

Sally and Shalika notation:

- k is a local field, \mathcal{O} its ring of integers, \mathfrak{p} its maximal ideal, $q = \|\mathcal{O}/\mathfrak{p}\|$ is assumed odd.
- Normalize the haar measure dx on $(k, +)$ so that \mathcal{O} has measure 1.
- Define the valuation on k via the equations $d(ax) = \|a\| dx$ for $a \in k$. Let $U = \{x \in k : \|x\| = 1\} = \mathcal{O}^\times$ and $U_n = 1 + \mathfrak{p}^n = \{x \in U : \|x - 1\| \leq q^{-n}\}$
- τ denotes a fixed uniformizer of \mathfrak{p} , ε is a fixed generator for the $q - 1$ th roots of 1 in U .

For V a quadratic extension of k , we let $\text{Nm}_{V/k}$ denote the norm map. Since k has odd residual characteristic, V is one of $k(\sqrt{\tau})$, $k(\sqrt{\varepsilon\tau})$, or $k(\sqrt{\varepsilon})$. The former two are ramified, and the latter is unramified. We will use θ to represent one of $\tau, \varepsilon\tau$, or ε .

For $V = V_\theta = k(\sqrt{\theta})$, we let $C_\theta = \ker \text{Nm}_{V/k} \leq V^\times$ and \mathfrak{p}_θ the prime ideal in V_θ . Set, in the unramified case, for $h \geq 1$:

$$C_\varepsilon^h = (1 + \mathfrak{p}_\varepsilon^h) \cap C_\varepsilon$$

and in either of the ramified cases, for $h \geq 0$:

$$C_\theta^h = (1 + \mathfrak{p}_\theta^{2h+1}) \cap C_\theta.$$

Then, in any case, $\{C_\theta^h\}$ constitute a neighborhood base about 1 in C_θ . For a character $\psi \in \hat{C}_\theta$, write $\text{cond}(\psi)$ for the largest subgroup in this filtration on which ψ is trivial. For each θ , C_θ has a unique character of order 2 which we denote by $\psi_o = \psi_{o,\theta}$.

Let $G = \text{SL}(2, k)$, $A = A(k)$ the diagonal subgroup (a maximal split torus). This subgroup is isomorphic to k^\times . We let A_d denote the image of U_d under this identification.

Let T be any compact cartan in G , which must be naturally isomorphic to some C_θ . Let T_d denote the image of C_θ^d under such an identification.

For a subset S of G , let $S^G = \{gsg^{-1} : g \in G, s \in S\}$, and let S' the subset of S consisting of regular elements (those with distinct eigenvalues).

For $f \in C_c^\infty(G)$, define, for γ noncentral, the orbital integral

$$I_f(\gamma) = \int_{G/G_\gamma} f(x\gamma x^{-1}) d\dot{x}$$