Line-Torus intersection

I have a vector equation for a line given by $\mathbf{r} = \mathbf{a} + \mathbf{b}t$. I would like to find the intersections that it makes with a torus given by

$$\left(\mathbf{r}\cdot\mathbf{r} - R^2 - r^2\right)^2 = 4R^2\left(r^2 - |\hat{\mathbf{n}}\cdot\mathbf{r}|^2\right)$$

where R is the radius of the torus, r is the radius of the 'tube' and $\hat{\mathbf{n}}$ is the direction of the hole of the torus. I came up with the equation of the torus as a generalisation of the standard implicit equation of a torus. I plotted many different directions for the torus in mathematica and they all plotted a rotated version correctly. So I am fairly sure that this is correct.

Now I should be able to get a quartic equation out of this by plugging in the equation for the line into the equation for the torus, expanding and collecting like terms.

I could not find anything on the internet about solving such an intersect in terms of vectors.

My solutions to the coefficients of t in the quartic equation $at^4 + bt^3 + ct^2 + dt + e = 0$ are:

$$\begin{aligned}
a &= (\mathbf{b}\cdot\mathbf{b})^2 \\
b &= 4(\mathbf{a}\cdot\mathbf{b})(\mathbf{b}\cdot\mathbf{b}) \\
c &= 4(\mathbf{a}\cdot\mathbf{b})^2 + 2(\mathbf{b}\cdot\mathbf{b})(\mathbf{a}\cdot\mathbf{a} - R^2 - r^2) + 4R^2(\mathbf{b}\cdot\hat{\mathbf{n}})^2 \\
d &= 4(\mathbf{a}\cdot\mathbf{b})(\mathbf{a}\cdot\mathbf{a} - R^2 - r^2) + 8R^2(\mathbf{a}\cdot\hat{\mathbf{n}})(\mathbf{b}\cdot\hat{\mathbf{n}}) \\
e &= (\mathbf{a}\cdot\mathbf{a} - R^2 - r^2)^2 + 4R^2((\mathbf{a}\cdot\hat{\mathbf{n}})^2 - r^2)
\end{aligned}$$

Is this correct?

(geometry) (proof-verification) (vectors) (analytic-geometry) (quartic-equations)

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Maple says your coefficients do match equation

$$\left(\vec{\mathbf{r}}\cdot\vec{\mathbf{r}} - R^2 - r^2\right)^2 - 4R^2\left(r^2 - \left(\hat{\mathbf{n}}\cdot\vec{\mathbf{r}}\right)^2\right) = 0$$

– Nominal Animal Jun 4 '16 at 17:42