Quantifying Chances

- This is a thinking lesson !!

Problem :

[illegible]

- A disease is prevalent in 0.2% of a population.
- We have a test that, given to a sick person, gives a +ve result 85% of the time.
- Of all the people ever tested, 8% were positive.

Q: If Nazo is tested and test comes back positive, what are the chances that she actually has the disease?

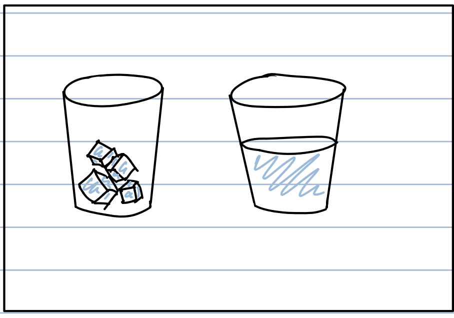
☐ 85% ☐ 77% ☐ 21% ☑ 2%

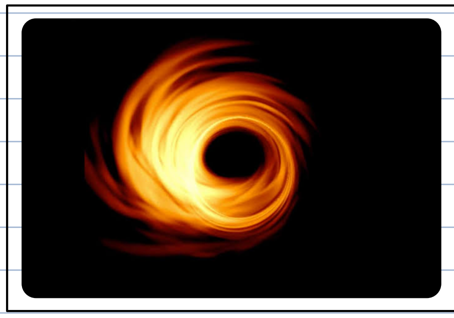
What is information?

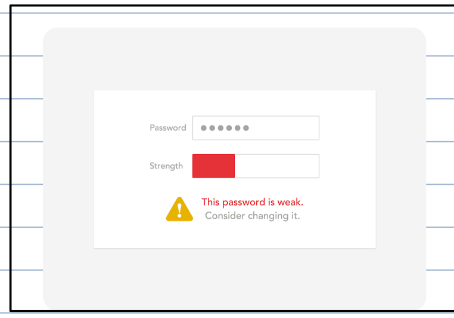
" Gooblede oompa googie " less

" I can speak Gibberish." more

quantification







Let's say you're doing cryptanalysis

Key 1 → " Gooblede oompa googie " → 29

✓ Key 2 → " I can speak Gibberish." → 1058

} Entropy

We can't eyeball 2.4 billion decryptions!

Chance → Certainty / Uncertainty → Entropy

- Video streaming — file size
- Password strength
- Hoffman's codes — compression
- Computer vision
- Machine learning — throw away data / information
- Bits — foundations of all computation

Quantifying Chances

- Flip a coin:

hard

Q: "What are the chances that it will land on its head?"

Not: "If I flip it 10 times, how many will be heads?"

easy

H , T → Events
(0 / 1)

Ω → universe

probability

0.5	0.5
H	T

$\Omega = \{H, T\}$

Chance of it being heads = $P(H) = 0.5$

" " " " tails = $P(T) = 0.5$

The two axioms of Probability:

- must lie in : $[0 - 1]$
- Sum of all events must be 1

$P(H) = 1.2$ ✗

$P(H) = 0.7 \quad P(T) = 0.2$

→ Flip it twice:

{ HH, HT, TH, TT }

H	H
H	T
T	H
T	T

The two flips are "independent".

P(HH)

"Both flips are heads"

0.25 HH	0.25 HT
0.25 TH	0.25 TT

$$P(\text{favorable event}) = \frac{\#\text{ of ways event can occur}}{\#\text{ of all possible outcomes}}$$

$$= \frac{1}{4} = 0.25 = 25\%$$

"Any of the flips is a head." } event

A= { HH, HT, TH }

$$P(A) = 3/4 = 0.75$$

Alternate way | Intuition

"Both flips are heads"

First flip is H and second flip is H

$$P(H) * P(H)$$

$$= 0.5 * 0.5 = 0.25$$

AND		
H 0	H 0	0
H 0	T 1	0
T 1	H 0	0
T 1	T 1	1

"Any of the flips is a head."

First flip is H or second flip is H

$$P(H) + P(H)$$

$$= 0.5 + 0.5$$

$$= 1$$

HH HT → 0.5

TH HH — 0.5

↳ 0.25

OR		
H 0	H 0	0
H 0	T 1	1
T 1	H 0	1
T 1	T 1	1

0.5 0.5

Remove the overlap

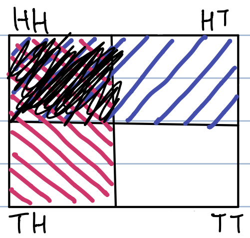
$$\underset{B}{P(H)} + \underset{A}{P(H)} - P(HH)$$

$$= 0.5 + 0.5 - 0.25$$

$$= 0.75$$

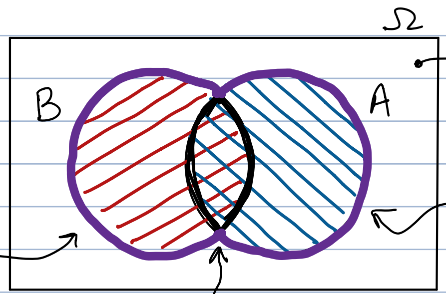
A and B are not independent!

Another view:



first flip H — A

Second flip H — B



Neither A nor B

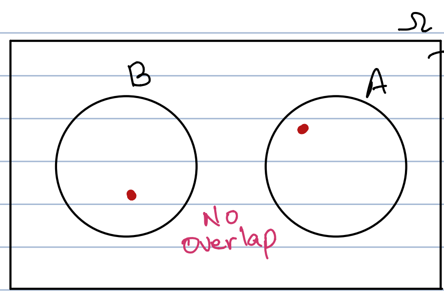
Venn

A but not B

B but not A

A and B

Single flip — independent events



this is φ

$$P(A \text{ or } B) = P(A) + P(B) + \overbrace{P(A \text{ and } B)}^{0}$$

Notation:

A and B $=$ $A \cap B$

A or B $=$ $A \cup B$

Let's scale this up to 1 million flips!

Jupyter notebook!