Weights for LATE conditional on covariates

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Deriving weights

Suppose there are covariates (X, V). For simplicity, suppose X and V are discrete. The $LATE$ for $z_1 \to z_2$ conditional on $X = x$ is

$$\begin{aligned}
&\mathbb{E}[LATE(X,V) \mid \text{Complier}, X = x] \\
&= \mathbb{E}[\mathbb{E}[Y(1) - Y(0) \mid \text{Complier}, X, V] \mid \text{Complier}, X = x] \\
&= \mathbb{E}\left[\int_{p(z_1,x,v)}^{p(z_2,x,v)} [m_1(u,x,v) - m_0(u,x,v)] \frac{1}{p(z_2,x,v) - p(z_1,x,v)}\, du \,\middle|\, \text{Complier}, X = x\right] \\
&= \sum_{v \in \text{supp}(V)} \int_{p(z_1,x,v)}^{p(z_2,x,v)} [m_1(u,x,v) - m_0(u,x,v)] \frac{1}{p(z_2,x,v) - p(z_1,x,v)}\, du \\
&\qquad \times \mathbb{P}\{V = v \mid \text{Complier}, X = x\}.
\end{aligned} \tag{1}$$

By Bayes' rule, $\mathbb{P}\{V = v \mid \text{Complier}, X = x\}$ can be written as

$$\begin{aligned}
\mathbb{P}\{V = v \mid \text{Complier}, X = x\} &= \frac{\mathbb{P}\{\text{Complier} \mid X = x, V = v\} \times \mathbb{P}\{V = v \mid X = x\}}{\mathbb{P}\{\text{Complier} \mid X = x\}} \\
&= \frac{[p(z_2,x,v) - p(z_1,x,v)] \times \mathbb{P}\{V = v \mid X = x\}}{\sum_{v' \in \text{supp}(V)} [p(z_2,x,v') - p(z_1,x,v')] \times \mathbb{P}\{V = v' \mid X = x\}}.
\end{aligned} \tag{2}$$

By substituting (2) into (1), we have

$$\begin{aligned}
&\mathbb{E}[LATE(X,V) \mid \text{Complier}, X = x] \\
&= \sum_{v \in \text{supp}(V)} \int_{p(z_1,x,v)}^{p(z_2,x,v)} [m_1(u,x,v) - m_0(u,x,v)] \frac{1}{p(z_2,x,v) - p(z_1,x,v)}\, du \\
&\qquad \times \frac{[p(z_2,x,v) - p(z_1,x,v)] \times \mathbb{P}\{V = v \mid X = x\}}{\sum_{v' \in \text{supp}(V)} [p(z_2,x,v') - p(z_1,x,v')] \times \mathbb{P}\{V = v' \mid X = x\}}. \\
&= \sum_{v \in \text{supp}(V)} \int_{p(z_1,x,v)}^{p(z_2,x,v)} [m_1(u,x,v) - m_0(u,x,v)]\, du \\
&\qquad \times \frac{\mathbb{P}\{V = v \mid X = x\}}{\sum_{v' \in \text{supp}(V)} [p(z_2,x,v') - p(z_1,x,v')] \times \mathbb{P}\{V = v' \mid X = x\}}.
\end{aligned}$$

Implementing weights

The `ivmte` package already estimates

$$\int_{p(z_1,x,v)}^{p(z_2,x,v)} m_1(u,x,v) - m_0(u,x,v)\ du \tag{3}$$

for every observation in the data. Then averaging the estimates of (3) across the subset of observations with $X = x$ provides an estimate of

$$\sum_{v \in \text{supp}(V)} \int_{p(z_1,x,v)}^{p(z_2,x,v)} [m_1(u,x,v) - m_0(u,x,v)]\ du \times \mathbb{P}\{V = v \mid X = x\}. \tag{4}$$

Then all that remains for estimating $\mathbb{E}[LATE(X,V) \mid \text{Complier} X = x\}$ is to divide (4) by an estimate of

$$\sum_{v \in \text{supp}(V)} [p(z_2,x,v) - p(z_1,x,v)] \times \mathbb{P}\{V = v \mid X = x\},$$

which is straightforward to obtain.