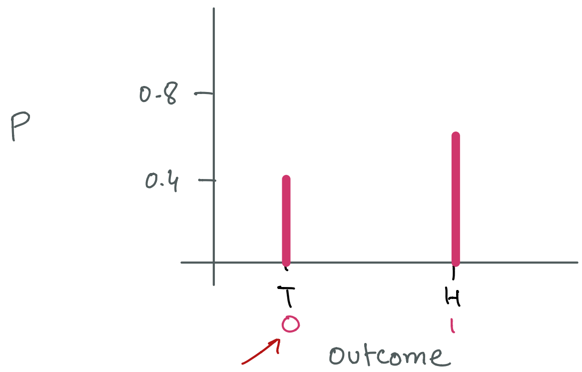
- Summarizing Distributions

- Consider a coin toss with $P(H) = 0.6$

- We can summarize this using a function:



$$P(n) = \begin{cases} 0.4 & \text{if } n=0 \\ 0.6 & \text{if } n=1 \end{cases}$$

$(P(A) + P(\bar{A}) = 1$ Law of total probability$)$

Generalize:

$$P(n) = \begin{cases} 1-p & \text{if } n=0 \\ p & \text{if } n=1 \end{cases}$$

Piecewise

Or, we can write this as:

$$P(n\,;\,p) = \underbrace{p^{n} \cdot (1-p)^{1-n}}_{\text{Probability density function (PDF)}} \quad \Big\} \text{ model}$$

Takes just one argument!

$\boxed{\text{called the model parameter}}$

Set the parameter: $p = 0.6$

The function becomes: $P(n) = 0.6^{n} \cdot 0.4^{1-n}$

Now: $P(0) = 0.6^{0} \cdot 0.4^{1}$
$= 0.4$

$P(1) = 0.6^{1} \cdot 0.4^{0}$
$= 0.6$

- A PDF calculates probabilities for any value of a RV.
- It is (often) parameterized
- Several common PDFs define well known "distributions"
- The one above is the "Bernoulli Distribution" — single experiment
- "n is Bernoulli distributed".

If we do 'n' experiments. Success chance $= p$

RV : X — number of successes in n experiments

If I flip it 10 times, it can have

0 — 10 successes

$P(0)$ — $P(10)$ needed!

Let's set $n=10$, $p = 0.6$

$$P(X=1) = (0.6^1 \times 0.4^9) + 0.4 \times 0.6^1 \times 0.4^8 + 0.4^2 \times 0.6 \times 0.4^7 + \dots = 10(0.6^1 \times 0.4^9)$$

How about $P(2)$?

But we know that this is a common pattern.

X is a RV which belongs to the "Binomial Distribution".

$$\boxed{P(X=k\ ;\ n, p) = \binom{n}{k} p^k (1-p)^{n-k}}$$

$\binom{n}{k} = \frac{n!}{k!(n-k)!}$

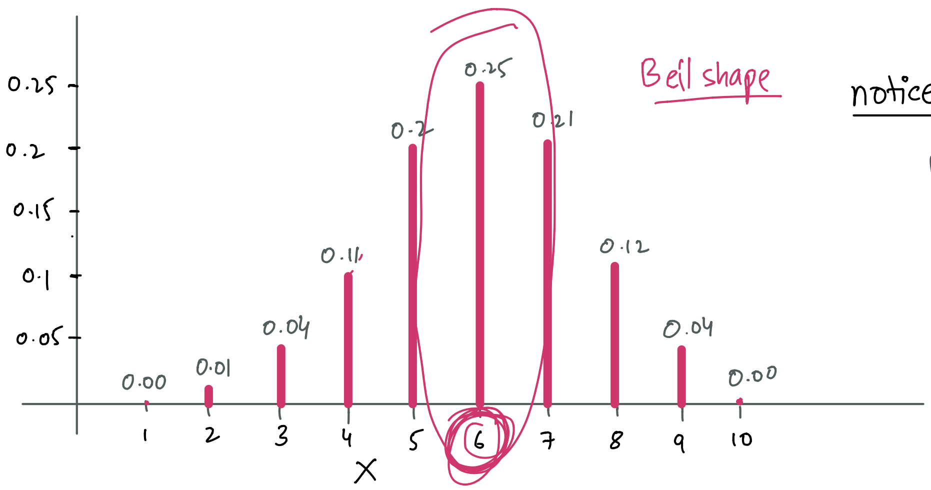
${}^nC_k$

Setting parameters : $n = 10$, $p = 0.6$

$$P(X=k) = \binom{10}{k} 0.6^k \, 0.4^{10-k}$$

$$P(X=2) = \binom{10}{2} 0.6^2 \cdot 0.4^8$$

$P(x)$



notice the shape!

$P(X=5.5) = ?$

Create variations to the model by varying the parameters.

Jupyter Lab!

- Practical applications.
  - Model the number of expected virus warnings
    - Don't want to annoy the user
    - Don't want to let viruses through

- Other distributions for discrete random variables
  - Poisson
  - Geometric