1. Let $R = \text{result}(A)$. If $0 \le R < 0.4$, $X = 1$

$0.4 \le R < 0.8$, $X = 0$

$0.8 \le R \le 1$, $X = -1$

2. Let A = the event that Alice draws a green ball.
Let B = the event that Bob draws a green ball.

$$A \rightarrow B$$

A	P(A)
t	5/12
f	7/12

A	P(B|A)
t	4/11
f	5/11

$$P(b) = \sum_a P(A, b)$$
$$= \sum_a P(A)\,P(b \mid A)$$
$$= \left(\frac{5}{12} \cdot \frac{4}{11}\right) + \left(\frac{7}{12} \cdot \frac{5}{11}\right)$$
$$= 0.4166$$

3. $P(F=\text{empty} \mid S=\text{false}) = \dfrac{P(F=\text{empty}, S=\text{false})}{P(S=\text{false})}$

$$P(F=\text{empty}, S=\text{false}) = \sum_b \sum_g \sum_t P(B, \overset{F=}{\text{empty}}, G, T, \overset{S=}{\text{false}})$$

$$= \sum_b \cancel{\sum_g} \sum_t P(B)\, P(\overset{F=}{\text{empty}})\, \cancel{P(G \mid B, \overset{F=}{\text{empty}})}\, P(T \mid B)\, P(\overset{S=}{\text{false}} \mid T, \overset{F=}{\text{empty}})$$

g doesn't influence S so it can be taken out

$$= \sum_b \sum_t P(B)\, P(F=\text{empty})\, P(T \mid B)\, P(S=\text{false} \mid T, F=\text{empty})$$

$$= P(F=\text{empty}) \sum_b \sum_t P(B)\, P(T \mid B)\, P(S=\text{false} \mid T, F=\text{empty})$$

$$= P(F=\text{empty}) \cdot \sum_b P(B)\big(P(T=\text{true} \mid B)\, P(S=\text{false} \mid T=\text{true}, F=\text{empty}) + P(T=\text{false} \mid B)\, P(S=\text{false} \mid T=\text{false}, F=\text{empty})\big)$$

$$= P(F=\text{empty}) \cdot \Big(P(B=\text{bad})\big(P(T=\text{true} \mid B=\text{bad})\, P(S=\text{false} \mid T=\text{true}, F=\text{empty}) + P(T=\text{false} \mid B=\text{bad})\, P(S=\text{false} \mid T=\text{false}, F=\text{empty})\big)$$
$$+ P(B=\text{good})\big(P(T=\text{true} \mid B=\text{good})\, P(S=\text{false} \mid T=\text{true}, F=\text{empty}) + P(T=\text{false} \mid B=\text{good})\, P(S=\text{false} \mid T=\text{false}, F=\text{empty})\big)\Big)$$

$$= 0.05\,(0.02\,(0.02 \cdot 0.92 + 0.98 \cdot 0.99) + 0.98\,(0.97 \cdot 0.92 + 0.03 \cdot 0.99))$$

$$= 0.0461715$$

$$P(F=\text{not empty}, S=\text{false}) = \sum_b \sum_g \sum_t P(B, \overset{F=}{\text{not empty}}, G, T, \overset{S=}{\text{false}})$$

$$= \sum_b \sum_t P(B)\, P(\overset{F=}{\text{not empty}})\, P(T \mid B)\, P(\overset{S=}{\text{false}} \mid T, \overset{F=}{\text{not empty}})$$

$$= P(\overset{F=}{\text{not empty}}) \cdot \Big(P(B=\text{bad})\big(P(T=\text{true} \mid B=\text{bad})\, P(S=\text{false} \mid T=\text{true}, F=\text{not empty}) + P(T=\text{false} \mid B=\text{bad})\, P(S=\text{false} \mid T=\text{false}, F=\text{not empty})\big)$$
$$+ P(B=\text{good})\big(P(T=\text{true} \mid B=\text{good})\, P(S=\text{false} \mid T=\text{true}, F=\text{not empty}) + P(T=\text{false} \mid B=\text{good})\, P(S=\text{false} \mid T=\text{false}, F=\text{not empty})\big)\Big)$$

$$= 0.95\,(0.02\,(0.02 \cdot 0.01 + 0.98 \cdot 1) + 0.98\,(0.97 \cdot 0.01 + 0.03 \cdot 1))$$

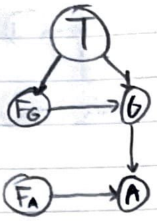
$$= 0.0555845$$

$$P(S=\text{false}) = P(F=\text{empty}, S=\text{false}) + P(F=\text{not empty}, S=\text{false})$$
$$= 0.101756$$

$$\Rightarrow P(F=\text{empty} \mid S=\text{false}) = \frac{0.0461715}{0.101756} = 0.453747199$$

4.

a.



b.

T	F_G	$P(G=\text{normal} \mid T, F_G)$	$P(G=\text{high} \mid T, F_G)$
normal	F	x	$1-x$
normal	T	$1-y$	y
high	F	$1-x$	x
high	T	y	$1-y$

c.

F_A	G	$P(A \mid F_A, G)$
T	normal	0
T	high	0
F	normal	0
F	high	1

4 d. Calculate $P(T=\text{high} \mid \neg F_A, \neg F_G, A)$:

$$= \frac{P(T=\text{high}, \neg F_A, \neg F_G, A)}{P(\neg F_A, \neg F_G, A)}$$

$$P(T=\text{high}, \neg F_A, \neg F_G, A) = \sum_g P(T=\text{high}, \neg F_A, \neg F_G, A, G)$$

$$= \sum_g P(T=\text{high}) P(\neg F_A) P(\neg F_G \mid T) P(A \mid F_A, G) P(G \mid F_G, T=\text{high})$$

$$= \big(P(T=\text{high}) P(\neg F_A) P(\neg F_G \mid T=\text{high})\big)\big(P(A \mid \neg F_A, G=\text{normal}) P(G=\text{normal} \mid \neg F_G, T=\text{high})$$
$$+ P(A \mid \neg F_A, G=\text{high}) P(G=\text{high} \mid \neg F_G, T=\text{high})\big)$$

$$= \big(P(T=\text{high}) P(\neg F_A) P(\neg F_G \mid T=\text{high})\big)\big(0 \cdot (1-x) + 1(x)\big)$$
$$= x\, P(T=\text{high}) P(\neg F_A) P(\neg F_G \mid T=\text{high})$$

$$P(\neg F_A, \neg F_G, A) = P(T=\text{high}, \neg F_A, \neg F_G, A) + P(T=\text{normal}, \neg F_A, \neg F_G, A)$$

$$= x P(T=\text{high}) P(\neg F_A) P(\neg F_G \mid T=\text{high}) +$$
$$\big(P(T=\text{normal}) P(\neg F_A) P(\neg F_G \mid T=\text{normal})\big)\big(P(A \mid \neg F_A, G=\text{normal}) P(G=\text{normal} \mid \neg F_G, T=\text{normal})$$
$$+ P(A \mid \neg F_A, G=\text{high}) P(G=\text{high} \mid \neg F_G, T=\text{normal})\big)$$

$$= x P(T=\text{high}) P(\neg F_A) P(\neg F_G \mid T=\text{high}) +$$
$$\big(P(T=\text{normal}) P(\neg F_A) P(\neg F_G \mid T=\text{normal})\big)\big(0 \cdot (x) + 1(1-x)\big)$$

$$= x P(T=\text{high}) P(\neg F_A) P(\neg F_G \mid T=\text{high}) +$$
$$(1-x) P(T=\text{normal}) P(\neg F_A) P(\neg F_G \mid T=\text{normal})$$

$$= P(\neg F_A)\big(x P(T=\text{high}) P(\neg F_G \mid T=\text{high}) + (1-x) P(T=\text{normal}) P(\neg F_G \mid T=\text{normal})\big)$$

$$\Rightarrow P(T=\text{high} \mid \neg F_A, \neg F_G, A) = \frac{x P(T=\text{high}) P(\neg F_G \mid T=\text{high})}{x P(T=\text{high}) P(\neg F_G \mid T=\text{high}) + (1-x) P(T=\text{normal}) P(\neg F_G \mid T=\text{normal})}$$

5.

a $P(c \mid s, r) = \alpha P(c)\, P(s \mid c)\, P(r \mid c) = \alpha \cdot 0.5 \cdot 0.1 \cdot 0.8$ $\rightarrow \alpha \langle 0.04, 0.05 \rangle$

$P(\neg c \mid s, r) = \alpha P(\neg c)\, P(s \mid \neg c)\, P(r \mid \neg c) = \alpha \cdot 0.5 \cdot 0.5 \cdot 0.2$ $= \langle 0.\overline{44}, 0.\overline{55} \rangle$

$P(c \mid s, \neg r) = \alpha P(c) P(s \mid c)\, P(\neg r \mid c) = \alpha \cdot 0.5 \cdot 0.1 \cdot 0.2$ $\rightarrow \alpha \langle 0.01, 0.2 \rangle$

$P(\neg c \mid s, \neg r) = \alpha P(\neg c)\, P(s \mid \neg c)\, P(\neg r \mid \neg c) = \alpha \cdot 0.5 \cdot 0.5 \cdot 0.8$ $= \langle 0.048, 0.952 \rangle$

$P(r \mid c, w, s) = \alpha P(r \mid c) P(w \mid r, s) = \alpha \cdot 0.8 \cdot 0.99$ $\rightarrow \alpha \langle 0.792, 0.18 \rangle$

$P(\neg r \mid c, w, s) = \alpha P(\neg r \mid c)\, P(w \mid \neg r, s) = \alpha \cdot 0.2 \cdot 0.90$ $= \langle 0.815, 0.185 \rangle$

$P(r \mid \neg c, w, s) = \alpha P(r \mid \neg c)\, P(w \mid r, s) = \alpha \cdot 0.2 \cdot 0.99$ $\rightarrow \alpha \langle 0.198, 0.72 \rangle$

$P(\neg r \mid \neg c, w, s) = \alpha P(\neg r \mid \neg c) P(w \mid \neg r, s) = \alpha \cdot 0.8 \cdot 0.90$ $= \langle 0.216, 0.784 \rangle$