Improving the Numerical Stability of `ivmte`: Adjusting the Condition Number

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January 27, 2021

- Denote the least squares criterion by

$$\hat{Q}(\theta) = \sum_{i=1}^{n} \left(Y_i - \sum_{k=1}^{K_0} \theta_{0k}(1 - D_i)B_{0ki} - \sum_{k=1}^{K_1} \theta_{1k} D_i B_{1ki} \right)^2 . \tag{1}$$

- Let's write this as a norm:

$$\hat{Q}(\theta) = \|Y - B\theta\|^2 \tag{2}$$

  where B is the design matrix constructed from the variables $\{(1 - D_i)B_{0ki}\}_{k=1}^{K_0}$ and $\{D_i B_{1ki}\}_{k=1}^{K_1}$. So B has n rows and $j \equiv K_0 + K_1$ columns.

- Let H be the square diagonal matrix with dimension equal to the number of columns of A and jth diagonal entry given by

$$h_{jj} = \begin{cases} \|B_j\|^{-1}, & \text{if } \|B_j\| \neq 0 \\ 1, & \text{if } \|B_j\| = 0 \end{cases} \tag{3}$$

  where B_j is the jth column of B. Note that H is clearly invertible.

- The idea is just to use the change of variables θ to $\tilde{\theta} \equiv H^{-1}\theta$. In terms of $\hat{Q}$ this produces

$$\tilde{Q}(\tilde{\theta}) \equiv \left\| Y - \tilde{B}\tilde{\theta} \right\|^2 , \tag{4}$$

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where $\tilde{B} \equiv BH$.

- It should be the case that $\tilde{B}$ has a smaller condition number than B, which could potentially improve stability.