THE LEMMA

JUSTIN KATZ

In this note $\mathfrak{k}$ is a $\mathfrak{p}$-adic field, with ring of integers $\mathfrak{o}$, maximal ideal $\mathfrak{p}$, and $k = \mathfrak{o}/\mathfrak{p}$ its residue field. We let V denote a 2-dimensional $\mathfrak{k}$-vectorspace, and $\mathcal{X}$ the Bruhat-Tits tree of $\mathfrak{k}$-homothety classes of $\mathfrak{o}$-lattices in V.

Two homothety classes $\Lambda, \Lambda' \in \mathcal{X}$ are adjacent, written as $\Lambda \sim \Lambda'$, if they have representative lattices $L \in \Lambda$ and $L' \in \Lambda'$ such that

$$\mathfrak{p}L \leq L' \leq L.$$

For a class $\Lambda \in \mathcal{X}$, let $N(\Lambda) = \{\Lambda' : \Lambda' \sim \Lambda\}$ denote its 1-neighborhood. There is

Lemma 1. Let $g \in \mathrm{SL}(2, \mathfrak{k})$ satisfy

(1) $(g) \in \mathfrak{o}$
(2) $x^2 - (g)x + 1$ is reducible modulo $\mathfrak{p}^j$.

Then g pointwise fixes a geodesic of length j in X.

Proof. v Since $(g) \in \mathfrak{o}$, it fixes some vertex $\Lambda_0 \in X$. Let $N_j(\Lambda)$ denote the j-neighborhood of Λ. We identify $N_j(\Lambda)$ with the projective line $\mathbb{P}^1(\mathfrak{o}/\mathfrak{p}^j)$ as follows. For each vertex $\Lambda' \in N_j(\Lambda)$, there is a sequence of □