Parameterizations for Bayesian nonlinear state-space models

# 2. Methods

## 2.1 Model specification

The nonlinear Bayesian state-space surplus production model is specified as a hierarchical model. The observed catch per unit effort at time is assumed to be proportional to the true abundance through the catchability coefficient . Further, catch per unit effort is assumed to be observed with log-normal error with variance parameter , so that:

|  |  |  |
| --- | --- | --- |
|  |  | (1) |

The Pella-Tomlinson surplus production model is used to model the population dynamics. The intrinsic growth rate is denoted and carrying capacity . The Pella-Tomlinson shape parameter is . consecutive years of catch data are assumed to be available, where is the catch observed at time . Biomass at time is . Depletion (, where ) is the unobserved state; this separates the estimation of dynamics from that of carrying capacity. Multiplicative (log-normal) process error with variance parameter is assumed for each year, with independence among years. Median depletion in year , is . Median depletion of the unfished population (i.e., ) is assumed to be . Independent log-normal process errors with variance parameter are included for each year. The population dynamics process is then:

|  |  |  |
| --- | --- | --- |
|  |  | (2) |
|  |  | (3) |
|  |  | (4) |

Priors match those used in Meyer and Millar (1999), which were based on a review of the literature for South Atlantic albacore, and an attempt to match particular quantiles (described in the appendix of Meyer and Millar (1999)). Catchability is given an improper, noninformative prior. The priors are

|  |  |  |
| --- | --- | --- |
|  |  | (5) |
|  |  | (6) |
|  |  | (7) |
|  |  | (8) |
|  |  | (9) |

|  |  |  |
| --- | --- | --- |
|  |  |  |

Models where the Pella-Tomlinson shape parameter ( in equation (3)) is estimated require an additional prior. This parameter can be difficult to estimate (Fletcher, 1978), and is often fixed by choosing a plausible relative biomass at which maximum sustainable yield is achieved, . The relationship between and is given by

Similar species have used a fixed shape parameter value corresponding to a depletion at maximum sustainable yield of (e.g. Winker et al., 2018). For this analysis a vaguely informative prior was placed on by examining the implicit prior on . It was parameterized such that the median of the implicit prior on is 0.4 and allows a wide range of values of while penalizing small values of that imply implausible growth rates at low population sizes. The prior is specified as

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| --- | --- | --- |
|  |  |  |

where the three parameters of the Skew Normal distribution are location, scale, and shape respectively. See Appendix B in the supplementary materials for more details. Here the posterior density is specified in terms of , but a prior distribution is specified for . Scalar transformation of a random variable requires a correction to the posterior density equal to the absolute value of the derivative of the transformation function (Gelman et al. 2014, pg. 21). Multivariate transformations use the determinant of the matrix of first partial derivatives of each transformation function with respect to each parameter, known as the Jacobian. The log-transformation here for example places all of the probability mass in the interval in the interval , while the probability mass in the interval ends up in the interval . This is accounted for by multiplying by a factor of . Note that linear transformations will have a correction factor that is constant, so no correction is required in the context of MCMC where posterior density is only required up to a constant of proportionality.

The posterior distribution of the parameter values combines the information from each of these components. Observation likelihoods are assumed independent, conditional on the process model, and process errors are also assumed independent. Although the statistical model is fully defined here, there are multiple parameterizations to consider. Each has specific performance characteristics of in terms of diagnostics and computational efficiency. The six parameterizations are summarized in Table [1](#tbl:param).

### 2.1.1 Centered model

The *centered* model corresponds directly to the model specified above. To avoid estimates of negative depletion and attempting to take the log of a negative number, a lower bound of is placed on the median depletion, i.e. equation (3) is modified to:

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| --- | --- | --- |
|  |  |  |

that the two *explicit F* parameterizations consistently exceeded the default maximum treedepth of 10, particularly at target acceptance rates closer to 1, so it was increased to 20. There was no evidence that the increased treedepth was used by the other parameterizations. Four chains of 30,000 iterations each were run, with the first 5,000 designated as warmup. This leaves 100,000 iterations for each fitted model. Each fit was performed using Stan v2.19.2 through the rstan interface (Stan Development Team 2019) in R v3.6.1 (R Core Team, 2019).

Betancourt, M., 2017. A conceptual introduction to Hamiltonian Monte Carlo. arXiv preprint. arXiv:1701.02434 [stat]. http://arxiv.org/abs/1701.02434

Table B. Quantiles of the Pella-Tomlinson shape parameter and associated depletion at MSY, . These cover a range of values and include the other surplus production model specifications used here, with of 0.4 and 0.5.

|  | 2.5% | 5% | 10% | 25% | 50% | 75% | 90% | 95% | 97.5% |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| $m$ | 0.60 | 0.63 | 0.68 | 0.83 | 1.19 | 1.92 | 3.14 | 4.32 | 5.71 |
| $P\_{MSY}$ | 0.28 | 0.29 | 0.30 | 0.34 | 0.40 | 0.49 | 0.59 | 0.64 | 0.69 |