Calculating Fin Flutter Velocity for Complex Fin Shapes

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In Peak of Flight Issue #615, I described how to estimate the fin flutter velocity for trapezoidal and elliptical fin shapes. This note describes how to handle triangular and other more complex fin shapes. In all examples shown, we will use arbitrary units of length, without concerning ourselves about the unit system in use.

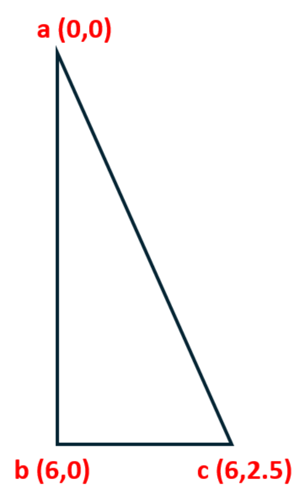
The fin properties that must be calculated from the provided fin geometry include:

> ***A***, the **fin "aspect ratio"**, equal to **[(fin semi-span length or height)^2 / fin area]** (dimensionless),
>
> ***t/c***, is the **fin "thickness ratio"**, equal to **[fin thickness / root chord length]** (dimensionless),
>
> **λ** (lambda), **the fin "taper ratio"** equal to **[tip chord length / root chord length]** (dimensionless),
>
> **Fin Area**, the area of one fin (not including the fin tab, if present) (length squared),
>
> ***ε*** ("epsilon") is the distance of the fin center of mass behind fin quarter-chord, expressed as a dimensionless fraction of the full chord, and
>
> **Cx,** the axial distance from the front of the fin to the fin centroid (length).

Let's examine how these values are calculated for fin shapes that are not trapezoidal or elliptical.

Triangular Fins

Although there are explicit formulae for the area and centroid of a triangle (see below), we don't have to use them here. Since a triangle can be considered as a degenerate form of trapezoid, the trapezoidal equations in the original article work fine for triangular fins; we just need to set the tip chord length to zero. This will also make **λ** equal to zero. The trapezoidal equations will produce correct results for triangular fins once this is done. Let's prove this to ourselves. Consider the following triangular fin:



The area of a triangle = **½ base * height**, or in this case, **3*2.5 = 7.5**. The area of a trapezoid with a zero-length tip chord reduces to the same formula:

$$Area = Height \times \frac{\cancel{Tip\,Chord\,Length +} Root\,Chord\,Length}{2}$$

As we will see below, the centroid of a triangle given its vertices is just the average of each of the vertex coordinates, i.e.,

Cx = **(0 + 6 + 6) / 3 = 4.**

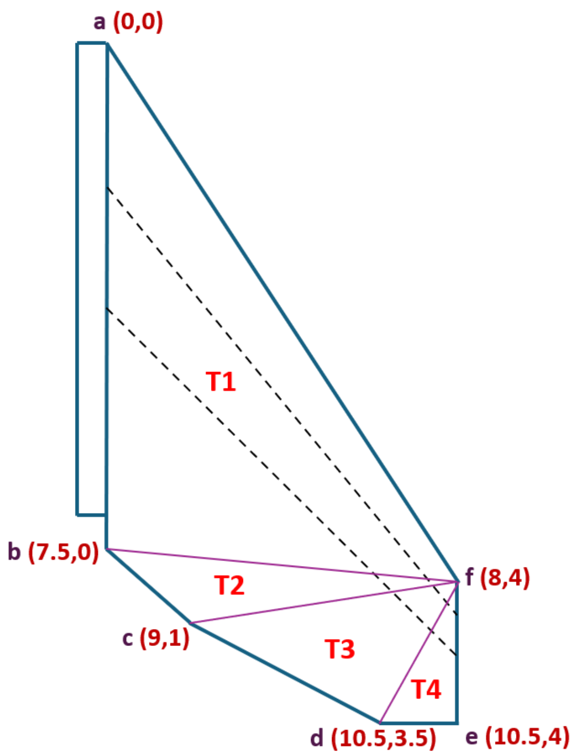
If we use our formula for the centroid of a trapezoid with the tip chord set to zero (note that m, the sweep length, is the same as RC in this example):

$$C_x = \frac{(2 \times TC \times m) + TC^2 + (m \times RC) + (TC \times RC) + RC^2}{3(TC + RC)}$$

We get **Cx** = **((0) + (0) + (6*6) + (0) + (6*6)) / (3*6) = 4**, which is the same answer.

<u>Other Polygon-Based Fin Shapes</u>

For more complex fin shapes, we can use a method called "geometric decomposition." This method divides the fin into a set of geometric objects (typically triangles or rectangles) that we know how to handle, and then combines the results. The easiest way to employ geometric decomposition is to assign **(x, y)** coordinates to each vertex of the fin polygon. For example, consider the swept fin pictured below.



The fin polygon vertices are labeled **a, b, c, d, e, and f**. We first determine the coordinates for each vertex (assigning **(0,0)** to the forward most vertex toward the nosecone). The vertex coordinates come from the fin design itself, or we can calculate them with a bit of geometry or trigonometry. Then we divide the fin polygon into a small number of triangles. The resulting four triangles: **abf**, **bcf**, **cdf**, and **def,** are demarked in the figure with purple lines. Let's label these triangles **T1**, **T2**, **T3**, and **T4**, respectively, as shown on the figure. If we calculate the areas of the four triangles, the area of the entire fin is just the sum of these areas. Let's see how this works. From geometry, we know that the area of any triangle, given its three vertex coordinates **(Ax, Ay)**, **(Bx, By)**, and **(Cx, Cy)** is:

$$Area = \left| \frac{A_x(B_y - C_y) + B_x(C_y - A_y) + C_x(A_y - B_y)}{2} \right|$$

Using this formula, we can calculate the areas of each of the four triangles are as follows:

$$Area(T_1) = \left|\frac{a_x(b_y - f_y) + b_x(f_y - a_y) + f_x(a_y - b_y)}{2}\right|$$

Area of T1 (abf) = **(0(0-4) + 7.5(4-0) + 8(0-0))/2** = **15**

$$Area(T_2) = \left|\frac{b_x(c_y - f_y) + c_x(f_y - b_y) + f_x(b_y - c_y)}{2}\right|$$

Area of T2 (bcf) = **(7.5(1-4) + 9(4-0) + 8(0-1))/2 = (-22.5+36-8)/2** = **2.75**

$$Area(T_3) = \left|\frac{c_x(d_y - f_y) + d_x(f_y - c_y) + f_x(c_y - d_y)}{2}\right|$$

Area of T3 (cdf) = **(9(3.5-4) + 10.5(4-1) + 8(1-3.5))/2 = (-4.5+31.5-20)/2** = **3.5**

$$Area(T_4) = \left|\frac{d_x(e_y - f_y) + e_x(f_y - d_y) + f_x(d_y - e_y)}{2}\right|$$

Area of T4 (def) = **(10.5(4-4) + 10.5(4-3.5) + 8(3.5-4))/2 = (0+5.25-4)/2** = **1.25**

The total fin area is therefore **15 + 2.75 + 3.5 + 1.25 = 22.5**. Now that we have the area, we can calculate **Cx**. To do this, we need to find the centroid of each of our triangles (recall that we only need the axial (parallel to the direction of flight) dimension of the centroid), and then geometrically average them to get the fin centroid. Let's start with the centroids. **Cx** of any triangle, given its coordinates, is just the average of the **x** coordinates of its vertices, i.e.,

$$C_x = \frac{(V1_x + V2_x + V3_x)}{3}$$

Therefore:

T1_Cx = (0 + 7.5 + 8)/3 = 5.17

T2_Cx = (7.5 + 9 + 8)/3 = 8.17

T3_Cx = (9 + 10.5 + 8)/3 = 9.17

T4_Cx = (10.5 + 10.5 + 8)/3 = 9.67

To calculate **Cx** of the entire fin, we calculate a weighted average by adding the products of each triangle's **Cx** and the area of the that triangle, and then divide the result by the total fin area as follows:

$$C_x = \frac{((T1\_Cx \times T1\_Area) + (T2\_Cx \times T2\_Area) + (T3\_Cx \times T3\_Area) + (T4\_Cx \times T4\_Area))}{TotalFinArea}$$

Therefore, **Cx** = ((5.7 * 15) + (8.17 * 2.75) + (9.17 * 3.5) + (9.67 * 1.25)) / 22.5

= ((85.5) + (22.47) + (32.1) + (12.09)) / 22.5

= **6.76**

Now we can calculate ***ε*** using the formula (recalling that ***ε*** is a measure of distance expressed as a fraction of the whole root chord):

$$\varepsilon = (\frac{C_x}{RC}) - 0.25$$

$\boldsymbol{\varepsilon}$ = 6.76/7.5 – 0.25 = **0.65**

Armed with these values, we can now calculate $\mathbf{V}_f$ in the usual way. Notice that the fin sweep length is just the **x** coordinate of vertex **f** (8 in this case).

Entering these data into the fin flutter velocity spreadsheet[1] (overriding the values for **Cx** and **Fin Area** to use those calculated above) allows us to estimate the flutter velocity for our polygonal fin.

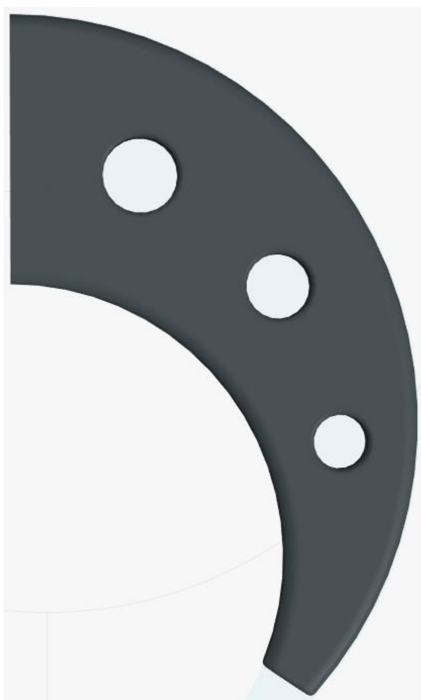
Polygonal fins such as these are also easy to simulate. Both RockSim and OpenRocket both provide the means to create custom fins by entering a set of points. In this case, we would just enter the vertices **a**, **b**, **c**, **d**, **e**, and **f**. See Peak of Flight Issue #488 (February 5, 2019)[2] for an example of how to do this in RockSim.

Complex Fins with Unusual Shapes

Geometric decomposition is a powerful tool, which can be applied in either an additive (as in the previous example) or subtractive manner, as we will see in the next example. If you have ever designed a part to be 3D printed by joining and cutting different geometric shapes, that is the same process we will use to obtain the area and centroid of a complex fin.

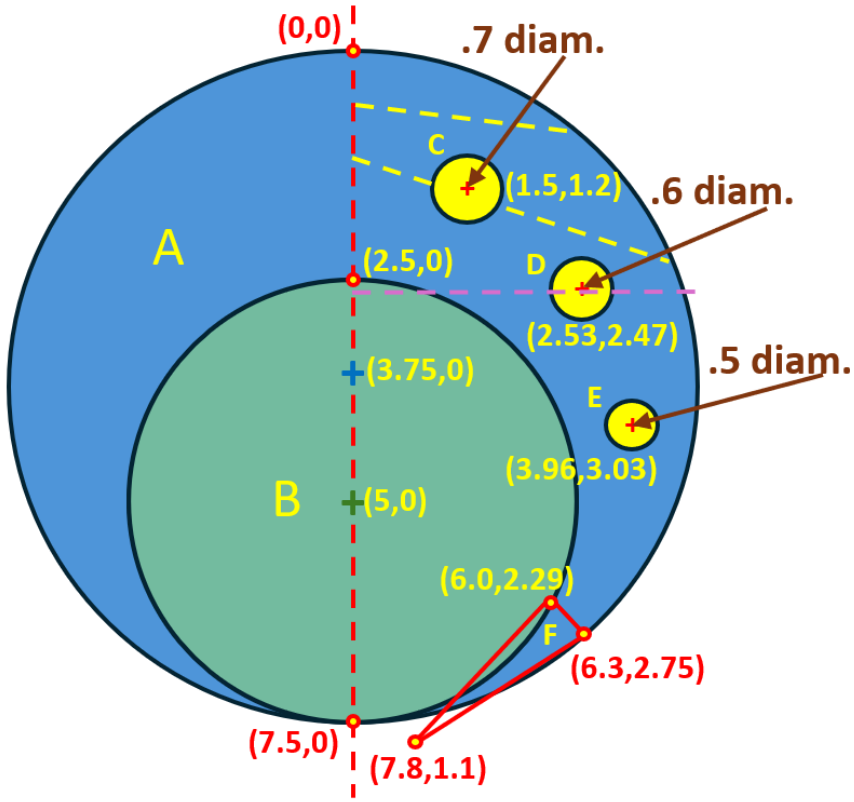
This example will explore how to estimate the flutter velocity of a "bat wing" fin with holes. For purposes of this example, we will ignore the question of the flight-appropriateness of such a fin, and just go with "hey, they look really cool." We will also ignore any aerodynamic concerns other than calculation of flutter velocity.

Consider the following fin shape:



Let's see how we can analyze this fin using geometric decomposition. We start by looking closely at how the fin image might be constructed (this approach was, in fact, the one used in Fusion 360 to design the fin depicted above), as shown in this (not quite to scale) figure:

1 https://github.com/jkb-git/Fin-Flutter-Velocity-Calculator
2 https://www.apogeerockets.com/education/downloads/Newsletter488_Large.pdf



This figure shows two large concentric circles (**A** and **B**), three small circles (**C**, **D**, and **E**), and triangle **F**. The red dashed line bisects the two large circles. The two dashed yellow lines dashed lines represent the quarter chord and half chord lines. We will come back to the dashed purple line in a bit. The coordinates show the location of all relevant points (color has no significance other than visibility). Triangle **F** is intended to represent the "straightened out" small, curved triangle at the bottom of the figure with an equivalent area triangle (this is just to save us a bunch of tedious math for a very small part of the problem). After we size triangle **F**, we can "cut off" the blue arced segment from our fin.

Calculation of Fin Area

Upon examination, we see that our bat fin image can be constructed by taking the right half of circle **A**, and subtracting the right half of circle **B**, as well as circles **C**, **D**, and **E**, and the triangle **F**. This is exactly how we can calculate the area of our fin, as follows:

The area of the right half of circle **A** is **(πr^2)/2 = (π*3.75*3.75) / 2** = 22.1

The area of the right half of circle **B** is **(πr^2)/2 = (π*2.5*2.5) / 2** = 9.82

The area of circle **C** is **(πr^2) = π*0.35*0.35** = .38

The area of circle **D** is **(πr^2) = π*0.3*0.3** = .28

The area of circle **E** is **(πr^2) = π*0.25*0.25** = .2

We use the formula for the area of a triangle given its vertices to calculate the area of triangle **F**, as follows:

$$Area = \left|\frac{A_x(B_y - C_y) + B_x(C_y - A_y) + C_x(A_y - B_y)}{2}\right|$$

The area of triangle **F** is **(7.8(2.75-2.29) + 6.3(2.29-1.1) + 6(1.1-2.75))/2 = (3.59+7.5-9.9)/2** = 0.6

Combining these terms, the **total area of the fin = 22.1 - 9.82 - .38 - .28 - .2 – 0.6** = **10.82**

Calculation of Cx

In order to calculate x dimension of the fin centroid, we need to calculate **Cx** (the **x** coordinate of the centroid) of each component for which we just found the area. For all of the circles, **Cx** is just the midpoint value, so:

A_Cx = **3.75**

B_Cx = **5**

C_Cx = **1.5**

D_Cx = **2.53**

E_Cx = **3.96**

For the triangle **Cx**, we average the **x** coordinates of its vertices:

$$C_x = \frac{(V1_x + V2_x + V3_x)}{3}$$

F_Cx = (7.8 + 6.3 + 6.0) / 3 = **6.53**

Cx of the fin is therefore (note that we are subtracting instead of adding):

$$Cx = \frac{((A\_Cx \times Half\_A\_Area) - (B\_Cx \times Half\_B\_Area) - (C\_Cx \times C\_Area) - (D\_Cx \times D_Area) - (E\_Cx \times E\_Area) - (F\_Cx \times F\_Area))}{TotalFinArea}$$

= **((3.75*22.1) - (5*9.82) - (1.5*.38) - (2.53*.28) - (3.96*.2) - (6.53*.6)) / 10.82**

= **((82.88) - (49.1) - (.57) - (.71) - (.79) - (3.92)) / 10.82**

= **2.57**

The dashed purple line on our figure shows the location of **Cx**. Now we can calculate ε using the formula:

$$\varepsilon = (\frac{C_x}{RC}) - 0.25$$

(recalling that ε is a measure of distance expressed as a fraction of the whole root chord).

ε = **2.57/2.5 – 0.25** = **0.78**

Now let's calculate this fin's flutter velocity. The only fin parameter we don't have at this point is the tip chord length. The conservative approach is to set this value to zero. A slightly less conservative approach would be to use the width of the end of the fin arc, which is:

$\sqrt{((6.3 - 6)^2 + (2.75 - 2.29)^2)}$ = **0.55**.

Let's go with zero for the tip chord length, and assume we want to use balsa fins that are 1/16" in thickness. For this simulation, assume that the launch altitude is 400 ft, launch temperature is 65 deg. F, and that the predicted height of our rocket at maximum velocity is 1000 ft. Summarizing the inputs to the fin flutter velocity calculator spreadsheet:

Data to be Entered			Imperial Units	
Launch Site Data				
AMaxV	=	predicted altitude at predicted maximum rocket velocity	1000	ft
LSA	=	launch site altitude (ASL)	400	ft
TLS	=	temperature at launch site	65	deg F
Use Default Temp?	=	Use Default Sea-Level Temp (**DST**) or Launch Site Temp (**LST**)	DST	DST or LST
Fin Geometry Data				
t	=	fin thickness	0.0625	in
m	=	fin sweep length	3.75	in
TC	=	tip chord length	0	in
RC	=	root chord length	2.5	in
SSL	=	semi span length (height)	3.75	in
G_E (shear modulus)		Shear Modulus (doubled in calculation if tip-to-tip reinforcement)	33359	psi
T2T	=	Tip-to-Tip reinforcing present?	NO	YES or NO

The $\mathbf{V}_f$ result given these inputs is 129 ft/sec (about 88 mph). If we double the balsa thickness (to 1/8”), $\mathbf{V}_f$ increases to 365 fps (249 mph). If we change the material to 1/16” birch aircraft plywood, $\mathbf{V}_f$ is 211 ft/sec (144 mph). Doubling the plywood thickness increases $\mathbf{V}_f$ to 596 ft/sec (406 mph). Small model rockets rarely exceed 250 mph, but larger and high power rockets may exceed Mach 1. Therefore, in order make the best choices for best fin material and thickness, we need to know how fast our rocket is expected to fly on the largest motor we intend to use. See the original article (PoF #615) for an example of how to make these choices.

End Notes

OpenRocket and RockSim both support custom fin shapes, but to my knowledge, neither simulator considers fin holes to have aerodynamic significance (if fin holes are allowed at all). We can manually adjust fin weight in the simulator, and if we are performing manual stability calculations, we can manually adjust fin area.

Polygonal fin shapes are supported in both simulators, and they are readily studied using both Barrowman’s and Martin’s analysis techniques. For rockets that are intended to fly fast and high, it’s probably best to design custom fins as polygons. For “fun” rockets that will only fly with small motors, there are many more possibilities.

That said, the aerodynamic effect of fin holes is unpredictable using simple analysis tools. Unusual fin shapes may have unexpected flight results. Novel fin designs should be flown with caution, and only when it is safe to do so. Also, it is a good idea to alert the RSO when testing a new fin design for the first time.

In this article, we calculated the centroids using first principles, but a lot of guidance can be found at Wikipedia’s “List of Centroids” URL: https://en.wikipedia.org/wiki/List_of_centroids.

Custom fin designs are readily designed using tools like Fusion 360, which can export either DXF (for laser cutting) or STL (for 3D printing) files.