Events vs Variables

- Assign outcomes of experiments to variables
- But why?
- Example : 6-side dice rolled

$$N \overset{\text{type}}{\in \mathbb{N}}, \quad \overbrace{1 \leq N \leq 6}^{\text{constraint}}$$

Maths —— Constant —— Variable

Programming —— Const —— Variable

Never changes

Assigned once. Never changes afterwards

Can always change

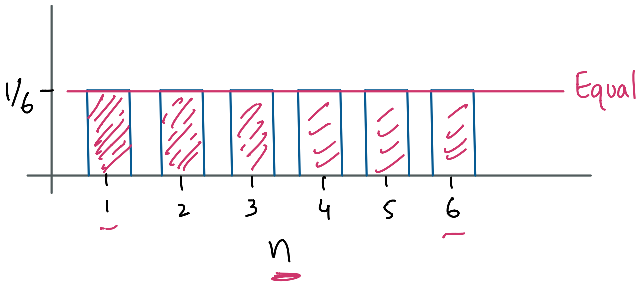
Random variables (RVs)

- A RV can take on a value from any set. Each <u>value</u> has a probability associated with it.

N	1	2	3	4	5	6
$P(N=n)$	$1/6$	$1/6$	$1/6$	$1/6$	$1/6$	$1/6$

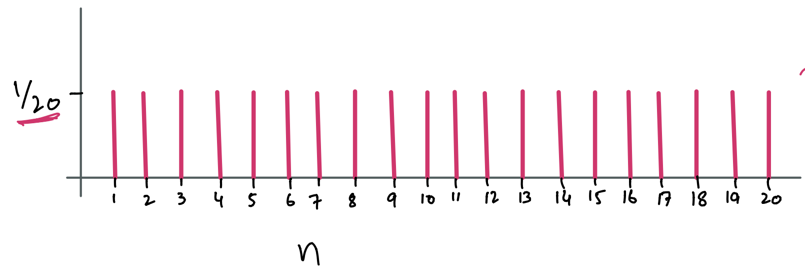
- Other examples :
  - Pick any person and measure their height
  - Pick any character and convert it to ASCII.

$P(N=n)$



– Similar to the histogram we saw earlier ...

$P(N=n)$



This is called a "distribution".

N is a 'discrete random variable'.

(side note: Another example)

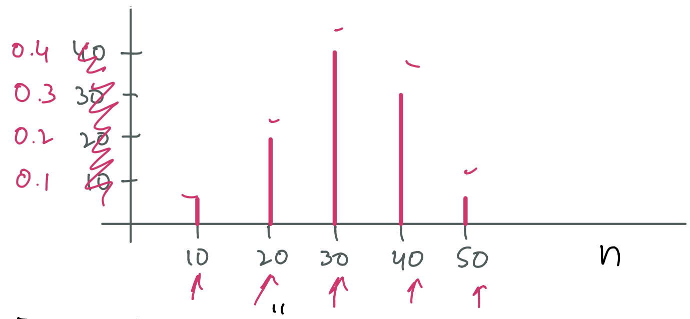
Measure ages of 100 people.

Frequency Table

Weight	Frequency
10	5 $/100 = 0.05$
20	20 $= 0.2$
30	40
40	30
50	5

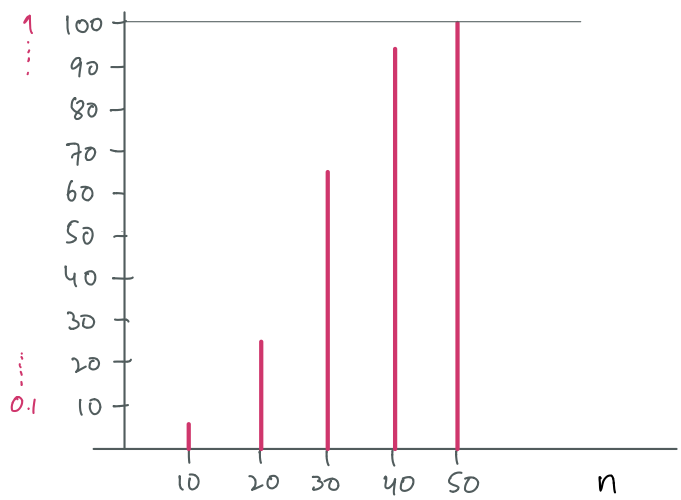
(frequentist view)

freq.



This is a "probability frequency distribution". (PFD)

Cumulative freq



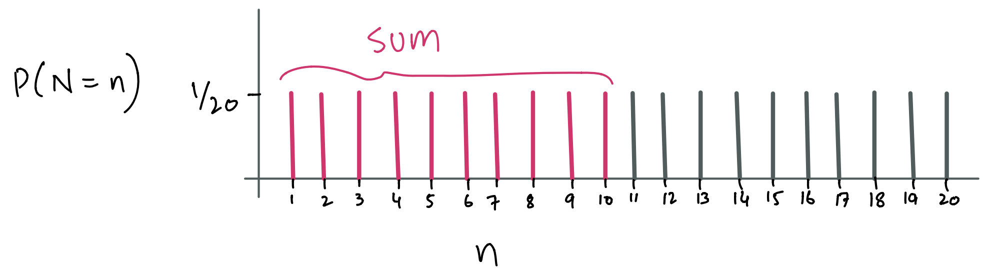
	Weight	Frequency
class 1	10	5
2	20	25
	30	65
	40	95
	50	100

This is "cumulative freq distribution" (CFD)

Back to distributions:

$P(N <= 10)$

1 D



$\underline{P(N \leq 10)} = \sum_{i=1}^{10} \underline{P(N=i)}$ (mutually exclusive)

$P(N \leq 20)$ must equal 1

$P(N < 1)$ " " 0

How about $\underline{2}$ random variables?

" Bag $\underline{i}$ has $\underline{i}$ blue balls and $\underline{2}$ green balls".

Say $i = 6$

Random variables: B = bag picked.
C = color of ball picked 1 = blue 2 = green

$P(B=b, C=c)$

$P(B=1, C=1)$

$\boxed{P(\Omega)=1}$

B \ C	C = 1	C = 2	P(B=b)
B = 1	$\frac{1}{6}\cdot\frac{1}{3} = \frac{1}{18}$	$\frac{1}{6}\cdot\frac{2}{3} = \frac{2}{18}$	$\frac{1}{6}$
B = 2	$\frac{1}{6}\cdot\frac{2}{4} = \frac{2}{24}$	$\frac{1}{2}\cdot\frac{2}{4} = \frac{2}{24}$	$\frac{1}{6}$
B = 3	$\frac{1}{6}\cdot\frac{3}{5} = \frac{3}{30}$	$\frac{1}{6}\cdot\frac{2}{5} = \frac{2}{30}$	$\frac{1}{6}$
B = 4	$\frac{1}{6}\cdot\frac{4}{6} = \frac{4}{36}$	$\frac{1}{6}\cdot\frac{2}{6} = \frac{2}{36}$	$\frac{1}{6}$
B = 5	$\frac{1}{6}\cdot\frac{5}{7} = \frac{5}{42}$	$\frac{1}{6}\cdot\frac{2}{7} = \frac{2}{42}$	$\frac{1}{6}$
B = 6	$\frac{1}{6}\cdot\frac{6}{8} = \frac{6}{48}$	$\frac{1}{6}\cdot\frac{2}{8} = \frac{2}{48}$	$\frac{1}{6}$
P(C=c)	0.594	0.405	1

A \ B	1	2
1		
2		
3		
	$P(B=1)$	$P(B=2)$

marginal probability of A

marginal probability of B

Joint probability distribution of A and B

Effect removed

$$\boxed{P(B=1) = \sum_i P(B=1, C=i)}$$

(same rule of summation of mutually exclusive events)

"sum over all possible values of C."

But we already knew the 1/6 for $P(B=1)$!!

why we do we need to do this summation?

B \ C	C = 1	C = 2	P (B=b)
B = 1	1/18	2/18	1/6
B = 2	2/24	2/24	
B = 3	3/30	2/30	
B = 4	4/36	2/36	
B = 5	5/42	2/42	
B = 6	6/48	2/48	

When we collect data from real world:

— Sensor (signal + noise)

We have important RVs and noise mixed together!

We are only able to measure joint probabilities.
But we are interested in marginals of one RV.