# Module 9 – Goal Programming: Emax Corporation

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## Problem Summary

We decide the production rates of three new products $x_1, x_2, x_3 \geq 0$.
Define goal deviation variables (nonnegative):

- Employment goal: $y_1^+$ = amount **over** the target, $y_1^-$ = amount **under** the target.
- Earnings goal: $y_2^+$ = amount **over** the target, $y_2^-$ = amount **under** the target.

Given unit contributions (per unit of production):

| Factor | Product 1 | Product 2 | Product 3 | Goal / Units |
|---|---|---|---|---|
| Profit $P$ | 20 | 15 | 25 | Maximize (millions of \$) |
| Employment level | 6 | 4 | 5 | = 50 (hundreds of employees) |
| Earnings next year | 8 | 7 | 5 | $\geq$ 75 (millions of \$) |

### Q1. Express $y_1^+, y_1^-, y_2^+, y_2^-$ and $P$ in terms of $x_1, x_2, x_3$.

Balance (goal) equations:

$$6x_1 + 4x_2 + 5x_3 \; + \; y_1^- \; - \; y_1^+ = 50,$$
$$8x_1 + 7x_2 + 5x_3 \; + \; y_2^- \; - \; y_2^+ = 75.$$

Hence,

$$y_1^+ = \max\{0,\ 6x_1 + 4x_2 + 5x_3 - 50\}, \quad y_1^- = \max\{0,\ 50 - (6x_1 + 4x_2 + 5x_3)\},$$
$$y_2^+ = \max\{0,\ 8x_1 + 7x_2 + 5x_3 - 75\}, \quad y_2^- = \max\{0,\ 75 - (8x_1 + 7x_2 + 5x_3)\}.$$

Total (discounted) profit over the products' lifetimes:

$$P \; = \; 20x_1 + 15x_2 + 25x_3.$$

## Q2. Management's objective in decision variables

Let $C$ be the change in employment (in either direction) from the current level (50 hundreds):

$$C \;=\; y_1^+ + y_1^-.$$

Let $D$ be the **decrease** in next year's earnings from the current level (75):

$$D \;=\; y_2^-.$$

The stated objective is

$$\max Z \;=\; P - 6C - 3D \;=\; (20x_1 + 15x_2 + 25x_3) \;-\; 6(y_1^+ + y_1^-) \;-\; 3y_2^-.$$

## Q3. LP formulation & solution

### Linear Program

$$\begin{aligned} \max\;\; & Z = 20x_1 + 15x_2 + 25x_3 - 6y_1^+ - 6y_1^- - 3y_2^- \\ \text{s.t.}\;\; & 6x_1 + 4x_2 + 5x_3 + y_1^- - y_1^+ = 50, \\ & 8x_1 + 7x_2 + 5x_3 + y_2^- - y_2^+ = 75, \\ & x_1, x_2, x_3, y_1^+, y_1^-, y_2^+, y_2^- \geq 0. \end{aligned}$$

### Solve in R (lpSolve)

```r
# install.packages("lpSolve")  # uncomment if needed
library(lpSolve)

# Decision variables: x1,x2,x3,y1p,y1m,y2p,y2m
# Objective (maximize): 20 15 25  -6 -6   0  -3
obj <- c(20, 15, 25, -6, -6, 0, -3)

# Constraint matrix (2 equalities)
A <- rbind(
  c(6, 4, 5, -1, 1,  0,  0),  # employment
  c(8, 7, 5,  0, 0, -1,  1)   # earnings
)
b <- c(50, 75)
sense <- c("=", "=")

res <- lp(direction = "max",
          objective.in = obj,
          const.mat = A,
          const.dir = sense,
          const.rhs = b,
          all.int = FALSE,
          compute.sens = FALSE)

res$status          # 0 = optimal
res$objval
```

```r
soln <- setNames(res$solution, c("x1","x2","x3","y1p","y1m","y2p","y2m"))
soln
```

**Expected optimal solution & interpretation**

The optimal plan is: - $x_1 = 0,\ x_2 = 0,\ x_3 = 15$ - $y_1^+ = 25,\ y_1^- = 0,\ y_2^+ = 0,\ y_2^- = 0$

Objective value: $Z = 225$.

**Interpretation.** Produce only Product 3 at rate 15.
This exactly meets next year's earnings target ($5 \cdot 15 = 75$), but **overshoots** the employment target by 25 (hundreds of employees). That overage is acceptable under the stated trade-off because the profit contribution from Product 3 more than offsets the employment-deviation penalty in the objective. No earnings shortfall occurs, so $y_2^- = 0$.

# Appendix: Model file (.lp) for solvers

The same model in CPLEX/GLPK ".lp" format is provided alongside this Rmd. "'