Conditional Probability:

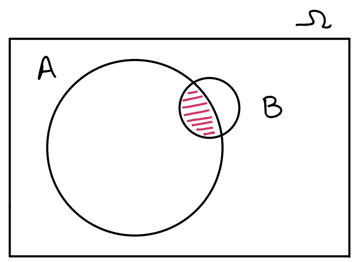
$$\boxed{P(A \mid B) = \frac{P(A \cap B)}{P(B)}}$$

$$P(A \mid B) \neq P(B \mid A)$$

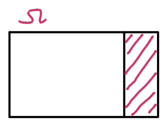
Interpretation

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)}$$

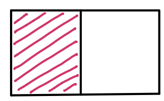


$P(B \mid A)$



≈ 0.2

$P(A \mid B)$



≈ 0.5

Problem:

- A disease is prevalent in 0.2% of a population.
- We have a test that, given to a sick person, gives a +ve result 85% of the time.
- Of all the people ever tested, 8% were positive.

Q: If Nazo is tested and test comes back positive, what are the chances that she actually has the disease?

- [ ] 85%
- [ ] 77%
- [ ] 21%
- [x] 2%

$$P(\text{Disease}) = 0.002$$

$$P(\text{Pos} \mid \text{Disease}) = 0.85$$

$$P(\text{Pos}) = 0.08$$

(*) Event "Disease" is difficult to measure directly!

Event "Pos" is relatively easier to measure

$$P(\text{Disease} \mid \text{Pos}) = ?$$

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)}$$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \cap B) = P(A \mid B)\, P(B)$$

$$P(B \mid A) = \frac{P(A \mid B)\, P(B)}{P(A)}$$

$$P(\overset{B}{\text{Disease}} \mid \overset{A}{\text{Pos}}) = \frac{P(\overset{A}{\text{Pos}} \mid \overset{B}{\text{Disease}})\, P(\overset{B}{\text{Disease}})}{P(\underset{A}{\text{Pos}})}$$

$$= \frac{(0.85)(0.002)}{0.08}$$

$$= 0.021 \qquad \underline{\underline{2.1\%}}$$

$$\underbrace{P(\text{Disease} \mid \text{Pos})}_{\substack{\text{Posterior} \\ \text{(After)}}} = \frac{\overbrace{P(\text{Pos} \mid \text{Disease})}^{\text{Likelihood}} \; \overbrace{P(\text{Disease})}^{\text{Prior (Before)}}}{\underbrace{P(\text{Pos})}_{\text{Normalizing factor}}}$$

$P(\text{Disease}) = 0.002$ — Prior belief

$P(\text{Pos} \mid \text{Disease}) = 0.85$ — Result of experiment

$P(\text{Pos}) = 0.08$

$P(\text{Disease} \mid \text{Pos}) = 0.21$ — Updated belief

* You start off with some belief and update it based on some experiment!

This is the "Bayes' Rule" of inference.

$$\rightarrow P(B \mid A) = \frac{P(A \mid B)\, P(B)}{P(A)} \leftarrow$$

Classical Statistics

Bayesial "

Applying Bayes' Rule to Spam Detection.

- "You have inherited a million dollars."
- "There will be a meeting at noon."
- Assumption: We have a dataset of spam emails.
- Need to find whether a piece of text is spam.

- Let's first consider a single word.

- "how frequently does this word appear in spam?"
- "How much spam is there in the world?"

- $$P(\text{Spam}|w) = \frac{P(w|\text{spam})\, P(\text{spam})}{P(w)}$$

- "Given that this word appears, how likely is it that the message is spam?"
- from dataset. "How frequent is this word?"

- $$P(\text{spam}) = \frac{\#\text{ of spam messages}}{\#\text{ of all messages}}$$

- $$P(w|\text{spam}) = \frac{\#\text{ of times this word appears in spam}}{\#\text{ of spam messages}}$$

- $$P(w) = \frac{\#\text{ of times this word appears}}{\#\text{ of total messages}}$$

- Now, do this for all words
- $P(spam \mid words) = P(spam \mid w_1) * P(spam \mid w_2) * \cdots P(spam \mid w_n$

$$P(spam \mid words) = \prod_{i=1}^{|words|} P(spam \mid w_i)$$

$$\sum_{i=1}^{n}$$