ON q, τ, AND s (OR: ON WHAT'S IN A NAME?)

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Cultural norms dictate that the complex variables q and τ should be related by an identity like

$$q = \exp(2\pi i \tau) \text{ or } \exp(\pi i \tau)$$

and that τ should run about the upper halfplane $\mathfrak{H}$.

Similarly, the variable s is typically reserved for the exponent in dirichlet series, e.g.

$$\zeta(s) = \sum_{n=1}^{\infty} n^{-s}.$$

The goal of this document is to explicate the 'nature' of the variables τ, q, and s: to describe an intrinsic description of their domain and codomain.

1. On s

- The variable s seems to live as a parameter on some manner of dual space: e.g. $\mathrm{Hom}(\mathbb{C}^{\times}, \mathbb{C}^{\times}) = \{\cdot \mapsto \cdot^s : s \in \mathbb{C}\}$.

References