Fin Flutter Analysis Revisited (Again)

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jkb@colorado.edu
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In Peak of Flight Issues #291 (2011)[1] and #411 (2016)[2], authors Howard and Sahr respectively proposed methods to calculate the velocity beyond which catastrophic fin failure was probable due to aerodynamically induced undampened fin oscillation. This article provides additional insight into the underlying calculations, allowing us to configure the calculation to more accurately match specific fin geometries, and corrects an error present in both previous articles that caused the flutter velocity to be overestimated by a factor of 1.414 (the square root of two).

The Howard and Sahr articles were both based on the much earlier work of Dennis J. Martin, who in 1958 synthesized a large body of previous analysis and experimental data into an empirical working tool for engineers[3]. As part of that work, Martin derived the following formula for bending-torsion flutter velocity (**V*f***):

$$V_f = a \times \sqrt{\frac{G_E}{\frac{39.3 \times A^3}{(\frac{t}{c})^3 \times (A+2)} \times (\frac{\lambda+1}{2}) \times (\frac{p}{p_0})}}$$

where (using Martin's notation),

- **V*f*** is the calculated **fin flutter velocity,**
- ***a*** is the **speed of sound at the altitude (above sea level) of maximum rocket velocity,**
- ***GE*** is the **shear modulus of the fin material** (in units of pressure),
- ***A*** is the **fin "aspect ratio"**, equal to **[(fin semi-span length or height)^2 / fin area]** (dimensionless),
- ***t/c*** is the **fin "thickness ratio"**, equal to **[fin thickness / root chord length]** (dimensionless),
- **λ** (lambda) is **the fin "taper ratio"** equal to **[tip chord length / root chord length]** (dimensionless),
- ***p/p*₀** is the ratio **[air pressure at the altitude where the speed of sound was determined / air pressure at sea level]** (dimensionless), and
- the constant "**39.3**" (whose value actually depends upon fin geometry) has units of pressure and is calculated as described below.

OpenRocket and RockSim use different nomenclature for the distance between the body tube and the tip chord. RockSim calls this distance "Semi-Span"; OpenRocket uses the more intuitive term "height."

[1] https://www.apogeerockets.com/education/downloads/Newsletter291.pdf
[2] https://www.apogeerockets.com/education/downloads/Newsletter411.pdf
[3] NACA TN 4917 - Dennis J. Martin, *Summary of Flutter Experiences as a Guide to The Preliminary Design of Lifting Surfaces on Missiles*, February 1958.

The main take-away from Martin's analysis is that V_f is equal to the speed of sound times a dimensionless factor calculated from the fin geometry and fin material physical properties, as well as relevant atmospheric conditions. As observed by Sahr, if we use systems of units consistently, we can perform this computation using either SI (meters/kilograms) or Imperial (feet/pounds) units.

There are more sophisticated (and more accurate) techniques for determining flutter velocity. A review of these techniques can be found in the graduate thesis of Joseph Simmons[4]. However, Martin's technique is great for amateur rocketry because it is based on numerical data that are reasonably accessible (and understandable), it doesn't require special software or an engineering degree to use, and, in a pinch, it can be employed using only a simple calculator. Let's drill down a bit and examine how Martin's calculation is performed.

Calculating the Speed of Sound

The speed of sound in air (***a***) depends only upon temperature (because, if we consider air as an ideal gas, the altitude-related effects of decreasing density and decreasing pressure tend to cancel each another out). In that case, the following equations produce the speed of sound:

$$a_{m/sec} = 20.05 \times \sqrt{273.16 + T_C}$$

$$a_{ft/sec} = 49.03 \times \sqrt{459.7 + T_F}$$

To compute the correct value for the speed of sound, we need to know the air temperature at the altitude (above sea level) where the rocket will be at maximum velocity, which for typical engine thrust curves is the altitude just before motor burn out. This altitude (AGL) can be found easily using RockSim or OpenRocket. Then add the launch site altitude to this predicted flight altitude to get altitude above sea level.

The Troposphere (up to about 12,000 meters / 37,000 ft) is heated by the Earth's surface, so as we go up, air temperature decreases. This decrease is approximately linear in the Troposphere, so air temperature will decrease by about 0.0065 °C per meter, or about 0.00356 °F per foot. Using these relationships, and a default sea level air temperature of 15 °C, or 59 °F, we get the following equations (which are only valid in the Troposphere):

$$T_C = 15 - (.0065 \times altitude_{meters})$$

$$T_F = 59 - (.00356 \times altitude_{ft})$$

These formulae are for dry air under "standard" atmospheric conditions. If the conditions on the ground are non-standard, we may want to make adjustments. For example, if our launch site is at 5000 ft., the temperature is 95°F

[4]Joseph R. Simmons III, *Aeroelastic Optimization of Sounding Rocket Fins*, Air Force Institute of Technology, 6-9-2009

on launch day, and the apogee of our rocket is 5000 ft. AGL, we might want to replace "59" with 95 in the second equation and use the AGL altitude of maximum velocity to compute T_F. In addition, the speed of sound in humid air is slightly higher than in dry air. This difference is negligible in cold air, and less than ½ % in warm air.

Determining the Shear Modulus

The shear modulus (***G_E***) (also called the *modulus of rigidity*) of a material, measured in units of pressure, is the ratio of shear stress to shear strain. Think of shear stress as the action of a pair of scissors ("shears"), and shear strain as the effect of the shear stress (paper is severed, leather is warped, steel may be unchanged). Thus, the shear modulus is a measure of the stiffness of a material in the presence of shear stress, i.e., how much pressure must be applied to deform the material (either temporarily or permanently). Rocket fins are made of many different types of material: balsa, plywood, fiberglass, carbon fiber, aluminum, and sometimes more exotic materials like titanium or magnesium. The shear modulus can vary widely across materials, or even different instances of the same material. In addition, composite materials like fiberglass and carbon fiber are *anisotropic*, meaning that the shear modulus (as well as other strength properties) will be different depending upon whether the shear stress is perpendicular to, or parallel to, the embedded glass of carbon cloth.

Since we are interested in what happens to the fins when they are bending and in torsion due to aerodynamically induced oscillation, the shear modulus of interest is parallel to the embedded cloth. In this axis, the shear modulus will be much more dependent on the epoxy than on the cloth, so there is little difference between carbon fiber and fiberglass when determining the ***G_E*** used to compute **V_f**.

Epoxy composite manufacturers rarely report the shear modulus directly, but they often report the Young's modulus (also called the *modulus of elasticity*) and Poisson's ratio, from which the shear modulus can be estimated using the formula:

$$G_E = \frac{E}{2 \times (1 + \nu)}$$

where E is the Youngs Modulus and ν ("nu") is the Poisson's ratio for the material in question. This formula assumes the material is isotropic, which is not actually true for most composites used to make rocket fins. However, it can be used for anisotropic composites, if we remember that this equation will tend to produce higher than warranted values for the shear modulus (and therefore for **V_f**) for anisotropic materials.

If we search the Internet for data pertaining to the shear modulus of G10 or G12 fiberglass, we will obtain values ranging from 425K to 1.7M psi (or the equivalent SI values). Experimental direct measurements report values near 775,000 psi (5,343,436 KPa). This number is also approximately the median of many reported and calculated values for both fiberglass and carbon fiber composites.

Some of the references that provide these data can be found here[5,6,7,8,9,10,11,12,13]. Allowing for margin, the following table contains suggested working values for G_E for various materials:

Fin Material	Shear Modulus (psi)	Shear Modulus (KPa)
Balsa Wood	**33,359**	**230,000**
Birch Aircraft Plywood	**89,000**	**613,633**
G10 & G12 Epoxy Fiberglass	**600,000**	**4,136,854**
Carbon Fiber Epoxy Composite	**600,000**	**4,136,854**
6061-T6 Aluminum	**3,800,000**	**26,200,078**
6M-4V Titanium	**6,200,000**	**42,747,495**
4130 Steel	**12,000,000**	**82,737,087**

Calculating Fin Geometry Ratios

The three ratios related to fin geometry are straight forward to calculate. Since these ratios are dimensionless, any unit system can be used, as long as the same units are used in the numerator and denominator of each ratio. Fin semi-span length (height), thickness, root chord length, and tip chord length come straight from the simulator values. Calculation of fin area depends upon the actual fin shape, but basic geometry provides the tools we need for many traditional fin shapes. If the fin is trapezoidal, area is calculated as follows:

$$Area = Height \times \frac{Tip\,Chord\,Length + Root\,Chord\,Length}{2}$$

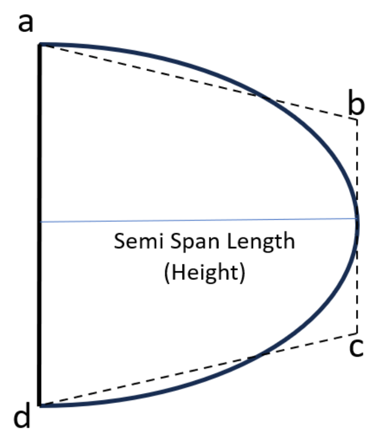
If the fin is elliptical, fin area is calculated as follows (assuming the fin is one half of an ellipse):

$$Area = \frac{Height \times \pi \times \frac{Root\,Chord\,Length}{2}}{2}$$

For elliptical fin shapes, there isn't an actual tip chord. To compute an approximate **λ**, we need to find the "pseudo" tip chord length of a trapezoidal fin that gives the same area as the elliptical fin. We do this by calculating the area

5 A. Iliopoulosa, J. Steubenb, and J. Michopoulosb; *Determination of Anisotropic Mechanical Properties of G-10 Composite via Direct Strain Imaging*, 2005, https://www.sciencedirect.com/science/article/abs/pii/S0142941815301094
6 https://pdf4pro.com/cdn/g-10-fr-4-g-11-glass-epoxy-dielectric-corp-4d23a7.pdf
7 https://www.usplastic.com/knowledgebase/article.aspx?contentkey=563
8 M. B. Kasen, G. R. MacDonald, D. H. Beekman, Jr., and R. E. Schramm, *Mechanical, Electrical, and Thermal Characterization of G-10CR and G-11CR Glass-Cloth/Epoxy Laminates Between Room Temperature and 4K*, National Bureau of Standards, Boulder, Colorado
9 http://k-mac-plastics.com/data-sheets/fiberglass_technical_data.htm
10 David S. Steinberg, *Vibration Analysis for Electronic Equipment*, Wiley-Interscience; 3rd edition, January 15, 2000, p. 131
11 https://apps.dtic.mil/sti/pdfs/ADA470630.pdf
12 https://www.imeko.org/publications/ysesm-2014/IMEKO-YSESM-2014-032.pdf
13 K. Ravi-Chandar and S. Satapathy, *Mechanical Properties of G-10 Glass–Epoxy Composite*, Institute for Advanced Technology, UT Austin, IAT.R 0466

of the elliptical fin, setting this value equal to the area of a trapezoidal fin with the same root chord and semi-span length (height), and then solving for tip chord length. This process is depicted in the figure below. Trapezoid ***abcd*** has the same area as the ellipse shown, and the "pseudo" tip chord length is ***bc***, calculated as shown.



$$bc = \left(\frac{Elliptical\,Fin\,Area}{Height} \times 2\right) - Root\,Chord\,Length$$

<u>Calculating the Air Pressure Ratio</u>

This ratio is the air pressure at the same altitude for which we determined the speed of sound, divided by the air pressure at sea level. In the Standard Atmospheric Model, air pressure at sea level is 101.325 Kpa (SI) or 14.696 psi (Imperial). To calculate air pressure at altitude, we use the temperature calculated previously in one of the following equations.

$$p_{(psi)} = 14.696 \times \left(\frac{T_F + 459.7}{518.7}\right)^{5.256}$$

$$p_{(KPa)} = 101.325 \times \left(\frac{T_C + 273.16}{288.16}\right)^{5.256}$$

To obtain the air pressure ratio, we divide this value by the appropriate $\boldsymbol{p_0}$, which has the effect of simply removing the first term of the equation above. Note that these equations are only valid under about 36,000 ft.

<u>Calculating the Denominator Constant</u>

The denominator constant ($\boldsymbol{DN}$) of "**39.3**" in Martin's equation is unusual in that it has the units of pressure. It is calculated as follows:

$$DN = \frac{(24 \times \varepsilon \times \kappa \times p_0)}{\pi}$$

where (again using Martin's notation),

the value of **24** is a dimensionless product of the whole number constants 4 and 6 found in Martin's derivation (Equations 9 and 10),

ε ("epsilon") is the distance of the fin center of mass behind fin quarter-chord, expressed as a dimensionless fraction of the full chord (see explanation below); For a symmetric fin, ***ε*** is 0.25,

κ ("kappa") is the dimensionless ratio of specific heats (also known as the *adiabatic index*) for air, equal to 1.4, regardless of unit system,

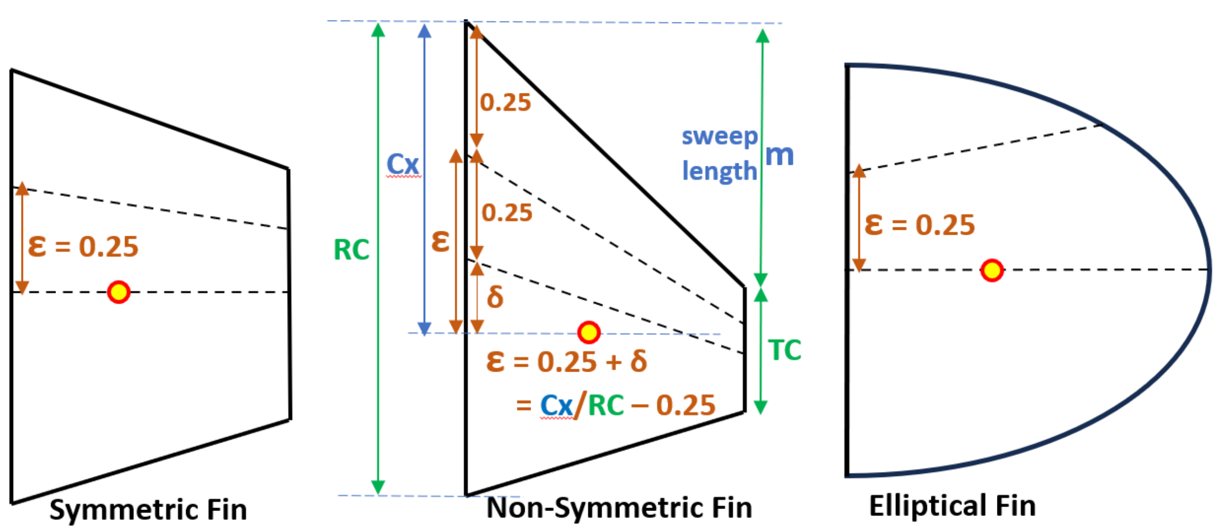
p_0 is air pressure at sea level in psi (14.696), or in KPa (101.325), and

π ("pi") is 3.14159 (dimensionless).

Since all components of DN are dimensionless except p_0, DN will have the units of pressure. <u>The pressure units chosen must match those of the units chosen for G_E</u>. Using the default value of 0.25 for ***ε***, DN = **39.294 (Imperial units) or 270.552 (SI units)**. Martin uses Imperial units throughout, and rounded DN up to 39.3.

In their respective articles, both Howard and Sahr compute DN incorrectly. Howard's PoF equation uses the constant 1.337, which is 39.3/14.696 (p_0: sea level absolute pressure), divided by 2, thus combining all constants in the denominator of Martin's original equation. However, Howard also included an additional factor of two in the denominator of his equation. This double division by two in the denominator results in the calculated value of V_f being overestimated by a factor of the square root of two (1.414). Sahr's article uses Howard's constant as given, thus propagating the error.

Martin's denominator constant of **39.3** assumes the value of ***ε*** is 0.25, but he notes that "for sections with the center of gravity far from the 50-percent chord position, a correction may be required." Let's see how this condition might arise. Consider the following fin shapes, where black dashed lines represent the quarter chord and half chord lines:



For symmetric fin shapes (including elliptical fins), ***ε*** is always 0.25. For non-symmetric fins, such as the middle fin shown above, ***ε*** is offset from the 50-percent chord by **δ** (delta). To determine ***ε*** in this case, we must first find **Cx**, which is the axial distance from the front of the fin to the fin centroid (shown as a red and yellow dot), or center of mass, or as Martin call it, the "center of gravity" of the fin. As shown above, the centroid is also offset toward the tip chord (by **Cy**), but we usually do not need to calculate this offset. To calculate **Cx** we use a formula from basic geometry for the centroid of a trapezoid:

$$C_x = \frac{(2 \times TC \times m) + TC^2 + (m \times RC) + (TC \times RC) + RC^2}{3(TC + RC)}$$

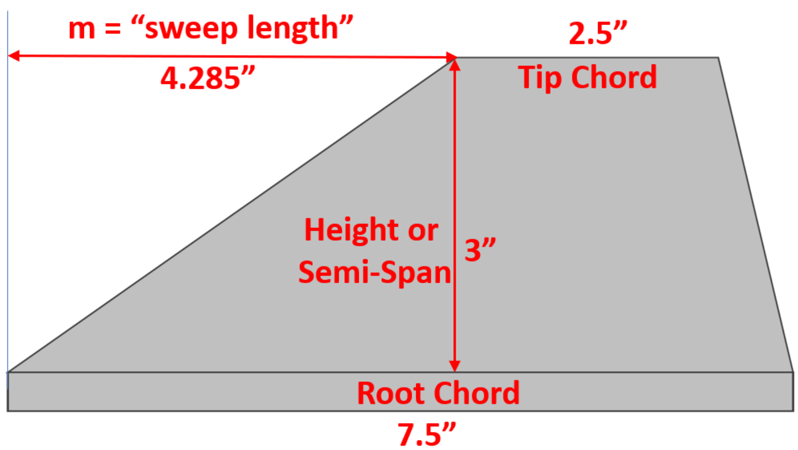
The value for **m** in this equation comes directly from the simulator as "sweep length." Once we have **Cx**, we can calculate ***ε*** using the formula:

$$\varepsilon = (\frac{C_x}{RC}) - 0.25$$

Recall that ***ε*** is a measure of distance expressed as a fraction of the whole root chord.

A Worked Example

Suppose we are designing a rocket intended to be capable of safe transonic and low supersonic flight, and we have decided that all structural components will be G10 or G12 fiberglass. We want our rocket to support 54mm motors up to K or small L. We have decided to use Imperial units. After working with our simulator, we have selected a 3" body tube, and designed the following fin shape (with through-the-wall fin mounting tabs shown):



The data in red is taken directly from the simulator. We now want to know the flutter velocity for varying choices of fin thickness, so we can choose an appropriate value. For now, assume a fin thickness of 1/8" (0.125").

Summarizing the given fin geometry:

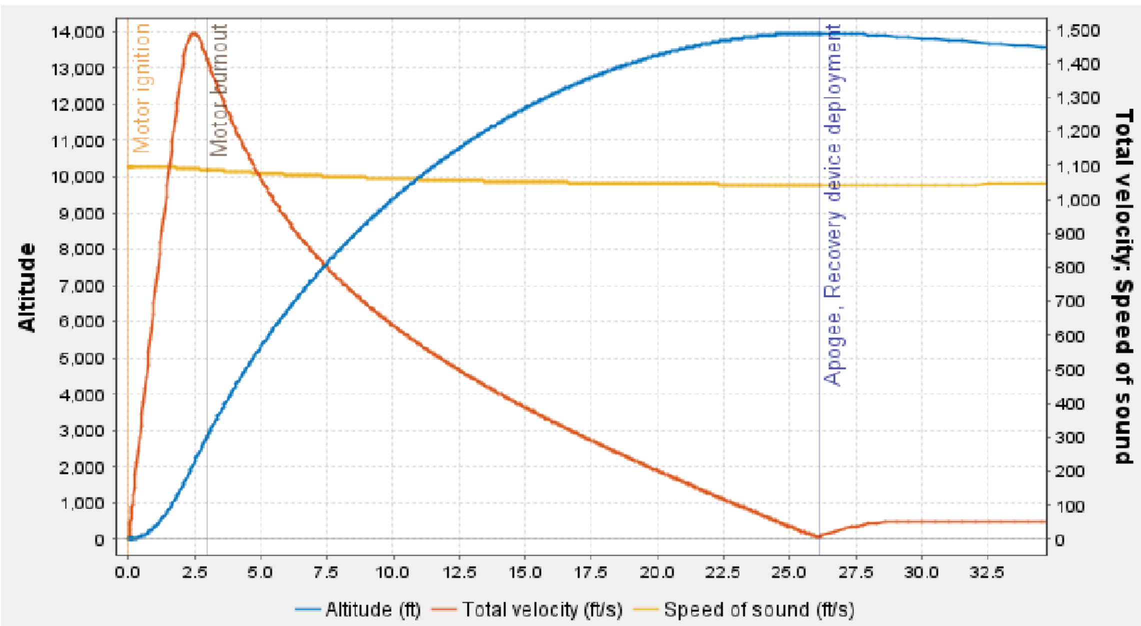
Fin Thickness: 0.125"
Semi-Span (Height): 3"
Tip Chord: 2.5"
Root Chord: 7.5"
Sweep Length (m): 4.285"

Let's starting by calculating DN.

From our formula for **Cx**, we find that **Cx** = 4.49".
From our formula for ***ε***, we find that ***ε*** = 0.349.
Therefore, DN = 54.88 (note that this is 40% more than Martin's default value of 39.3).

Now we determine the altitude of maximum velocity for the largest motor we intend to use. In our case, this is the Aerotech L1090W. Let's assume we are launching from the Brothers site in Oregon (elevation 4500 ft) and that the temperature will be 65°F on a clear, relatively calm (wind speed in the range 2.5 – 7.5 mph) day. This is cool enough that we will use the standard atmospheric model without adjustment. When we simulate (using OpenRocket) the L1090W motor in our rocket under these conditions, we see the following results (excerpted here):

Configuration	Velocity off rod	Apogee	Velocity at deploy...	Optimum delay	Max. velocity
[L1090W-0]	95.6 ft/s	13967 ft	43.4 ft/s	23.2 s	1491 ft/s



Based upon this simulation, our rocket will reach about 1500 fps (Mach 1.33) at an altitude of about 14,000 ft. AGL (18,500 ft ASL), just before motor burnout[14]. Now we can calculate the air temperature, air pressure, and speed of sound at that altitude using the standard atmosphere barometric model embodied in the equations above.

<u>At 18,500 ft.:</u>

T_F = -6.86°F
a (speed of sound) = 1043.36 fps
p (air pressure) = 7.2 psi
p/p_0 = 0.49

[14] RockSim predicts slightly lower apogee and Vmax for same rocket and launch conditions.

Now let's calculate the fin geometry ratios:

Fin Area = 15 in^2
t/c (thickness ratio) = 0.0167
λ (lambda) (taper ratio) = 0.3333
A (aspect ratio) = 0.6

We are now ready to choose G_E. Since our fins are fiberglass epoxy, we select a G_E of 600,000 psi from our table of suggested values.

Finally, put everything together and compute V_f:

V_f = 1425 fps

Since our rocket's maximum velocity is 1500 fps, this rocket is a likely candidate for failure due to fin flutter. And there is no safety margin.

Let's try increasing the fin thickness to 5/32" (0.156").

Now, V_f = 1991.7 fps, giving us a 32% margin.

For the flight conditions given, this is probably OK, but on a 95° day, V_f would be 1580.2 fps (only a 5% margin). That's not good, so let's try increasing the fin thickness to 3/16" (0.1875").

Now, V_f = 2618.1 fps (74% margin) under the conditions given, and 2077 fps (38% margin) on a 95° day.

This is a safety factor likely appropriate for any launch conditions, so we choose 3/16" thick fins.

OK, now that we know how to perform this calculation, wouldn't it be great if we could automate some of the process? Your prize for reading this far is a link to a spreadsheet that you can use to analyze your own fin designs for flutter risk. You can download this spreadsheet from GitHub at this URL: https://github.com/jkb-git/Fin-Flutter-Velocity-Calculator. An image of this spreadsheet in use is shown in the Appendix.

<u>Some Final Thoughts</u>

1. It is important that the choice of unit system (Imperial or SI) be consistent at several points in the calculation. In addition to being consistent with each numerator/denominator pair, the choice of unit system for V_f, the speed of sound, G_E, and air pressure must also be the same.

2. Martin's work was only intended to serve as a "guide in the preliminary design of lifting surfaces on missiles." Several potentially important details have been abstracted away. For this reason, a significant safety margin should be included if these calculations will be used in go/no-go flight decisions. I would consider a safety margin under 25% for actual flight conditions a poor risk, and one under 20% as potentially unsafe.

3. Martin's work does not consider fins whose thickness tapers toward the ends. Using the average fin thickness value is one possibility. Another possibility is to use the fin thickness at the fin centroid or mid-height. The best choice will likely depend upon the linearity (or non-linearity) of the taper.

4. Tip-to-tip reinforcing, a common practice for transonic and supersonic amateur rockets, is also not considered directly. If using tip-to-tip reinforcement, a reasonable approximation that takes this reinforcement into account would be to double $\boldsymbol{G_E}$. The accompanying spreadsheet does this.

5. The selection of $\boldsymbol{G_E}$ itself has a fair bit of windage. Virtually none of the materials in common use for amateur rocket fins come with guaranteed manufacturing specifications. This is especially true for composite materials. The margins in the suggested $\boldsymbol{G_E}$ values found here attempt to take this fact into account.

6. The equations as presented are only valid in the Troposphere (under 36,000 ft. or so). It would be relatively straight forward to use a more sophisticated atmospheric model that handles higher altitudes. A good starting point for this endeavor might be found here[15] or here[16]. On the other hand, this effort is likely unwarranted. First, Martin's analysis tells us that fin flutter is most likely to occur in dense air at transonic speeds. Second, $\mathbf{V_f}$ increases as we gain altitude because air temperature and air pressure decrease. For example, if we use NASA's RocketModeler high altitude temperature and pressure models,[17] our example rocket (with 3/16" fin thickness) has a predicted $\mathbf{V_f}$ of 5025 ft/sec at 50,000 ft, 9157 ft/sec at 75,000, and 16,726 ft/sec at 100,000 ft. Unless we are deep into the hypersonic range (where many other issues are likely to control rocket design decisions), fin flutter is not likely to be a problem at high altitude. Finally, Martin's methods were only validated to about Mach 1.5, so if we want to design a hypersonic rocket, we need to be using more sophisticated design tools.
7. This method for fin analysis does not address the need for strong fin attachment, but any attachment method will benefit from not being stressed by fin flutter.

John Bennett is an engineering Professor Emeritus at the University of Colorado Boulder, and a TRA and NAR member in OregonRocketry. Using Brinley's book as a guide, he built and flew many Zn/S and KNO_3/sugar rockets as a teenager. In retirement, he is now rediscovering his love of amateur rocketry, this time with better supervision.

[15] https://en.wikipedia.org/wiki/International_Standard_Atmosphere
[16] https://en.wikipedia.org/wiki/U.S._Standard_Atmosphere
[17] https://www1.grc.nasa.gov/beginners-guide-to-aeronautics/rocketmodeler/

Appendix

Calculation of Fin Flutter Velocity v1.1

December 2023

Primary reference: NACA TN 4917 - Dennis J. Martin, *Summary of Flutter Experiences as a Guide to The Preliminary Design of Lifting Surfaces on Missles* , February, 1958

This spreadsheet estimates the fin flutter velocity using Martin's method. See accompanying article *Fin Flutter Analysis Revisited (Again)* for implementation details and limitations.

Due to slight differences in atmospheric model constants, as well as rounding differences, Imperial and SI calculations for the same geometry and conditions may differ slightly.

Enter the data highlighted in green in either Imperial or SI Units (but not both)

		Data to be Entered	**Imperial Units**		**SI Units**	
Launch Site Data						
MaxV	=	maximum predicted rocket velocity	1500	ft/sec	457.2	meters/sec
AMaxV	=	predicted altitude at predicted maximum rocket velocity	14000	ft	4267.2	meters
LSA	=	launch site altitude (ASL)	4500	ft	1371.6	meters
TLS	=	temperature at launch site	65	deg F	18.3	deg C
Use Default Temp?	=	Use Default Sea-Level Temp (**DST**) or Launch Site Temp (**LST**)	DST	DST or LST	DST	DST or LST
Fin Geometry Data						
t	=	fin thickness	0.1875	in	0.4763	cm
m	=	fin sweep length	4.285	in	10.88	cm
TC	=	tip chord length	2.5	in	6.35	cm
RC	=	root chord length	7.5	in	19.05	cm
SSL	=	semi span length (height)	3	in	7.62	cm
G_E (shear modulus)		Shear Modulus (doubled in calculation if tip-to-tip reinforcement)	600000	psi	4136854	KPa
T2T	=	Tip-to-Tip reinforcing present?	NO	YES or NO	NO	YES or NO
		Calculated Values				
Cx (for trapezoidal fins) (edit formula for other fin shapes)		$C_x = \frac{(2\times TC\times m)+TC^2+(m\times RC)+(TC\times RC)+RC^2}{3(TC+RC)}$	4.49	in	11.41	cm
t/c (thickness ratio)		**fin thickness / root chord length**	0.0250		0.0250	
λ (lambda) (taper ratio) (create "pseudo" tip chord if nec.)		**tip chord length / root chord length**	0.3333		0.3333	
Fin Area (trapezoidal fin) (edit formula for other fin shapes)		$Area = Height \times \frac{Tip\ Chord\ Length + Root\ Chord\ Length}{2}$	15.000	in sq	96.774	cm sq
A (aspect ratio)		**(semi-span length or height)^2 / fin area**	0.6000		0.6000	
ε (epsilon)		$\varepsilon = (\frac{C_x}{RC}) - 0.25$	0.3492		0.3492	
DN (denominator constant)		$DN = \frac{(24\times\varepsilon\times\kappa\times p_0)}{\pi}$	54.88	psi	378.39	KPa
Tf or Tc (air temp at AMaxV)		$T_C = 15 - (.0065 \times altitude_{meters})$ $T_F = 59 - (.00356 \times altitude_{ft})$	-6.86	deg F	-21.65	deg C
a (speed of sound at AMaxV)		$a_{m/sec} = 20.05 \times \sqrt{273.16 + T_C}$ $a_{ft/sec} = 49.03 \times \sqrt{459.7 + T_F}$	1043.36	ft/sec	317.97	meters/sec
p (air pressure at AMaxV)		$P_{(psi)} = 14.696 \times (\frac{T_F + 459.7}{518.7})^{5.256}$ $P_{(KPa)} = 101.325 \times (\frac{T_C + 273.16}{288.16})^{5.256}$	7.20	psi	49.57	KPa
First Term		$\frac{DN \times A^3}{(\frac{t}{c})^3 \times (A+2)}$	291797.59	psi	2011866.56	KPa
Second Term		$(\frac{\lambda+1}{2})$	0.67		0.67	
Third Term		$(\frac{p}{p_0})$	0.49		0.49	
Vf		$V_f = a \times \sqrt{\frac{G_E}{\frac{DN\times A^3}{(\frac{t}{c})^3\times(A+2)} \times (\frac{\lambda+1}{2}) \times (\frac{p}{p_0})}}$	2618.13	ft/sec	798.43	meters/sec
Safety Margin			1118.1	ft/sec	341.2	meters/sec
Safety Margin %		**>= 25% margin is good; 20-25% margin is fly-with-caution; and < 20% margin is no-fly**	74.5	%	74.6	%