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## **Line-Torus intersection**

I have a vector equation for a line given by  $\mathbf{r} = \mathbf{a} + \mathbf{b}t$ . I would like to find the intersections that it makes with a torus given by

$$\left(\mathbf{r}\cdot\mathbf{r}-R^2-r^2
ight)^2=4R^2\left(r^2-|\hat{\mathbf{n}}\cdot\mathbf{r}|^2
ight)$$

where R is the radius of the torus, r is the radius of the 'tube' and  $\hat{\mathbf{n}}$  is the direction of the hole of the torus. I came up with the equation of the torus as a generalisation of the standard implicit equation of a torus. I plotted many different directions for the torus in mathematica and they all plotted a rotated version correctly. So I am fairly sure that this is correct.

Now I should be able to get a quartic equation out of this by plugging in the equation for the line into the equation for the torus, expanding and collecting like terms.

I could not find anything on the internet about solving such an intersect in terms of vectors.

My solutions to the coefficients of t in the quartic equation  $at^4 + bt^3 + ct^2 + dt + e = 0$  are:

$$\begin{split} &a = (\mathbf{b} \cdot \mathbf{b})^2 \\ &b = 4(\mathbf{a} \cdot \mathbf{b})(\mathbf{b} \cdot \mathbf{b}) \\ &c = 4(\mathbf{a} \cdot \mathbf{b})^2 + 2(\mathbf{b} \cdot \mathbf{b})(\mathbf{a} \cdot \mathbf{a} - R^2 - r^2) + 4R^2(\mathbf{b} \cdot \hat{\mathbf{n}})^2 \\ &d = 4(\mathbf{a} \cdot \mathbf{b})(\mathbf{a} \cdot \mathbf{a} - R^2 - r^2) + 8R^2(\mathbf{a} \cdot \hat{\mathbf{n}})(\mathbf{b} \cdot \hat{\mathbf{n}}) \\ &e = (\mathbf{a} \cdot \mathbf{a} - R^2 - r^2)^2 + 4R^2((\mathbf{a} \cdot \hat{\mathbf{n}})^2 - r^2) \end{split}$$

Is this correct?

(geometry) (proof-verification) (vectors) (analytic-geometry) (quartic-equations)

edited Jun 9 '16 at 20:26

asked Jun 4 '16 at 16:09

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3,542 5 26 54

Maple says your coefficients do match equation

$$\left(\vec{\mathbf{r}}\cdot\vec{\mathbf{r}}-R^2-r^2
ight)^2-4R^2\left(r^2-\left(\hat{\mathbf{n}}\cdot\vec{\mathbf{r}}
ight)^2
ight)=0$$

- Nominal Animal Jun 4 '16 at 17:42