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A pedagogical potential of one mathematical inaccuracy

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Dear editor of Educational Studies in Mathematics,

This letter follows up the article by Fan, Qi, Liu, Wang, and Lin (2017) "Does a transformation approach improve students' ability in constructing auxiliary lines for solving geometric problems? An intervention-based study with two Chinese classrooms," which appeared in ESM 96(2). It appears that the article contains a task with a mathematical inaccuracy (Fig. 1). In what follows, I first describe the inaccuracy and then outline the pedagogical opportunity that it provides.

$MP = NQ$ does not imply $MP \perp NQ$ (see Fig. 2). In Fig. 2a, $NQ = MP = MP_1$, MP is perpendicular to NQ, but MP_1 is not perpendicular to NQ. Moreover, for some choices of M, N, and Q, it is impossible to obtain a right angle between MP and NQ at all (e.g., Fig. 2b).

I wrote a letter to the editor, driven by a belief that this task, like many others, published in ESM may become a source of reflection for researchers, as well as for the future activities of teachers and student learning. As a researcher interested in the topic of introducing auxiliary lines in situations of proof and problem solving (Palatnik & Dreyfus, 2018; Palatnik & Sigler, 2018), I found it necessary to refer to the study by Fan et al. (2017). As a practitioner, I liked the task in question, which the authors used for the intervention and decided to use it with my undergraduate students. I noticed the counterexamples mentioned above using the dynamic sketch that I prepared for a lesson.

Thus, a sequence of an original paper and pencil task and a DGE task (Fig. 3) provides an opportunity for students' exploration activity with a surprising twist. For instance, first, the original statement is proved, and then its shortcomings are discovered.[1]

Among the possible conjectures to discover and questions to explore in a due course of this activity are

[1]The readers are invited to use applets, which were prepared for the lessons, or to create similar ones: https://www.geogebra.org/classic/fpuaceds; https://www.geogebra.org/classic/huvywfzq; https://www.geogebra.org/m/arq9frkj; https://www.geogebra.org/classic/gp4bpk2q.

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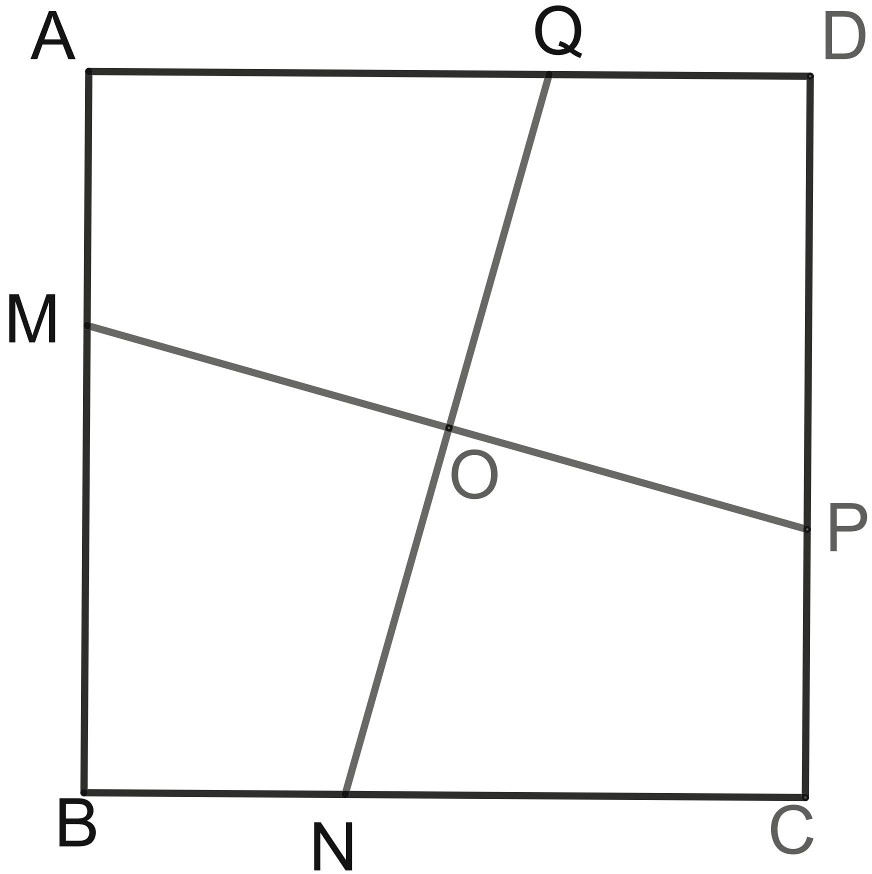

Fig. 1 Task IntQ6 from pp. 242-243 in Fan et al. (2017). ESM 96, 229-248. In square ABCD, M, N, P and Q are the points on sides AB, BC, CD and DA,respectively. MP = NQ. Prove MP⊥NQ

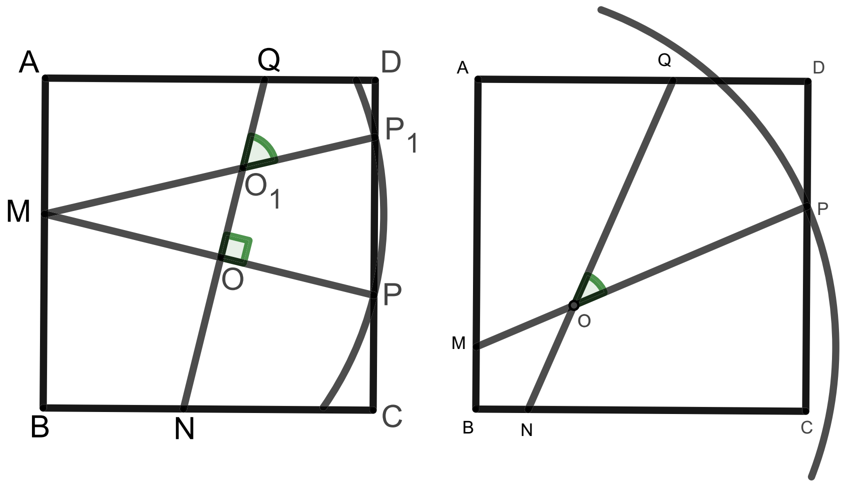

Fig. 2 Counterexamples

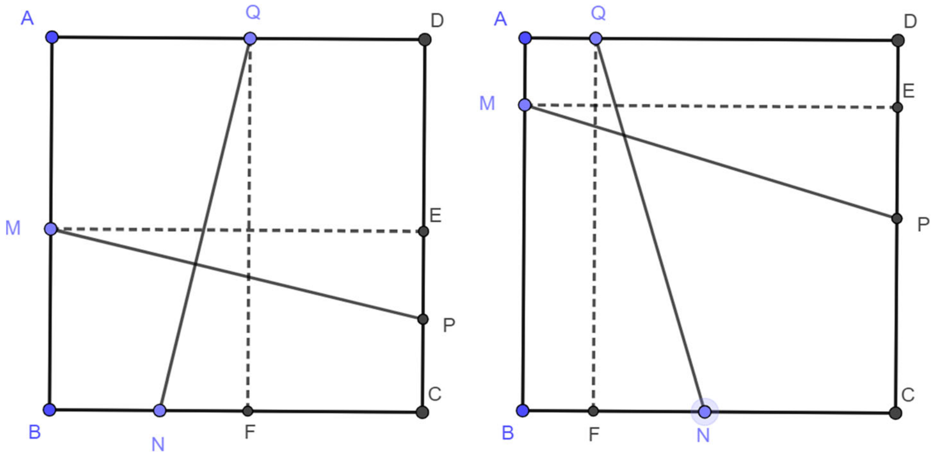

Fig. 3 DGE activity enables to explore various positions of the points

- What is the magnitude of the "non-right" angle?
- An alternative formulation of the problem. In square ABCD, M and Q are the points on sides AB, and DA, respectively, and isosceles triangles MPP_1 and QNN_1 are congruent (with vertexes and bases on respective adjusted sides). What are the properties of a quadrilateral formed by the intersection of isosceles triangles, as shown in Fig. 4?
- A converse statement: In square ABCD, M, N, P, and Q are the points on sides AB, BC, CD, and DA, respectively. $MP \perp NQ$. Prove $MP = NQ$.

The given examples in no way exhaust the pedagogical potential of the situation, which is manifold. First, students face a mathematical problem where an overreliance on the drawing may lead them to incomplete evaluation of a geometric configuration. In the context of the ongoing discussion on the development of critical thinking by means of mathematics education and proving in particular, this activity allows us to discuss an often overlooked question of how we chose which statement to prove. Secondly, students experience an unusual (for school mathematics) task to "patch up" a mathematical statement to prove. In this process, they may

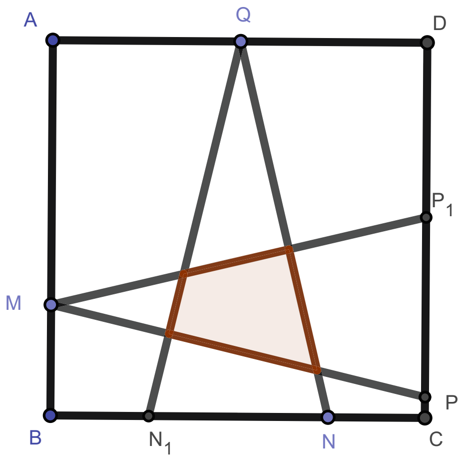

Fig. 4 Alternative formulation of the problem: a quadrilateral formed by an intersection of isosceles triangles

discover nontrivial features of familiar geometric objects. Thirdly, DGE exploration is essential in this case, since it reveals inaccuracy through the generation of new knowledge by modifying a given diagram.

References

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