QERM APPLIED QUALIFYING EXAMINATION INSTRUCTIONS
Spring Quarter 2013 (June 17 - 24, 2013)

This exam includes 4 examination questions and 2 data sets (text file attachments have been emailed to you). Please open immediately and confirm by email that you are able to open all attachments.

INSTRUCTIONS:

1. Your unique ID number is: *(emailed to you)*. Please list this ID number on the top of each page.
2. **DO NOT INCLUDE YOUR NAME ON THE EXAM.**
3.
4. Number each page of the exam. Appendices should also be numbered so that any reference to the appendix refers to a particular set of pages.
5.
6. Provide a 1-3 page summary at the beginning of your exam.
7.
8. Your write-up should include a narrative (including any pasted in tables, graphics, etc.) for each question showing the thought process of the analysis, the results at each step, and how that leads to the next step.
9. Clearly describe in your text the points that any diagrams or tables demonstrate.
10. All graphs should be clearly labeled and computer output annotated as need be.
11.
12. Please send an electronic copy of your exam (PDF format) and any appendices, *directly to Joanne* at jbesch@u.washington.edu, **no later than 12:00 p.m. on Monday, June 24th**. She will forward an electronic copy to each exam committee member.
13.

LIBRARY HOURS:

REMEMBER libraries will most likely be CLOSED Saturday and Sunday, so any reference materials you may need should be checked out before then.

QUESTIONS?

Vincent Gallucci (Question #1): By email at vgallucc@u.washington.edu, Office phone: 206-543-1701; Cell Phone: 206-612-4333; Home phone: 206-525-3842. **FYI:** If you don't have any luck reaching Vince on his cell phone feel free to try his home number, office phone or email. He sometimes has reception difficulties with his cell phone.

Vladimir Minin (Question #2): By email at vminin@u.washington.edu. Cell phone: 310-254-6101

Loveday Conquest (Question #3):

- On Monday, Tuesday & Wednesday (June 17-19th) between 8:00 a.m. and 5:00 p.m. email Loveday (conquest@u.washington.edu) or call her office phone (206-543-1708). On those dates after 5:00 p.m. **it is best to reach her at her home number** (206-325-7237) since she does **not** have email at home. She does not have a cell phone.
- Loveday will be out of town Thursday through Monday (June 20 – 24th) and the best way to reach her during that time is by email at conquest@u.washington.edu. (*NOTE: On Thursday, June 20th she will not be checking her email until later that evening*).

Sandor Toth (Question #4): By email at toths@u.washington.edu, office phone: 206-616-2738, cell: 206-518-4978.

If you need to reach Joanne you may email her at jbesch@u.washington.edu, office phone (206-616-9571), or cell phone 206-661-5054.

Good luck!

QUESTION 1.

One of the consequences of a temperature dependent system follows from the fact that infectious disease intensity is also a function of temperature. Therefore with salmonids moving from pool to pool, infection will likely follow. It is not clear how all the physiological responses relate to disease, but for the moment, consider the responses to disease as separate from other responses to temperature increase. This subject remains to be studied. But, our interest here is not the connection but rather how disease spreads in a salmonid population.

Assume that a disease is present in a population of size N(t) and that the salmon population can be divided into three classes:

$S(t), I(t), R(t)$

Where S(t) is the number at time t of susceptibles, I (t) is the number of infected at t that can transmit the disease, and R(t) is the number of recovered at t and immune to further infection.

The disease spreads according to the mass-action law. Therefore each susceptible becomes infected at a rate proportional to the number of infectious, I, present. And each infected recovers at a constant rate ***a*** and becomes infected at a rate ***b***, times the density of both susceptibles and infected, and so on.

a) Write the three dynamical equations for
dS/dt , dI/dt, dR/dt

where, e.g., the first one is:

$dS/dt = - b S I$

b) Show that

$dS/dt + dI/dt + dR/dt = 0$

c) Show analytically that

$S(t) + I(t) + R(t) =$ a constant which was called N (t) above.

Now, consider the system of three simultaneous interacting equations above with the following initial conditions:

$S(0) > 0$; $I(0) > 0$ and $R(0) = 0$

d) Clearly a condition for the spread of infection is that

$$I(t) > I(0),\ t>0$$

Use $d I/dt$ to show that the condition is

$$[bS(0) / a] > 1.$$

e) Show that

$$S(t) = S(0) \exp \{ - (b/a) R(t) \}$$

By solving the differential equation that follows from dividing the differential equation for the rate of change of susceptibles by the rate of change of recovered.

f) Show that the number of infectious satisfies

$$I(t) = N - S(t) + (a/b) \ln (S(t)/S (0)$$

g) It can be shown that $I(t)$ is a function with a maximum. Show that when the max occurs the number of susceptibles is a constant,

$$S = a/b$$

h) Derive the maximum number of infectives (noted above, denoted as I_{max}) . Note that it will be a function of a/b over a restricted range of its possible values.

i) Show that I_{max} is a decreasing function of parameter a/b when $a/b < S(0)$

j) Interpret as best as you can the applicability of these results to a captive population of salmon.

QUESTION 2.

Suppose we are interested in studying movement of chinook salmon in free-flowing segments of the Snake River. It has been observed that salmon tends to stay in the same location for some time before moving further downstream. We model this behavior by a two state discrete-time Markov model X_k with states 0=decision to stay, 1=decision to move one distance unit. For example, transition $0 \to 1$ means that after having made a decision to stay, the fish decided to move. We parameterize the transition probability matrix $\mathbf{P}$ in terms of transition probabilities p_{00} and p_{11}:

$$\mathbf{P} = \begin{pmatrix} p_{00} & 1 - p_{00} \\ 1 - p_{11} & p_{11} \end{pmatrix}.$$

Since we are interested in the behavior of a fish after it moves to a new location, we always assume that $X_0 = 1$.

(a) Let W be the number of decisions (time steps) fish makes to move one unit of distance. For example, if $(X_0, X_1) = (1,1)$, $W = 1$. Similarly, if $(X_0, X_1, X_2, X_3, X_4) = (1, 0, 0, 0, 1)$ then $W = 4$. Prove that

$$\mathrm{E}(W \mid X_0 = 1) = p_{11} + \frac{1 - p_{11}}{1 - p_{00}} \times (2 - p_{00}).$$

There are multiple ways to show this, with all solutions being not very difficult or long, but do make sure that all steps are carefully explained.

(b) Write a computer program to simulate realizations of the random variable W. For at least two sets of values for p_{00} and p_{11}, generate 1000 realizations of W, plot the corresponding histograms and confirm that the empirical means match the theoretical means of W derived in part (a).

(c) Assume p_{11} is known. Suppose we collect independent and identically independent observations $w_1, \ldots, w_n$ by radio tagging n chinook salmon. The method of moments prescribes estimating p_{00} by matching the empirical expectation of the data, $\bar{w} = (\sum w_i)/n$, and the theoretical expectation that depends on p_{00} (and the known constant p_{11}). Derive the method of moments estimator of p_{00}.

(d) Using $p_{00} = 0.84$ and $p_{11} = 0.88$, simulate behavior of 150 chinook salmon and record their corresponding numbers of decisions taken to move one distance unit: $w_1, \ldots, w_{150}$. Holding p_{11} fixed, estimate p_{00} from your simulated data.

(e) Using Monte Carlo, estimate the variance of your estimator $\hat{p}_{00}$.

QUESTION 3.

Assessing the distribution of salmon during the "holding stage" in a river, with respect to river habitat characteristics. (Note: During the holding stage, salmon stay put and do not move around.)

Investigators anticipate that salmon may respond to a combination of river habitat characteristics. They carried out the following study:

At each river kilometer (rkm), fish counts were taken. (We have to assume that all fish were counted.) In addition to the fish counts, other variables included: temperature (Celsius, cooler is better for salmon), the number of pools associated with that rkm (more pools are better), width-to-depth ratio (a lower ratio is better), gradient (a shallower gradient generally means more salmon), and pools volume (more volume is better).

In order to characterize the patchy spatial environment, variables were also recorded as being in a "state favorable to salmon" (coded with a 1) or in a "state unfavorable to salmon" (coded with a 0). Favorable states are defined as follows: temperature < 24 degrees; number of pools > 2.2; width-to-depth ratio < 29.9; gradient < 0.59; pools volume > 316.1. The data set shows the stream characteristics with their actual measurements (temp.a, pool.a, wd.a, grad.a, pvol.a). These columns are followed by the same stream characteristics being coded as being in an "unfavorable state" [0] or a "favorable state: [1]. The response variable, "fish", is the count of fish at that particular rkm location. For purposes of the analysis, assume all the data are independent. Use significance level alpha = 0.10.

a. Conduct an analysis and find the best model you can that relates the response variable y=salmon counts to the habitat characteristics. Justify your choice of model. The predictor variables come in two forms: [1] the ones that are measured (or counted, in the case of pools), or [2] the 0/1 coded predictors that simply indicate "favorable" or "unfavorable" states for salmon. To keep things from getting too complicated, it is *not* necessary to try to "mix" the predictor variables from the two sets.

b. Interpret your final model by putting an interpretation on each coefficient (including the intercept if there is one) that ends up in your final model.

QUESTION 4.

Chinook salmon (*Oncorhynchus tshawytscha*) is very sensitive to stream temperatures. Being a cold-water species, Chinook faces a challenging future due to habitat degradation, dams, water withdrawal and climate change. Since *anadromous* fish (fish that live in the sea but migrate to fresh water to breed), such as the Chinook, need to travel between habitats at different life stages, it is important to consider if there are any obstacles to these movements. One critical obstacle for Chinook is streches of fresh water that are too hot (>= 24 centigrades) for long-term survival. Suppose that you were asked by a conservation organization to help them identify land parcels along a river in Oregon that, if purchased and/or restored, can provide temperature refuges for the salmon to travel through to reach the breeding grounds. You classified each parcel based on whether purchasing them for conservation alone (and letting the vegetation grow) would be sufficient to reduce stream temperatures ("adequate if protected"), or if restoration action(s) are also needed ("restorable"), or if the parcel was "unrestorable" (See Table 1. below). Suppose further that the Chinook salmon cannot travel more than 500 meters in "hot" water. In other words, stretches of hot water longer than 500 m must be interrupted by restored, cool or protected "adequate if protected" segments/parcels. What is the minimum cost of a network of parcels/segments that would allow the salmon to travel to all 4 reaches of the river (see Fig. 1 and Table 1 below). Use mixed integer programming as a planning tool. Clearly list and define the parameters, decision and auxiliary or indicator variables that you used. Please also explain the objective function, the constraints, and any pre-processing algorithms that you used. Lastly, please note that there are 5 parcels that contain 2 disjoint segments: 2, 15, 17, 22 and 29. The associated pairs of segments (2.1 & 2.2, 15.1 & 15.2, etc.) are listed as separate entries in Table 1. Note that if one of the segments in a pair is selected for purchase, and possibly for restoration, the other segment must also be selected since they belong to the same parcel. One can only buy parcels (and not segments). Parcel/segment 1 is in the estuary.

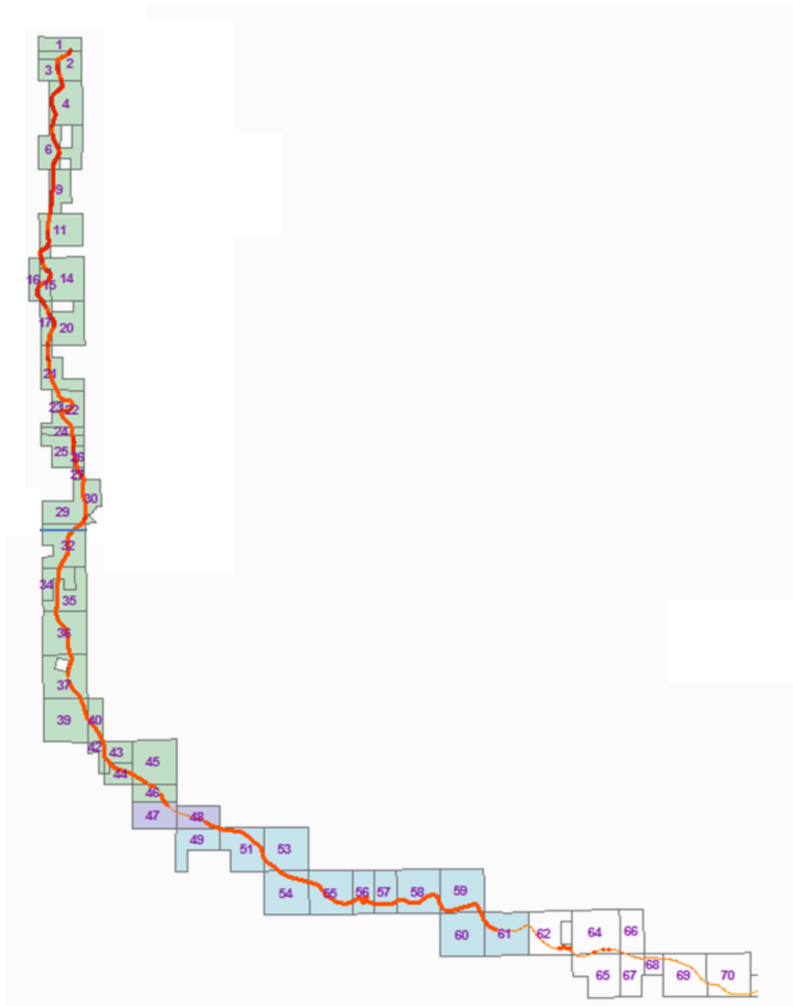

Fig. 1. The spatial configuration of parcels and river segments

Parcel no.	Parcel ID	Purchase price ($)	Restoration cost ($)	Restoration status	Length of river segments that are too hot for salmon (m)	Reach	Temperature category
1	1	140,120	7,061	Restorable	32	1	Hot
2	2.1	150,488	N/A	Adequate if protected	491	1	Hot
3	3	0	165,715	Restorable	744	1	Hot
4	2.2	32,842	N/A	Adequate if protected	107	1	Hot
5	4	335,558	N/A	Adequate if protected	1,853	1	Hot
6	6	330,713	386,609	Restorable	1,737	1	Hot
7	9	0	364,896	Restorable	1,639	1	Hot
8	11	358,379	N/A	Adequate if protected	1,765	1	Hot
9	14	0	N/A	Unrestorable	432	1	Hot
10	15.1	71,887	205,015	Restorable	921	1	Hot
11	16	111,143	N/A	Adequate if protected	513	1	Hot
12	15.2	11,864	N/A	Unrestorable	152	1	Hot
13	17.1	63,473	147,802	Restorable	664	1	Hot
14	20	0	133,386	Restorable	599	1	Hot
15	17.2	49,323	114,853	Restorable	516	1	Hot
16	21	252,722	N/A	Adequate if protected	1,721	1	Hot
17	22.1	174,262	N/A	Adequate if protected	1,389	1	Hot
18	23	28,177	N/A	Unrestorable	125	1	Hot
19	22.2	82,615	146,560	Restorable	658	1	Hot
20	24	76,965	N/A	Unrestorable	295	1	Hot
21	25	220,664	129,116	Restorable	580	1	Hot
22	26	0	149,892	Restorable	673	1	Hot
23	27	27,564	N/A	Adequate if protected	481	1	Hot
24	29.1	218,541	N/A	Adequate if protected	799	1	Hot
25	30	132,500	150,233	Restorable	675	1	Hot
26	29.2	54,634	N/A	Unrestorable	200	1	Hot
27	32	412,431	432,665	Restorable	1,944	1	Hot
28	34	164,743	N/A	Adequate if protected	416	1	Hot
29	35	0	274,219	Restorable	1,232	1	Hot
30	36	0	385,974	Restorable	1,734	1	Hot
31	37	0	345,205	Restorable	1,551	1	Hot
32	39	0	171,655	Restorable	771	1	Hot
33	40	0	219,605	Restorable	987	1	Hot
34	42	0	13,850	Restorable	62	1	Hot
35	43	0	180,475	Restorable	811	1	Hot
36	44	126,270	204,812	Restorable	920	1	Hot
37	45	449,585	N/A	Adequate if protected	601	1	Hot
38	46	177,476	N/A	Adequate if protected	498	1	Hot
39	47	267,001	N/A	Adequate if protected	141	1	Hot
40	47	0	0	Safe - no action needed	428	2	Cool
41	48	0	0	Safe - no action needed	1,060	2	Cool
42	48	224,399	N/A	Adequate if protected	575	3	Hot
43	49	279,922	N/A	Adequate if protected	28	3	Hot
44	51	389,219	468,094	Restorable	2,103	3	Hot
45	53	450,333	172,761	Restorable	776	3	Hot
46	54	452,479	N/A	Adequate if protected	1,172	3	Hot
47	55	440,870	N/A	Adequate if protected	1,903	3	Hot
48	56	215,999	N/A	Adequate if protected	1,264	3	Hot
49	57	224,315	187,844	Restorable	844	3	Hot
50	58	439,272	428,682	Restorable	1,926	3	Hot
51	59	440,540	N/A	Adequate if protected	1,754	3	Hot
52	60	444,926	N/A	Unrestorable	289	3	Hot
53	61	434,864	120,555	Restorable	542	3	Hot
54	61	0	0	Safe - no action needed	0	4	Cool
55	62	0	0	Safe - no action needed	473	4	Cool
56	64	0	0	Safe - no action needed	217	4	Cool
57	65	0	0	Safe - no action needed	0	4	Cool

Table 1. The spatial configuration of parcels and river segments

2013 Qualifying Exam Data Set – Question 3 (data set will also be emailed to you)

rkm	fish	temp-a	pool-a	wd-a	grad-a	pvol-a	temp01	pools01	wd01	grad01	pvol01
79	1	24.7	5	38	0.4	874	0	1	0	1	1
80	2	24.8	4	38	0.4	1146	0	1	0	1	1
81	5	24.9	4	33	0.4	1353	0	1	0	1	1
82	1	25	1	62	0.6	73	0	0	0	0	0
83	2	25	2	64	0.6	406	0	0	0	0	1
84	0	24.6	0	47	1	0	0	0	0	0	0
85	0	24.7	0	44	0.7	0	0	0	0	0	0
86	3	25.1	2	40	0.5	180	0	0	0	1	0
87	0	24.7	1	54	0.7	376	0	0	0	0	1
88	1	23.9	1	54	0.9	122	1	0	0	0	0
89	7	23.5	4	38	0.6	827	1	1	0	0	1
90	3	23.6	3	27	0.6	945	1	1	1	0	1
91	3	23.9	6	23	0.6	1163	1	1	1	0	1
92	0	24.2	2	28	0.6	99	0	0	1	0	0
93	2	24.3	2	26	0.4	66	0	0	1	1	0
94	1	24	1	32	0.4	27	0	0	0	1	0
95	0	23.7	0	31	0.6	0	1	0	0	0	0
96	0	23.5	0	33	1.5	0	1	0	0	0	0
97	1	23.3	1	34	1.3	37	1	0	0	0	0
98	0	23	0	36	1.5	0	1	0	0	0	0
99	5	22.7	3	30	0.6	213	1	1	0	0	0
100	4	22.8	3	33	0.3	196	1	1	0	1	0
101	7	23.2	6	25	0.3	417	1	1	1	1	1
102	4	23.5	7	25	0.3	822	1	1	1	1	1
103	7	23.4	5	26	0.4	355	1	1	1	1	1
104	5	23.6	3	14	0.5	213	1	1	1	1	0
105	2	23.7	1	22	0.6	83	1	0	1	0	0
106	6	23.8	3	24	0.6	315	1	1	1	0	0
107	0	24.1	2	15	0.7	103	0	0	1	0	0
108	1	23.3	0	18	0.9	0	1	0	1	0	0
109	1	22.7	1	29	0.8	20	1	0	1	0	0
110	2	23.5	2	31	0.7	253	1	0	0	0	0
111	0	24.2	1	26	0.6	47	0	0	1	0	0
112	2	22.9	1	30	0.6	0	1	0	0	0	0
113	6	20.5	4	22	0.4	326	1	1	1	1	1
114	2	24.4	4	33	0.4	585	0	1	0	1	1
115	1	24.6	3	36	0.5	494	0	1	0	1	1
116	5	24.5	6	22	0.4	739	0	1	1	1	1

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35	43	0	180,475	Restorable	811	1	Hot
36	44	126,270	204,812	Restorable	920	1	Hot
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