1 Setup

This part is just a review of the notation in your writeup.

We have that

$$\Gamma_s \equiv \mathbb{E}\left[\begin{array}{c} s(0,X_i,Z_i)\int_0^1 b_{0,1}(u,X)\,\mathbb{1}[u>p(X_i,Z_i)]\;du \\ s(0,X_i,Z_i)\int_0^1 b_{0,2}(u,X)\,\mathbb{1}[u>p(X_i,Z_i)]\;du \\ \vdots \\ s(0,X_i,Z_i)\int_0^1 b_{0,K_0}(u,X)\,\mathbb{1}[u>p(X_i,Z_i)]\;du \\ s(1,X_i,Z_i)\int_0^1 b_{1,1}(u,X)\,\mathbb{1}[u\le p(X_i,Z_i)]\;du \\ s(1,X_i,Z_i)\int_0^1 b_{1,2}(u,X)\,\mathbb{1}[u\le p(X_i,Z_i)]\;du \\ \vdots \\ s(1,X_i,Z_i)\int_0^1 b_{1,K_1}(u,X)\,\mathbb{1}[u\le p(X_i,Z_i)]\;du \end{array}\right] \equiv \mathbb{E}[G_{i,s}].$$

For each s function in $\mathcal{S}$, we have

$$\mathbb{E}[Y_i s(D_i,X_i,Z_i) - \theta' G_{i,s}] = 0.$$

Let S_i denote the $d_s \times 1$ vector formed by stacking the s functions in $\mathcal{S}$. Let G_i denote the matrix with dimension $d_S \times d_\theta$ that has the s^{th} row $G'_{i,s}$. Then we can stack the moment conditions into the following system:

$$h(\theta) \equiv \mathbb{E}[h_i(\theta)] \equiv \mathbb{E}[Y_i S_i - G_i\theta] = 0.$$

2 Counting moments

The moments determine $h_i(\theta)$ solely through the weights S_i, so it should suffice to only look at the s-weights across all $i = 1, \ldots, n$ individuals to determine the number of independent moments.

Each element of h_i can be written as a linear combination of

$$\vec{S}_{i,s} = \left[\begin{array}{c} s_s(0,X_i,Z_i) \\ s_s(1,X_i,Z_i) \end{array}\right].$$

Specifically, $\theta' G_{i,s}$ can be written as

$$
\begin{aligned}
\theta' G_{i,s} &= s_s(0, X_i, Z_i) \sum_{k=1}^{K_0} \theta_{0,k} \int_0^1 b_{0,k}(u, X)\, \mathbb{1}[u > p(X_i, Z_i)]\; du \quad + \\
&\qquad s_s(1, X_i, Z_i) \sum_{k=1}^{K_1} \theta_{1,k} \int_0^1 b_{1,k}(u, X)\, \mathbb{1}[u \le p(X_i, Z_i)]\; du \\
&= \underbrace{\begin{bmatrix} \sum_{k=1}^{K_0} \theta_{0,k} \int_0^1 b_{0,k}(u, X)\, \mathbb{1}[u > p(X_i, Z_i)]\; du \\ \sum_{k=1}^{K_1} \theta_{1,k} \int_0^1 b_{1,k}(u, X)\, \mathbb{1}[u \le p(X_i, Z_i)]\; du \end{bmatrix}'}_{\vec{G}_i(\theta)'} \underbrace{\begin{bmatrix} s_s(0, X_i, Z_i) \\ s_s(1, X_i, Z_i) \end{bmatrix}}_{\vec{S}_{i,s}}.
\end{aligned}
$$

Similarly, $Y_i s_s(D_i, X_i, Z_i)$ can be written as

$$
\begin{aligned}
Y_i s_s(D_i, X_i, Z_i) &= Y_i(1 - D_i) s_s(0, X_i, Z_i) + Y_i D_i s_s(1, X_i, Z_i) \\
&= \underbrace{\begin{bmatrix} Y_i(1 - D_i) \\ Y_i D_i \end{bmatrix}'}_{\vec{Y}_i'} \underbrace{\begin{bmatrix} s_s(0, X_i, Z_i) \\ s_s(1, X_i, Z_i) \end{bmatrix}}_{\vec{S}_{i,s}}.
\end{aligned}
$$

Thus,

$$
h_i(\theta) = \begin{bmatrix} \vec{S}_{i,1}' \\ \vdots \\ \vec{S}_{i,d_s}' \end{bmatrix} \left[\vec{Y}_i - \vec{G}_i(\theta) \right],
$$

and

$$
\begin{aligned}
H &= \begin{bmatrix} h_1(\theta)' \\ \vdots \\ h_n(\theta)' \end{bmatrix} \\
&= \underbrace{\operatorname{diag}\left(\left\{ \vec{Y}_i' - \vec{G}_i(\theta)' \right\}_{i=1}^{n} \right)}_{n \times (2 \times n) \text{ block diagonal matrix}} \underbrace{\begin{bmatrix} \vec{S}_{1,1} & \dots & \vec{S}_{1,d_s} \\ \vdots & \ddots & \vdots \\ \vec{S}_{n,1} & \dots & \vec{S}_{n,d_s} \end{bmatrix}}_{(2 \times n) \times d_s \text{ matrix.}}
\end{aligned}
$$

Each column of the right matrix in the final equality corresponds to a single moment. Its construction simply requires us to compute the weights for each moment and individual, which we always have to do whether or not we have point identification. The rank of this matrix corresponds to the number of independent moments.