Weights for LATE conditional on covariates

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Deriving weights

Suppose there are covariates (X, V). For simplicity, suppose X and V are discrete. The $LATE$ for $z_0 \to z_1$ conditional on $V = v$ is

$$\begin{aligned}
&\mathbb{E}[LATE(X,V) \mid \text{Complier}, V = v] \\
&= \mathbb{E}[\mathbb{E}[Y(1) - Y(0) \mid \text{Complier}, X, V] \mid \text{Complier}, V = v] \\
&= \mathbb{E}\left[\int_{p(X,V,z_0)}^{p(X,V,z_1)} [m(1 \mid u, X, V) - m(0 \mid u, X, V)] \frac{1}{p(X,V,z_1) - p(X,V,z_0)} \, du \,\middle|\, \text{Complier}, V = v \right] \\
&= \sum_{x \in \text{supp}(X)} \int_{p(x,v,z_0)}^{p(x,v,z_1)} [m(1 \mid u, x, v) - m(0 \mid u, x, v)] \frac{1}{p(x,v,z_1) - p(x,v,z_0)} \, du \\
&\qquad \times \mathbb{P}\{X = x \mid \text{Complier}, V = v\}. \qquad (1)
\end{aligned}$$

By Bayes' rule, $\mathbb{P}\{X = x \mid \text{Complier}, V = v\}$ can be written as

$$\begin{aligned}
\mathbb{P}\{X = x \mid \text{Complier}, V = v\} &= \frac{\mathbb{P}\{\text{Complier} \mid V = v, X = x\} \times \mathbb{P}\{X = x \mid V = v\}}{\mathbb{P}\{\text{Complier} \mid V = v\}} \\
&= \frac{[p(x,v,z_1) - p(x,v,z_0)] \times \mathbb{P}\{X = x \mid V = v\}}{\sum_{x' \in \text{supp}(X)} [p(x',v,z_1) - p(x',v,z_0)] \times \mathbb{P}\{X = x' \mid V = v\}}. \qquad (2)
\end{aligned}$$

By substituting (2) into (1), we have

$$\begin{aligned}
&\mathbb{E}[LATE(X,V) \mid \text{Complier}, V = v] \\
&= \sum_{x \in \text{supp}(X)} \int_{p(x,v,z_0)}^{p(x,v,z_1)} [m(1 \mid u, x, v) - m(0 \mid u, x, v)] \frac{1}{p(x,v,z_1) - p(x,v,z_0)} \, du \\
&\qquad \times \frac{[p(x,v,z_1) - p(x,v,z_0)] \times \mathbb{P}\{X = x \mid V = v\}}{\sum_{x' \in \text{supp}(X)} [p(x',v,z_1) - p(x',v,z_0)] \times \mathbb{P}\{X = x' \mid V = v\}}. \\
&= \sum_{x \in \text{supp}(X)} \int_{p(x,v,z_0)}^{p(x,v,z_1)} [m(1 \mid u, x, v) - m(0 \mid u, x, v)] \, du \\
&\qquad \times \frac{\mathbb{P}\{X = x \mid V = v\}}{\sum_{x' \in \text{supp}(X)} [p(x',v,z_1) - p(x',v,z_0)] \times \mathbb{P}\{X = x' \mid V = v\}}.
\end{aligned}$$

Implementing weights

The `ivmte` package already estimates

$$\int_{p(x,v,z_0)}^{p(x,v,z_1)} m(1 \mid u, x, v) - m(0 \mid u, x, v) \, du \tag{3}$$

for every observation in the data. Then averaging the estimates of (3) across the subset of observations with $V = v$ provides an estimate of

$$\sum_{x \in \operatorname{supp}(X)} \int_{p(x,v,z_0)}^{p(x,v,z_1)} [m(1 \mid u, x, v) - m(0 \mid u, x, v)] \, du \times \mathbb{P}\{X = x \mid V = v\}. \tag{4}$$

Then all that remains for estimating $\mathbb{E}[LATE(X, V) \mid \text{Complier} V = v]$ is to divide (4) by an estimate of

$$\sum_{x \in \operatorname{supp}(X)} [p(x, v, z_1) - p(x, v, z_0)] \times \mathbb{P}\{X = x \mid V = v\},$$

which is straightforward to obtain.