Module 6 — Transportation Model (Heart Start AED)

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Problem summary

We have two plants (A, B) that produce automated external defibrillators (AEDs) and ship to three wholesalers (W1, W2, W3). We must **minimize total cost = production cost (by plant) + shipping cost (by route)** while meeting each warehouse's demand and not exceeding each plant's capacity.

Data

	W1 ship	W2 ship	W3 ship	Unit Prod. Cost	Capacity
Plant A	22	14	30	600	100
Plant B	16	20	24	625	120

Warehouse monthly demand: **W1 = 80**, **W2 = 60**, **W3 = 70** (total 210). Total available capacity is 220, so we will only produce what we need.

Linear program (LP) formulation

Decision variables (units):
- x_{A1}, x_{A2}, x_{A3}: units from Plant A → W1, W2, W3
- x_{B1}, x_{B2}, x_{B3}: units from Plant B → W1, W2, W3

Objective (production + shipping, \$/unit shown in parentheses):
min $622x_{A1} + 614x_{A2} + 630x_{A3} + 641x_{B1} + 645x_{B2} + 649x_{B3}$

Subject to demands:
$x_{A1} + x_{B1} = 80$

$x_{A2} + x_{B2} = 60$

$x_{A3} + x_{B3} = 70$

Plant capacities:
$x_{A1} + x_{A2} + x_{A3} \leq 100$

$x_{B1} + x_{B2} + x_{B3} \leq 120$

Nonnegativity: all $x \geq 0$.

Solve in R (`lpSolve`)

```r
# install.packages("lpSolve") # uncomment if needed
library(lpSolve)

# Cost vector (production + shipping)
# order: A1, A2, A3, B1, B2, B3
costs <- c(600+22, 600+14, 600+30, 625+16, 625+20, 625+24)

# Constraints
# Demands (equalities)
A_eq <- rbind(
  c(1,0,0, 1,0,0),   # W1: x_A1 + x_B1 = 80
  c(0,1,0, 0,1,0),   # W2: x_A2 + x_B2 = 60
  c(0,0,1, 0,0,1)    # W3: x_A3 + x_B3 = 70
)
b_eq <- c(80, 60, 70)

# Capacities (inequalities)
A_ub <- rbind(
  c(1,1,1, 0,0,0),   # A capacity
  c(0,0,0, 1,1,1)    # B capacity
)
b_ub <- c(100, 120)

# Combine to a single lp() call:
# We'll build 'const.mat', 'const.dir', 'const.rhs'
const.mat <- rbind(A_eq, A_ub)
const.dir <- c("=", "=", "=", "<=", "<=")
const.rhs <- c(b_eq, b_ub)

# Solve
sol <- lp(direction = "min", objective.in = costs,
          const.mat = const.mat, const.dir = const.dir, const.rhs = const.rhs,
          all.int = FALSE)  # continuous variables

sol$status          # 0 means optimal
sol$objval          # minimal total cost
sol$solution        # decision variables

# Tidy output
```

```r
vars <- data.frame(
  variable = c("x_A1","x_A2","x_A3","x_B1","x_B2","x_B3"),
  value = round(sol$solution, 4)
)
vars

# Production by plant
prod_A <- sum(sol$solution[1:3])
prod_B <- sum(sol$solution[4:6])
c(PlantA = prod_A, PlantB = prod_B)
```

Expected optimal solution

The optimal shipping plan is:

- $x_{A1} = 40$, $x_{A2} = 60$, $x_{A3} = 0$
- $x_{B1} = 40$, $x_{B2} = 0$, $x_{B3} = 70$

Production totals: **Plant A = 100**, **Plant B = 110**.
Minimum total cost: **$132,790**.

Files

The CPLEX LP version of the model is provided alongside this Rmd as `module6_model.lp`.