QERM 598 - HW 5 Due February 20, 2008 Eli Gurarie

## Stochastic processes and Markov chains

## 1 Hoops



Ray Allen is considered to be one the best jump shooters in the NBA. When he practices his jump shot, he likes playing a game called "Plus One, Minus Two" with very simple rules: You start with zero points and every time you make a basket, you get one point. Every time you miss a basket, you lose two points. If you get to -10 before you get to +10, you lose.

- a. Assuming a constant probability p of making a shot, construct a Markov matrix that models "Plus One, Minus Two". Be clear about the meaning and domain of the state space. The matrix might be too big to fit on a sheet of paper, and it is very sparce (meaning there are many zeroes) so be prepared to describe in notation:  $M_{ij} = ....$
- b. Create a function in R that generates this matrix for an arbitrary point threshold N and arbitrary plus and minus rules. For the following questions, use the matrix you constructed and provide answers based on simulation.
- c. Ray Allen's career free thrown percentage is 0.889. At that probability of success, what are his chances of winning the game?
- d. Given that he wins the game, how many shots on average does that take?

- e. Given that he loses the game, how many shots on average does that take?
- f. Ray Allen's career three point shooting percentage is 0.445. Answer questions c-e based on that percentage.
- g. What shooting percentage would you need to break even playing this game?

## 2 Breezes

Another national organization, NOAA, deploys a great number of Buoys that collect data at different locations on oceanographic parameters like wind speed and direction, temperatures, pressures, wave heights etc. The list of buoys is available here: http://www.ndbc.noaa.gov/stndesc.shtml. If you search the Buoy number in the Search window on the upper right end of the screen, you will be given access to data files. We are interested in learning a little bit about the nature of wind direction changes at various locations.

- a. Choose a buoy and find its archived data. Download a dataset from the "continuous winds data" set from a recent year, save it on to your computer, and load it into R. Summarize some information about the buoy. Note that these are large datasets: measurements are taken every 10 minutes throughout the duration of the year.
- b. The wind directions in these datasets are given in degrees from true North. Convert a series of wind directions to categorical variables "North", "East", "South" and "West". FOr example, everything between 45° and 135° will be considered East, up to 215 degrees is South, and so on.
- c. Construct a Markovian transition matrix that models the probabilistic way winds change at your buoy. Comment.
- d. Perform the same operation on data from 1997 (the last major El Niño event). Do the results appear different?
- e. How would you guess inference is done on the estimates parameters of a Markov matrix? How would you suggest testing whether the Markovian model is an appropriate one in the first place?

