The background of the slide features a close-up photograph of a large colony of Steller sea lions (Eumetopias jubatus) resting on a dark, craggy rock face. Many of the seals are a reddish-brown color, while others are dark grey or black. They are piled together in various positions, some facing the camera and others with their backs to it.

# Making Sense of Movement, Part III

## A robust method for identifying behavioral signals in individual animal track data

Eli Gurarie

Quantitative Ecology and Resource Management  
University of Washington - Seattle

April 30, 2008

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- There's lots of data
- There's lots of really interesting questions to ask

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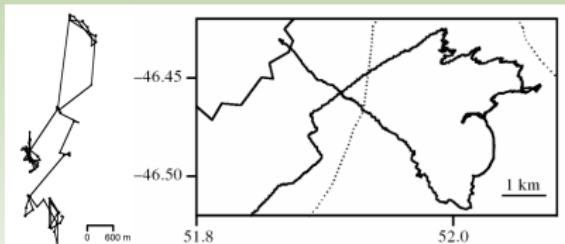
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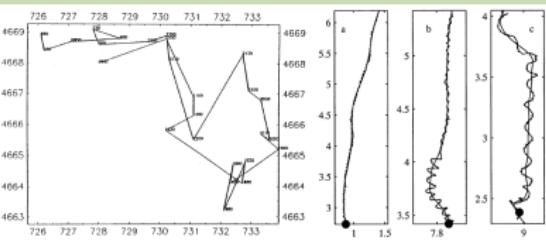
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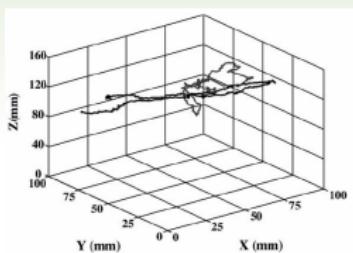
Tree creeper (Doerr 2004)  
*Heterosigma* (Bearon 2003)



Albatross (Fritz 2002)



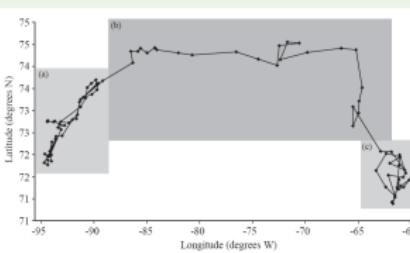
Iberian wolf (Bascompte 1997)



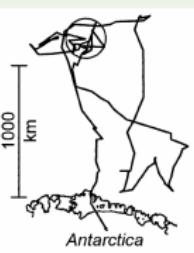
Daphnia Pulex (Uttieri 2005)  
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Cebus monkey (Wentz 2003)

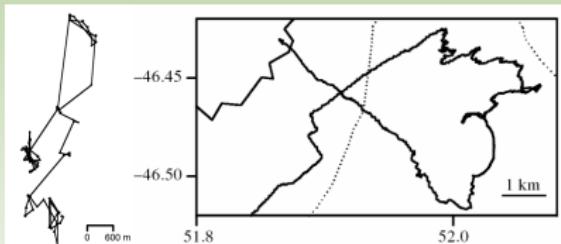


Narwhal (Laidre 2004)



Antarctica

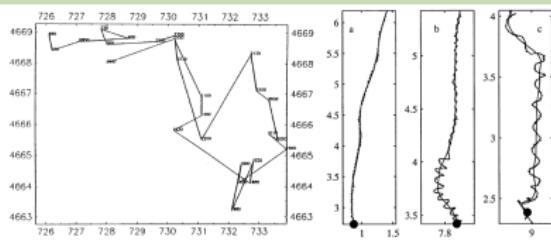
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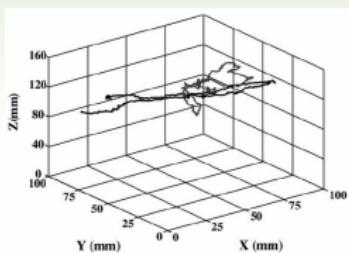
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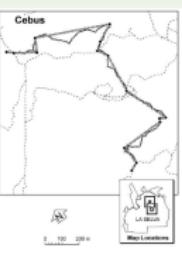
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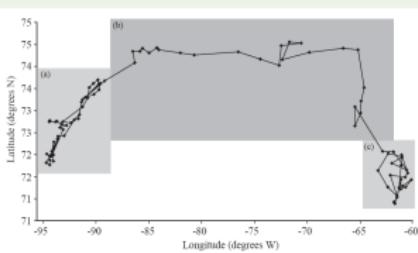
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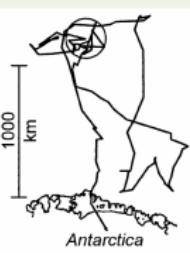
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Antarctica

What kind of information is hiding in this data?

# Common and Somewhat Inconvenient Features of Track Data

- Multi-dimensional
- Auto-correlated
- Error-ridden
- Gappy
- Heterogeneous

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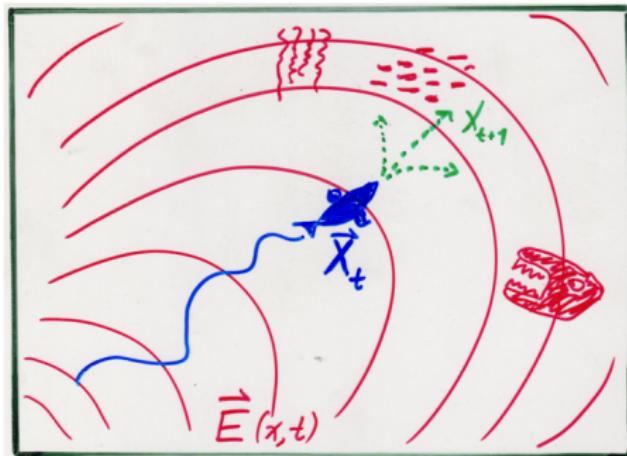
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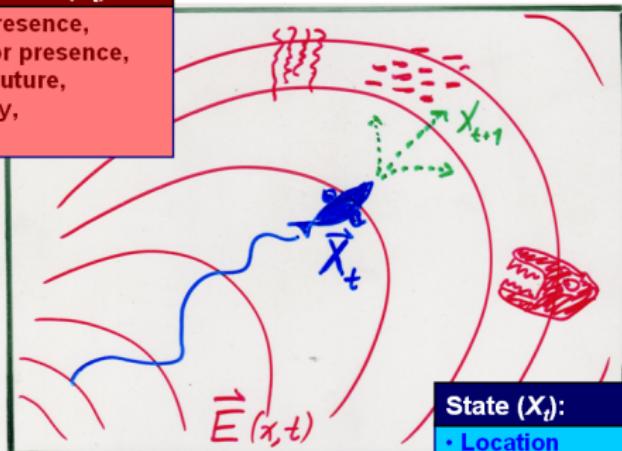
# Conceptual model of Behavior



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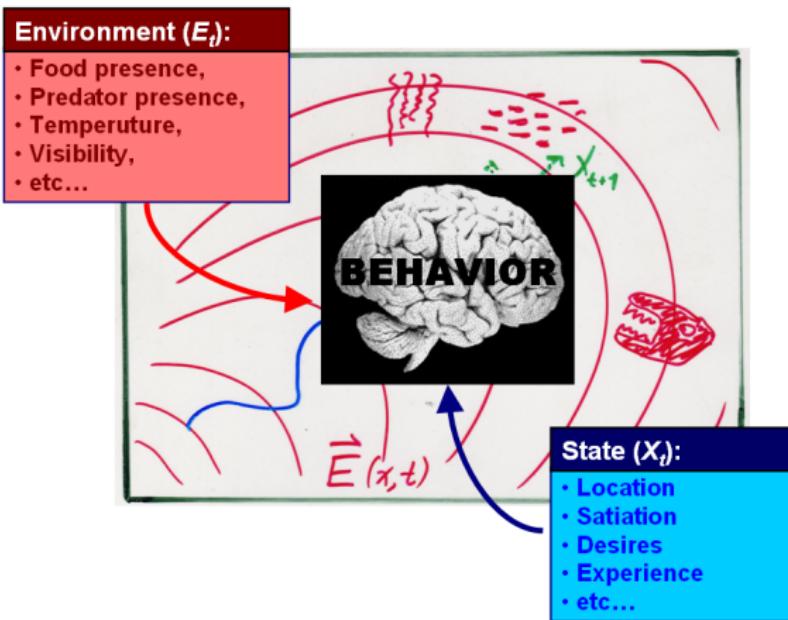
**Environment ( $E_t$ ):**

- Food presence,
- Predator presence,
- Temperature,
- Visibility,
- etc...

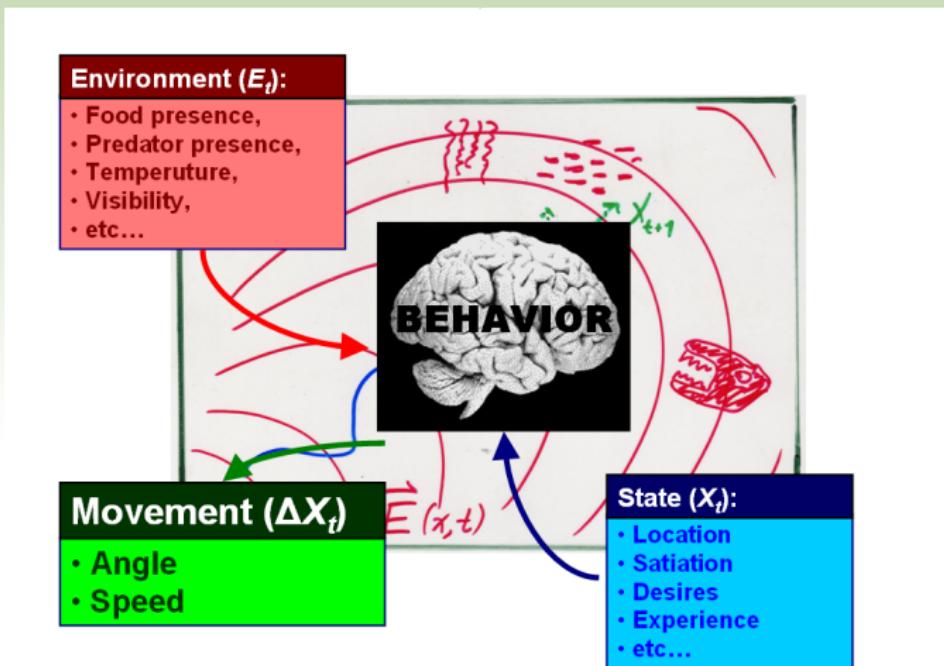
**State ( $X_t$ ):**

- Location
- Satiation
- Desires
- Experience
- etc...

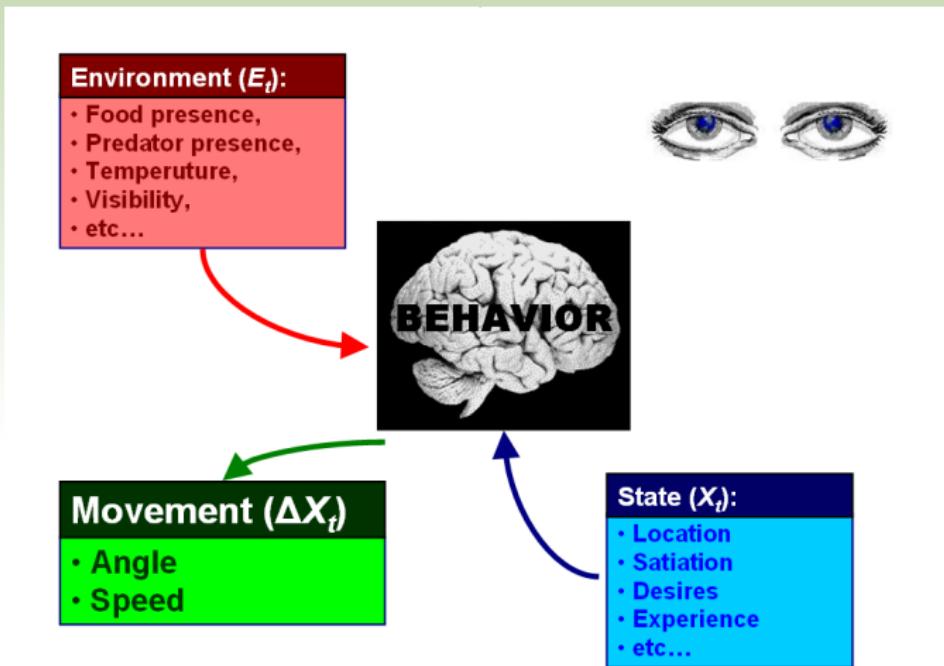
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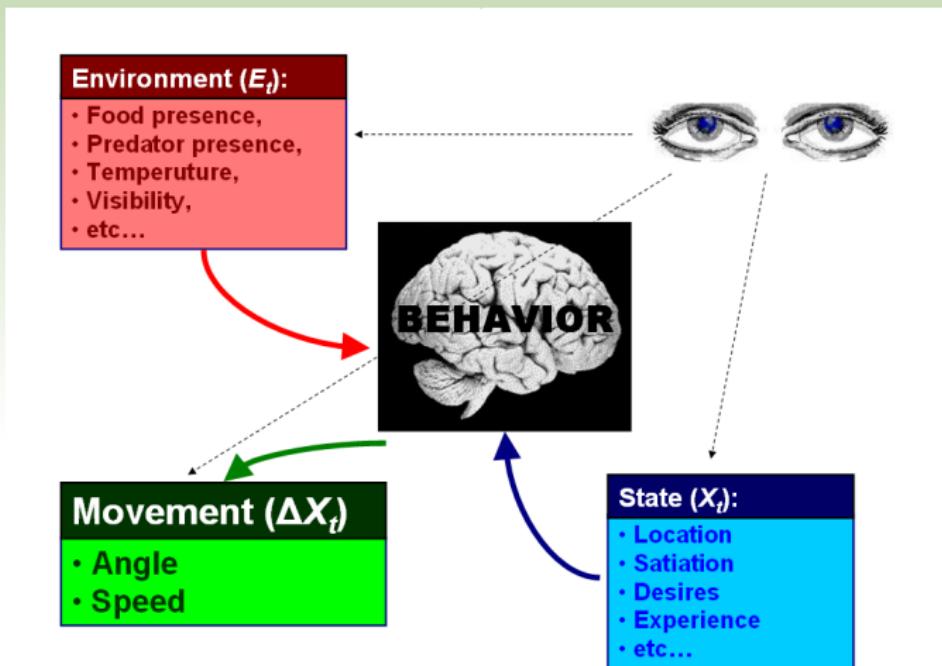
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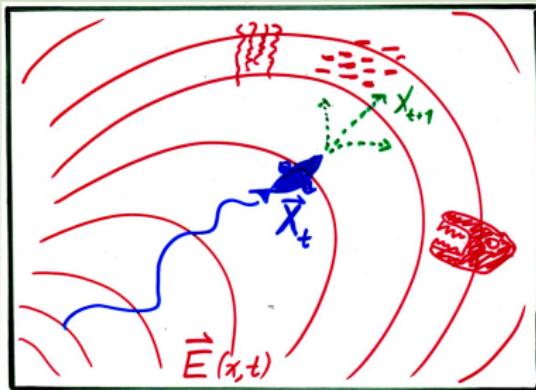
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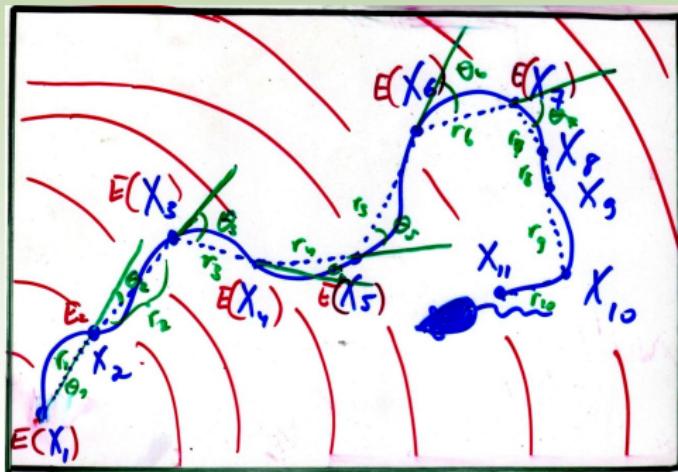
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**In Math:**  $\Delta X_t = f(X_t, E_t)$

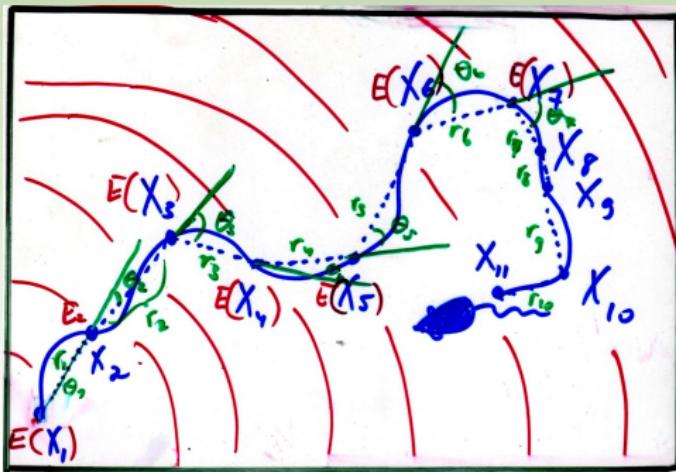
**In English:** Behavior ( $f$ ) is a process which transforms the state of an organism ( $X_t$ ) and the local environment ( $E_t$ ) into Movement ( $\Delta X_t$ ).

# Decomposing Tracks



Strategy:

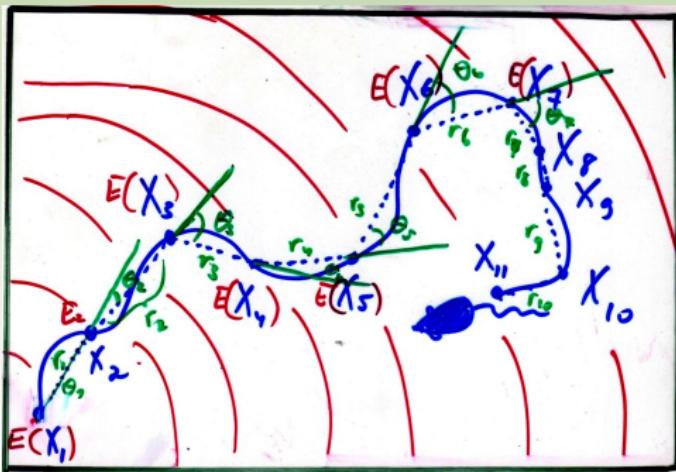
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- Obtain positions  $X_t$  and times of observation  $T_t$

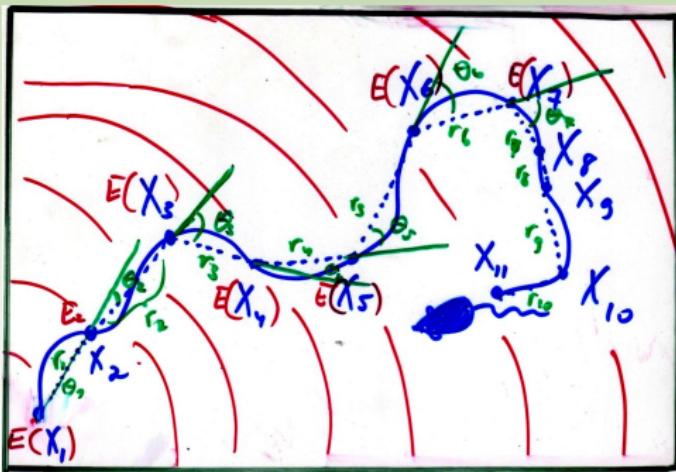
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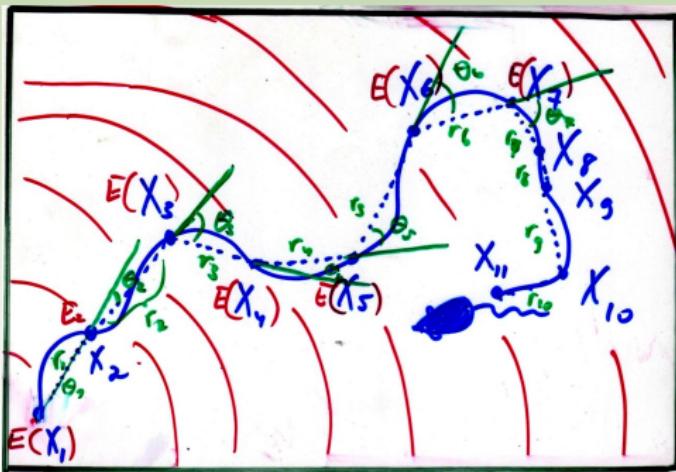
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- Analyze and Interpret!

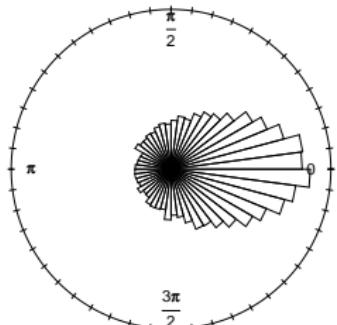
# Correlated Random Walk (CRW) Model

- $\theta_t \sim \text{Some Circular Distribution}$
- $V \sim \text{Some Skewed Unimodal Positive Distribution.}$

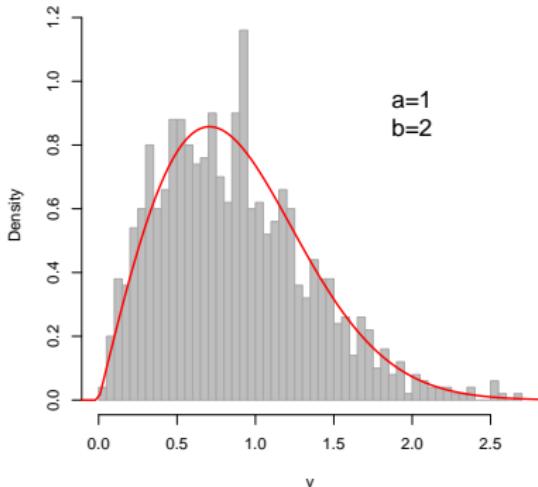
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Wrapped Cauchy  
 $\kappa = 0.6$



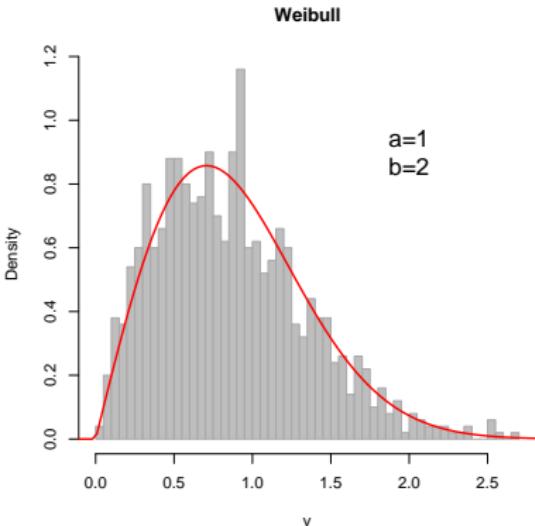
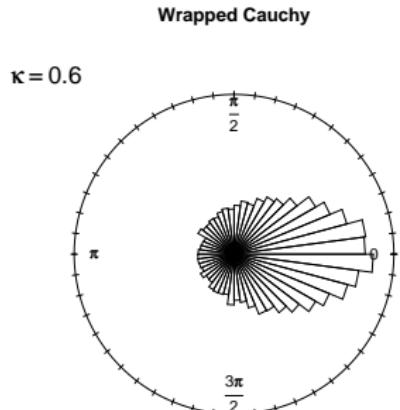
Weibull



BUT ... these distributions depend on the time interval!

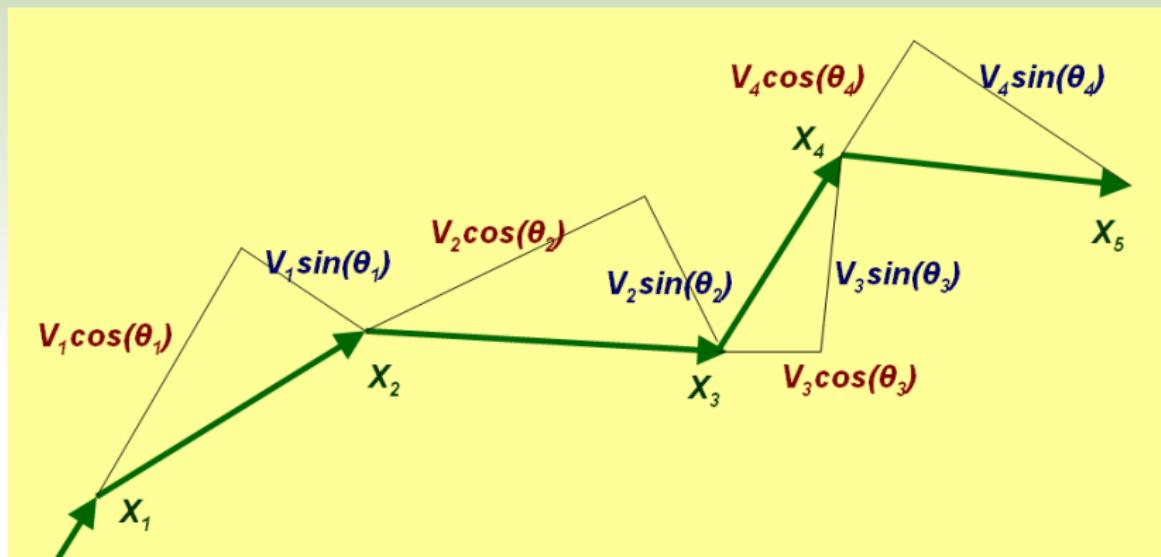
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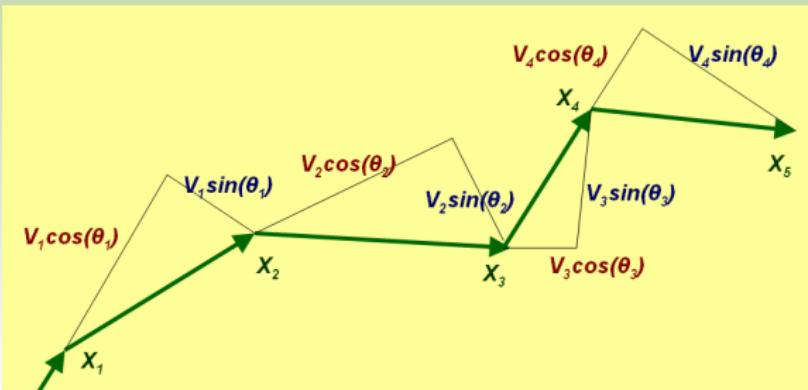


BUT ... these distributions depend on the time interval!  
AND ... this is a slightly narrow “rendering” of correlation

# Orthogonal decomposition



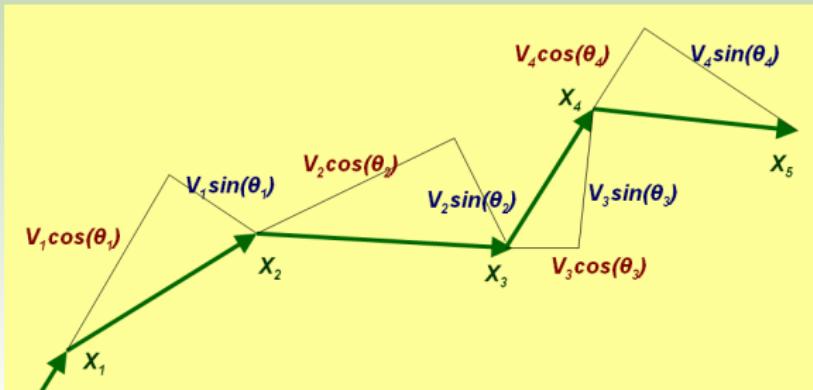
# Orthogonal decomposition



Persistence Velocity Component:  $V_p = V \cos(\theta)$

- Captures tendency and speed of persistence:
  - high **mean** indicates speed and consistent orientation
  - high **variance** indicates variable behaviors (stopping and going, slowing down and speeding)
  - high **auto-correlation** indicates behavioral changes occur more slowly than the sampling interval

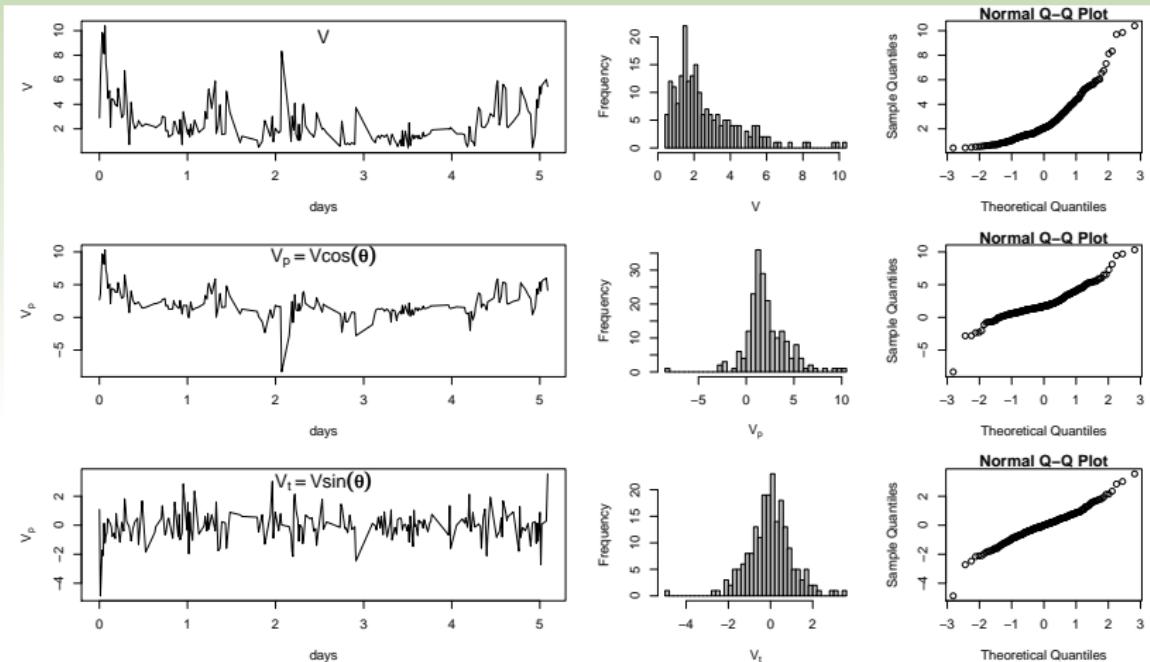
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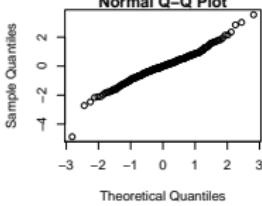
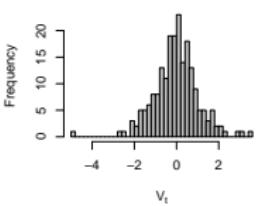
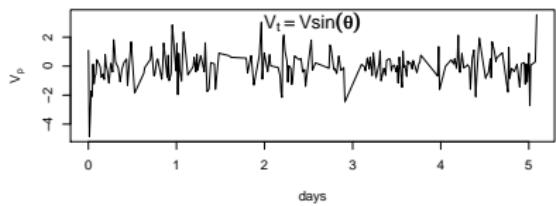
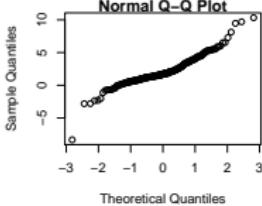
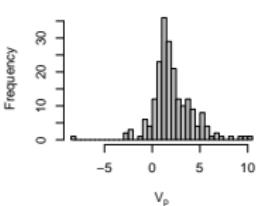
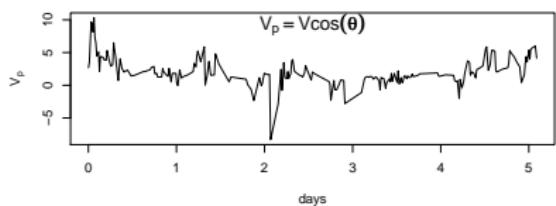
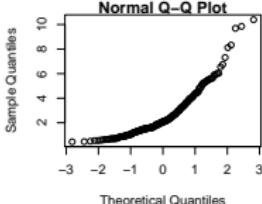
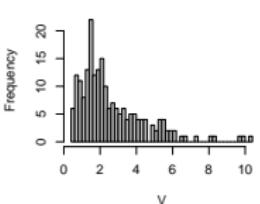
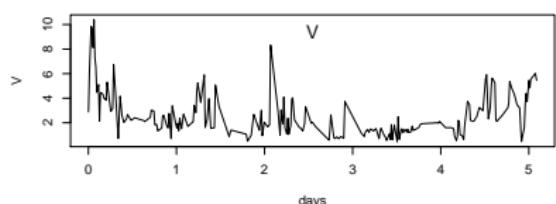


Orthogonal Component of Velocity:  $V_t = V \sin(\theta)$

- Captures tendency and speed of turning (displacement at a perpendicular direction)
  - **mean** can be fixed at zero.
  - high **variance** indicates combination of high speeds and “sharp” turns
  - high **auto-correlation** indicates turning radius larger than the scale of measurement.

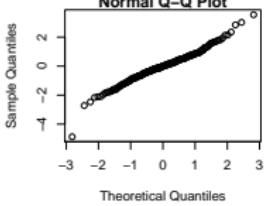
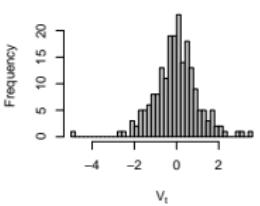
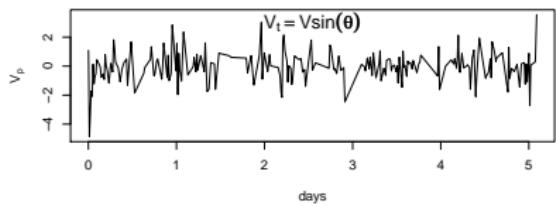
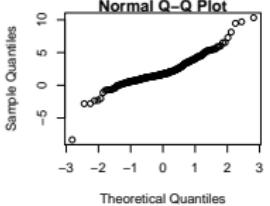
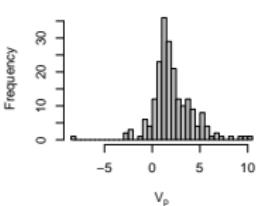
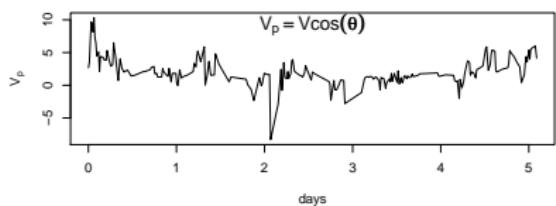
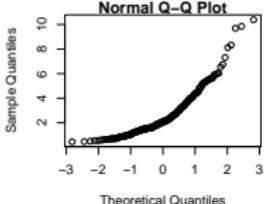
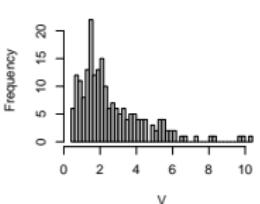
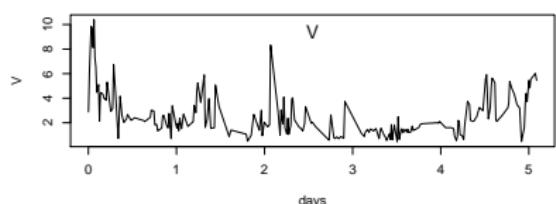
# Actual Data Decomposed (northern fur seal)





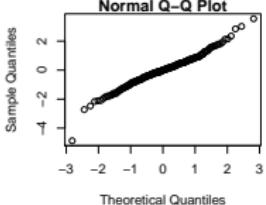
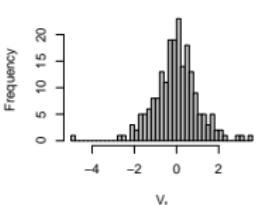
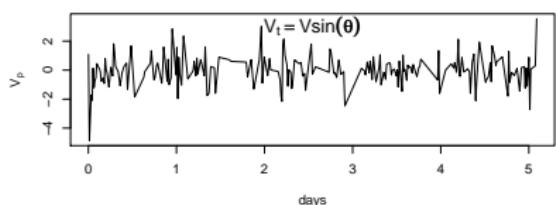
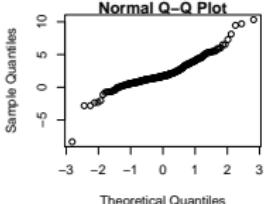
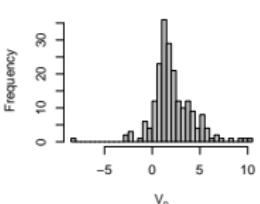
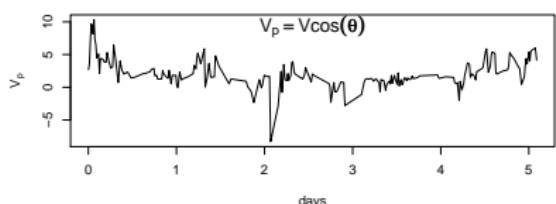
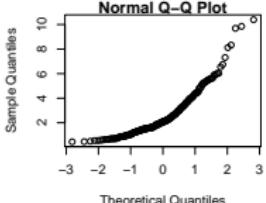
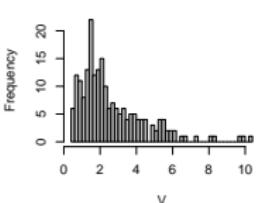
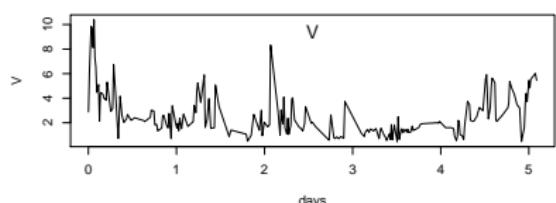
## The real trump card

- Stationary
- Gaussian
- Modellable using standard time-series techniques



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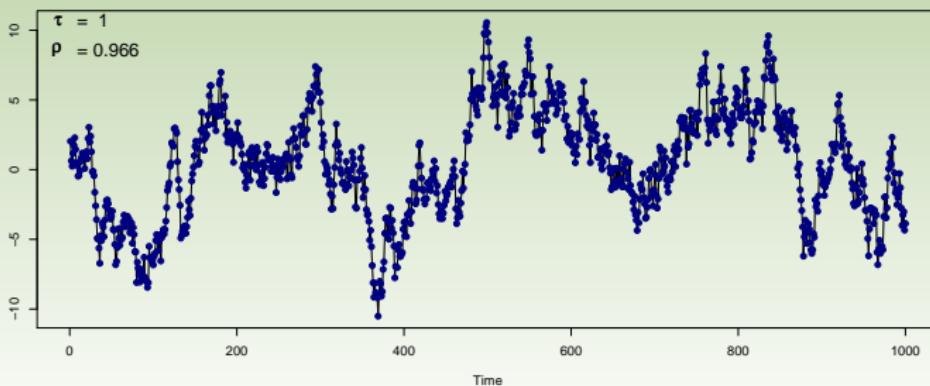
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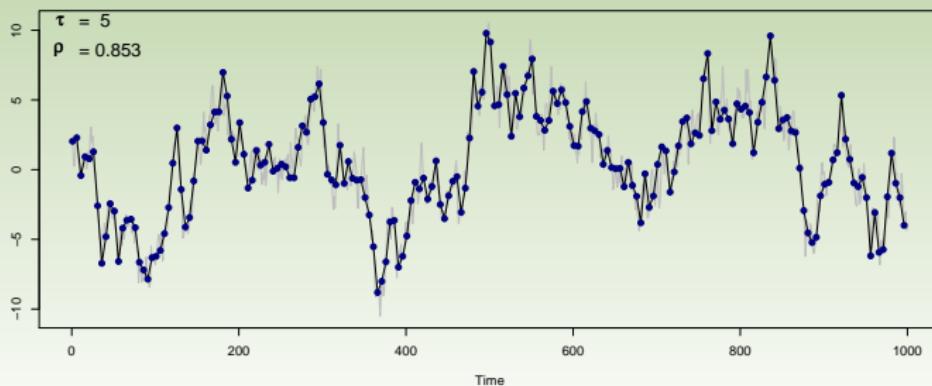
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# Properties of AR(1) time-series



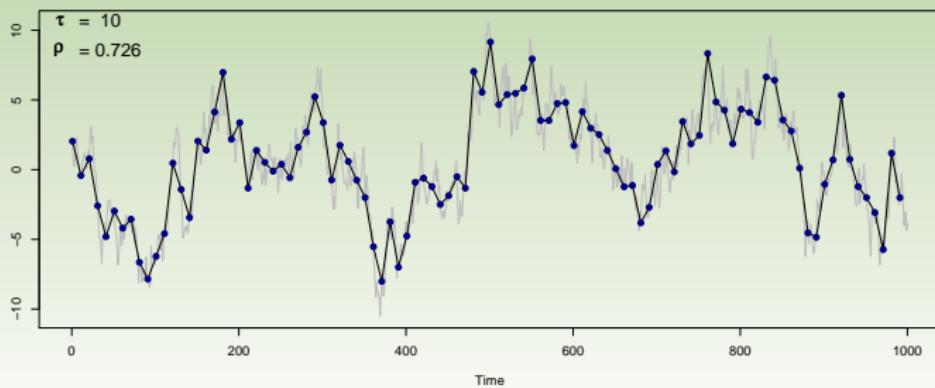
$$\begin{aligned}X_t &= \rho(X_{t-1} - \mu) + \mu + \epsilon \\ \epsilon &\sim N(0, \sigma^2)\end{aligned}$$

# Properties of AR(1) time-series



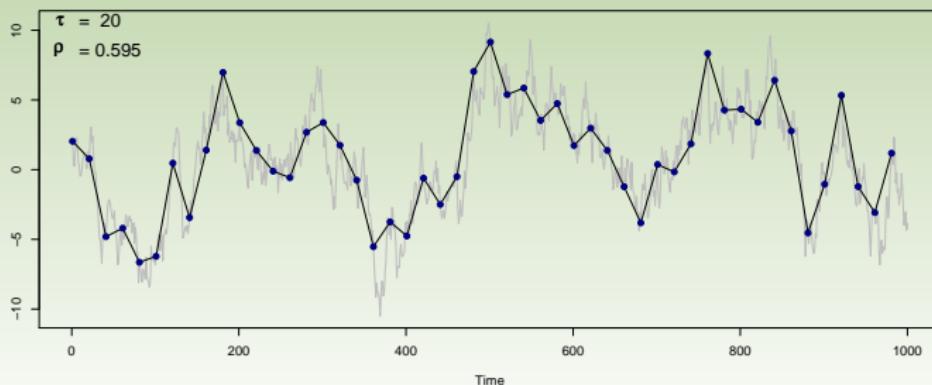
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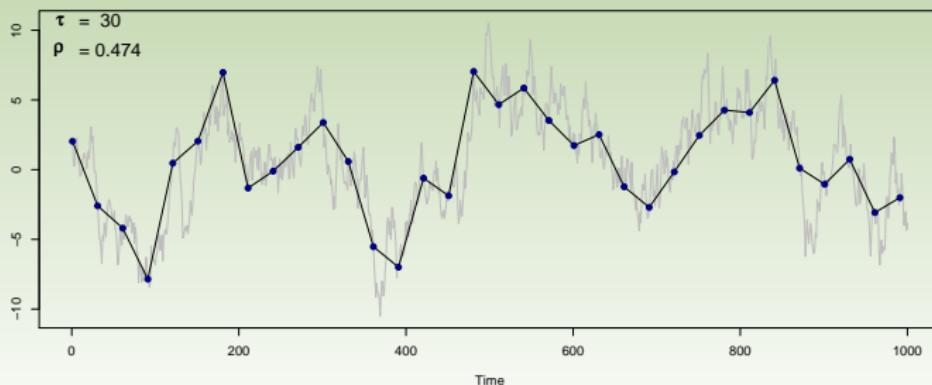
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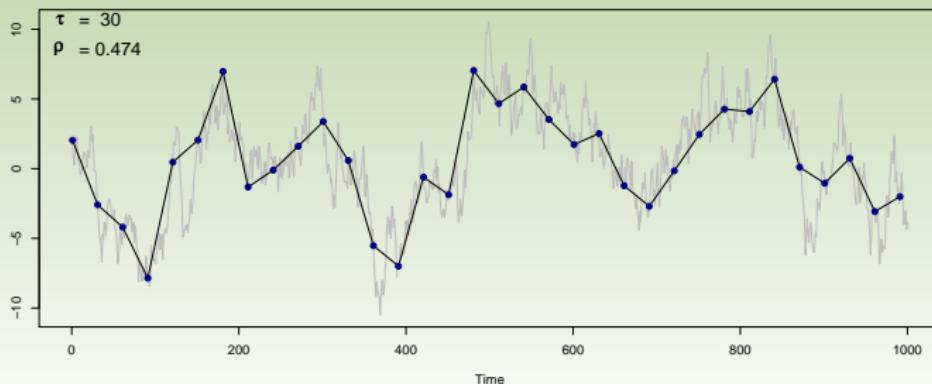
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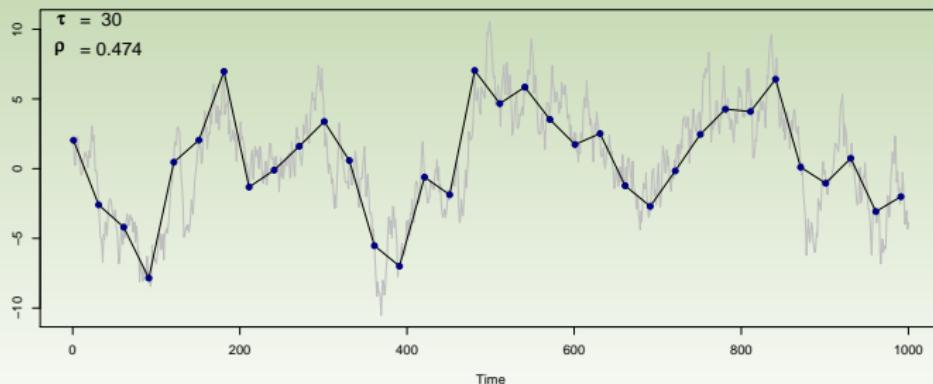
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# Properties of AR(1) time-series



$$\begin{aligned} E[X(t)] &= \mu \\ \text{Var}[X(t)] &= \sigma^2 \\ \text{Corr}[X(t), X(t - \tau)] &= \rho^\tau \end{aligned}$$

# Properties of AR(1) time-series



$$f(X(t)|X(t-\tau)) \sim \text{Gaussian} [\rho^\tau X(t-\tau), \sigma^2(1-\rho^{2\tau})]$$

# Estimating $\rho$

Conditional Likelihood:

$$L(\rho | \mathbf{X}, \mathbf{T}) = \prod_{i=1}^n f(X_i | X_{i-1}, \tau_i, \rho),$$

where:

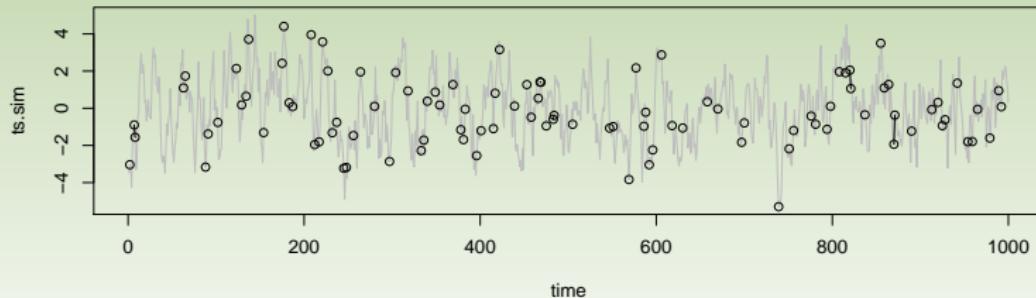
$$f(X_i | X_{i-1}) = \frac{1}{\sigma \sqrt{2\pi(1 - \rho^{2\tau_i})}} \exp \left( \frac{(X_i - \rho^{\tau_i}(X_{i-1} - \mu))^2}{2\sigma^2(1 - \rho^{2\tau_i})} \right).$$

then:

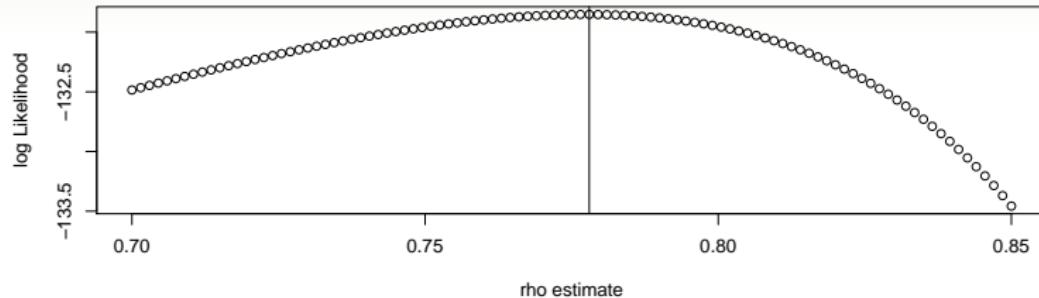
$$\hat{\rho} = \operatorname{argmax}_{\rho} L(\rho | \mathbf{X}, \mathbf{T})$$

# Estimating $\rho$

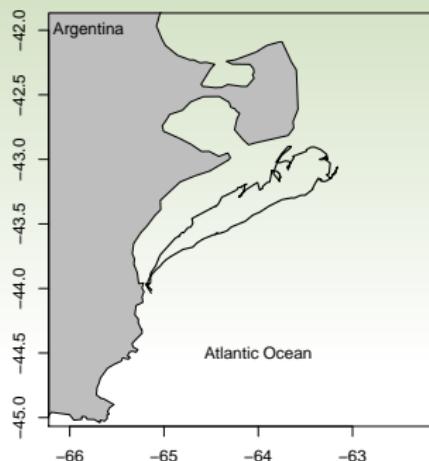
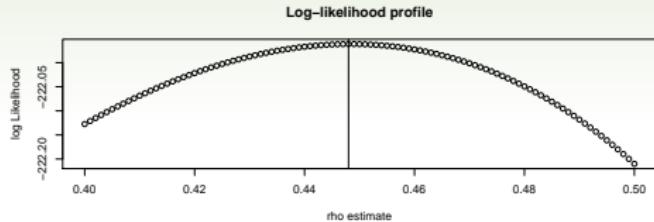
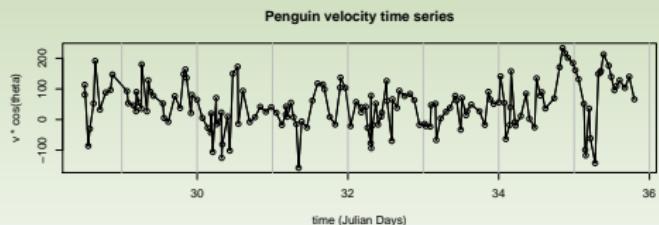
## Simulated Gappy Time Series



Log-likelihood profile



# Estimating $\rho$



# Structural shifts

$$\Theta(t) = \begin{cases} \Theta_1 & \text{if } 0 < t \leq \tau \\ \Theta_2 & \text{if } \tau < t \leq T \end{cases}$$

$$L(\Theta|\mathbf{X}, \mathbf{T}) = \prod_{i=1}^n f(X_i|X_{i-1}, \Theta_1) \prod_{j=n+1}^N f(X_j|X_{j-1}, \Theta_2)$$

All estimates:

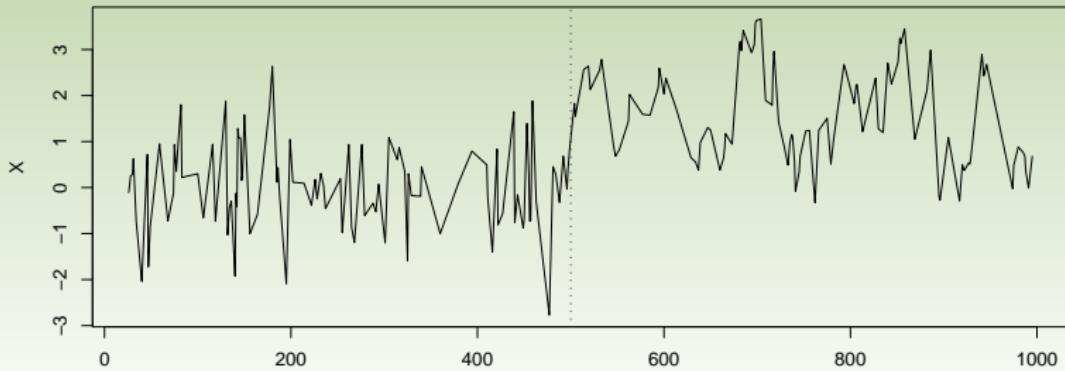
$$\hat{n} = \operatorname{argmax}_n L(\Theta|\mathbf{X}, \mathbf{T})$$

$$\hat{\mu}_j = \bar{X}_j$$

$$\hat{\sigma}_j = S_j$$

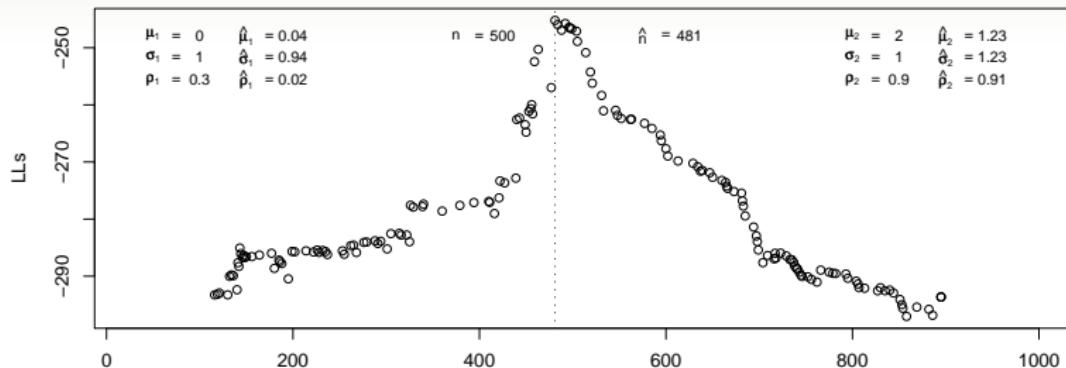
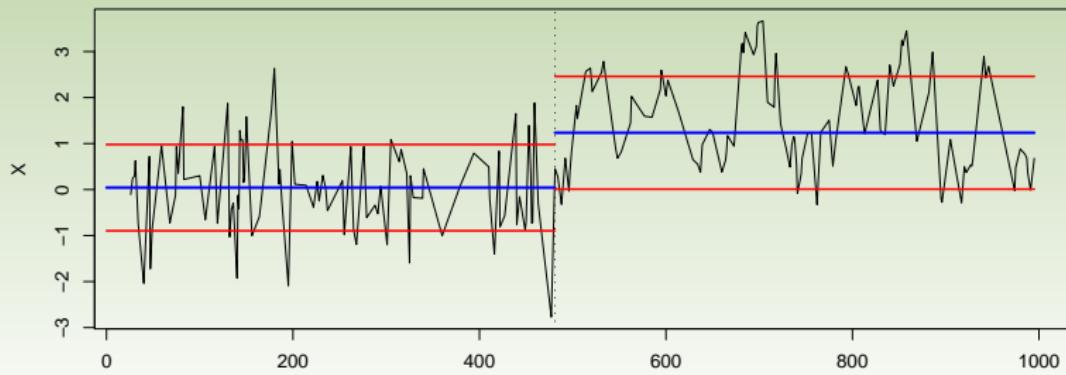
$$\hat{\rho}_j = \operatorname{argmax}_{\rho} L(\rho|\mathbf{X}_j, \mathbf{T}_j, \hat{\mu}_j, \hat{\sigma}_j)$$

# Identifying Change Point

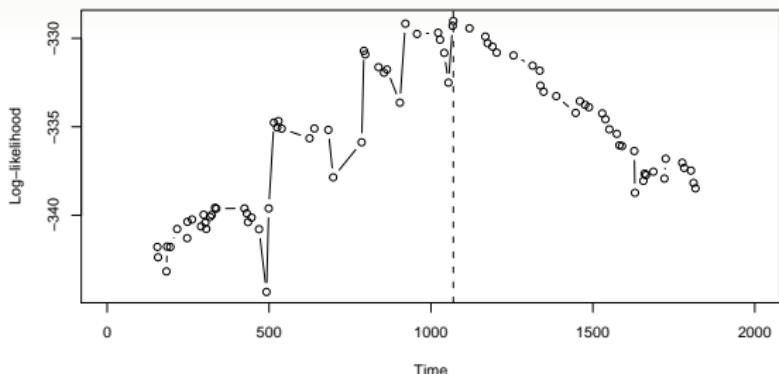
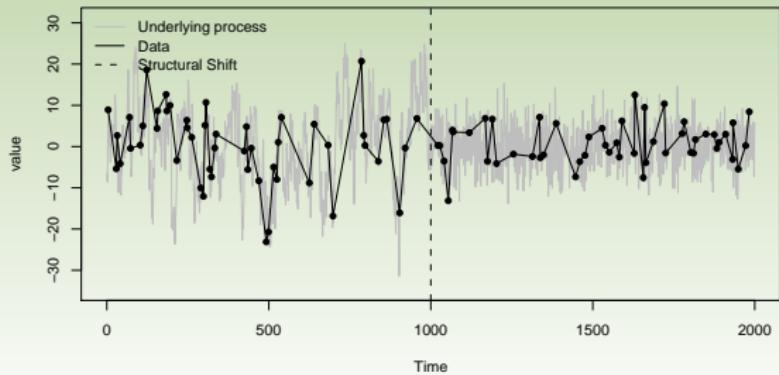


# Identifying Change Point

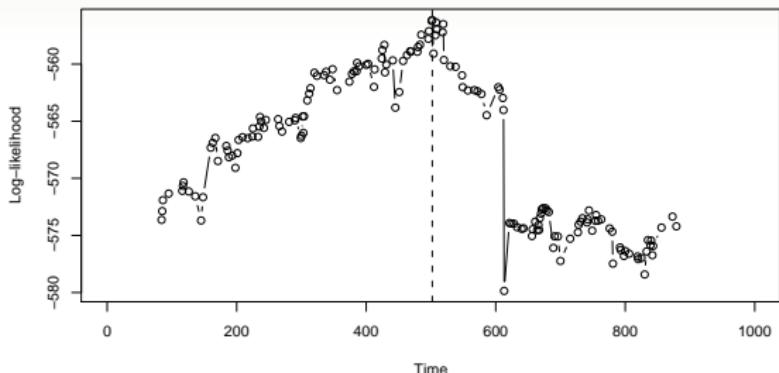
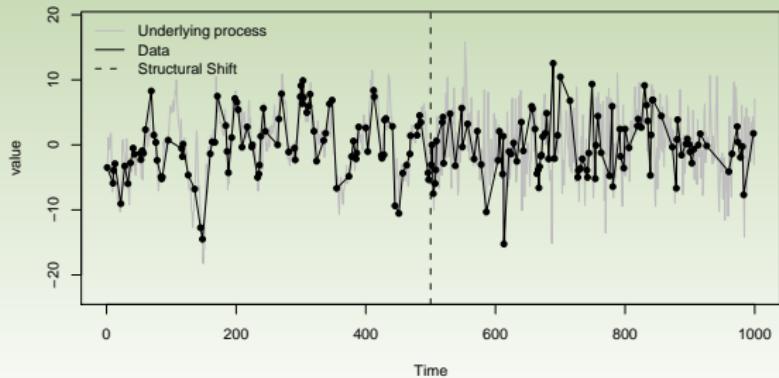
# Identifying Change Point



# Identifying Change Point, sparse data



# Identifying Change Point, different $\rho$ 's



# Identifying Models

Model 0	$\mu_1 = \mu_2$	$\sigma_1 = \sigma_2$	$\rho_1 = \rho_2$
Model 1	$\mu_1 \neq \mu_2$	$\sigma_1 = \sigma_2$	$\rho_1 = \rho_2$
Model 2	$\mu_1 = \mu_2$	$\sigma_1 \neq \sigma_2$	$\rho_1 = \rho_2$
Model 3	$\mu_1 = \mu_2$	$\sigma_1 = \sigma_2$	$\rho_1 \neq \rho_2$
Model 4	$\mu_1 \neq \mu_2$	$\sigma_1 \neq \sigma_2$	$\rho_1 = \rho_2$
Model 5	$\mu_1 \neq \mu_2$	$\sigma_1 = \sigma_2$	$\rho_1 \neq \rho_2$
Model 6	$\mu_1 = \mu_2$	$\sigma_1 \neq \sigma_2$	$\rho_1 \neq \rho_2$
Model 7	$\mu_1 \neq \mu_2$	$\sigma_1 \neq \sigma_2$	$\rho_1 \neq \rho_2$

# Identifying Models

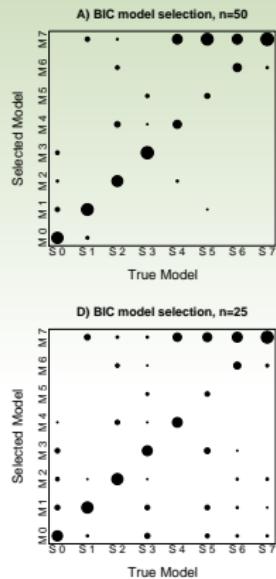
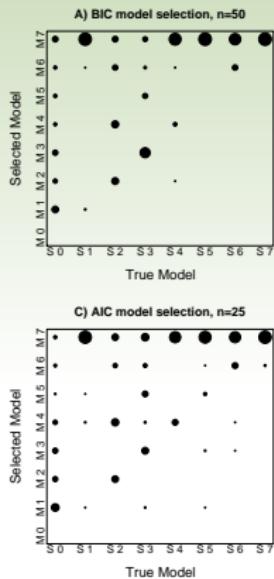
Model 0	$\mu_1 = \mu_2$	$\sigma_1 = \sigma_2$	$\rho_1 = \rho_2$
Model 1	$\mu_1 \neq \mu_2$	$\sigma_1 = \sigma_2$	$\rho_1 = \rho_2$
Model 2	$\mu_1 = \mu_2$	$\sigma_1 \neq \sigma_2$	$\rho_1 = \rho_2$
Model 3	$\mu_1 = \mu_2$	$\sigma_1 = \sigma_2$	$\rho_1 \neq \rho_2$
Model 4	$\mu_1 \neq \mu_2$	$\sigma_1 \neq \sigma_2$	$\rho_1 = \rho_2$
Model 5	$\mu_1 \neq \mu_2$	$\sigma_1 = \sigma_2$	$\rho_1 \neq \rho_2$
Model 6	$\mu_1 = \mu_2$	$\sigma_1 \neq \sigma_2$	$\rho_1 \neq \rho_2$
Model 7	$\mu_1 \neq \mu_2$	$\sigma_1 \neq \sigma_2$	$\rho_1 \neq \rho_2$

How to choose?

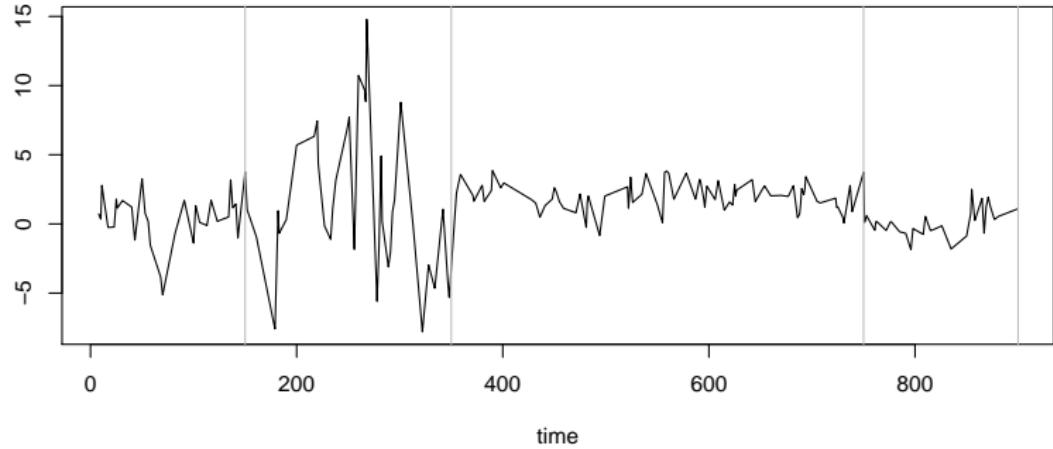
$$\text{AIC} : I_A(\mathbf{X}, \mathbf{T}) = -2n \log(L(\hat{\theta}|\mathbf{X}, \mathbf{T})) + 2d$$

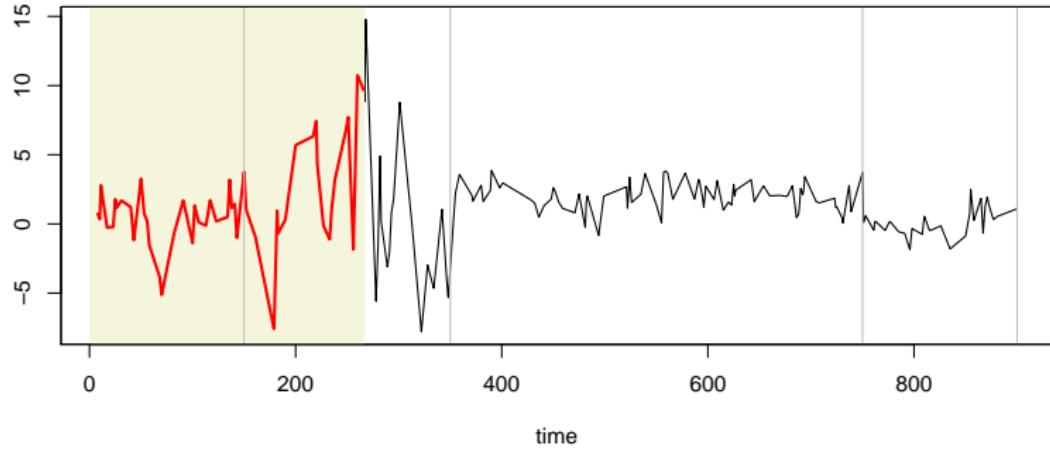
$$\text{BIC} : I_B(\mathbf{X}, \mathbf{T}) = -2n \log(L(\hat{\theta}|\mathbf{X}, \mathbf{T})) + d \log(n)$$

# Identifying Models



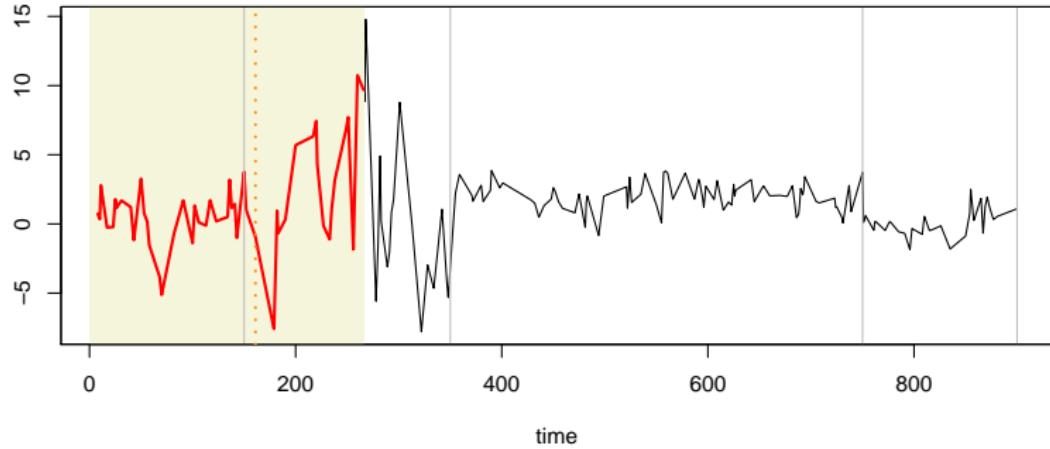
	$\mu_1$	$\mu_2$	$\sigma_1$	$\sigma_2$	$\rho_1$	$\rho_2$
S0	0	0	1	1	0.5	0.5
S1	-1	1	1	1	0.5	0.5
S2	0	0	0.5	2	0.5	0.5
S3	0	0	1	1	0.2	0.9
S4	-1	1	0.5	2	0.5	0.5
S5	-1	1	1	1	0.2	0.9
S6	0	0	0.5	2	0.2	0.9
S7	-1	1	0.5	2	0.2	0.9





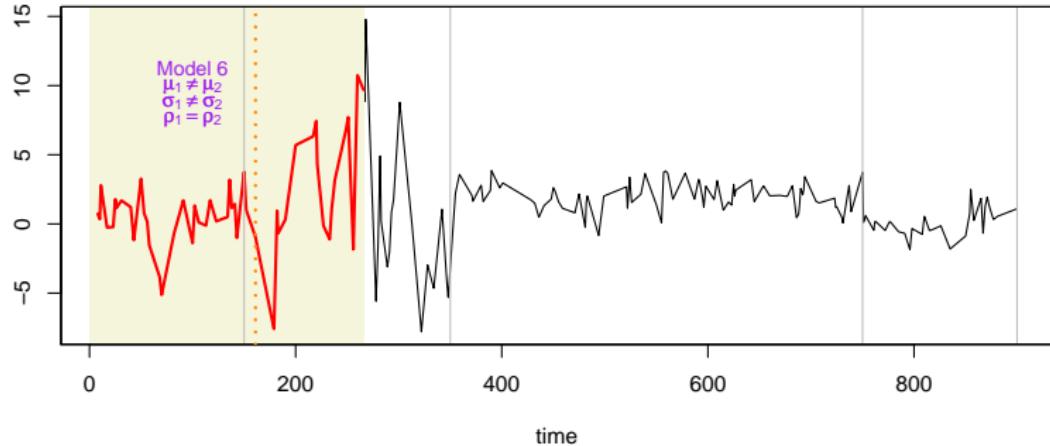
### Algorithm for Identifying Multiple Changepoints

- Select Window
- Find MLBP
- Identify Model
- Record estimates based on model selected.
- Move window forward and repeat



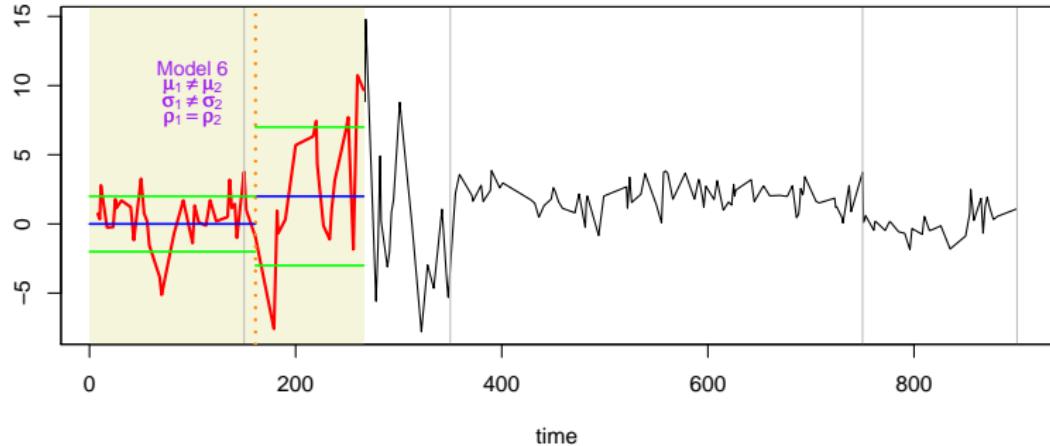
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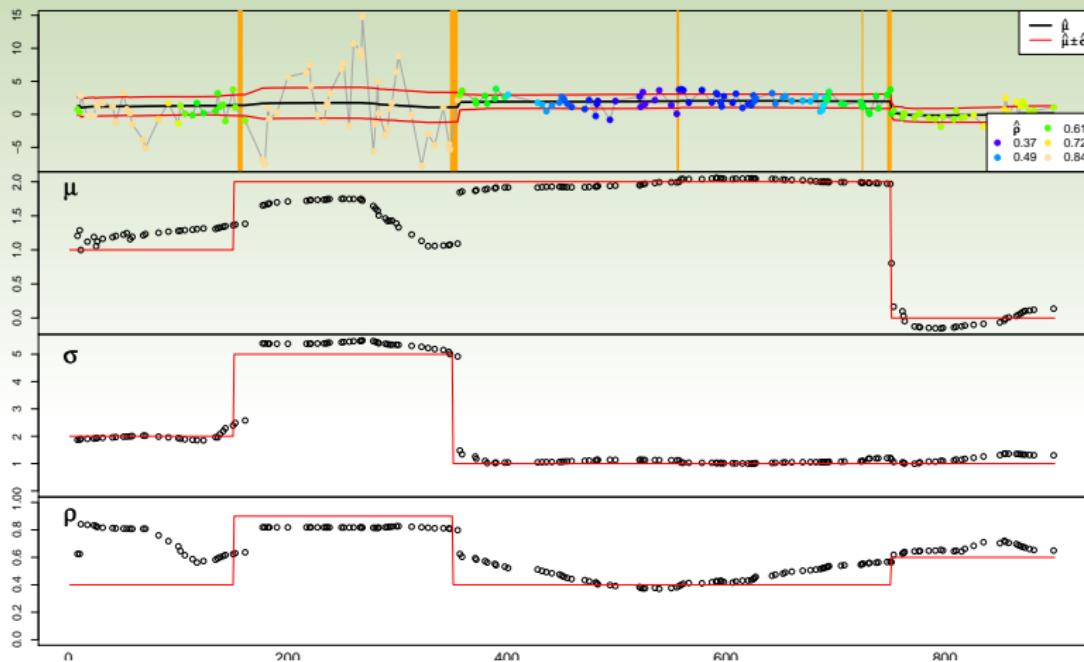
### Algorithm for Identifying Multiple Changepoints

- Select Window
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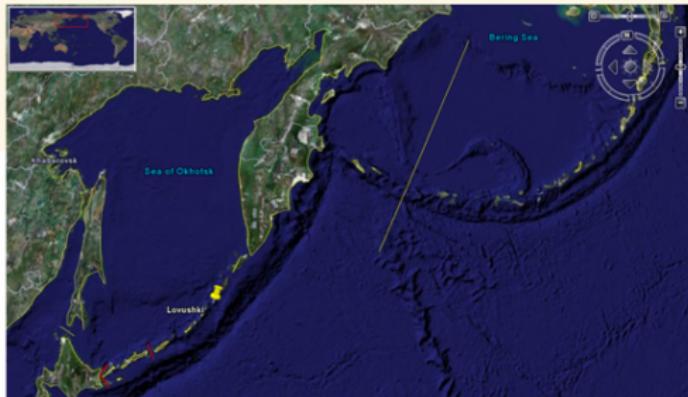
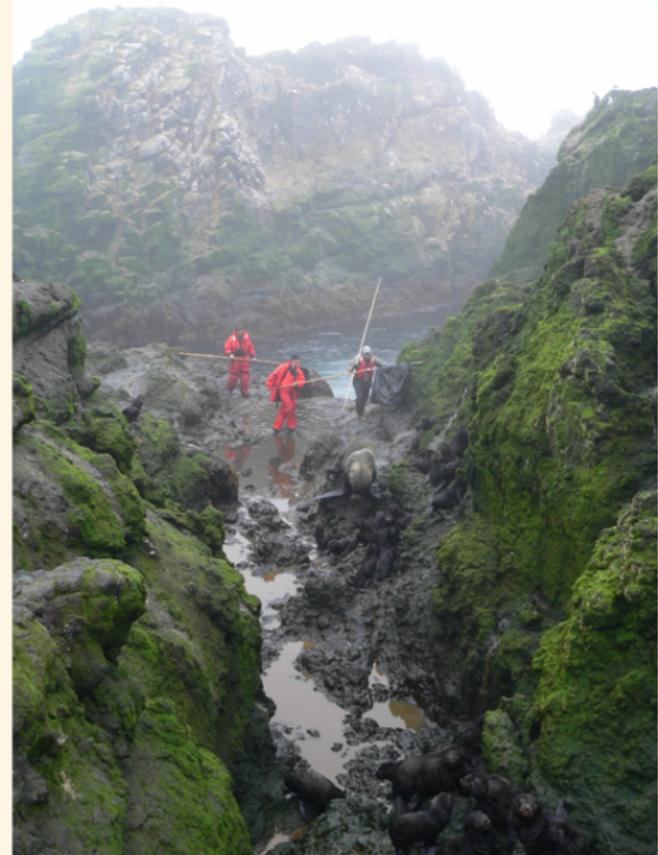
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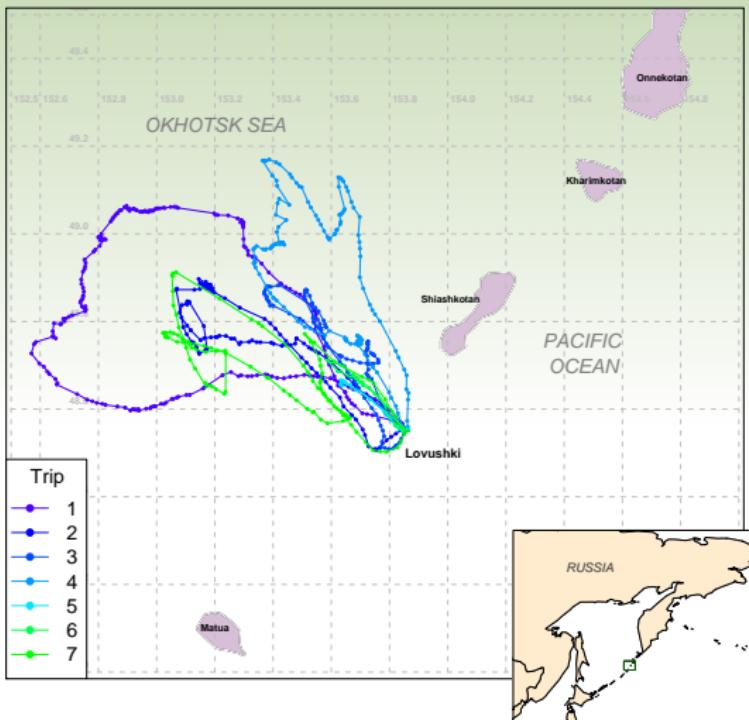
# Movement analysis output



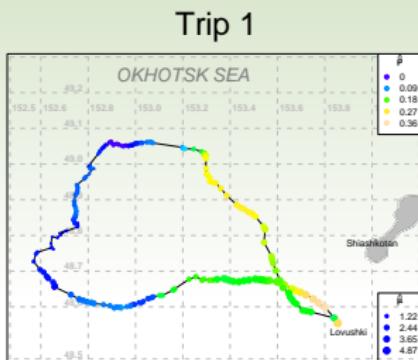
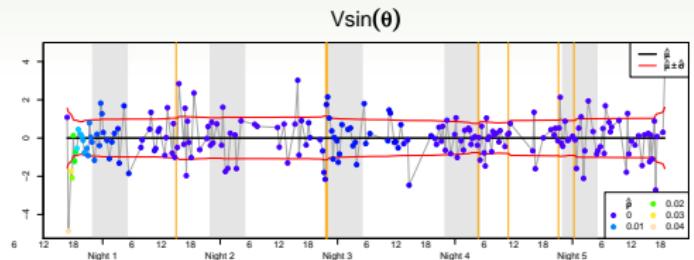
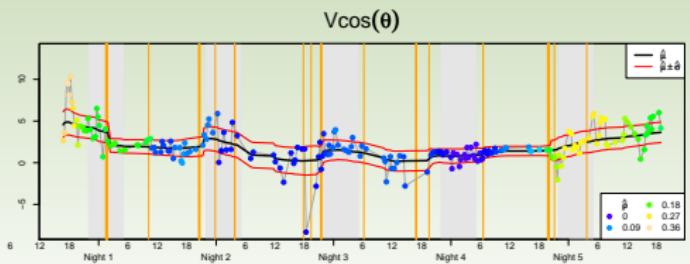
# Northern Fur Seal (*Callorhinus ursinus*)



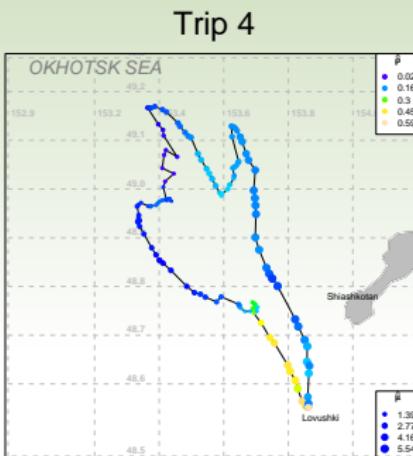
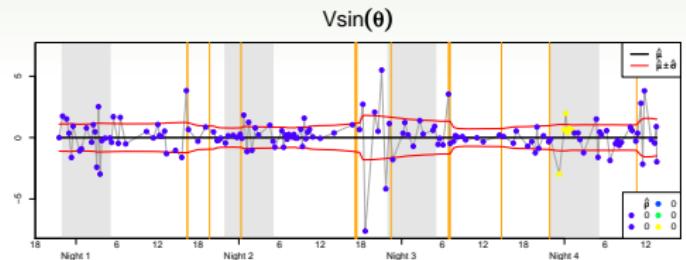
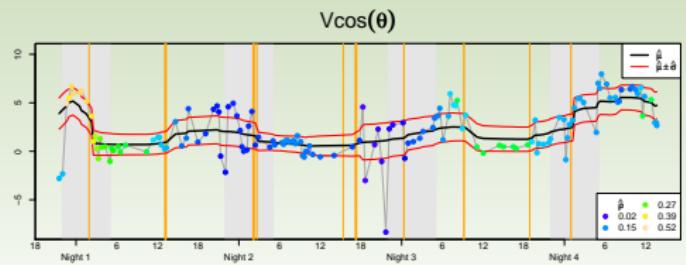
# Map of all tracks



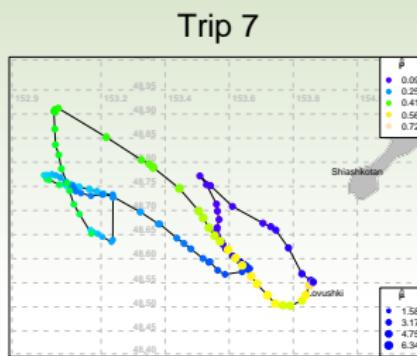
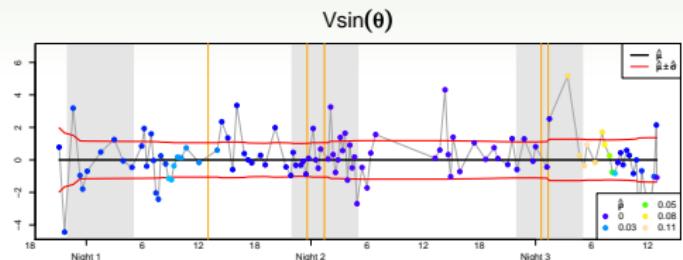
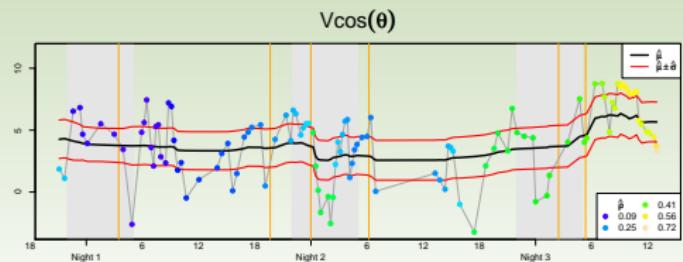
# Track Analysis



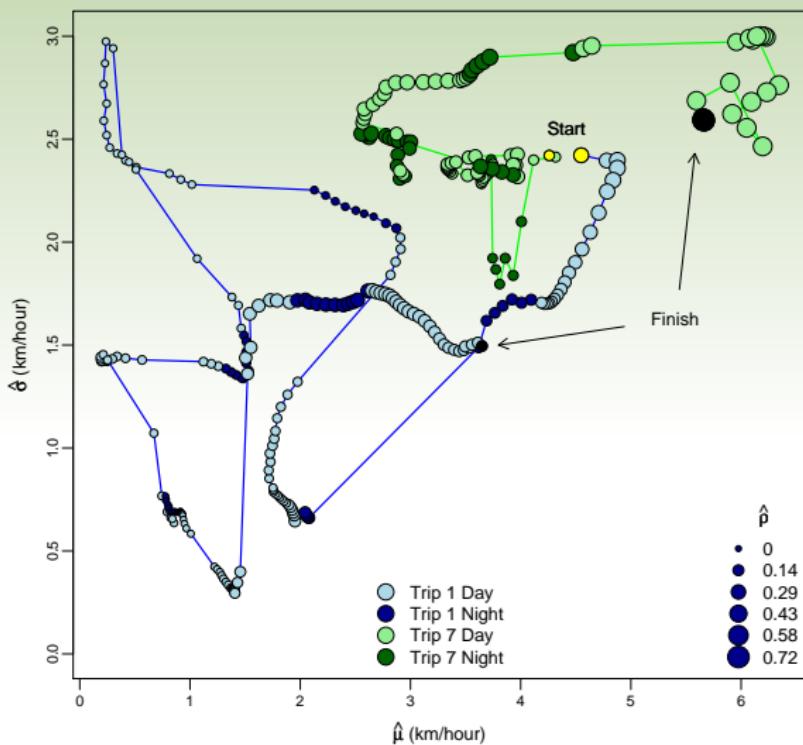
# Track Analysis



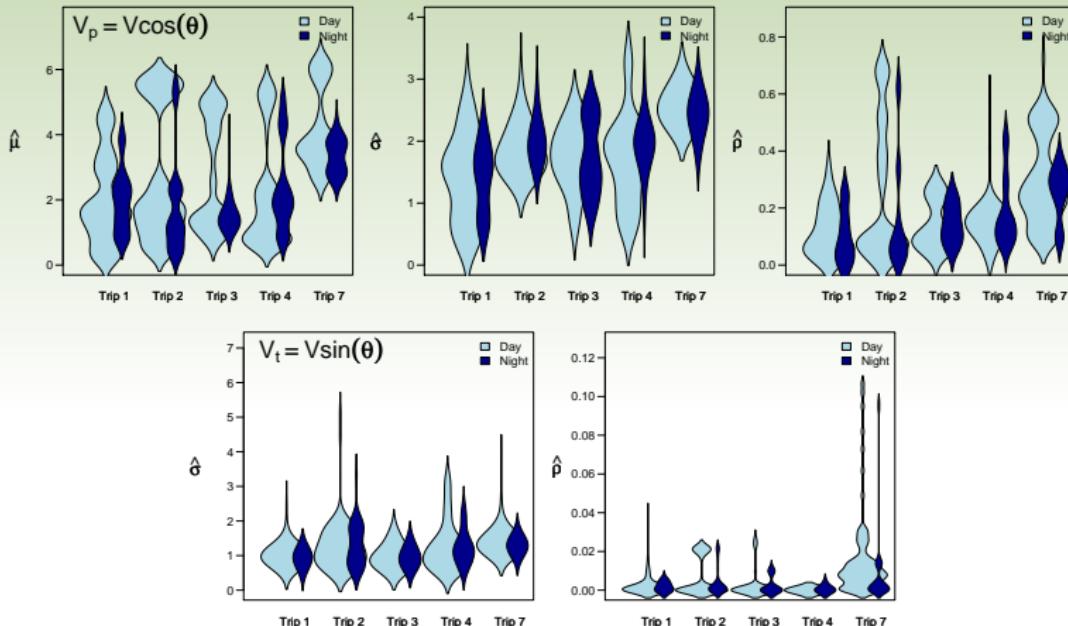
# Track Analysis



# Behavioral Phaseplot



# Violin plots



## Conclusions:

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- Behavior can be very complex!
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A photograph of a man with glasses and a cap, smiling while holding a small, dark-furred fur seal pup. They are in an industrial-looking setting with pipes and metal structures in the background.

## Acknowledgements

### Data:

- Fur Seals: R. Andrews, V. Burkanov, Russian Far East Marine Mammal Project
- Penguin: Elizabeth Skewgar, P. Dee Boersma

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- University of Washington School of Arts and Sciences
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- Colleagues and friends at QERM, SAFS and beyond
- Metapopulation Research Group in Helsinki, Finland