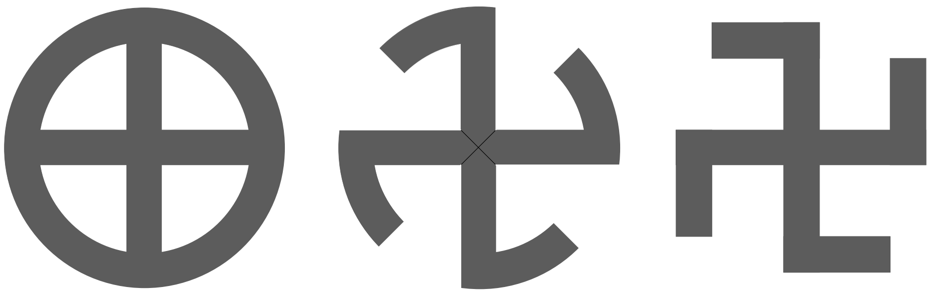
Hindu yantras *are highly patterned and usually highly colored religious emblems that serve a host of purposes from talismans to protect the owner to tools of reflection on the religious symbolism of their geometry. There are as many* yantras as there are purposes, but each shares a number of common properties in their combination of square circular and triangular motifs. Lotus petals also abound.

The yantras *also serve as layout templates for* stupas, *a common temple structure in the south and east of Asia. The* stupa *extends the inner circles into a layered dome that gradually tapers to a small spire. This is built on a square base, sometimes walled, with four entrances and follows the form of the* yantras' *outer square.*

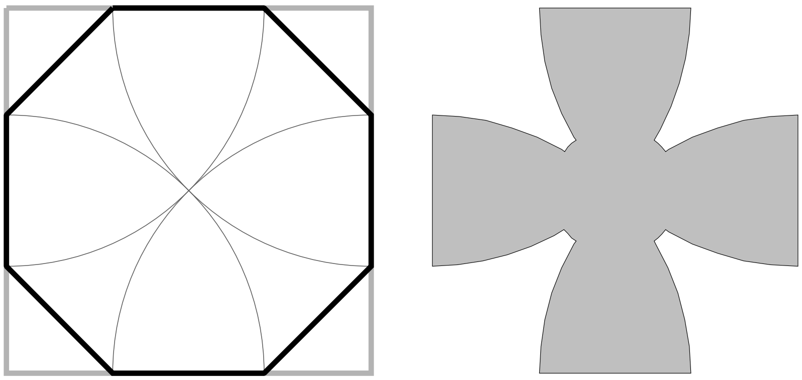
There is little if any interior space in the stupa *and what little there may be is often taken up by another* stupa *inside.*

Almost all religions feature symbols that meld aspects of the square and the circle into one form. The most universal of these is the crossed circle, a cosmograph that integrates symbols of earth and sky in order to acknowledge the fundamental unity of the universe.

In its simplest form this symbol is simply a circle divided by crossing its horizontal and vertical diameters, but it comes in many varieties. The most well known variation is the bent-arm cross. In this form a right-angled extension attaches to the left of the end of each of the cross arms, creating the effect of bent arms. This also creates the effect that the cross is rotating clockwise in the direction of the sun and stars across the sky and adds the implication of movement to its meanings. Best known by its Sanskrit name, *swastika*, this symbol was found in places as varied as Navaho rituals, Buddhist shrines and the ancient temple of Jerusalem.



Another integration of the circle and square is the octagon. Made by cropping the corners of a square the octagon shares qualities with the square, but rounds off to convey properties of the circle.

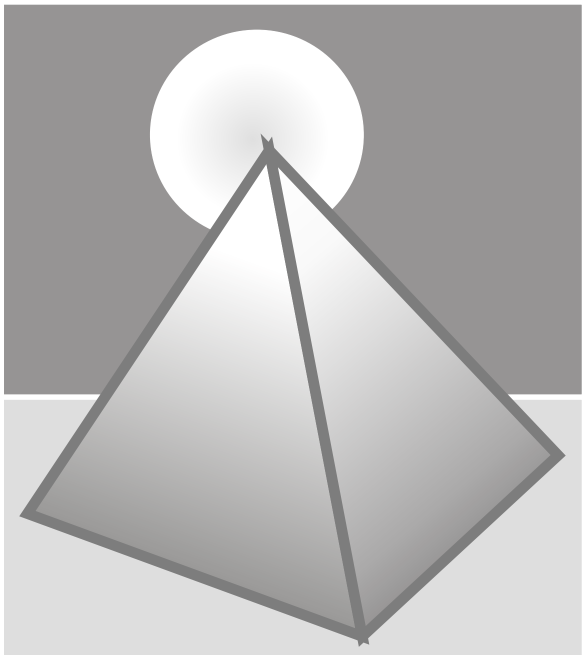


This drawing of the Maltese cross begins with the classic construction of the octagon and retains the geometry of the construction lines. This cross is a widening of the Greek cross, distinguished from the Latin cross by its equal horizontal and vertical arms.

Though they dominate religious symbolism, the circle and the square by no means hold the exclusive franchise. All of the familiar polygons to some degree share the task of expressing the metaphysical tenets of religious thought.

The triangle often appears, for instance, as a symbol of directed energy. Hindu *yantras* regard the energy as funneled to the apex: pointed downward the triangle suggests energy from the heavens, while pointed upward the energy flows from the earth. Such meanings, though, have a lot to do with the physical and cultural context in which they are used.

The ancient Egyptians originally covered their pyramids with a veneer of polished white stone intended to catch the light of the sun and transform the pyramid into a giant ray of light sourced in the heavens. In this case the triangular faces of the pyramid still express energy, but emanating from the apex and spreading to the base.



The Pyramid of Khafre, second largest of the Egyptian pyramids still displays some of the limestone that once smoothed its surface. The nearby city of Cairo owes its much of its architecture to the stone appropriated from the pyramid field at Giza. On the whole pulling stone from the pyramids was easier than quarrying new stone, until the stone gatherers neared the pyramid's lofty peak.

mystical geometry

One polygon that has an exceptionally fascinating history in sacred geometry is the pentagon. The geometry of the pentagon and the relationship of that geometry to the mathematics of nature and the built world are rife with coincidence and remarkable patterns.

The more the pentagon was studied, the more remarkable its properties seemed. For these reasons many practitioners of the priestly traditions of geometry regarded the pentagon as more than a symbol. Rather they believed that the study of the pentagon and related geometry, especially the circle and the square, could give insight into the purer dimensions of reality that underlie the corrupted physical dimensions of this world. To understand something of these relationships some background is in order, beginning with one of history's most famous thinkers.

The most influential priestly geometer of the ancient world was Pythagoras of Samos. Pythagoras left his home island off the coast of Turkey about 600 BC to spend over 20 years studying the esoteric traditions of Babylonian *magi* and the Egyptian priesthood. There is some indication that he may have been exposed to Vedic geometry as well.

Pythagoras believed that reality was ultimately founded in numbers. He developed that belief in part through his study of musical harmony. He found, for example, that taut strings of simple whole number increments of length could account for harmonic progressions of tones and convinced him and his followers of the truth of his reverence for number.

His study of right triangles, however, uncovered a more disturbing truth: the length of most hypotenuses of right triangles relative to their legs could not be expressed in numbers as he understood them.

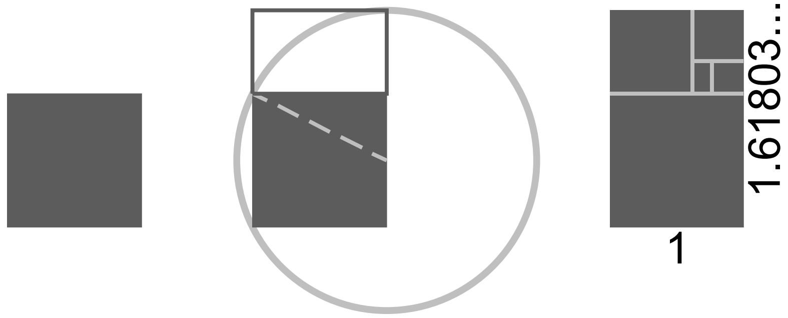
With only a few exceptions (like the 3-4-5 right triangle) the hypotenuse of triangles, whose legs were drawn in whole number increments, were always irrational numbers. This was a new concept to a world where numbers were whole or expressed fractionally as the ratio of whole numbers. Irrational numbers cannot by definition be written as such a ratio, but can only be approximated. Written decimally an irrational number appears as an infinite, non-repeating sequence of digits. The decimal value for *pi* can be written 3.14159, or 3.14159… with the ellipsis indicating progression to infinity. Aristotle later proved that any whole number without a whole number square root must have an irrational root. No fractions need apply.

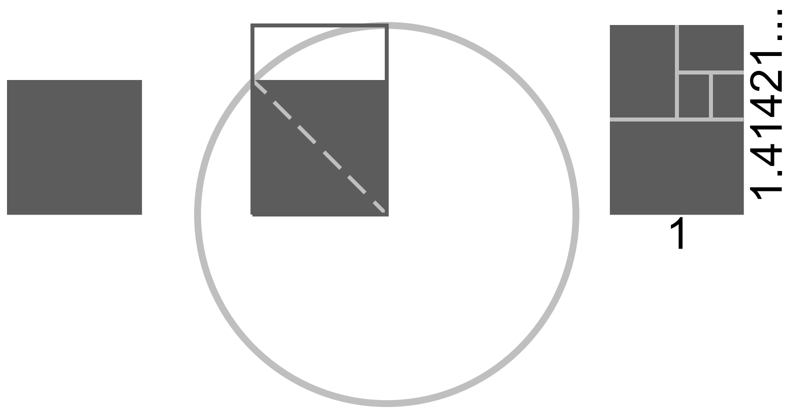
A number of important relationships between the circle and the square also yield irrational numbers. Two of these appear below:

If a square portion is removed from the top rectangle the remainder is a rectangle of exactly the same proportion as the original. This means that the smaller rectangle has the same properties: remove a square and a yet smaller copy of the original appears.

For ancient designers this was a good device for laying out a page of text or the facade of a building. Quickly doable with a compass and straightedge, this method yielded a pattern with a built in proportional and rhythmic breakdown.

The √2 rectangle offers a similar practical breakdown. Folded in half it yields two pages with the exact same proportions of the larger; fold againand the pages double, still the same proportion. This is a popular proportion for printing papers. In book publishing, for example, sixteen pages are printed at a time on a single large sheet and then folded three times. This group of pages, called a signature, is then trimmed, collated with other signatures and sewn into a book. othertrimmewhere pages pageA similar





These constructions yielded rectangles that could be subdivided into elegant proportional rhythms, and so were useful systems for laying out architecture and text. While designers and craftsmen applying these systems knew these as useful construction methods, there is no evidence they understood their fascinating mathematics.

By coincidence the second construction above gives a proportion identical to the Golden Mean, which is presented by Euclid as the division of a line such that the shorter segment is related to the longer segment by the same ratio as the larger segment is to the whole line. The longer line segment is the *mean* average between the short segment and the whole line.

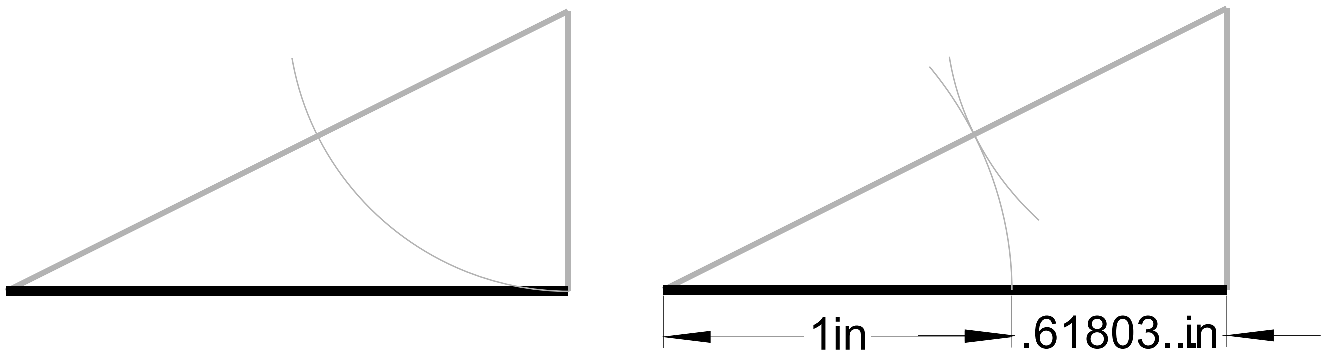
Along with *pi* this number 1.61803 dubbed *phi*, is one of the great irrational numbers. It describes growth progressions found all throughout nature and in a large array of man-made patterns. The

Renaissance Italian mathematician Leonardo Fibonnaci, who calculated the growth of populations of animals, discovered that the ratio of growth from breeding cycle to breeding cycle approached the value of the Golden Mean as the population grew larger and larger.

Euclid applied the Golden Mean chiefly to create the construction of the pentagon. By another remarkable coincidence the ratio of the side of a pentagon to its diagonal is also 1.61803. The earliest construction of the pentagon required the construction of the Golden Mean first.

36) Find the Golden Mean of a given line.

This is the classic construction for the Golden Mean. Like so many features of this curious proportion, it is not intuitively grasped. At the end of a given line segment draw a perpendicular whose length is equal to half of the given line. Draw a line connecting the open ends of the two lines to complete a right triangle. From the apex of this triangle draw an arc to intersect the hypotenuse with a radius equivalent to the vertical leg:

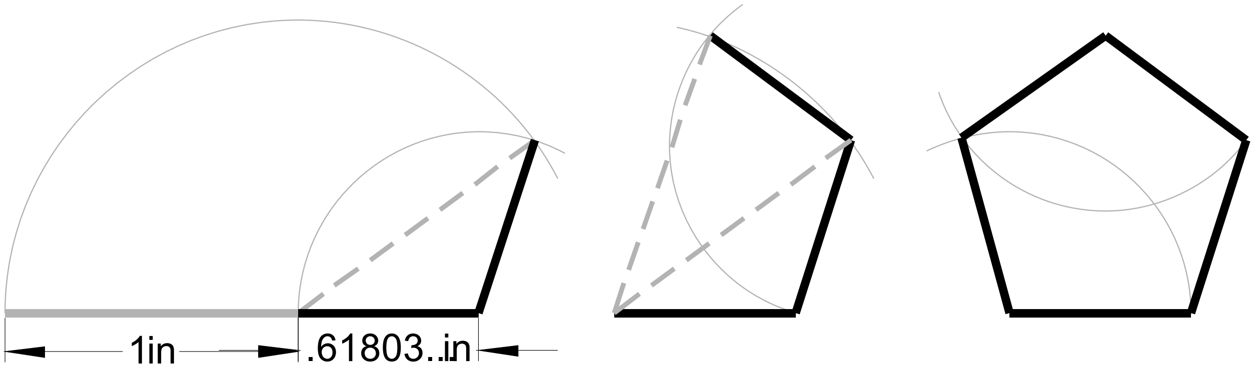


37) Draw a pentagon from the Golden Mean.

Begin with the shorter line segment as the base for the pentagon. From one end of the base strike an arc whose radius is the length of the base and from the other end strike an arc whose radius is the length of the longer segment. Connect the intersection of the arcs to the nearest end of the base to draw the second side of the pentagon.

From the open end of the second side strike a third arc with a radius equal to the side of the pentagon and intersect the larger arc. A line from this intersection to the open end of the second side will form the third side of the pentagon.

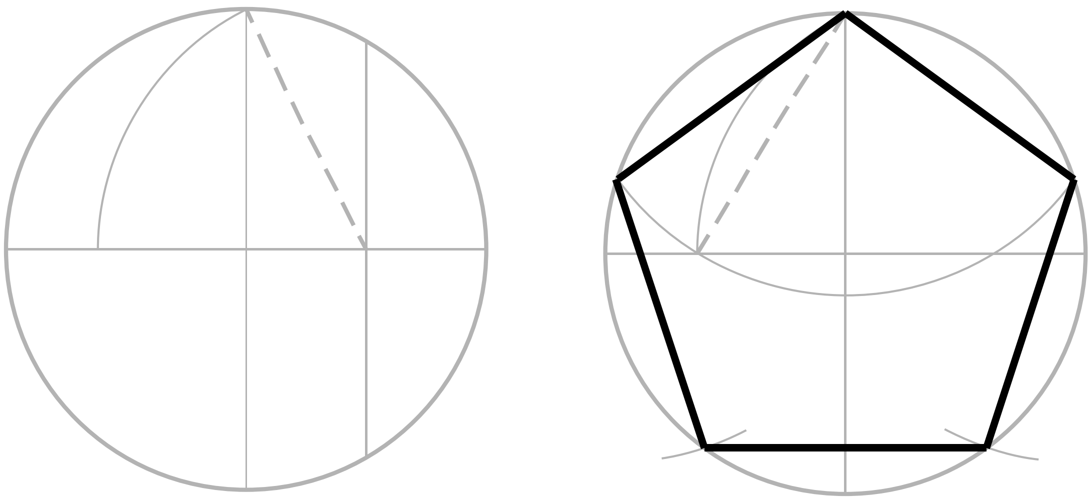
To close the pentagon: position the compass on the open end of the first and the third side and draw two intersecting arcs with the radius of the sides. The point of intersection of these two arcs will determine the last vertex of the pentagon.



38) Inscribe a pentagon in a circle.

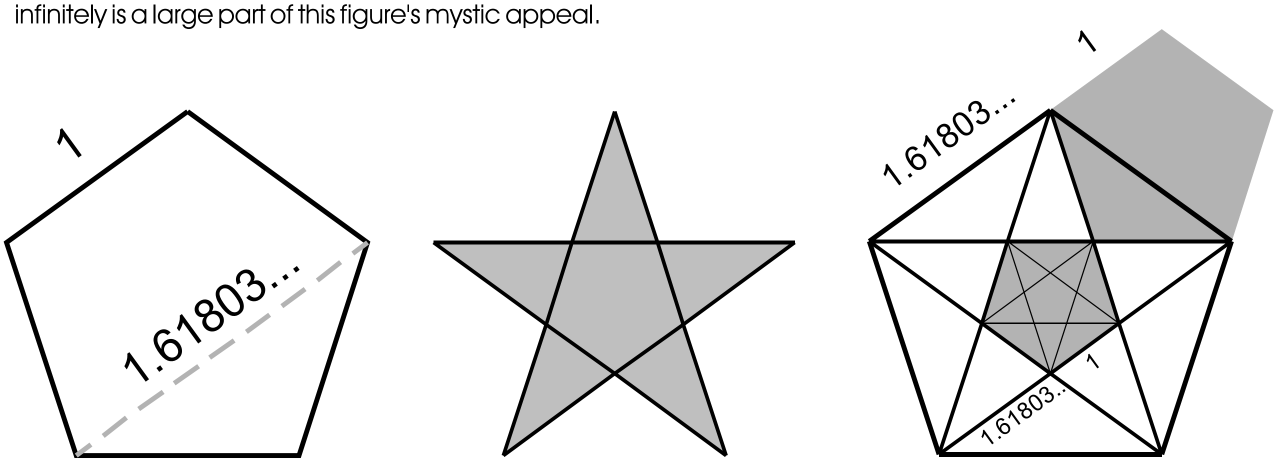
A later construction of the pentagon merges the two previous constructions into one procedure.

Draw two diameters of the circle perpendicular to one another and bisect one horizontal arm of the cross they form. Set the point of the compass on the point of bisection and the pencil on an endpoint of the vertical diameter. Draw an arc to intersect the horizontal diameter.



The distance from this intersection to the endpoints of the vertical diameter is equal to the side of the inscribed pentagon. Set the compass to this distance and divide the circle five times. Connect the points of division to draw the pentagon.

The pentagon's diagonals form a five-pointed star, or pentagram. This familiar figure is today a clichéd symbol of witchcraft and magical rituals and was probably the most used of mystical figures related to the pentagon, frequently appearing in illustrations for alchemical and astrological texts. At the center of the pentagram is a smaller inverted pentagon, whose diagonals will create yet another pentagram. The fact that this procedure can repeat infinitely is a large part of this figure's mystic appeal.



In addition to his discovery of growth ratios Fibonnaci also introduced the decimal system and the algebraic computation of Arabic mathematics to the European continent, thus opening up whole new avenues of computational possibilities. (Fibonnaci's father had been the financial legate of the Italian government in North Africa and had his son trained by Arab mathematicians. Fibonacci introduced their methods of computation to the financial establishment where their popularity for keeping books spread quickly.)

Ancient geometers understood *phi* primarily through geometric manipulation, but during the Renaissance curious geometers, armed with this new mathematics, could uncover more and more aspects of *phi*.

Luca Pacioli, who was first mentioned in chapter 5, published a book entitled "The Divine Proportion". Illustrated by his friend Leonardo Da Vinci, Pacioli's book apotheosized the Golden Mean, tying its host of intricate properties to a supra-natural realm beyond causal, physical properties. Revered as the father of modern accounting Pacioli was more than familiar with Fibonacci's work.

Pacioli was a trained *abaci,* as those professionals trained in Arabic methods were known, and a student of Piero Della Francesca. Most of Pacioli's published writings were expositions of his mentor's ideas on art and mathematics and "The Divine Proportion" was no exception. As one of the Renaissance's greatest artists and greatest mathematicians,Della Francesca knew both the craftsman's use of the Golden Ratio for layout and studied its role in pentagonal structures.

Some accuse Pacioli of gross plagiarism, but without his writings the ideas of Della Francesca, who never published, would not have come to public light.

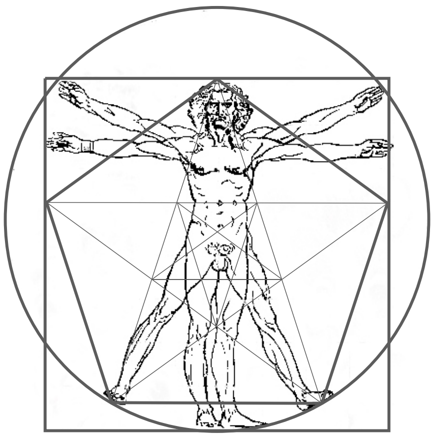
moral geometry

One cultural outgrowth of this mysticizing of geometry was a perception of geometry as a character-building tool. The English language reflects this with more than its share of metaphors for expressing moral behavior in terms of geometry. Trustworthy people are "straight" talkers and "upright" citizens who are "on the level", while those who talk in "circles" lead us to wonder what their "angle" is. To be prepared is to be "squared' away; to be exact is to be "plumb".

Many of these metaphors are physical, that is, they refer to the use of geometry in building. The overarching metaphor is that the construction of behavior and personality can be referenced in the building of one's physical reality. Another overarching metaphor insinuates that logical thought and supra-logical ideas are governed by overt and tacit principles to be found in geometry.

Some educational theories of the 19th century held geometry to be a tool for engineering thought as well as comprehension of the physical world. Early in the century Friederick Froebel , the German educator who founded kindergarten, introduced his noted set of geometric blocks for early childhood education. Later in the century, as new forms of geometry proliferated, a push toward phasing out the teaching of constructive methods in England was thwarted by the impassioned argument that its elimination would damage the moral character of British schoolchildren.

Today this geometry is under the purview of design fields, where, its practical value remains intact. In general education geometry is the field of mathematics where proofs are taught and stressed. Along with algebra it is a step toward more computational methods for dealing with form and motion.



DaVinci's depiction of the ideal proportions of the human body can be broken down into a relationship between the square, the circle and the pentagon. The urge was strong, even to one of DaVinci's genius, to establish a connection between humans and an ideal, divine structure to underlie reality. Today such an arbitrary connection seems ludicrous: there is no ideal human form, just an anthropometric average of human dimensions. In DaVinci's time, however, the universe was not perceived as a physical entity, but as a geometric construct bound by a hypothetical element, the quintessence.