

- Let $G \leq \text{Aut}_{\mathbb{Q}_l}(V)$ with $\dim(V) = 2$, and $n \geq 1$. Suppose that G is compact and that $\det(1 - g) = 0 \pmod{l^n}$ for all $g \in G$. Then there exist G stable lattices $L' \leq L$ such that L/L' has order ℓ^n and G acts trivially on L/L' . Note though, that we cannot say a-priori whether this quotient is cyclic.
- Equivalently, this says that there is a basis v_1, v_2 for V and $a, b \geq 0$ with $a + b = n$ such that every element of G takes the form $\begin{pmatrix} 1 + l^a \mathbb{Z}_l & l^a \mathbb{Z}_l \\ l^b \mathbb{Z}_l & 1 + l^a \mathbb{Z}_l \end{pmatrix}$ (where we interpret $1 + l^0 \mathbb{Z}_l = \mathbb{Z}_l^\times$).