My research

1 past

In my first paper, in collaboration with et al., I produced lots of examples of blah blah. In order to state our result, we need some definitions.

Given a closed Riemannian manifold M, one associates its Laplace-Beltrami operator Δ_M: an essentially self adjoint unbounded operator acting on a dense subspace of the Hilbert space of L^2 k-forms $\Omega^k(M)$ on M. We denote the eigenvalue spectrum of Δ_M acting on $\Omega^2(M)$ by $\mathscr{E}^k(M)$. The spectrum $\mathscr{E}^k(M)$ is a well studied invariant of the Riemannian metric on M and is known to determine its dimension, volume, and total scalar curvature [CITE].

A related geometric invariant is the **primitive geodesic length spectrum** $\mathscr{L}_p(M)$ of M. Assuming for simplicity that M is negatively curved, each free homotopy class of closed curve on M has a unique geodesic representative. Then $\mathscr{L}_p(M)$ is the set of lengths (counted with multiplicty) of each geodesic representative in each free homotopy class.

We denote by $H^k(M,\mathbb{Z})$ the **kth singular cohomology group** of M with trivial $\mathbb{Z}$–coefficients. Given a finite cover $M' \to M$, we have induced homomorphisms $\mathrm{Res}\colon H^k(M,\mathbb{Z}) \to H^k(M',\mathbb{Z})$ and $\mathrm{Cor}\colon H^k(M',\mathbb{Z}) \to H^k(M,\mathbb{Z})$. For a pair of finite covers $M_1, M_2 \to M$, we say that a morphism $\psi_k\colon H^k(M_1,\mathbb{Z}) \to H^k(M_2,\mathbb{Z})$ is **compatible** if the diagram

$$\begin{tikzcd}
 & H^k(M,\mathbb{Z}) \arrow[dl, bend right, "\mathrm{Res}"'] \arrow[dr, bend left, "\mathrm{Res}"] & \\
H^k(M_1,\mathbb{Z}) \arrow[ur, bend right, "\mathrm{Cor}"'] \arrow[rr, bend right, "\psi_k"'] & & H^k(M_2,\mathbb{Z}) \arrow[ul, bend left, "\mathrm{Cor}"]
\end{tikzcd} \tag{1}$$

commutes.

Finally, M is called **large** if there exists a finite index subgroup $\Gamma_0 \leq \pi_1(M)$ and a surjective homomorphism of Γ_0 to a non-abelian free group.

We now state our first result and refer the reader to §2 for a brief review of real/complex hyperbolic manifolds and the definition of non-arithmetic manifolds.

Theorem 1. Let M be a closed hyperbolic n-manifold which is large and arithmetic. Then for each $j \in \mathbb{N}$ there exist pairwise non-isometric, finite Riemannian covers $M_1, \ldots, M_j$ such that

- $\mathscr{E}_k(M_i) = \mathscr{E}_k(M_{i'})$ for all k, and all i, i'
- $\mathscr{L}_p(M_i) = \mathscr{L}_p(M_{i'})$ and all i, i'

- There exist compatible isomorphisms $\psi_k : H^k(M_i, \mathbb{Z}) \to H^k(M_{i'}, \mathbb{Z})$ for all k, and all i, i'.

2 Present

In this section, I will summarize my two current projects.

The first is the subject of my dissertation, and regards the spectral rigidity of a substantial collection of hyperbolic surfaces. First we define arithmetic Fuchsian groups, which are a special type of lattice in $\mathrm{SL}_2(\mathbb{R})$.

Let k be a totally real number field, and B be an indefinite quaternion algebra over k split over a unique real place. Using this place, we have a unique embeding $B^\times \to \mathrm{GL}_2(\mathbb{R})$ such that the restriction to the subgroup B^1 of **units of reduced norm** 1 embeds into $\mathrm{SL}_2(\mathbb{R})$. Let $\mathscr{O}$ be a maximal order in B, and $\Gamma(\mathscr{O}) \leq \mathrm{SL}_2(\mathbb{R})$ be the image of $\mathscr{O}^1 = \mathscr{O} \cap B^1$ under the above embedding. We say that a subgroup Λ of $\mathrm{SL}_2(\mathbb{R})$ is an **arithmetic fuchsian group** if it is commensurable to a $\Gamma(\mathscr{O})$, for (some $\mathscr{O}$ in some B over some k as above.)

Remark 1. Recall that two subgroups H_1, H_2 of an ambient group G are **commensurable in** G if their intersection $H_1 \cap H_2$ has finite index in both H_1 and H_2. When the H_i arise as the fundamental groups $\pi_1(M_i)$ of locally isometric manifolds M_i embedded in the common isometry group G of their universal cover, commensurability translates to the existence of a common finite Riemannian cover M over both M_1 and M_2.

We say that B has **type number one** if it has a unique conjugacy class of maximal order.