Modeling Chinook Growth and Mortality

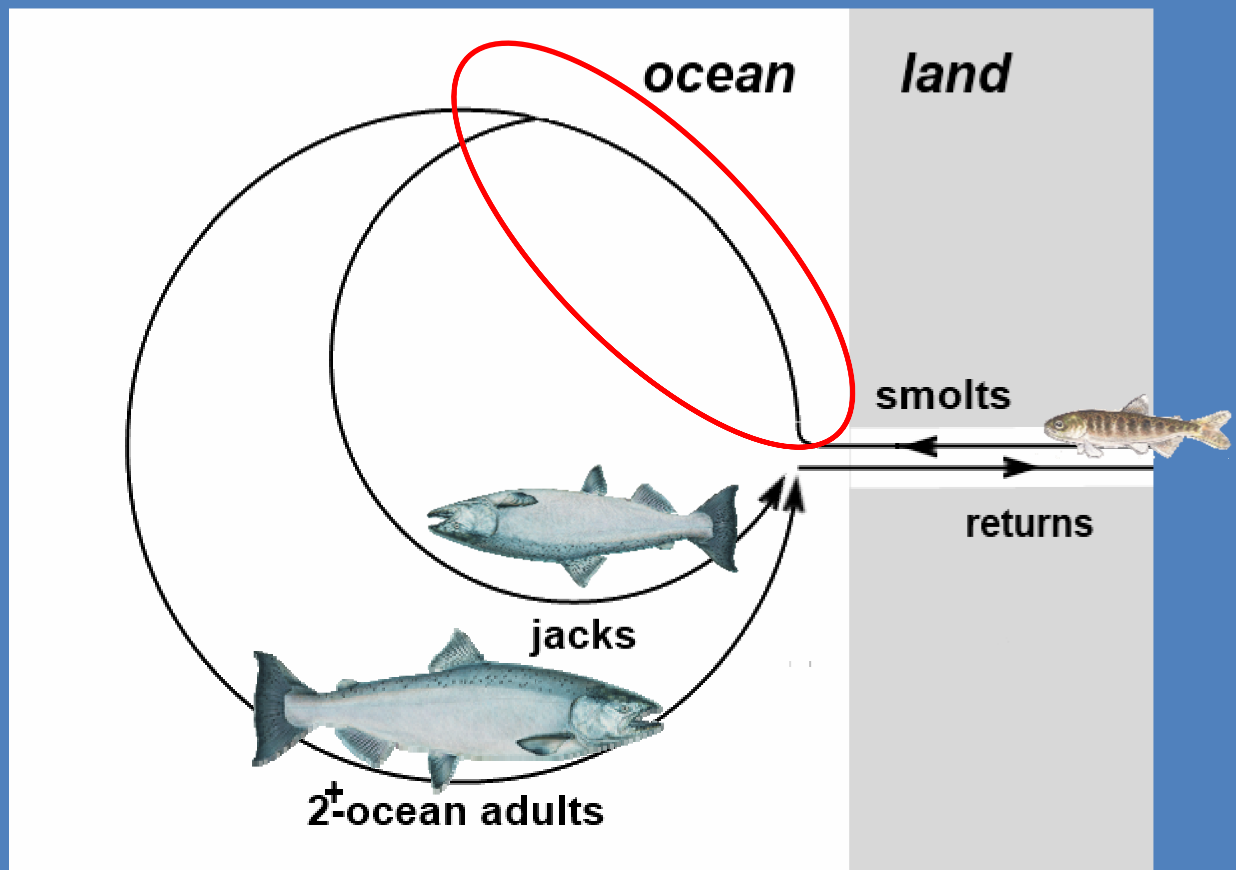
And the implications of

size-selective culling

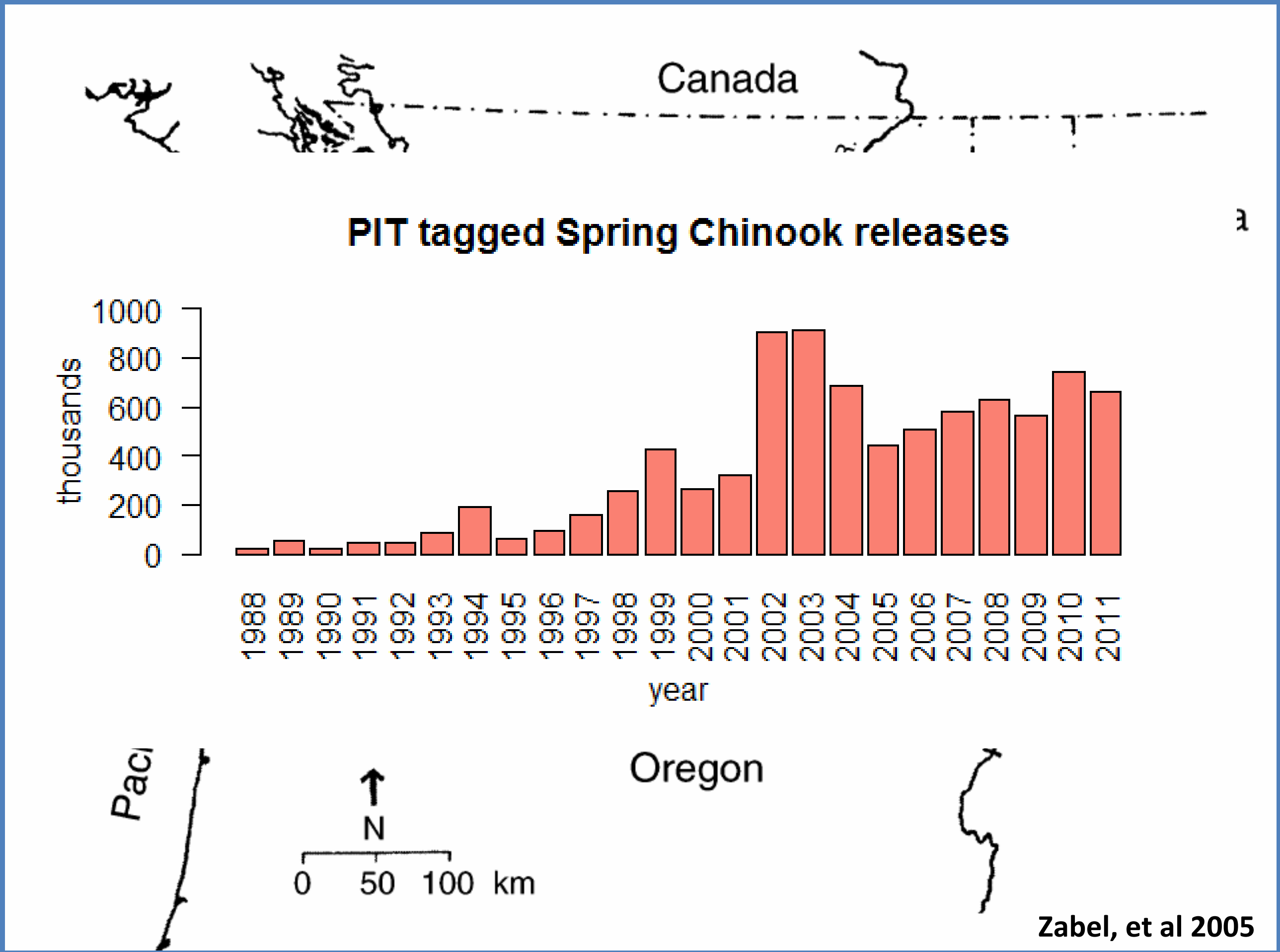
Growth and Mortality

- Background
- Mathematical Models
- Measuring Growth
- Maturation Models

Chinook Salmon Lifecycle



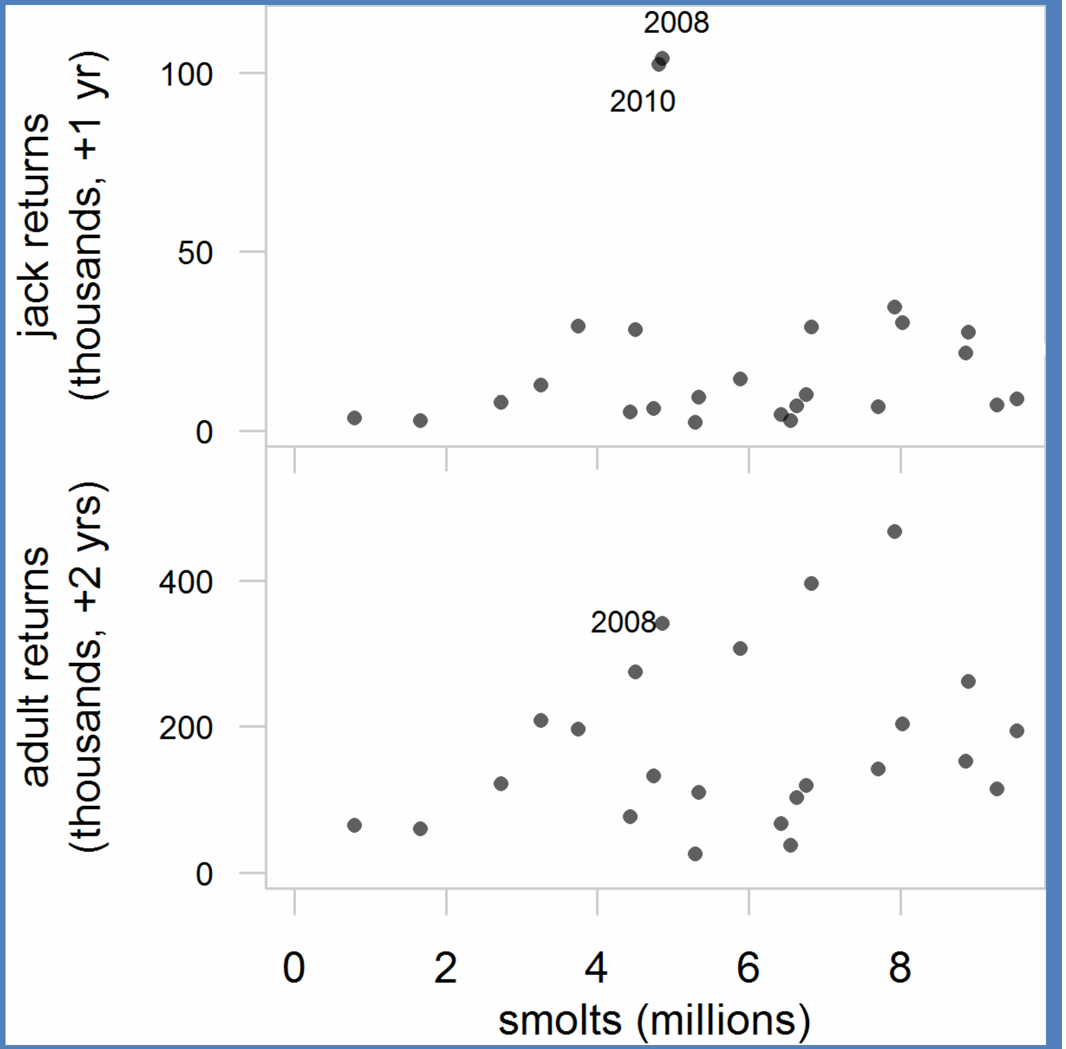
Columbia River Basin Salmon



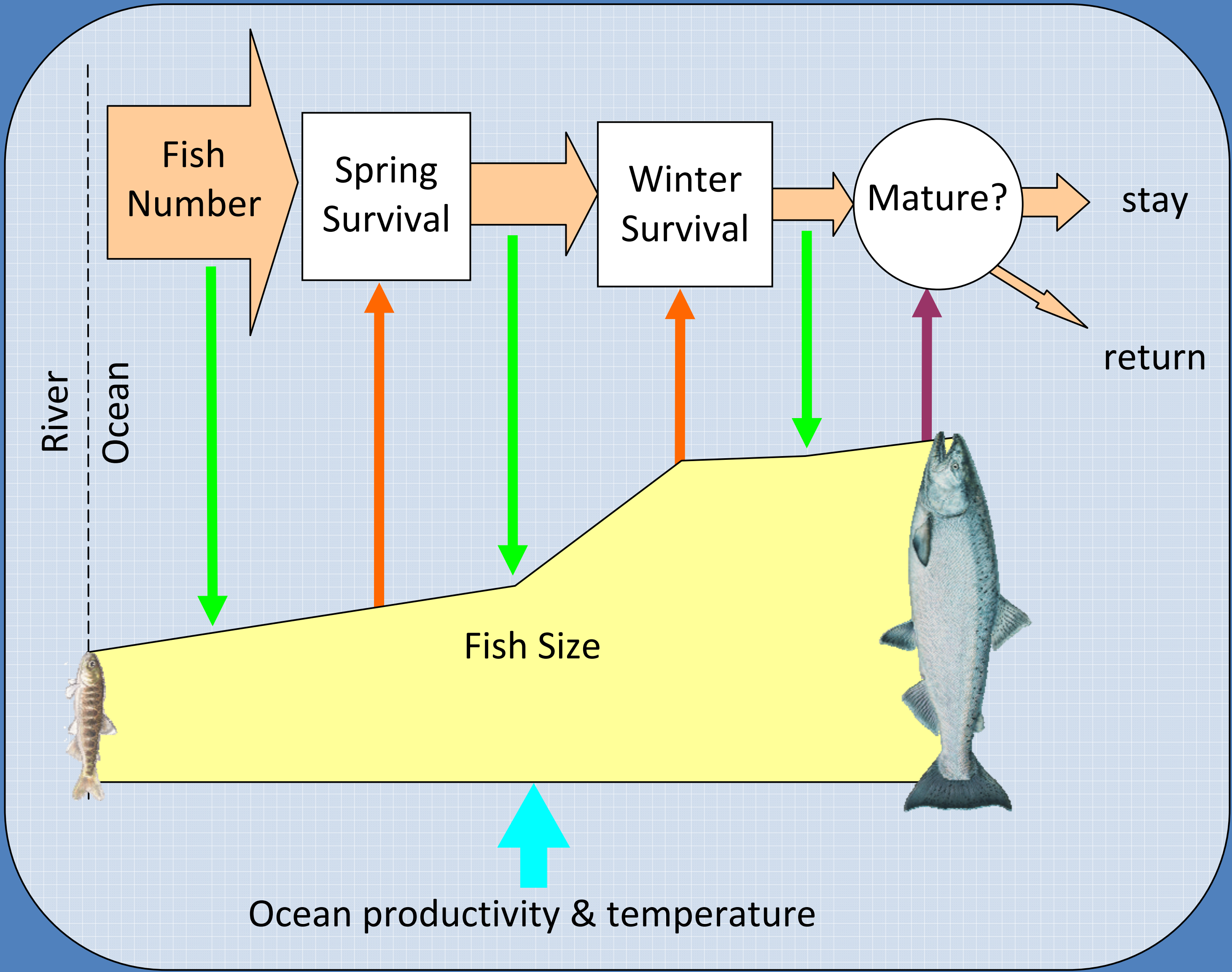
Zabel, et al 2005

The Puzzle

- CRB Chinook
  - Saturation
  - DD in Freshwater
  - Diminishing returns
  - DD in ocean



First Ocean Year



Basic Growth & Mortality Model

- Mortality rules:
  - Larger → greater survival capacity
- Growth rules:
  - Variable food → Variation in growth
    - ↑: Low, patchy, competition
    - ↓: High, well-distributed, no competition
    - Genetics
  - Density dependence:
    - •Fewer salmon → more food per → faster growth

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McKendrick - von Foerster PDE

$$\frac{\partial N}{\partial t} = -\frac{\partial}{\partial x}(g\,N) - m\,N$$

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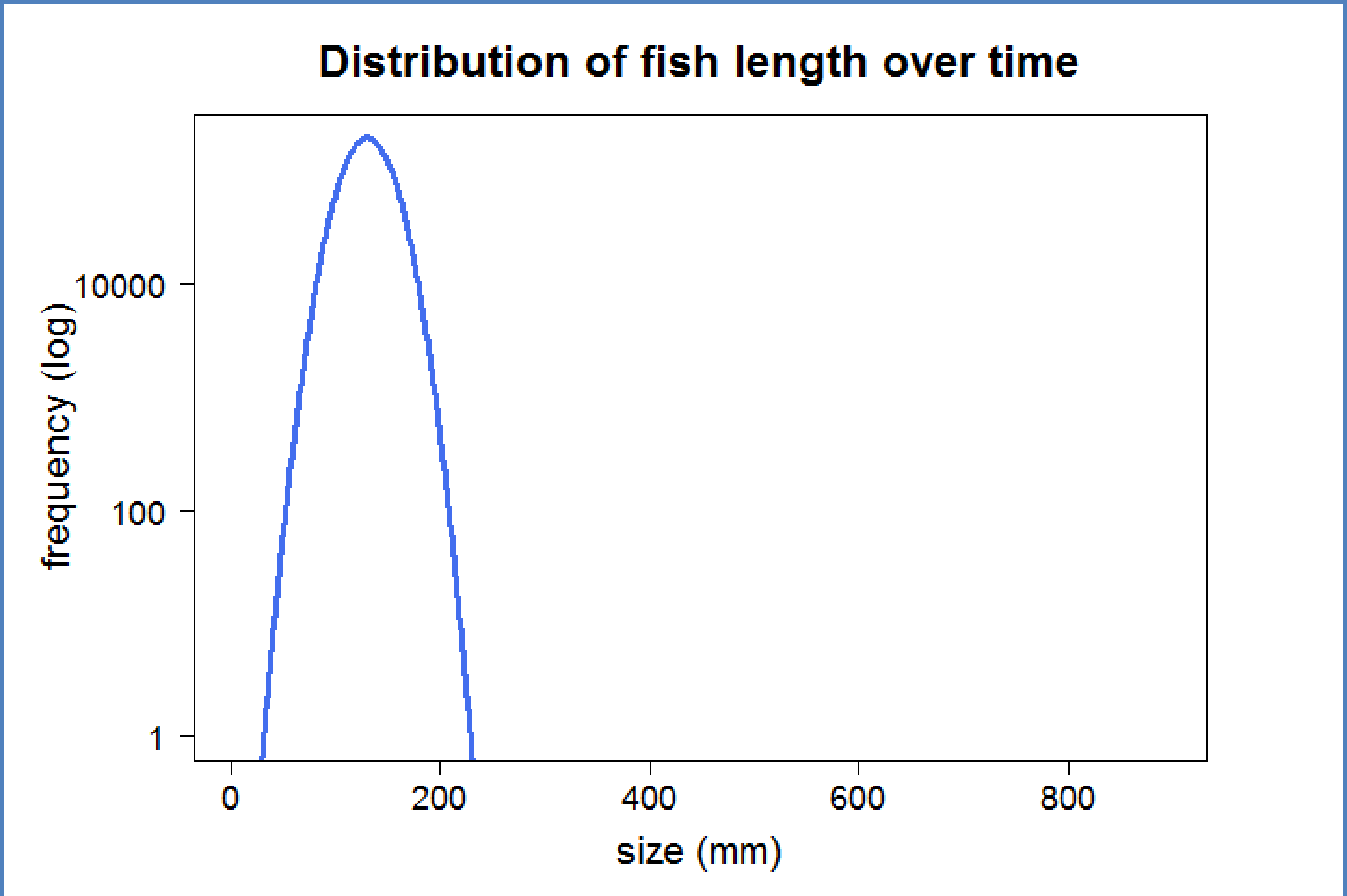
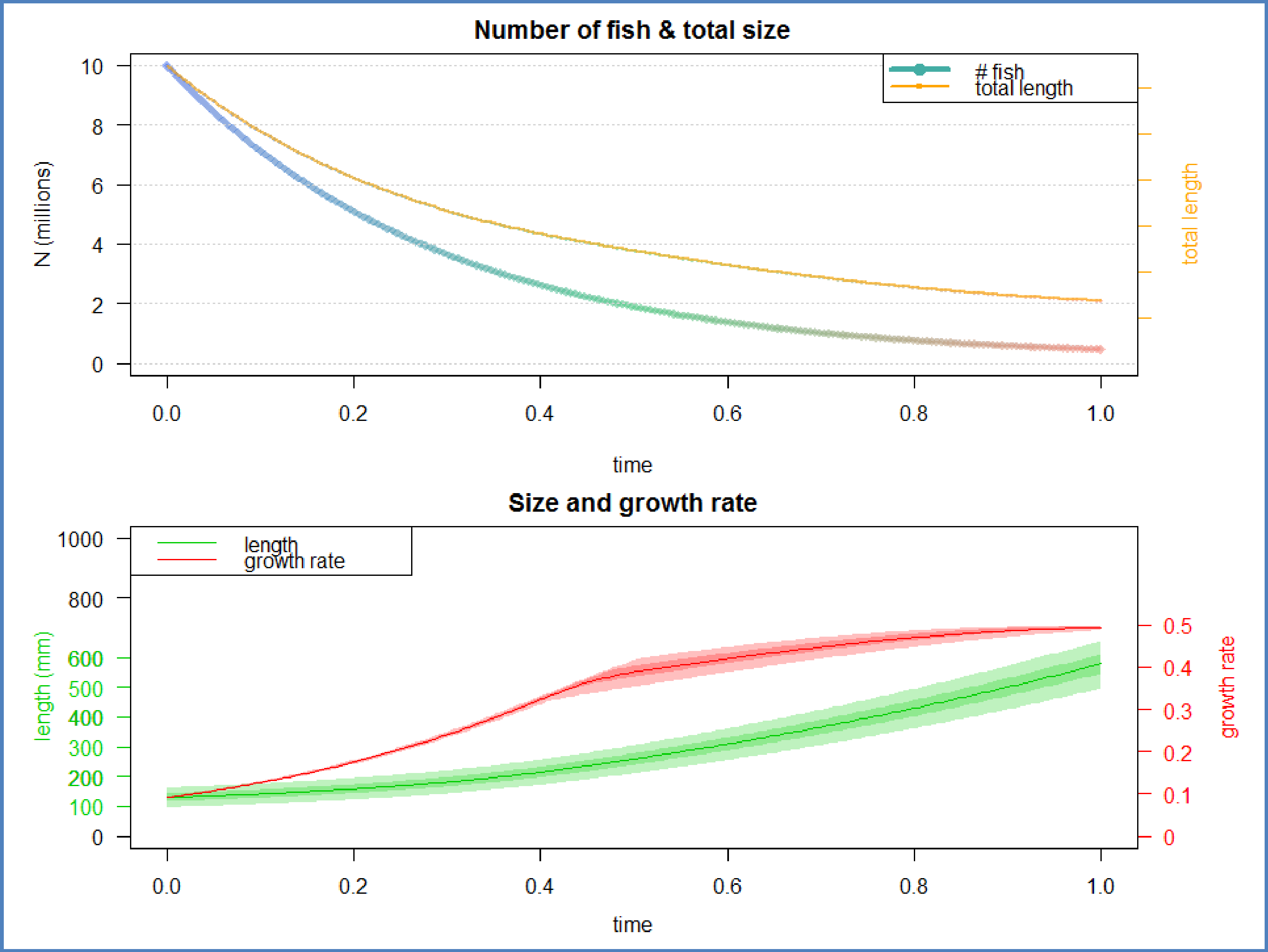
MKVF example



MKVF example



Munch formulation (2003)

$$\frac{dN}{dt} = -m(x)N \qquad \frac{dx}{dt} = g(x)$$

where

$x(t)$ is the size of a fish at time t (which was size x_0 at time 0)

$N(t)$ is the number fish at time t (which were size x_0 at time 0 and size x_t at time t)

$g(x)$ is the growth rate for fish of size x $\left(\frac{dx}{dt}\right)$

$m(x)$ is the mortality rate for fish of size x $\left(\frac{1}{N}\frac{dN}{dt}\right)$

Munch formulation (2003)

$$\frac{dN}{dt} = -m(x)N \qquad \frac{dx}{dt} = g(x)$$

$$\int \frac{dN}{N} = -\int m(x)dt \qquad \int_{x_0}^{x_t} \frac{dz}{g(z)} = \int_0^t ds$$

$$\frac{N(x_t,t)}{N(x_0,0)} = \exp\left(-\int_0^t m(x_s)\,ds\right) \qquad \varphi^{-1}(x_t) \equiv t$$

$$\frac{N(x_t,t)}{N(x_0,0)} = \exp\left(-\int_{x_0}^{x_t} \frac{m(x)}{g(x)}\,dx\right) \qquad x_t \equiv \varphi(t;x_0)$$

Munch formulation (2003)

$$\frac{N(x_t, t)}{N(x_0, 0)} = \exp\left(- \int_{x_0}^{x_t} \frac{m(x)}{g(x)}\, dx \right)$$

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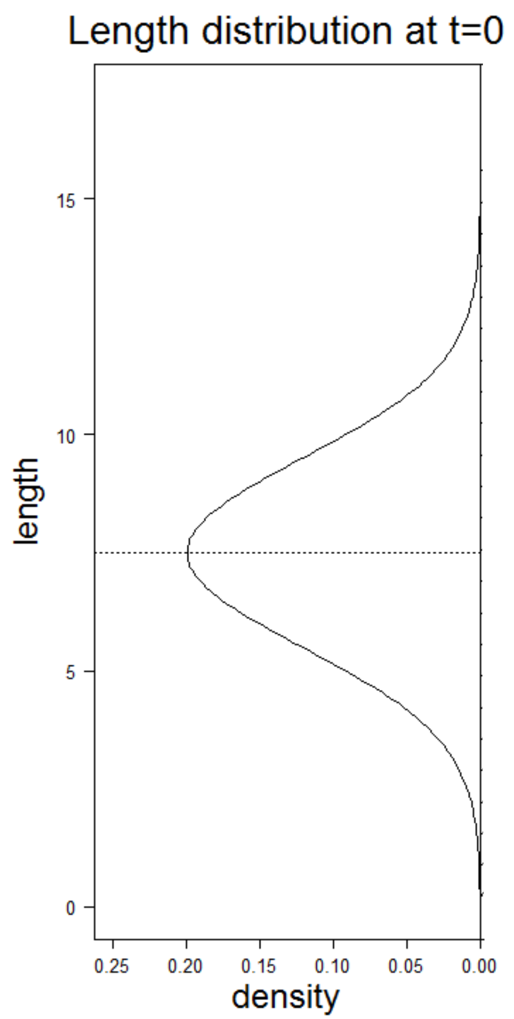
Example of Munch formulation



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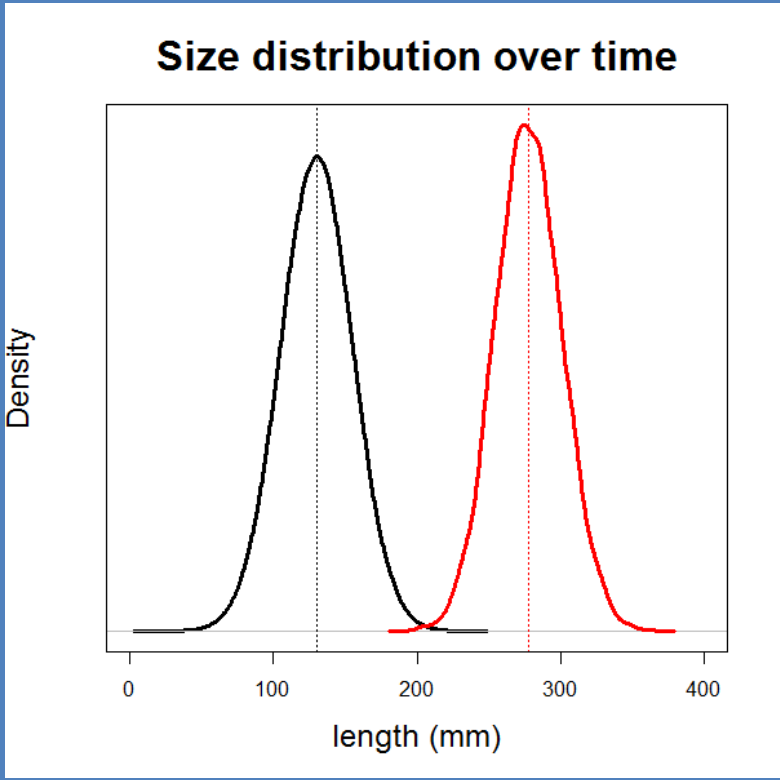
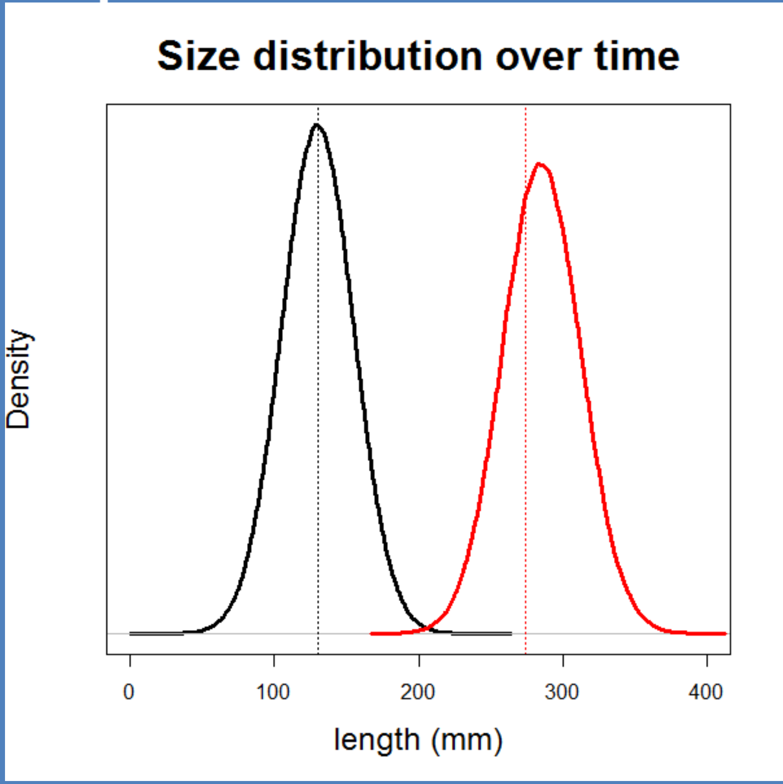
Measuring Growth and the Impact of Size-Selective Culling

- **Not size-selective**
  - Size distribution translates



- **Size selective**
  - Size distribution "pushed"



Estimating Growth

- Samples of size at two times
- Growth ≠ difference in means
- Estimate parameters
  - growth and mortality
- Growth computable
  - Error bounds?

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Maturation Biology

- Age – size/growth
  - Critical size at age
  - Most growth is marine
- Initiation
  - 6-12 months prior
  - Hormonal cues detectable in some outmigrants!
- Genetics
  - Faster/slower growers

Potential Factors influencing jack returns

Freshwater conditions

- Size at tagging
- Growth index
- Flows
- PNI
- Location
- Water temperatures

Ocean conditions

- Ocean upwelling
- Copepod
- SST (by season)
- PDO (by season)
- ONI (by season)

What fraction of returns will be jacks?

Linear regression model

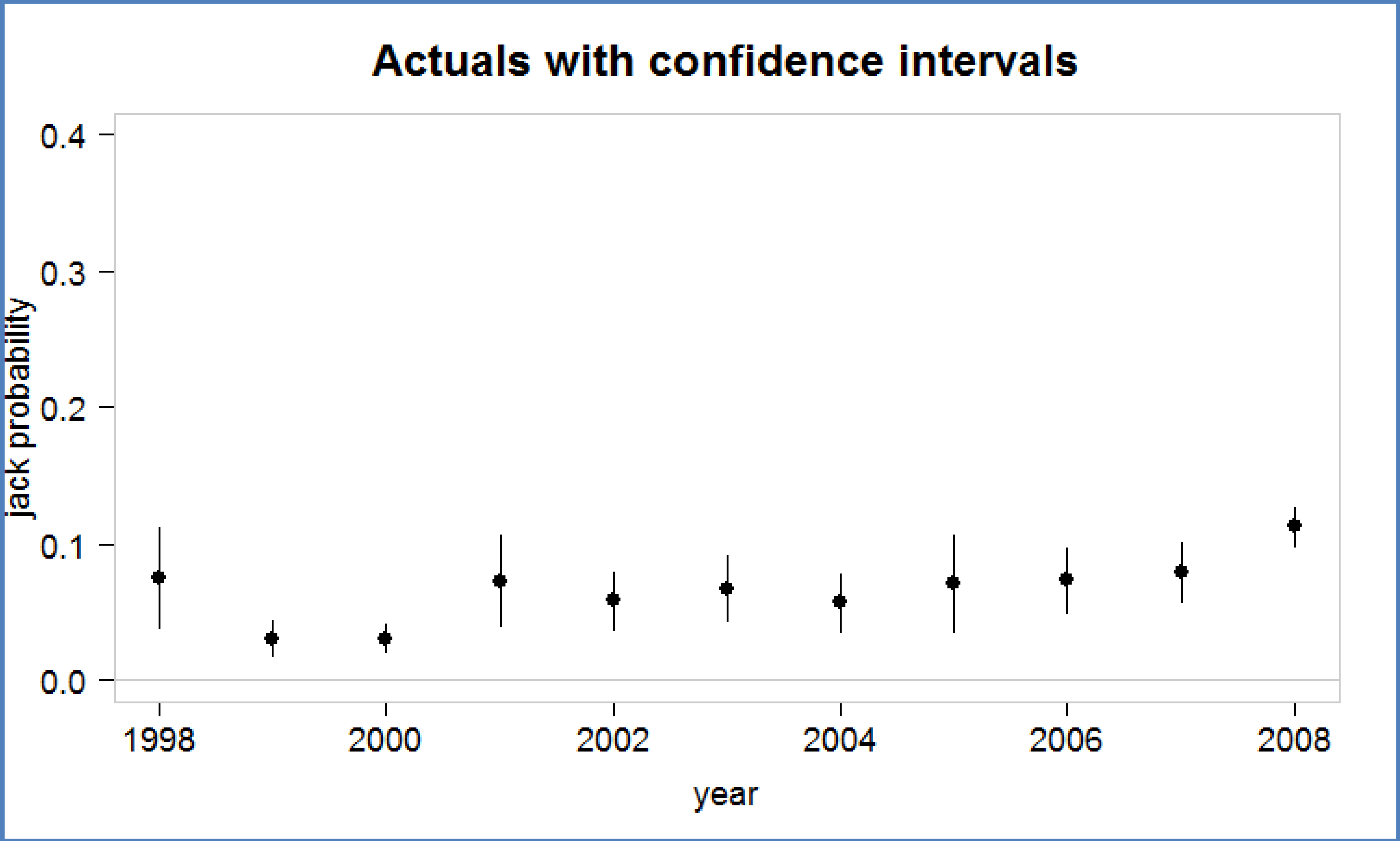
Freshwater conditions

- Important:
  - Growth index
  - Size at tagging
- Influential:
  - Water temperatures
- Not:
  - Flows
  - PNI
  - location

Ocean conditions

- Important:
  - Ocean upwelling
  - PDO (summer)
- Not:
  - SST
  - PDO (winter, spring, fall)
  - ONI
  - Copepod

Model Results



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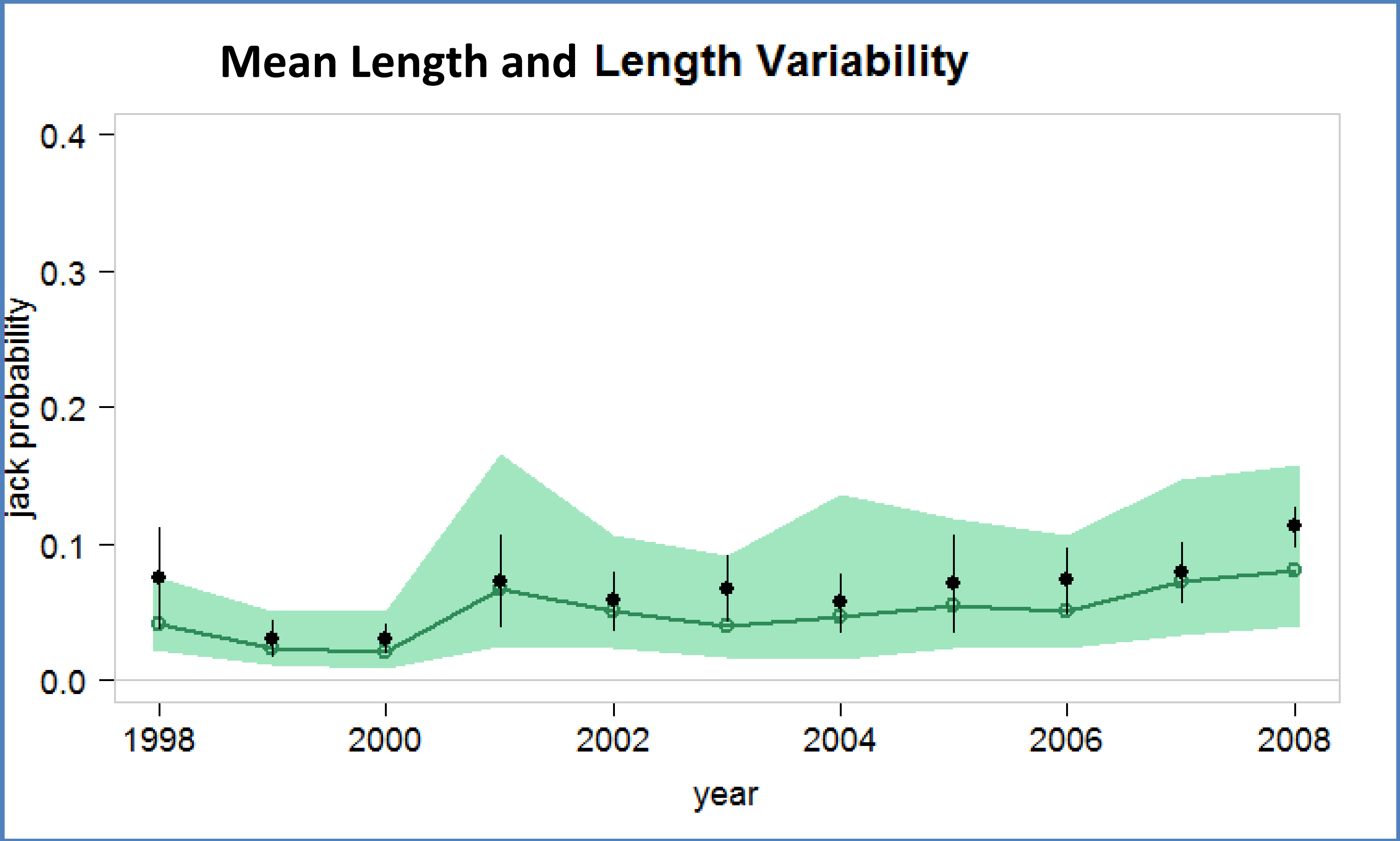

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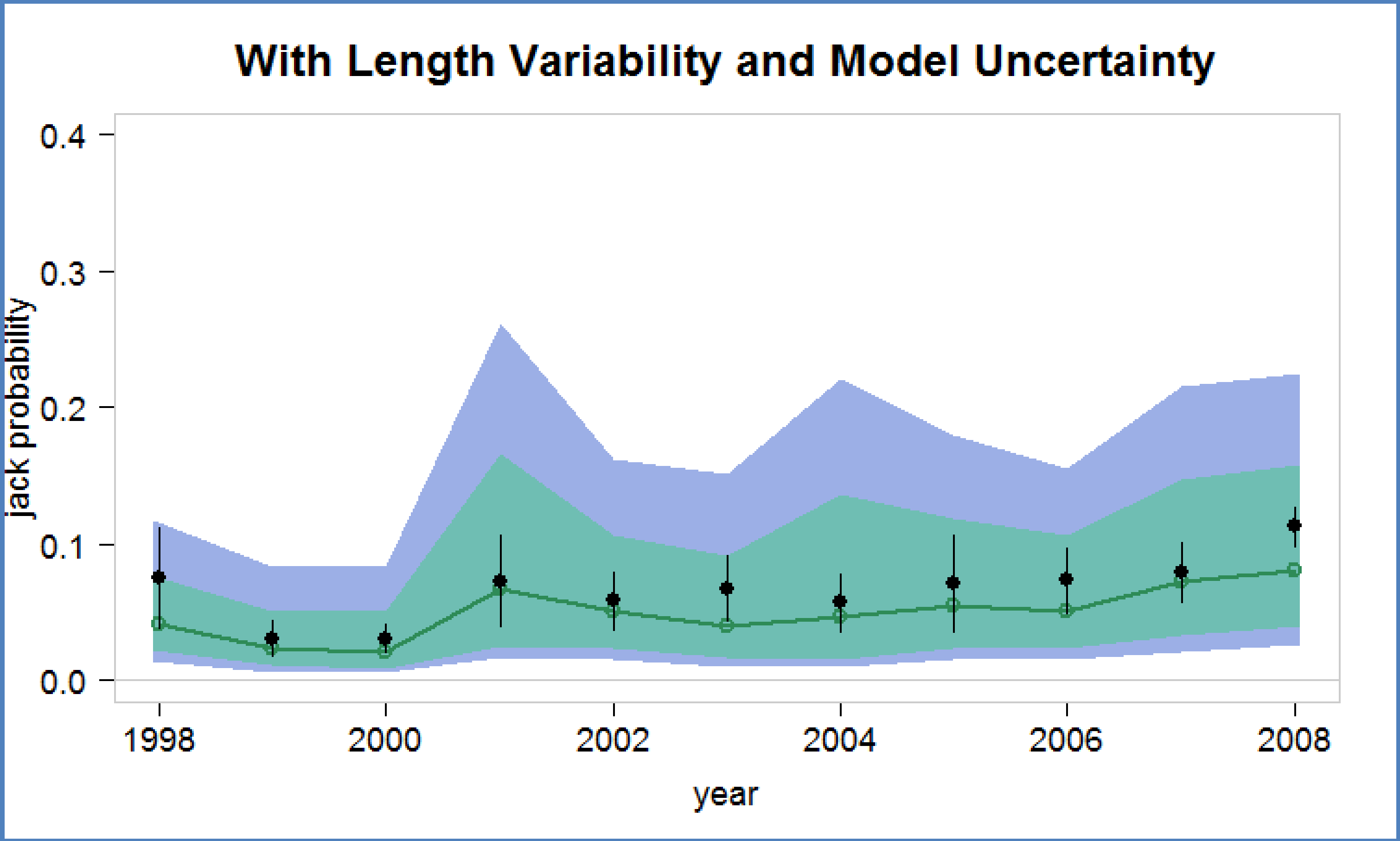

Summing up & moving on

- Model development
  - Environmental Covariates
  - Interacting Populations
    - Interspecific
    - Wild & hatchery
    - Fast/slow growers
- Growth Measurement
- Jack Returns
  - Freshwater Development
  - Variables Missing?

Thank

You

!

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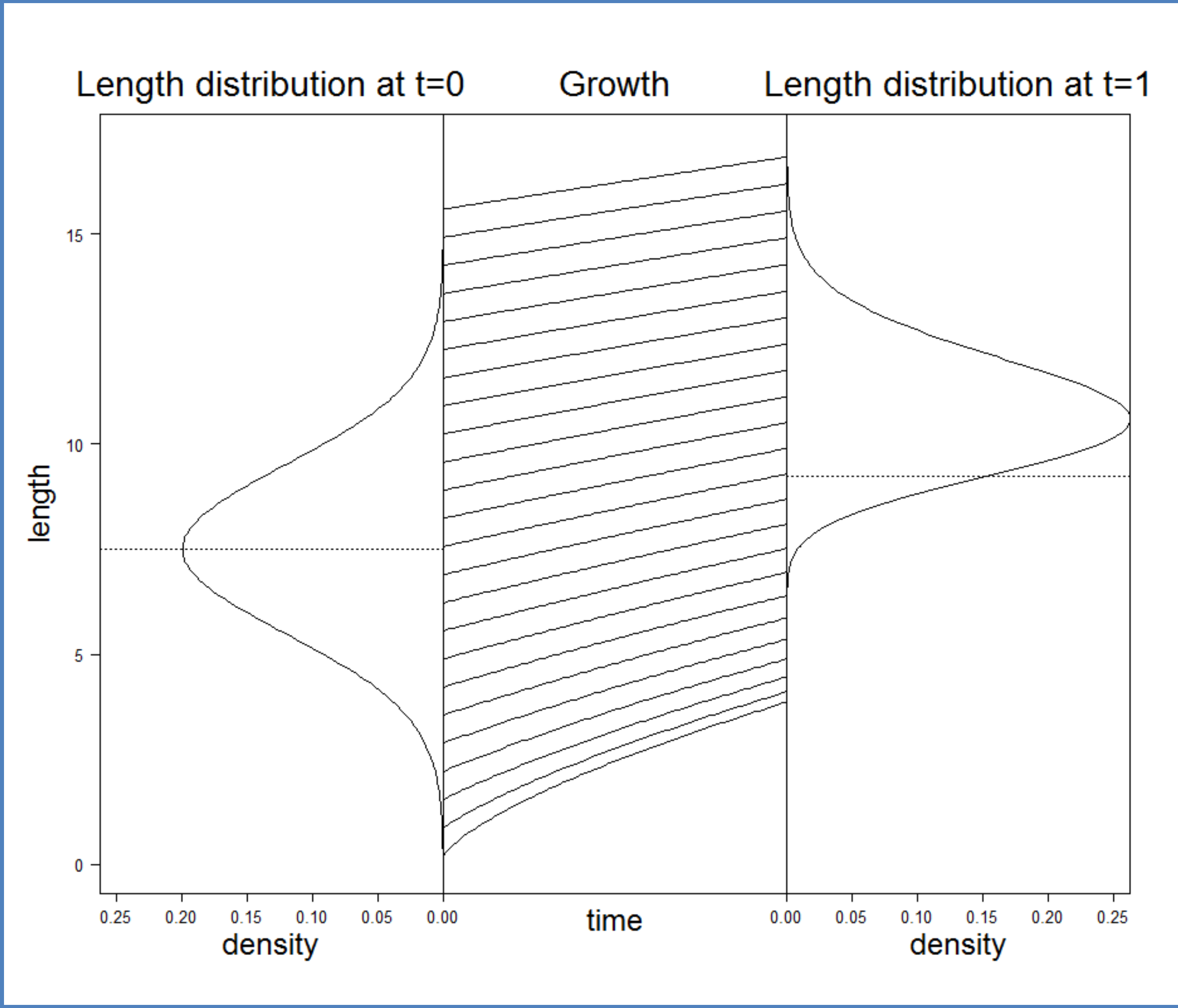

Estimating params

- Measurements: time 0 & t
- Algorithm
  - guess θ (growth & mortality params)
  - construct bins for time 0
  - Repeat until MLE found
    - translate bins to time t using growth params
    - solve for MLE of nuisance parameters
    - compute likelihood
    - systematically explore parameter space