Archimedean Nightlight:
A Creative Investigation of Spherical Polyhedrons

Materials: 110 lb or heavier index board, small metal ruler with non-slip backing, Exacto knife with #11 blade, white glue.
Equipment: Inkjet printer
Programs: PolyPro, TurboCad Learning Edition. (Both are available for free download)

This tutorial focuses on the design and implementation of a beautiful and functional object -- a nightlight based on the symmetries of a spherical polyhedron. With the exception of the final assembly, the design and production of the nightlight will feature the use of computer programs freely available on the Internet. This tutorial offers practice in using constructive principles along with CAD tools for creating polygons, as well as editing procedures using symmetry transformations.

Spherical Polyhedrons

A polyhedron is a 3-D geometric figure enclosed only by flat surface planes. These planes, called faces, meet at straight edges and these edges meet at corners called vertices. A polyhedron is spherical if all of is vertices lie on the surface of a sphere.

There are several categories of spherical polyhedrons, the most famous of which are the five Platonic solids. Named after their ancient Greek advocate, these comprise the set of all regular polyhedrons, that is, these are the only polyhedrons for which all faces, edges and angles are equal. This includes the angles between the faces. By inference all of the faces of a Platonic solid are the same regular polygon.

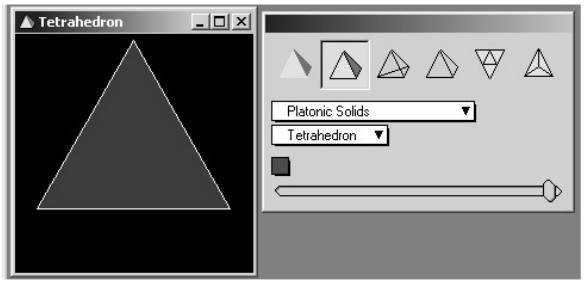
There are many more semi-regular polyhedrons. These, too, are spherical, but in contrast to the Platonic solids this group has faces comprising more than one regular polygon. The edges remain equal, while the angles vary with the face polygon. Of these the best known are the thirteen Archimedean solids. The form of the nightlight will derive from one of this set of polyhedrons.

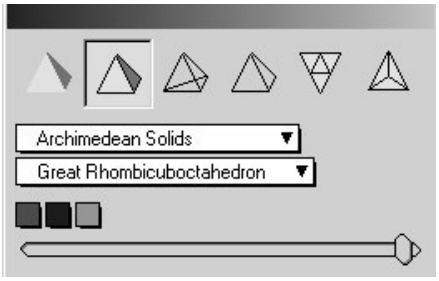
Part 1. Set-Up

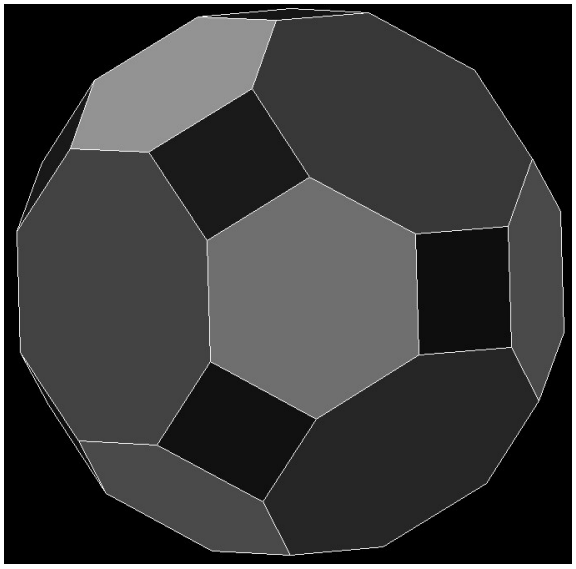
The demo version of PolyPro, a program for creating and studying polyhedrons, offers a broad array of elegant polyhedrons to view in various formats. Among these are the Archimedean solids.

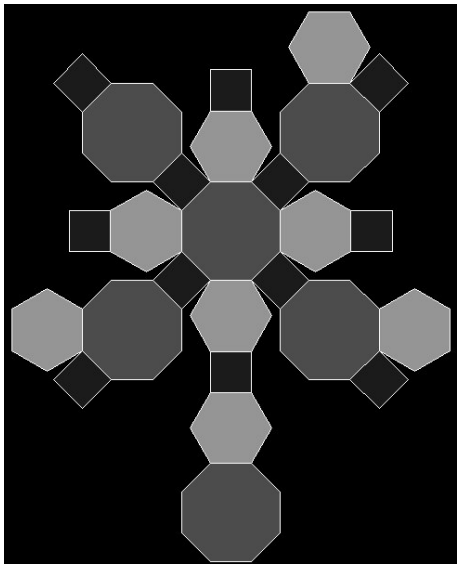
1. Open PolyPro and maximize the window, in the upper left, which displays the polyhedron. The menu form will automatically dock in the upper right of the screen. The first drop-down menu lists the Polyhedron Categories. Select Archimedean Solids from this menu. The second drop-down menu names the various polyhedrons in that category. A mouse click on each name will cause that polyhedron to pop up on screen.

   Once the polyhedron is presented, depressing the right mouse button and dragging will rotate the object in the direction of the drag. Examine the various Archimedean solids and settle on one for the design.







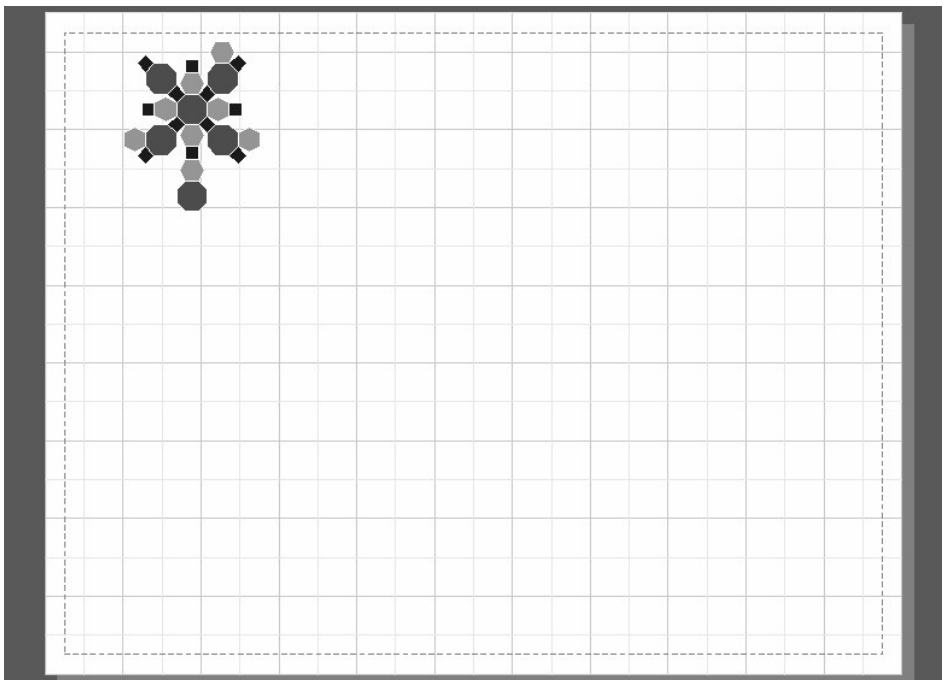


2. For the sake of example, this tutorial uses the Great Rhombicuboctahedron.

 Note that several View Mode icons line the top of the menu form. Click on the Two-Dimensional Net button and the polyhedron will unfold into a flat pattern. If the icon is not present, go to File, Preferences and select Available View Modes. Check the desired boxes.

Hit Control + C to copy the unfolded pattern. This will capture the screen as a .wmf (Windows Metafile).

3. OpenTurboCAD. Select New from Template and double click on the Normal template to set up the Drawing Sheet. Paste the .wmf pattern onto the drawing. Scale the pattern down to a thumbnail size (about 0.3 in the Scale boxes on the Edit bar below the Drawing Sheet) and move to a corner of the drawing sheet. This image is for reference and will not be a part of the finished design.



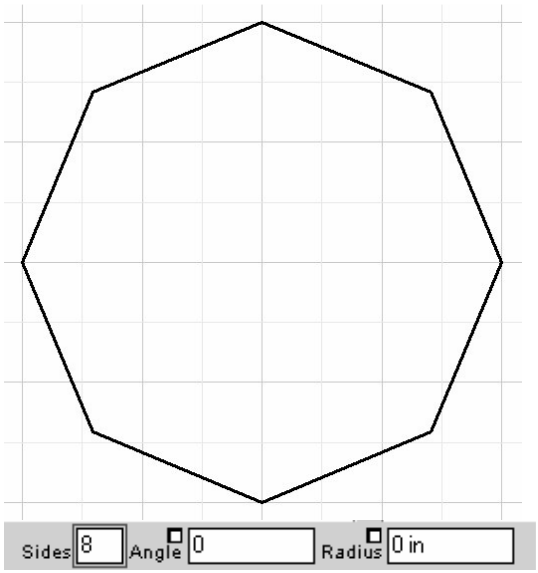
Part II. Laying Out the Polyhedron

This section of the tutorial runs through methods for laying out the polyhedron pattern in TurboCad. It covers the insertion of regular polygons and the editing procedures, which incorporate mirror, radial and translation symmetries.

4. <u>Inserting and orienting polygons.</u>

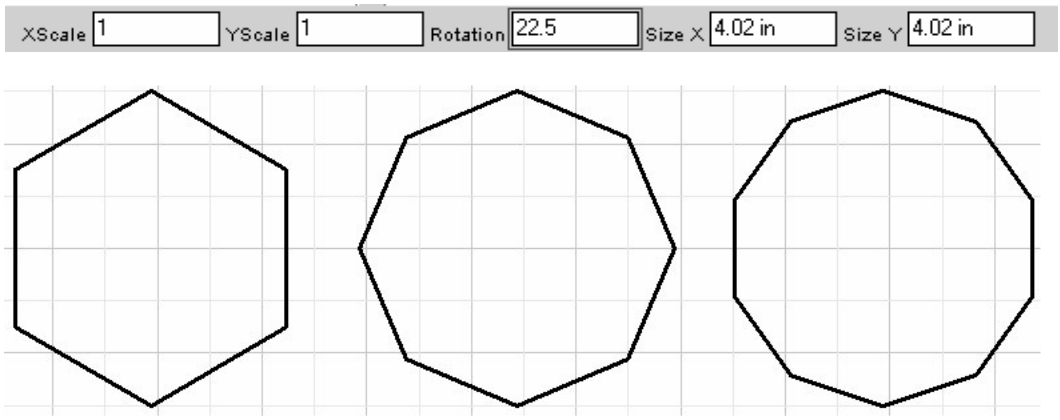
 Go to the Insert menu and select Line, Polygon or click on the Polygon button from the drop-down Line menu on the Standard toolbar. This tool only

inserts regular polygons -- those with all sides and angles equal. The Edit bar under the drawing sheet will prompt for the number of sides on the polygon. This example illustrates the insertion of an octagon, an 8-sided figure, to begin the layout of the polyhedron's pattern.



Activate the Grid snap. Click on one grid point to set the center of the polygon and click on another point on the same grid line to set the radius of the polygon. The radius is the distance from the center to the vertices of a regular polygon.

With this procedure polygons having an odd number of sides will automatically orient with a horizontal base. To similarly orient the octagon, rotate so the bottom and top sides are horizontal. Select the polygon and type 22.5 in the Rotation box on the Edit bar. To determine the proper rotation: divide 360 by the number of sides of the polygon and then halve that figure.



360/6 = 60 | 60/2 = 30 360/8 = 45 | 45/2 = 22.5 360/10 = 36 | 36/2 = 18

5. <u>Attaching a polygon to the side of another.</u>

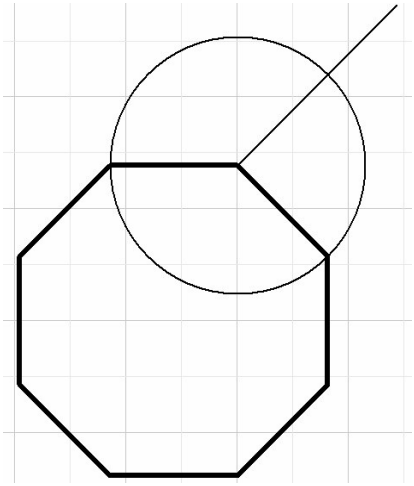
a) Attaching a square.

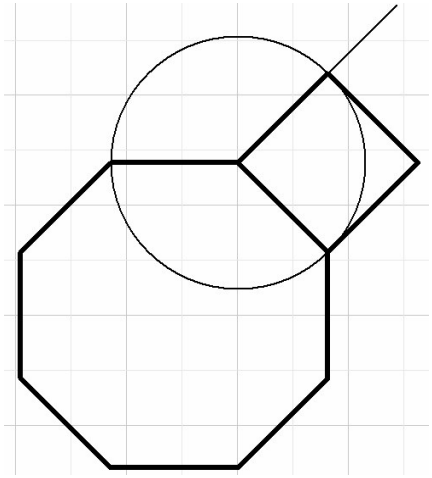
b)

Activate the Vertex snap from the Snap toolbar to the left of the Drawing Sheet and select the Perpendicular tool from the Insert, Line menu. Click on one side of the host polygon. This selects the line on which to build the perpendicular. Now click on a vertex of

the polygon to set the base of the perpendicular. Drag the mouse to extend the perpendicular an approximate distance greater than the side of the polygon. Hit the **s** key to set the end of the perpendicular.

 With the Vertex snap still active, select Insert, Circle, Center and Radius, or click on the Center and Radius button from the drop-down Circle menu on the Standard toolbar.



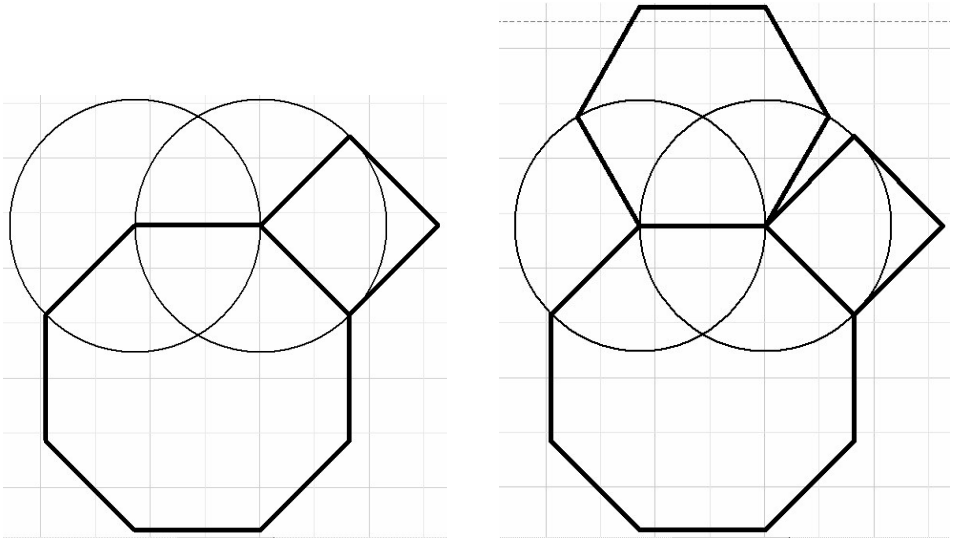


Click at the base of the perpendicular to set the center of the circle and then click on the next vertex to set the radius and complete the circle.

 Choose Insert, Line, Rotated Rectangle, or activate the Rotated Rectangle button on the drop-down Line menu of the Standard toolbar. Also activate the Intersection snap. Click first on one vertex of the host side and then on the other vertex of that side to set the length and position of the rectangle. The third click on the intersection of the circle and the perpendicular will complete the rectangle, which will be a square.

b) Attaching a hexagon.
Activate the Vertex snap and, using the Center and Radius tool, draw a circle from each of two adjacent vertices: click on one vertex to set the center and then on the other to set the radius. The intersection of the circles will be the center of a hexagon sharing that side.

With the Intersection snap active, use the Polygon tool with the number of sides at **6** to draw the hexagon. Click on the intersection of the two circles to set the center and on one of the vertices of the host polygon to set the radius of the hexagon.



c) To attach an equilateral triangle.
Begin as with the hexagon, by drawing the two intersecting circles and keeping the Vertex and Intersection snaps active.

 Choose Insert, Line, Multiline or click the Multiline button in the drop-down Line menu on the Standard toolbar. Click on the vertices on each end of the host side and on the intersection of the two circles. Right click on the mouse and from the pop-up menu click on Close. This will complete the Multiline as a polygon. The equivalence of the radii of the circles with the side of the host polygon ensures that the triangle is equilateral.

d) To "attach" other regular polygons.
The remaining three regular polygons found in the Archimedean solids -- the octagon (8 sides), the pentagon (5 sides) and the decagon (10 sides) -- are much more difficult to attach. A better strategy would be to insert these polygons first and then attach the equilateral triangle, the square and the hexagon as needed.

6. Copy polygons using symmetry transformations.

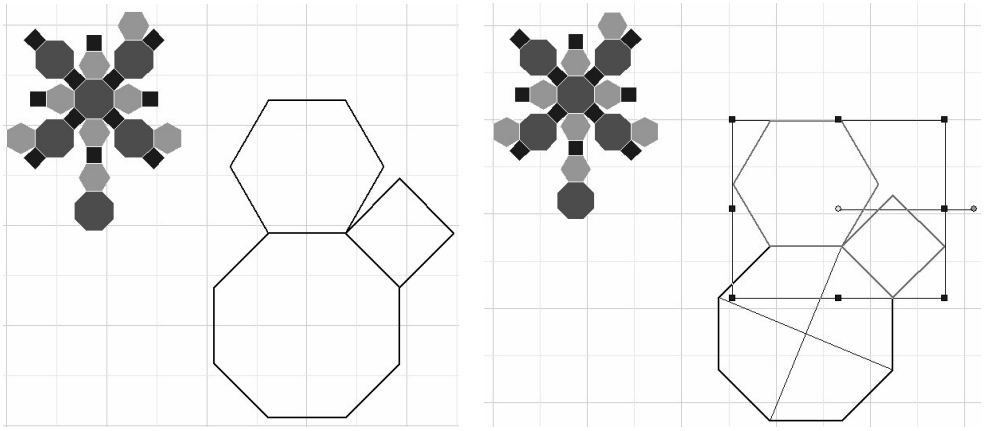
The many edit tools available in CAD and other graphic programs use the geometric principles of transformation, that is, they are programmed to move, rotate or scale the graphic object. Most of these tools involve multiple transformations and symmetry operations that permit copying in very regular arrays. As a result, it is usually only necessary to insert and attach one of each polygon in the patterns of the Archimedean solids, and then multiply these with the edit operations.

The following steps apply some of these editing tools to the construction of the Great Rhombicuboctahedron.

a) Radial Copy

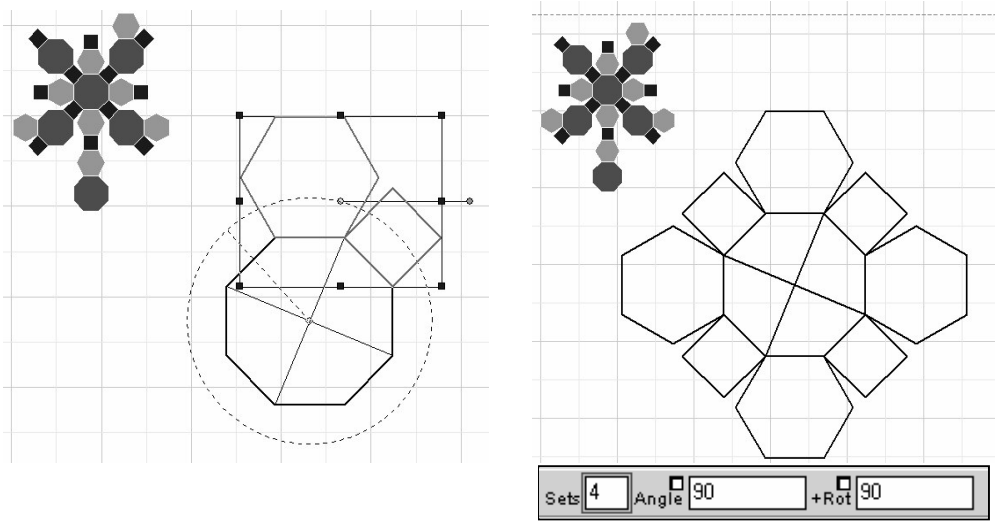
A radial copy involves the transformations of rotation and move. The graphic object moves to a series of points equidistant from a center point and equidistant from its respective copies. As it does so it rotates on angles radial to that center.

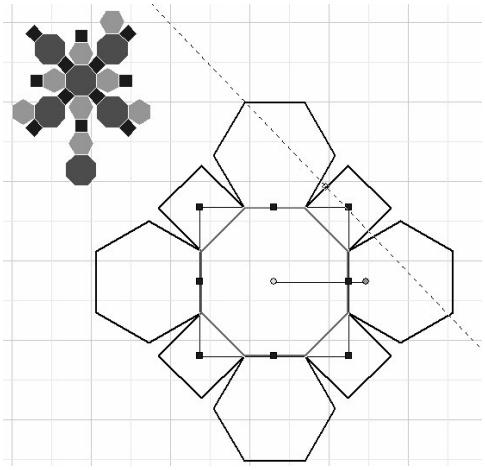
In this example the octagon had been inserted first and then the square and hexagon attached:

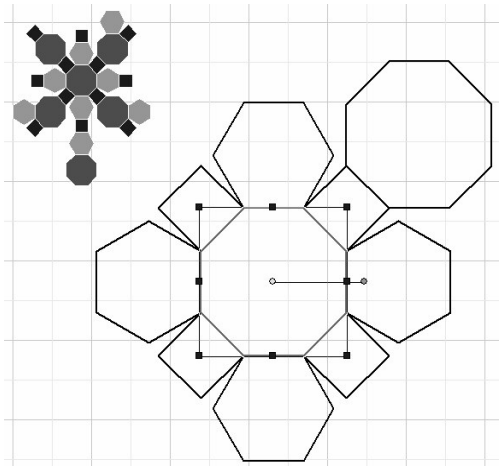


Again activate the Vertex and Intersection snaps and draw two lines connecting the octagon's diametric vertices. Their intersection marks the octagon's center.

Select the hexagon and the square by clicking on each while holding down the **Shift** key, or by dragging a window. From the Menu bar choose Edit, Copy Entities, Radial. A dashed circle will appear with a dashed radius line. With the Intersection snap active, click on the intersection at the center of the octagon, and on the Edit bar type **4** into the Sets box. Hit Enter to complete the copy.





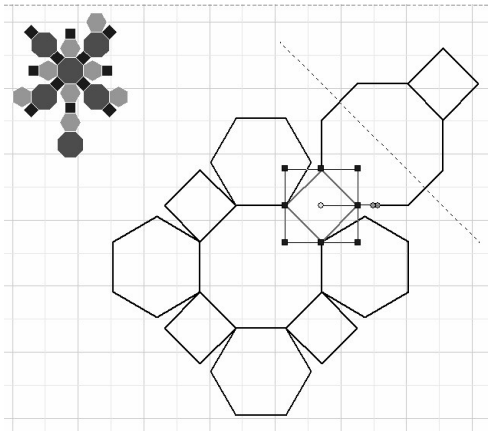


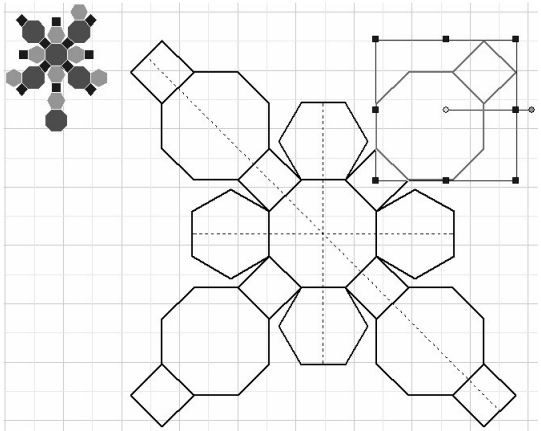
b) Mirror Copy

Mirror copy creates a replica of the graphic object across an axis of reflection. For symmetrical objects, like the regular polygon there is no discernible difference in appearance.

Right click over the drawing sheet and on the pop-up menu de-select Open Window Mode. Select the octagon by dragging a window completely around it. Activate the Middle Point snap. Choose Edit, Copy Entities, Mirror and click on opposite sides of a square to mirror the octagon across that square.

c) Select the same square and mirror it across this exterior octagon. This new octagon and square should form an arm extending from the pattern. Select the newly mirrored octagon and square and copy this to the other three corners of the pattern. Use the Radial copy tool or the Mirror copy tool. Note the mirror axes and the center point of the radial copies in the illustration below, right.



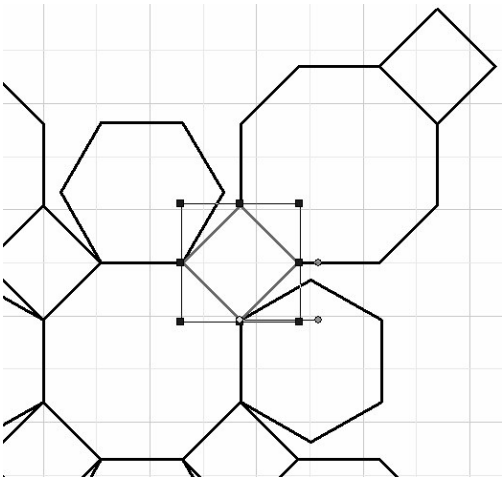


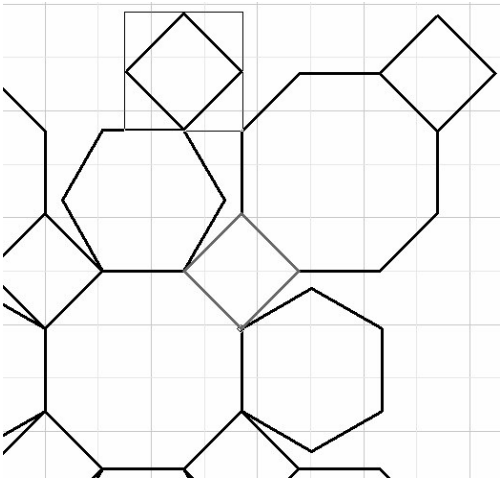
d) Move Copy

A graphic object can be copied to a specific point in space by combining the snap tools with modifications of its reference point. When selecting an object for editing a yellow dot appears, by default, at the center of the selection box. When rotating the object, this is the point around which it rotates. When moving the object this is the point that is repositioned in space.

With Vertex snap active, select the square adjacent to the top right side of the octagon, and then hit the "**D**" key. A small icon of a hand holding a small dot appears. Click on the lower corner of the square and the reference point re-locates to that point.

Right click and select Rubber Stamp from the pop-up menu. A small icon of a rubber stamp appears. Click on the top right vertex of the upper hexagon to place a copy of the square.

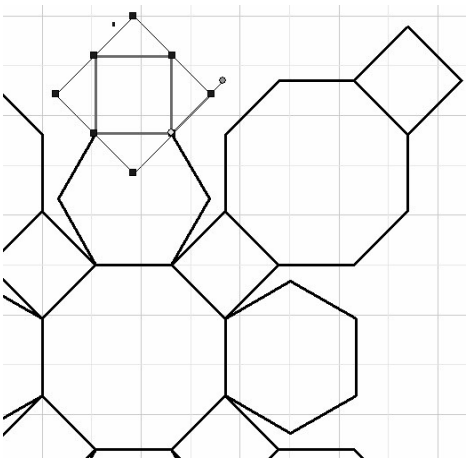


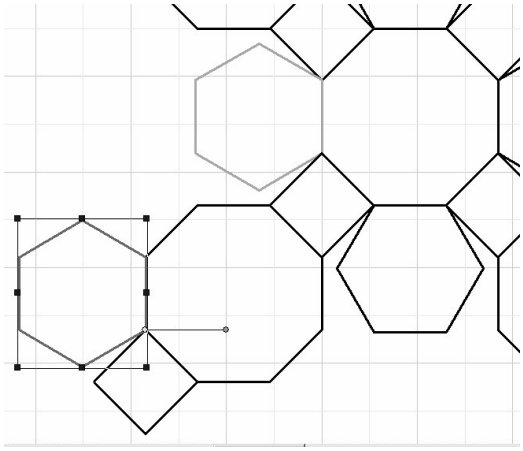


The square is canted at a 45°angle. Since the reference point is located on the coinciding vertices of the two polygons, it is only necessary to rotate the square 45° to bring it in alignment with the hexagon. To do this, select the square and type 45 in the Rotation box on the Edit bar. Hit **Enter**.

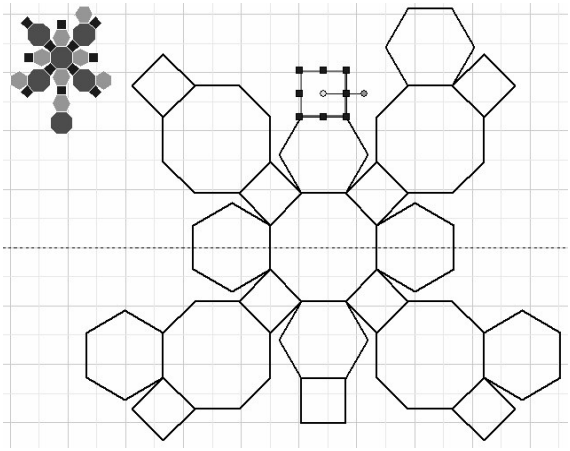
This square can then be copied to the other three open hexagons by using either Radial copy or Mirror copy.

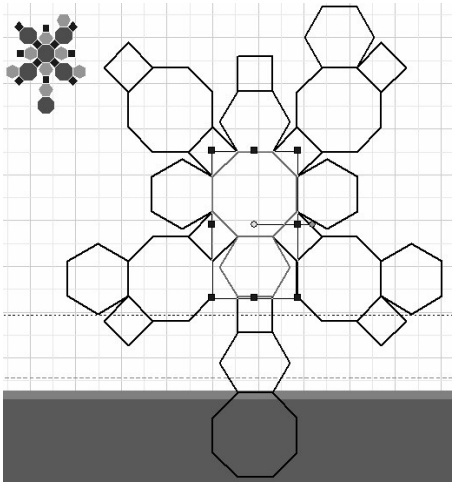
Moving and copying the remaining hexagons in the pattern follows the same procedure minus the rotation.





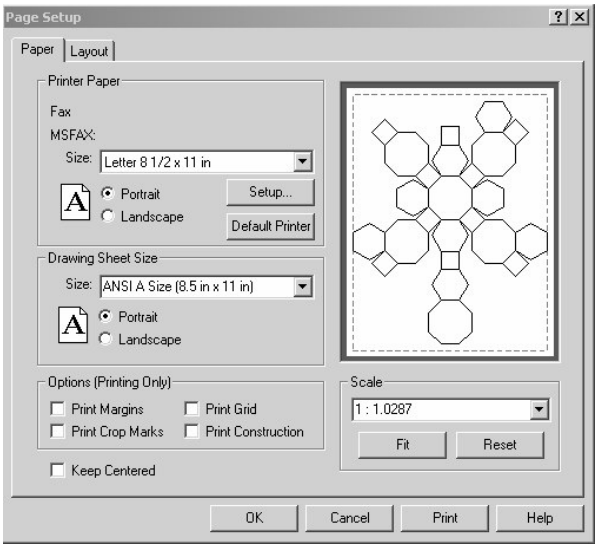
A succession of Mirror copies will situate the polygons making up the lower extension of this pattern.





7. <u>Fitting the pattern to the drawing sheet.</u>

Delete the thumbnail pattern. Choose File, Page Setup to open the Page Setup window. Make sure that both the Printer Paper Size and Drawing Sheet Size are both 8-1/2 x 11 and that both are in Portrait orientation. Go to Scale and click Fit. Click OK and return to the drawing.



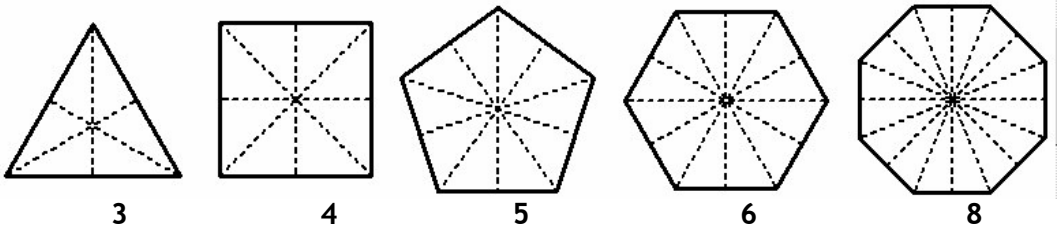
Part III. Creating the Surface Graphics

The finished nightlight will be a closed polyhedron with openings for light to escape out into the room. The paper used in construction is slightly translucent so that some light escapes in a soft glow. To block this glow and to stand in shadowed contrast to it, imprint areas of the surface with black ink. To allow light to escape and project into the room, cut perforations into the surfaces of the polyhedron. These are the three basic options for manipulating light into elegant patterns.

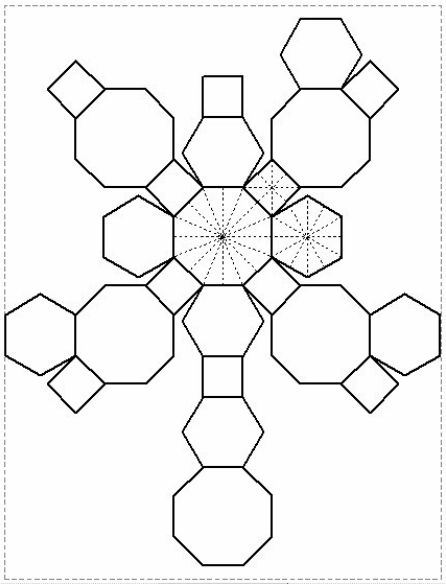
The next stage of this tutorial is the most essentially creative part of the process. Shapes and patterns will disperse across the surface to take advantage of the light that is alternately blocked and released by means of the surface design.

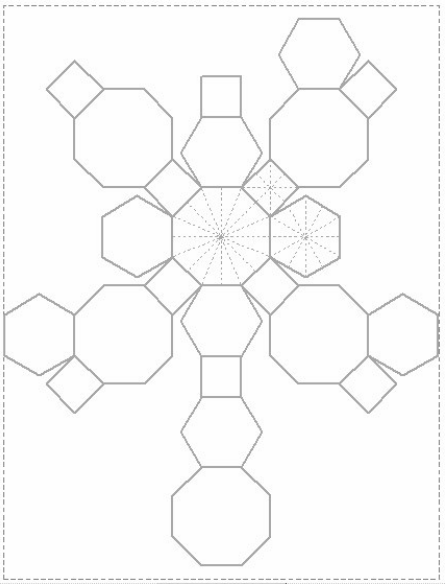
8. Setting up a symmetry template.

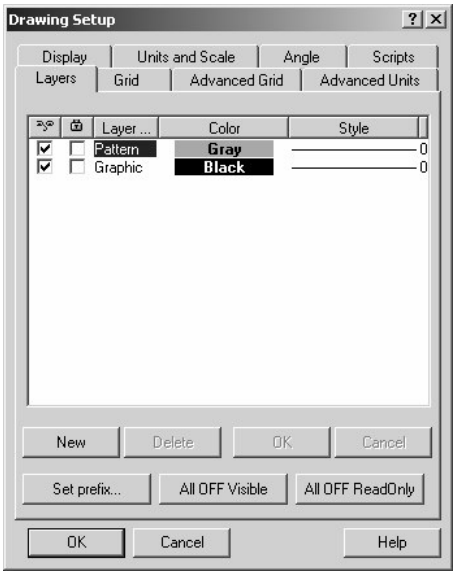
Each regular polygon has the same number of axes of symmetry as it has sides:

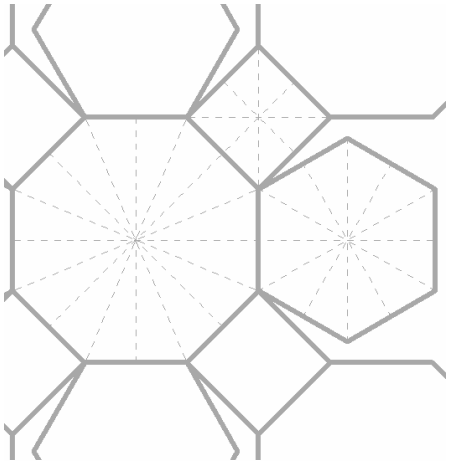


a) Activate Vertex and Middle Point snaps. Activate the Line tool and select the dashed line from the drop-down Pen Style menu in the Properties bar. Draw the axes of symmetry in one of each polygon in the pattern.

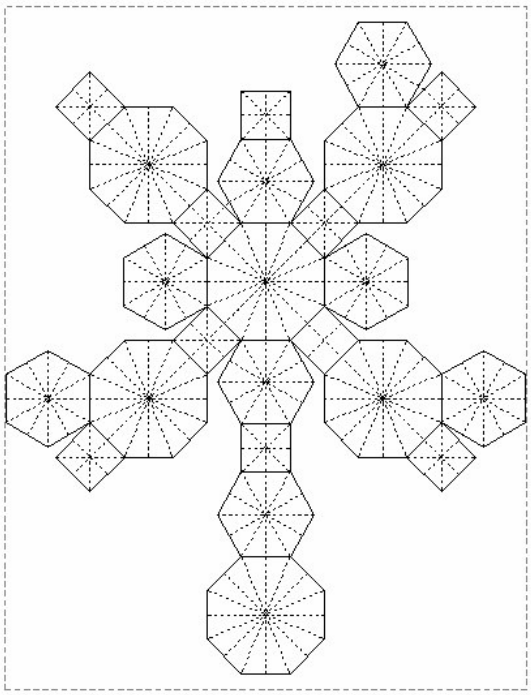








b) Type Ctrl + A to select the entire pattern and select Gray from the drop-down Pen Colors menu in the Properties bar. Choose Options, Layers to access Layers editing in the Drawing Setup window. Only one layer is displayed and its visibility box (indicated by the blue icon of the wide-open eyes) is checked and active. Click on the 0 in the Layer column and type in **Pattern** as the layer's name. Click on New and another layer will appear. By default the visibility box is checked and active and the name defaults to Layer 1 while Color defaults to Black. Change "Layer 1" to **Graphic**.



A layer imitates the drawing technique of creating complex drawings by placing related parts on separate sheets of tracing paper and then laying these on one another. In the design of a building the floor plan might be drawn first then layered with a sheet of tracing paper in order to draw the plumbing relative to this plan. Another sheet, or layer, might hold the wiring diagram, another heating, etc.

With the Pattern layer locked and the Graphic layer colored Black, the lines of the pattern will serve as guides on which to draw the surface graphics of the polyhedron. When locked, the objects in a layer cannot change. However, all snaps remain in effect to facilitate the overdrawing.

9. Designing a face of the polyhedron

The drawing to the right depicts all of the axes of symmetry for all of the polygons in the pattern. Note how each axis connects to an axis of an adjacent polygon or to an edge to trace a continuous web over the entire surface. A design built upon this web and with consistent motifs from face to face will be highly organized with respect to the geometry of the polyhedron.

A design built on this symmetry boasts another advantage: it is only necessary to create a design for one of each type of face polygon. Each can then copy to the other parts of the pattern by the same copy edits used to layout the original pattern.

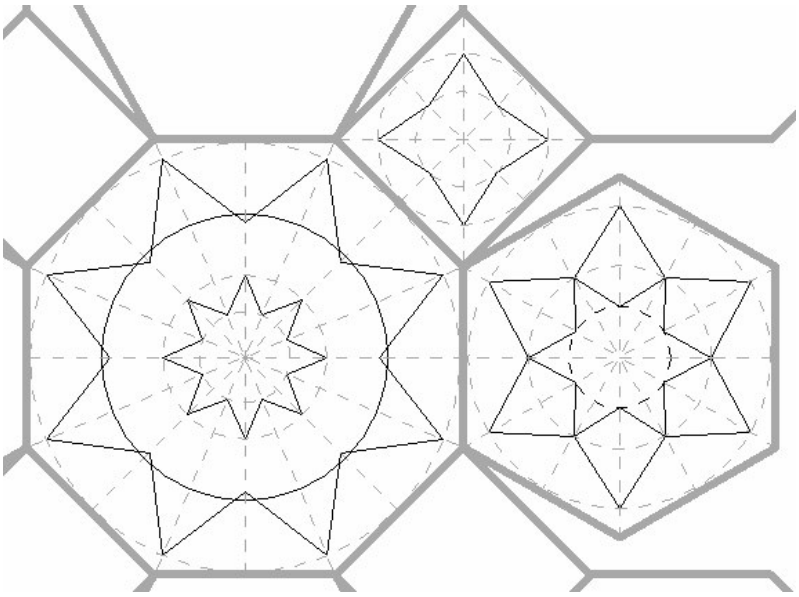
a) Zoom in on the divided polygons.

Insert any additional guides that the design may require. In this example the designer inserted concentric circles to use as guides for creating star patterns: two in the square, three in the hexagon and four in the octagon. Like the axial lines these are also dashed and gray. Make sure that these lines are added to the Pattern layer. As the first layer it is the default layer, but check the Property bar anyway. The drop-down Layers menu will appear when one of the line tools is selected. If this does not show Pattern in the box, then go to the menu and activate that layer.

b) Go to Options, Layers and lock the Pattern layer. Click OK to return to drawing.

In this example the next step was to select the Multiline to draw the stars. With the Intersection Snap active, click on the intersections of the circles and the axial lines. Work around the circle and back to the starting point. Right click over the drawing sheet and click Close on the pop-up menu. This will tell the program that this is not just a line, but a shape for filling later.

c) With Intersection and Vertex snap active draw over these polygons with the Polygon tool. Snap on the center and then on a vertex to draw each polygon.

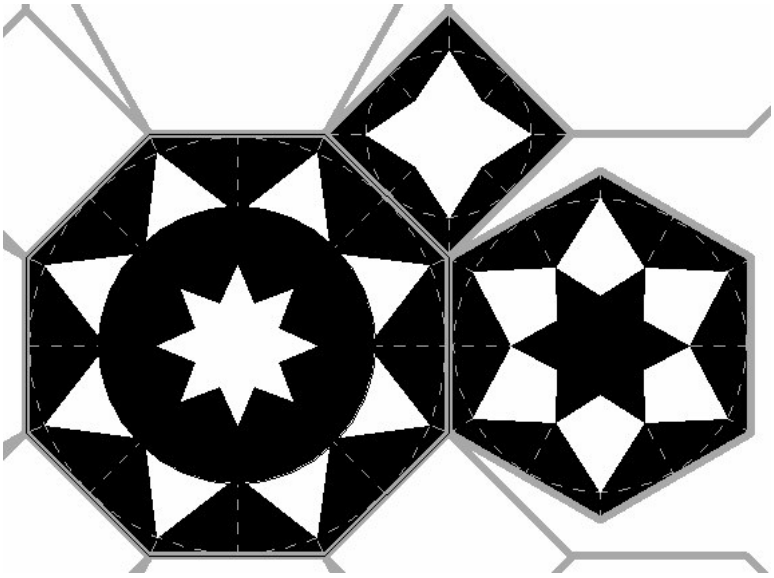


d) Within each polygon in this example is a sequence of shapes that converge to the center. In the octagon, for instance, this starts with the octagon itself, then the large star, then the circle and finally the small star. To achieve the alternating black/white graphics shown below, these shapes must alternate between a black fill and a white fill: octagon = black; large star = white; circle = black; small star = white. The shapes must also proceed from back to front in that same order, as if they were pieces of black and white paper laid one atop the other.

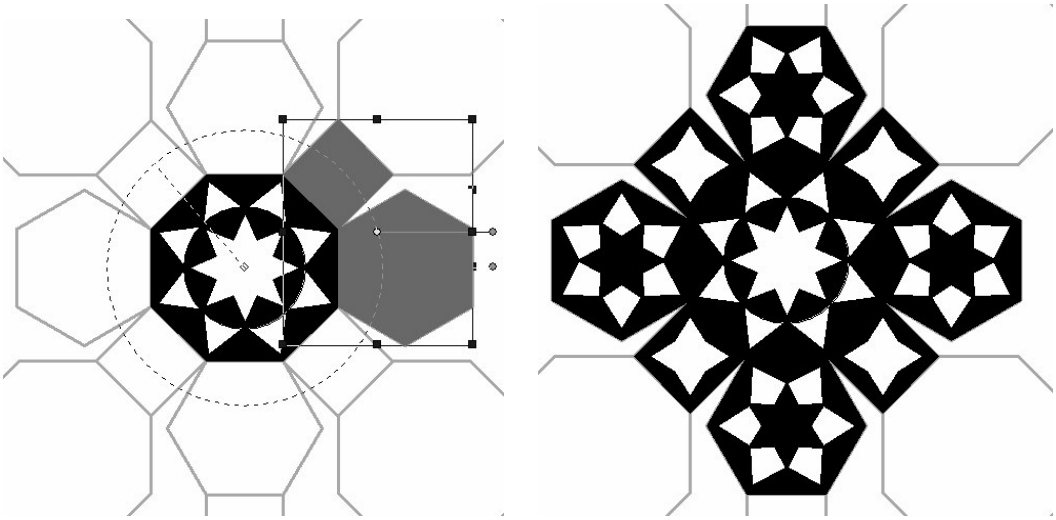
Select the three shapes inside the octagon and change their Pen Color to White. Choose Format, Send to Front to place these in front of the octagon. Select the octagon and from the drop-down Brush Pattern menu in the Properties bar choose the solid pattern. The inside shapes will be outlined in white against a solid black octagon.

Next select the circle and the small star and change their Pen Color to Black and choose Format, Send to Front. Select the large star and choose the solid pattern to fill it with white. The Brush Pattern color will automatically match the Pen Color. The two inner shapes will now appear outlined in black against white. The last step is to select the small star and change its color to white, fill it and send it to the front. Click on the circle and fill with black.

Follow this procedure to complete the hexagon and square patterns.

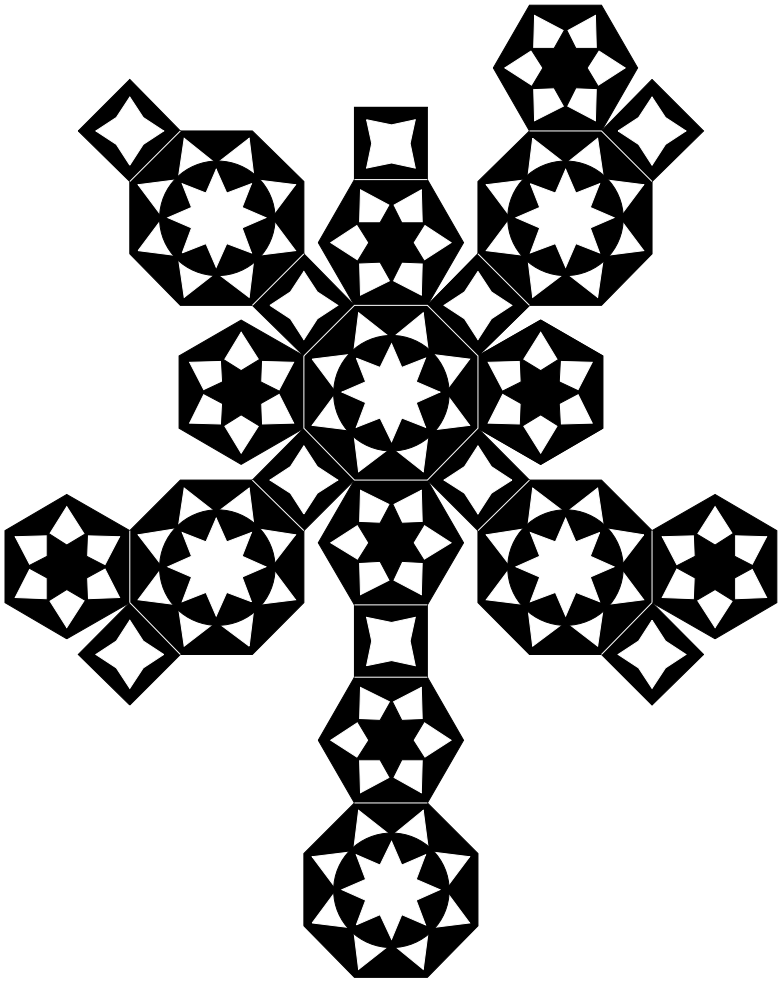


e) Once all three are completed, some evaluation is usually in order and the design invariably adjusted. In this example the central stars of the octagon and hexagon were enlarged. The former to balance it against its larger encircling kin and the latter to provide structural connection in anticipation of the white areas being cut out.



f)  To complete the graphics just copy the colored polygons to follow the polyhedron's pattern. Use the same procedure as when initially laying out the pattern, but first be sure to group each one. Select a polygon and all shapes within the polygon and then click the Create Group button on the Standard tool bar.

Below is the completed graphic. Note that some white lines have been added to clarify the division between polygons.



Part IV. Structural Design

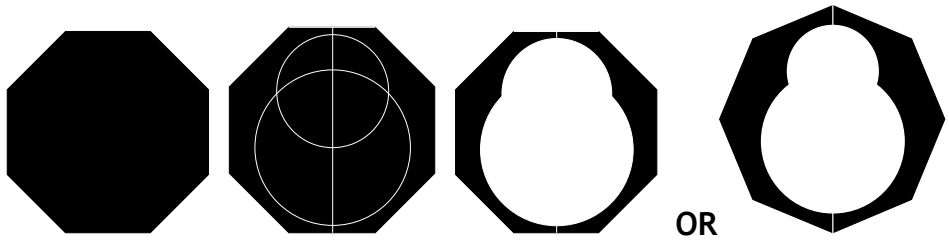
To function as a nightlight two structural features are necessary: 1) a means to mount the polyhedron on the bulb and 2) a secure and stable method of assembly.

10. Key slot mount:

One face of the polyhedron will need an opening to insert the light bulb into the polyhedron and to hold the polyhedron to the light fixture. This face will not need a design, but could use some reinforcement. The mounting face, then, will have double thickness and with an additional polygon laminated to the face. This face can contain the cutting diagram for the mount.

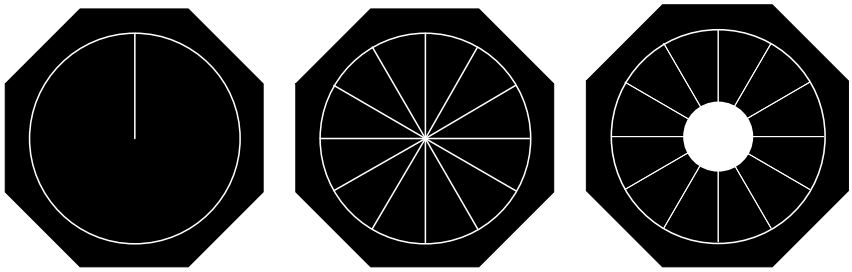
a)  Copy one of the octagons from the pattern onto an open area of the drawing. Remove the Group formatting by selecting and exploding the copy by clicking the Explode button on the Standard toolbar. Delete all inside shapes, leaving only the black octagon.

b) Begin the key slot with two overlapping circles: one whose diameter equals that of the bulb's stem and the other whose diameter equals the widest flare of the bulb. The circles should overlap, and rest symmetrically on the polygon. Use White as the Pen Color. Fill the circles.



11. Diaphragm mount

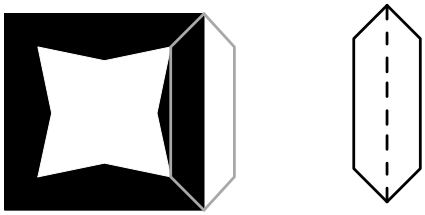
a) In the octagon center a circle whose diameter is about 1/8" larger than that of the light bulb. Activate the Arc Center snap and choose the Line tool. Click on the circle and start of the line will snap to the circle's center. Activate the Quadrant Point snap and click on the circle to complete the line. Select the line and copy it around the circle about 8 to 12 times. Insert a filled circle whose diameter is about 1/3 that of the larger circle.



3. Joining tabs

The best means to join the parts of a paper model is to use folded tabs along with white glue, such as Elmer's Glue. Do not use stick glues or paste. The tabs should be scaled so that they are large enough to fold and handle easily, but small enough that they do not interfere with the desired lighting effects. Usually this means that they should fit within the black printed areas of the polyhedron pattern.

a) Scaling the tab.
The tab may be scaled according to the smallest area of black into which it must fit. In the case of the demo pattern, this is along the edge of the square polyhedron.

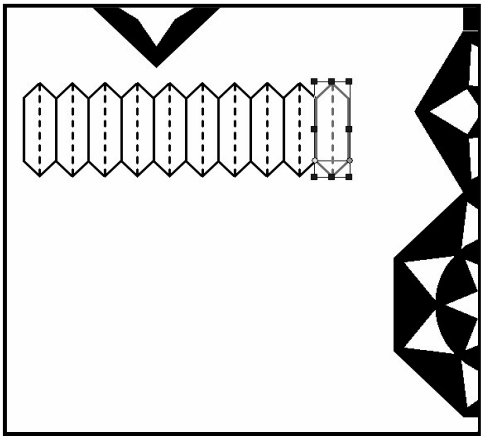


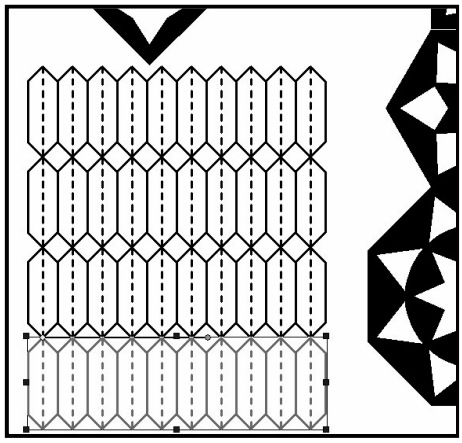
With the Vertex snap active and the Pen Color set to Gray, draw a Multiline from a corner of the square to the nearest point of the star, then to the next point and then to the nearest corner of the square. Mirror this line to outline the full tab and move it away from the square. Draw a dashed fold line down the length of the tab.

b) It will take a good number of tabs to join all of the edges of the Great Rhombi-cuboctahedron, so it would speed up the assembly process if they could be quickly and easily cut out. Copying them in a regular pattern will help.

Move the tab to an open part of the drawing. Select the tab, group and re-set the reference point into the lower left corner (the Vertex snap should still be active). Using the Rubber Stamp tool copy the tab to itself by clicking on the lower corner of the tab. Continue copying each successive tab to its previous copy to create a row of these copies.

Select this entire row and move the reference point to the top vertex of the first tab. Using the Rubber Stamp tool copy the row to itself by clicking on the bottom vertex of the first tab. Continue to fill the available space.





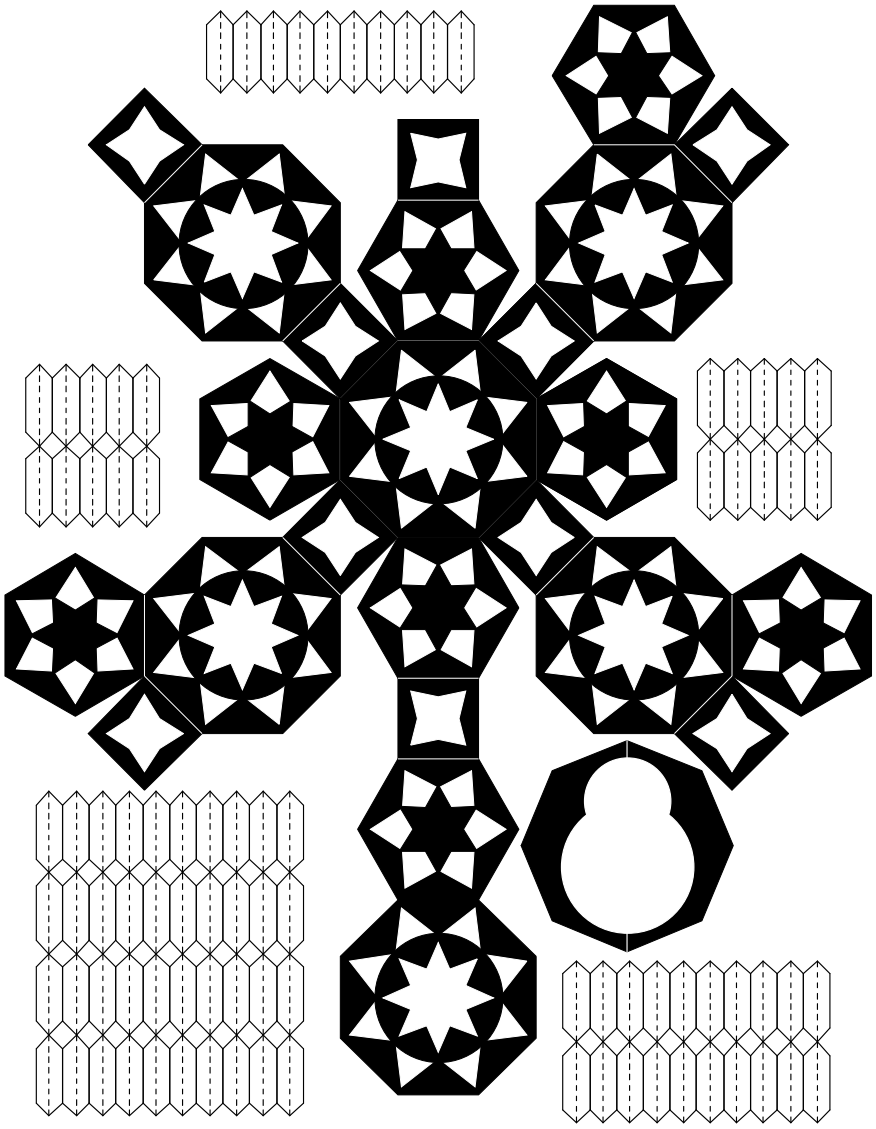
c) The completed pattern ready for printing and assembly is shown on the following page.

Part V. Printing and Assembly

Print on heavy white index stock. Usually only a back feeding inkjet printer can handle such heavy stock. Most printers sold are of this type.

12. <u>Cutting out the pattern</u>

a) Cut out the pattern with an Exacto or similar knife, sometimes sold under the name craft knife. Use a stainless steel rule with a non-slip cork backing. Such a ruler will greatly reduce error from slipping or from the knife nicking the edge.



Begin each cut with a light pass along the edge of the rule to create a pilot cut. This is a tracking cut that reduces the possibility of the knife blade swerving as it bites into the cut. Cut through the paper on the second pass.

b) Glue the mounting polyhedron on the face that will turn to the wall and that will mount on the light.

c) Cut out all white areas that are to become perforations for projecting the light into the room. Cut out the mounting openings.

d) Score along all fold lines. A score is a light cut that just breaks the surface of the index stock and ensures a straight fold. Always score to the outside of the fold.

e) Score the tabs on the dashed lines prior to cutting them out. This goes quickly, since they are all in alignment. The same goes for cutting out the tabs.

13. Assembling the pattern.

a) Before assembling pre-fold all of the scored fold lines to ensure straight and accurate folded edges. This is far more difficult to do once assembly has begun.

b) Each pair of mating edges will need a tab to hold them together. Carefully glue the tab to one of the pair with white glue. Wait a few moments to allow the glue to set a bit before gluing to the other edge. Use the waiting time to glue down tabs on three or four subsequent edges. By keeping around four tabs ahead, wait time is virtually eliminated.

c) The trickiest part is closing the final face. It is important that all of the tabs are in place around the edges of the last opening and glue applied before folding in the final face.