Matrix decomposition comparisons

August 5, 2022

1 Introduction

This document compares various estimates returned by the `ivmte` package when using various matrix decompositions.

There are four test cases:

1. Original example from GitHub issue #209. The issue was raised because Gurobi was frequently running into numerical issues when solving the QCQP problems. The issue was resolved by (i) implementing the Cholesky decomposition for the QCQP problem; (ii) avoiding the QCQP problem altogether using the normal equations approach.

   Example uses the AE data set. Design matrix contains 120 variables and has a rank of 59.

2. Original example from GitHub issue #225. The issue was raised because Gurobi was returning a lower bound that exceeded the upper bound. Gurobi Support explained this was due to numerical instabilities and suggested tightening the tolerance parameters.

   Example uses the AE data set. Design matrix contains 66 variables and has a rank of 44.

3. Original example from GitHub issue #198. The issue was raised because Gurobi warned that the model was poorly scaled and Gurobi was unable to satisfy the optimality tolerances (see GitHub issue #196). The issue was resolved by implementing the `rescale` option.

   Example uses the AE data set. Design matrix contains 11 variables and has a rank of 8.

4. A new example involving a spline of degree 3. The example uses example data from `ivmte` (generated using `ivmte:::gendistCovariates()`). Design matrix contains 80 variables and has a rank of 59.

For each test case, I did the following:

- Estimate the bounds using different values of `criterion.tol`, different methods of matrix decompositions, and both the moment- and conditional moment-based approaches (LP and QCQP approaches, respectively).

  `ivmte` allows for five different approaches to matrix decompositions. The approaches can be selected by passing the following arguments to the `direct` option.

  1. `lp0`/`qp0`: Gram matrix $A'A$ is treated as $A'A$.
  2. `lp1`/`qp1`: By the QR decomposition, $A = QR$ and $A'A = R'Q'QR = R'R$. No new variables are introduced to the LP/QCQP problem.
  3. **[No longer considered!]** `lp2`/`qp2`: By the QR decomposition, $A = QR$, and the new variable $y = QRx$ is introduced to the LP/QCQP problem. The number of new variables is thus equal to the number of observations. **This approach runs into memory issues when using large data sets with rich MTR specifications.**
  4. `lp3`/`qp3`: By the QR decomposition, $A = QR$, and the new variable $y = Rx$ is introduced to the LP/QCQP problem.
  5. `lp4`/`qp4`: By the Cholesky decomposition, $A = C'C$ and the new variable $y = Cx$ is introduced to the LP/QCQP problem.

- For each combination of `criterion.tol`, matrix decomposition, etc., I estimated the bounds on 200 random samples of the full data. Each random sample consisted of 10,000 randomly drawn (with replacement) observations.

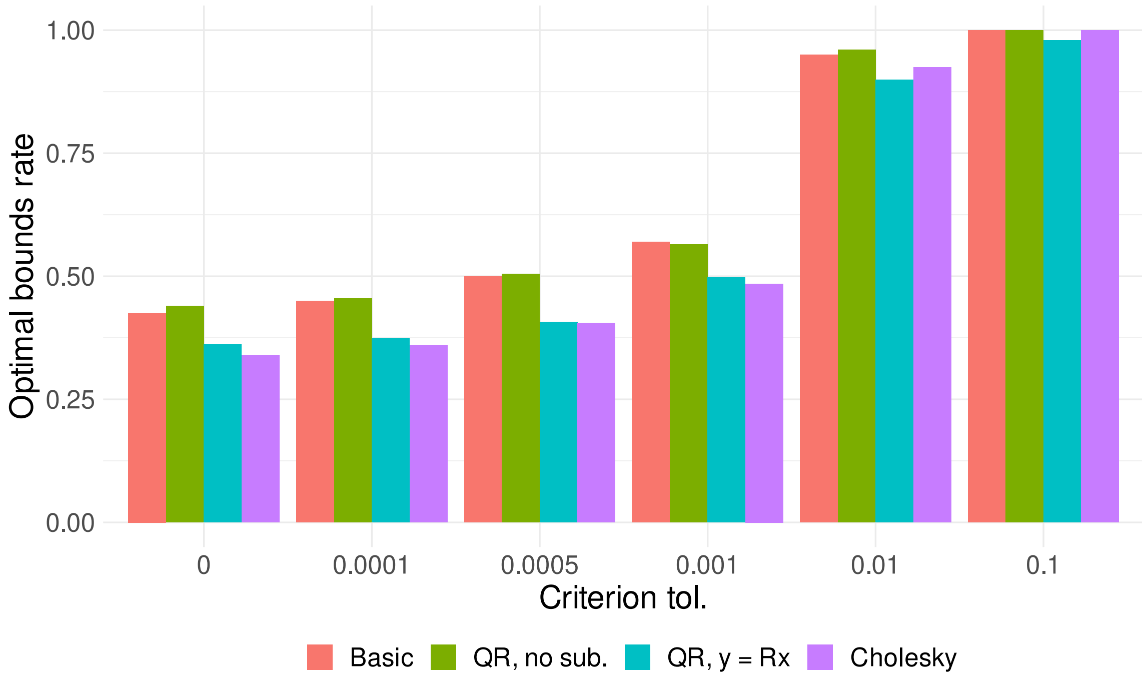
- The option `rescale` is always set to `FALSE`.

- Whenever splines were involved (test cases 1 and 4), I included the intercept terms in `uSplines`. Without these collinear intercepts, the original instability issues could not be replicated.

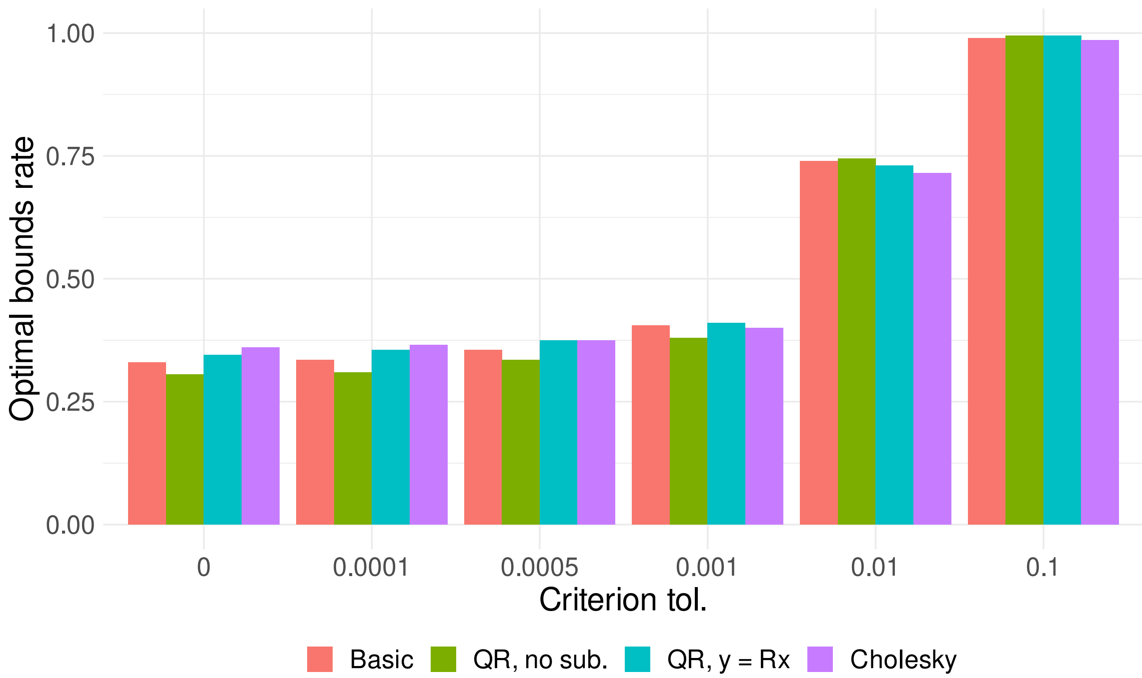
The code for the simulation is available on the GitHub issue #227 thread.

2 Optimal upper and lower bounds

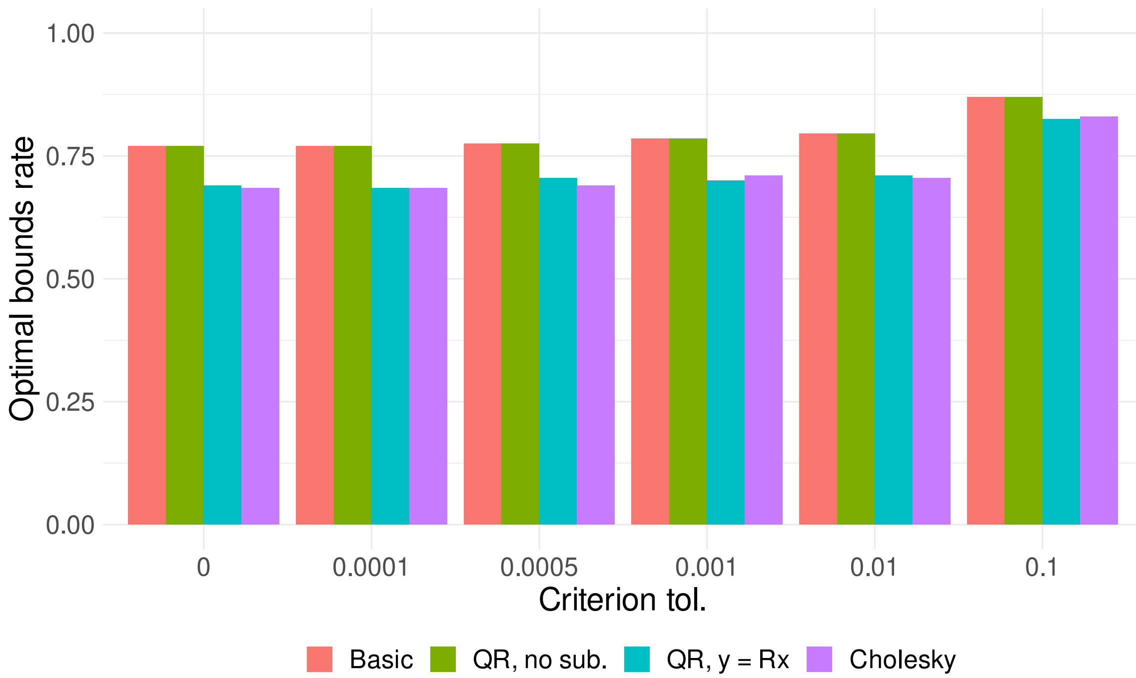
2.1 Case 1, LP



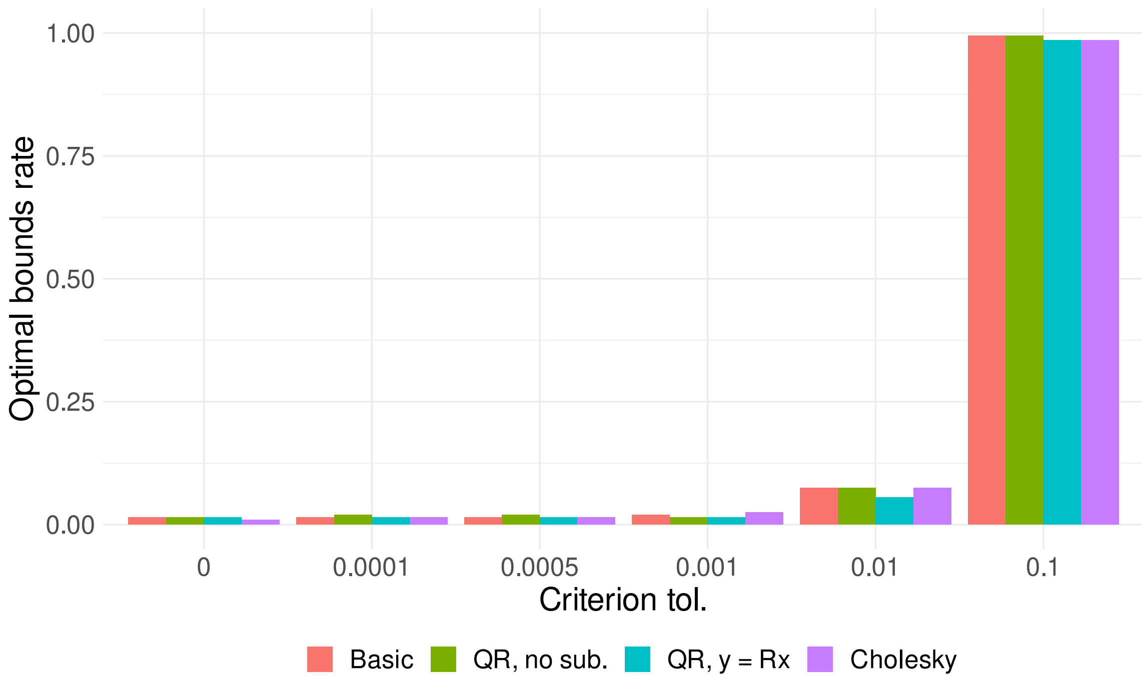
2.2 Case 2, LP


Optimal bounds rate
1.00
0.75
0.50
0.25
0.00
0
0.0001
0.0005
0.001
0.01
0.1
Criterion tol.
Basic
QR, no sub.
QR, y = Rx
Cholesky

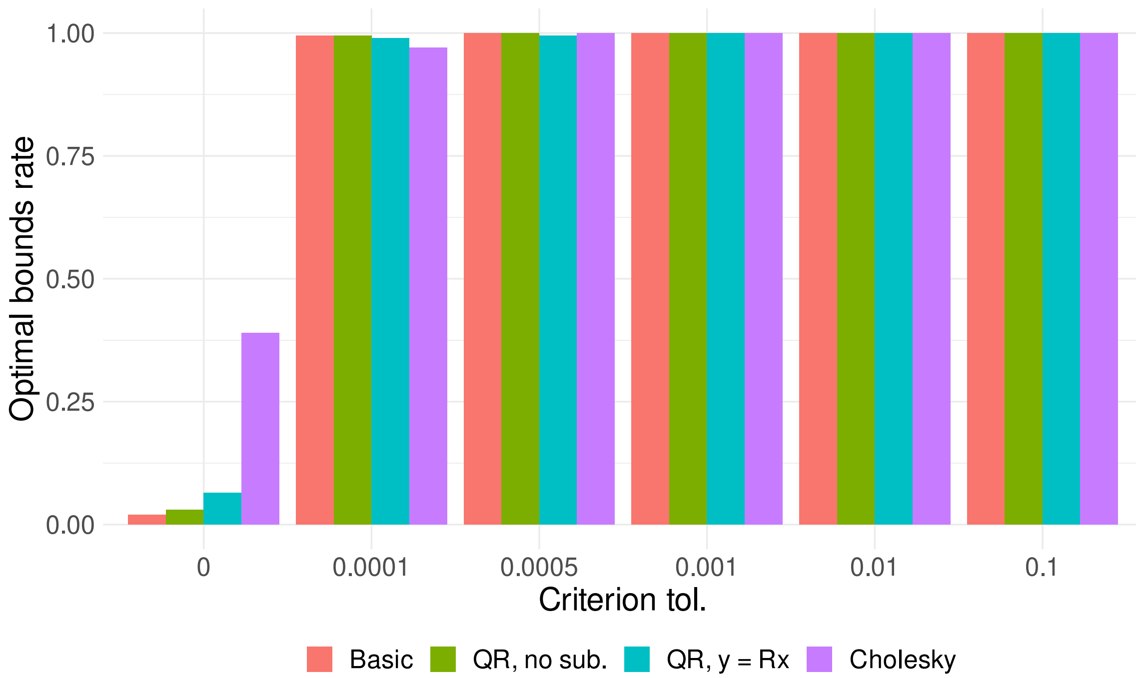
2.3 Case 3, LP


1.00
0.75
0.50
0.25
0.00
Optimal bounds rate
0
0.0001
0.0005
0.001
0.01
0.1
Criterion tol.
Basic
QR, no sub.
QR, y = Rx
Cholesky

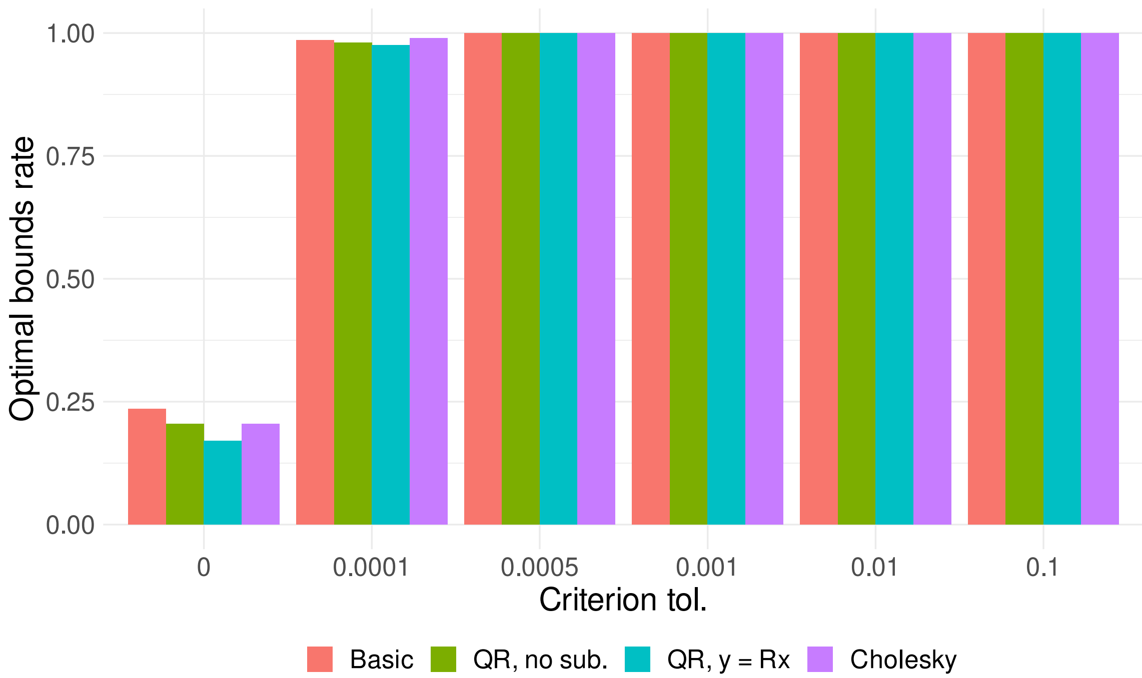
2.4 Case 4, LP


Optimal bounds rate
1.00
0.75
0.50
0.25
0.00
0
0.0001
0.0005
0.001
0.01
0.1
Criterion tol.
Basic
QR, no sub.
QR, y = Rx
Cholesky

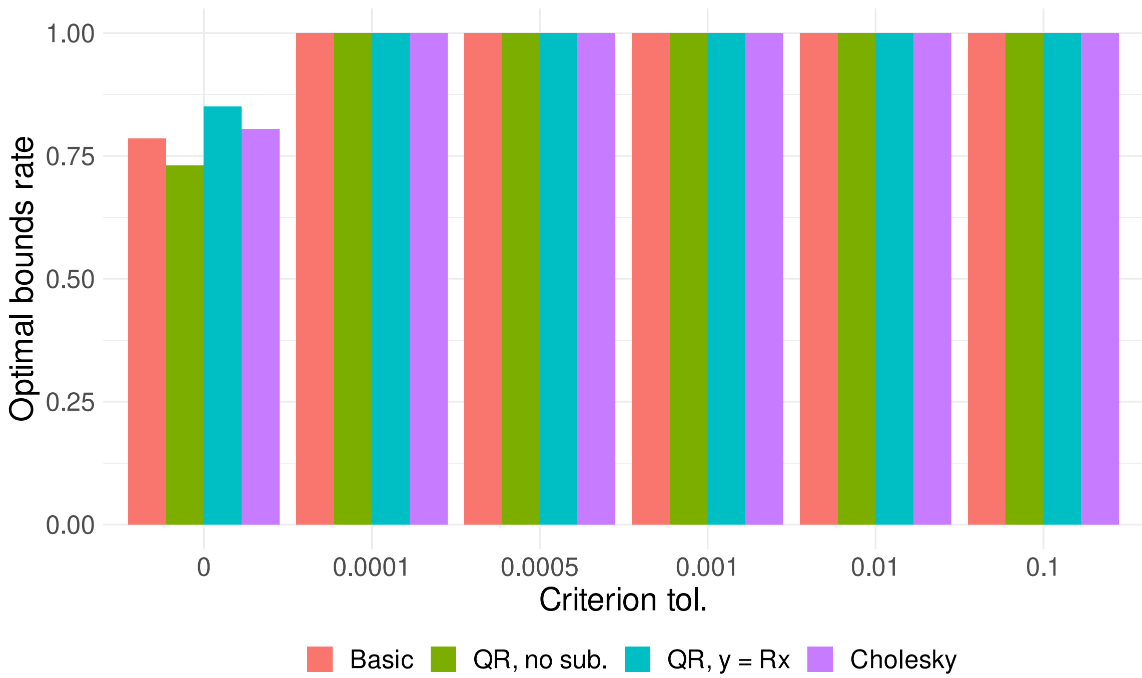
2.5 Case 1, QCQP



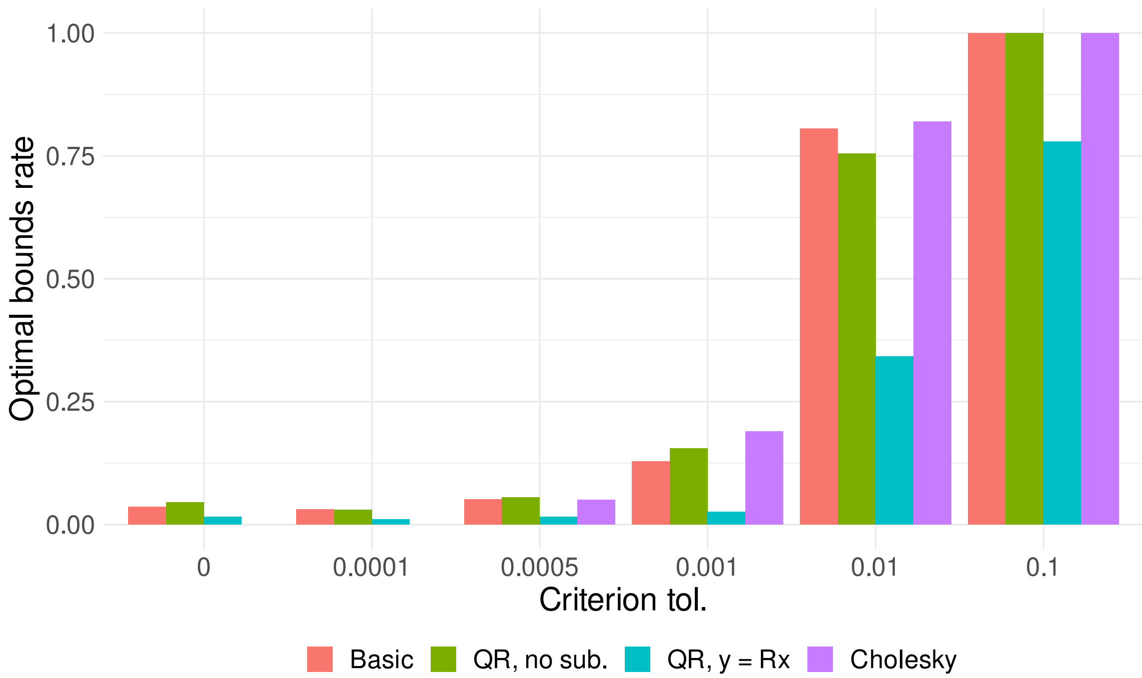
2.6 Case 2, QCQP



2.7 Case 3, QCQP

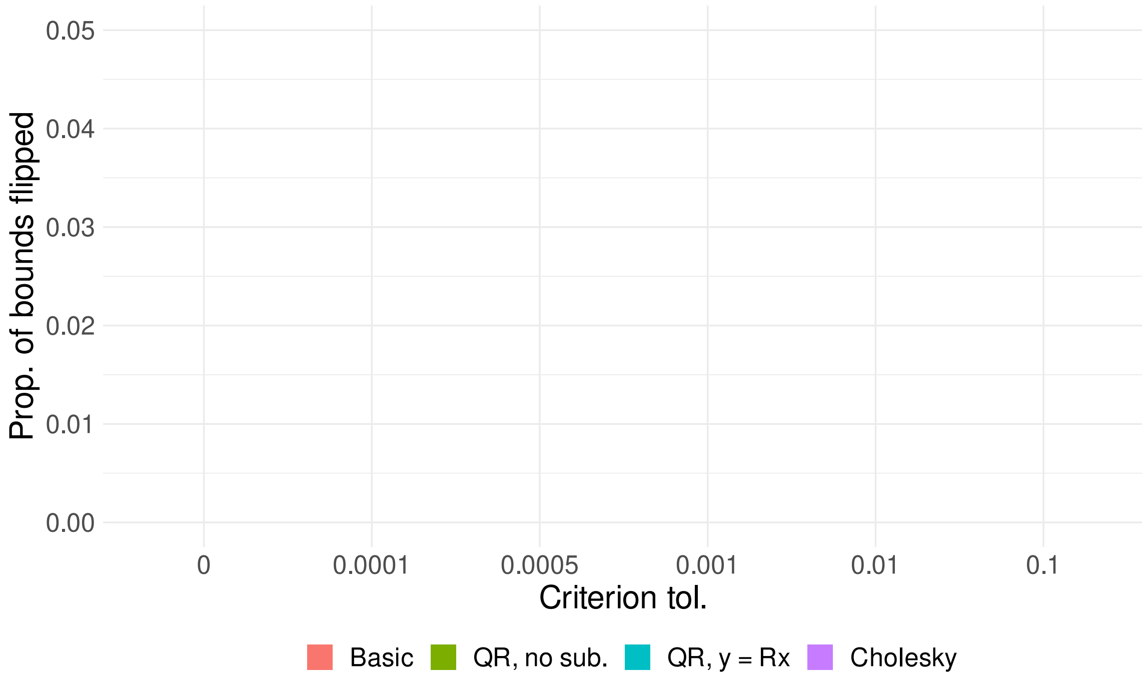


2.8 Case 4, QCQP

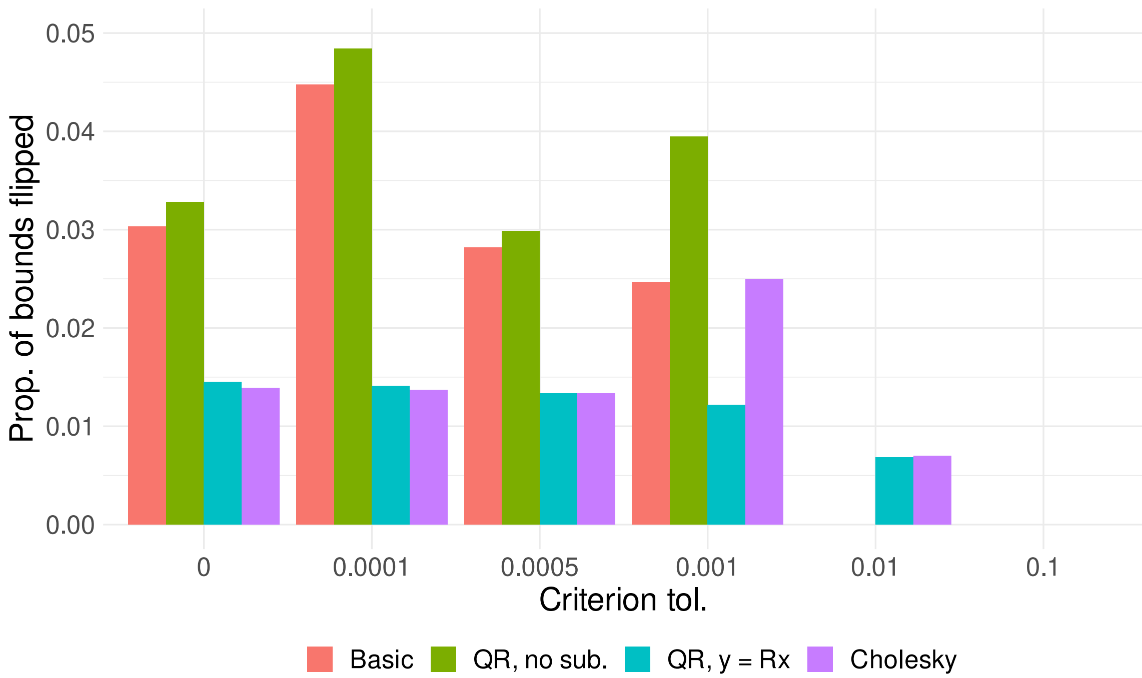


3 Upper bound exceeds lower bound

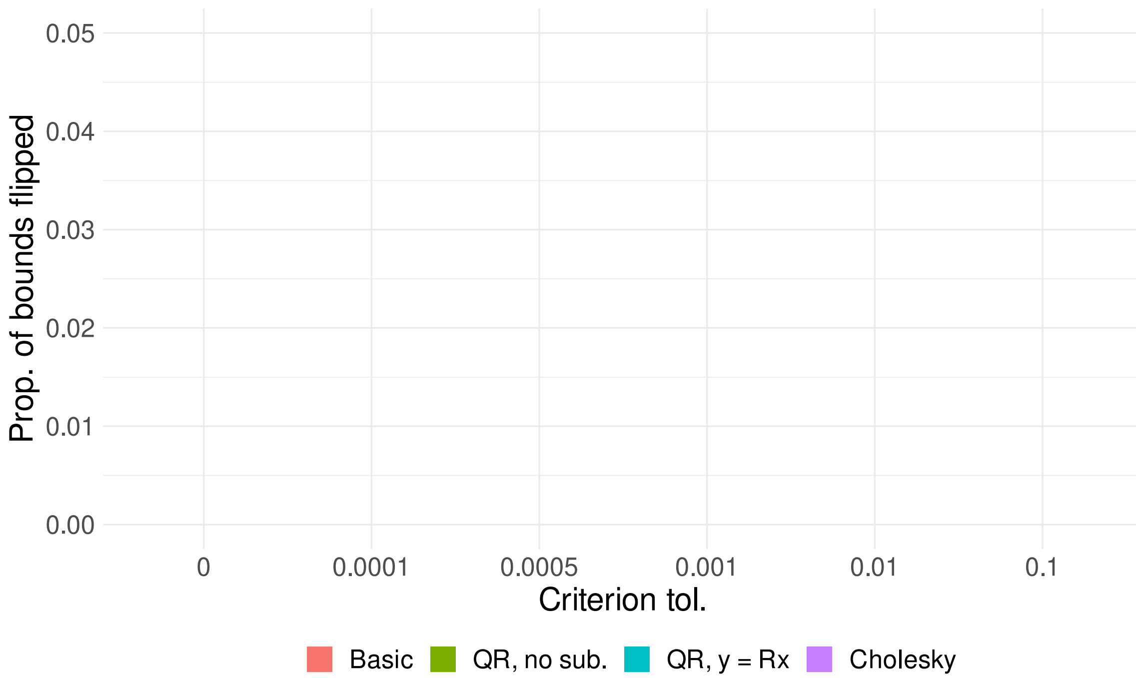
3.1 Case 1, LP



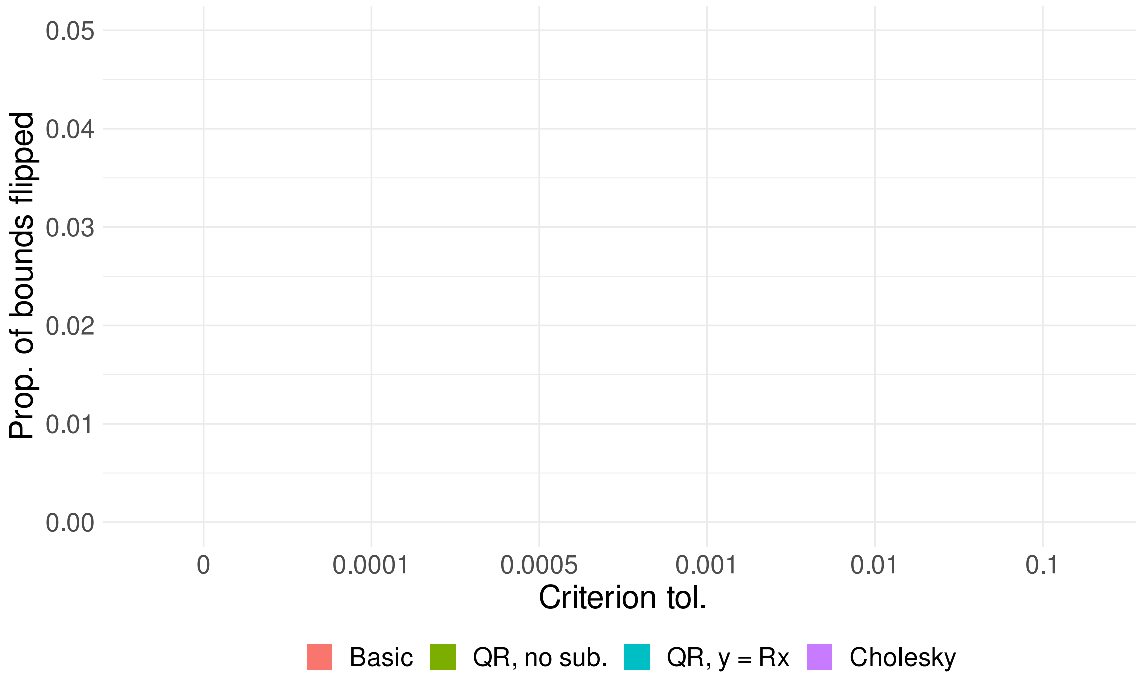
3.2 Case 2, LP


0.05
0.04
0.03
0.02
0.01
0.00
Prop. of bounds flipped
0
0.0001
0.0005
0.001
0.01
0.1
Criterion tol.
Basic
QR, no sub.
QR, y = Rx
Cholesky

3.3 Case 3, LP


Prop. of bounds flipped
0.05
0.04
0.03
0.02
0.01
0.00
0
0.0001
0.0005
0.001
0.01
0.1
Criterion tol.
Basic
QR, no sub.
QR, y = Rx
Cholesky

3.4 Case 4, LP


0.05
0.04
0.03
0.02
0.01
0.00
Prop. of bounds flipped
0
0.0001
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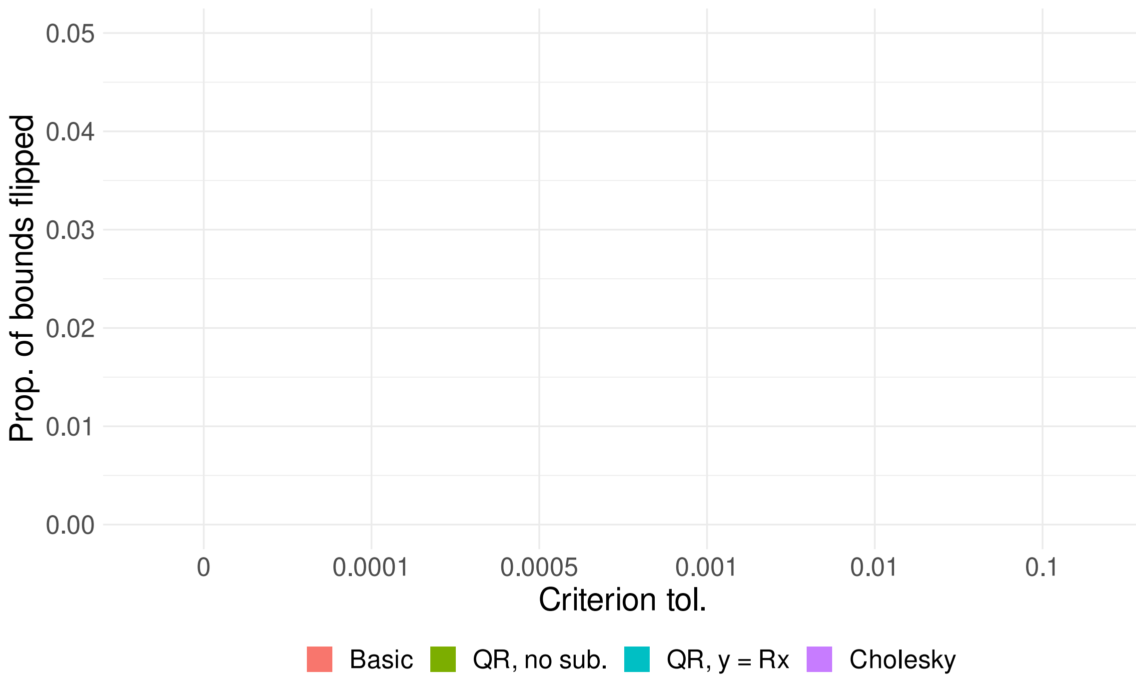
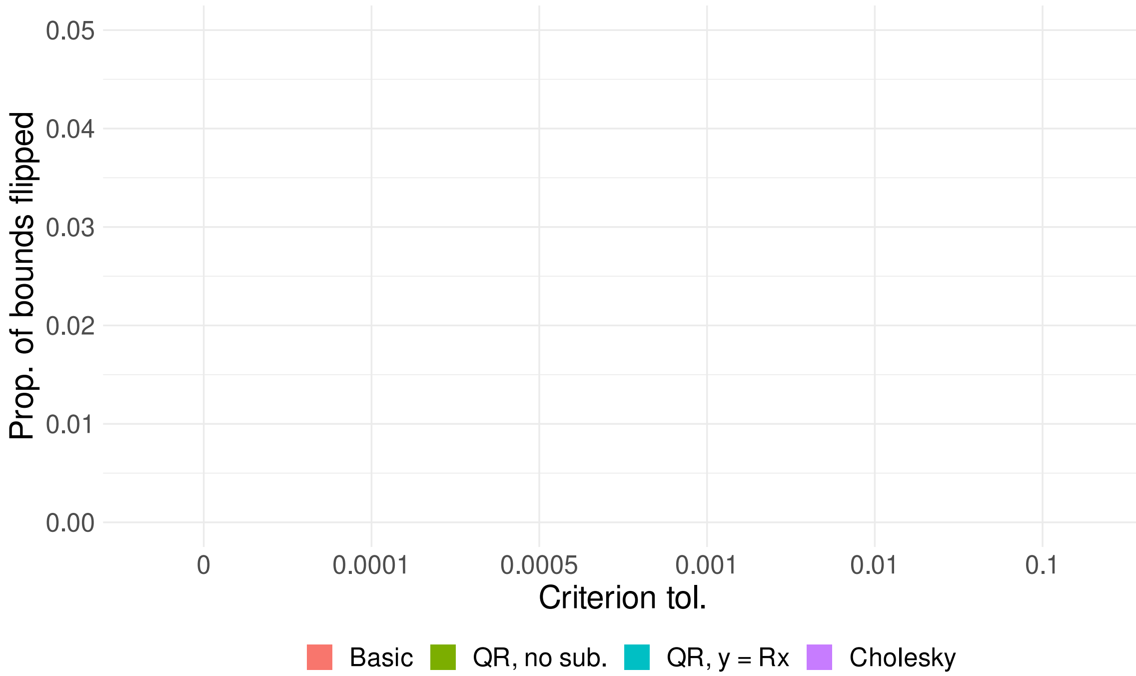
0.01
0.1
Criterion tol.
Basic
QR, no sub.
QR, y = Rx
Cholesky

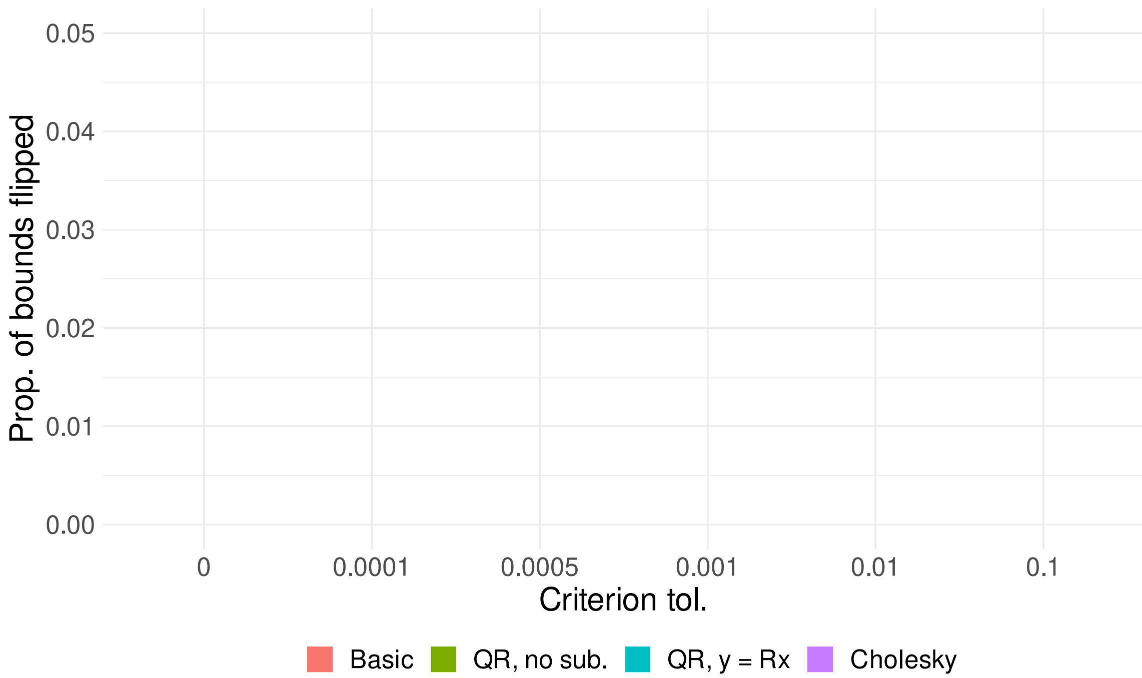
3.5 Case 1, QCQP



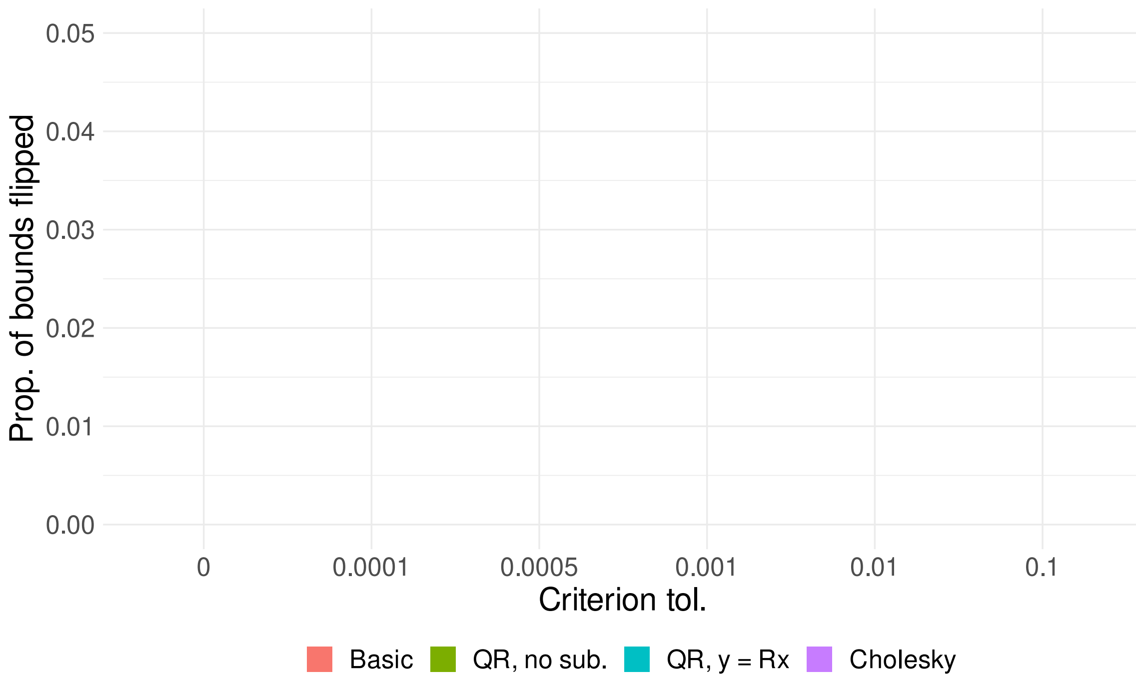
3.6 Case 2, QCQP



3.7 Case 3, QCQP

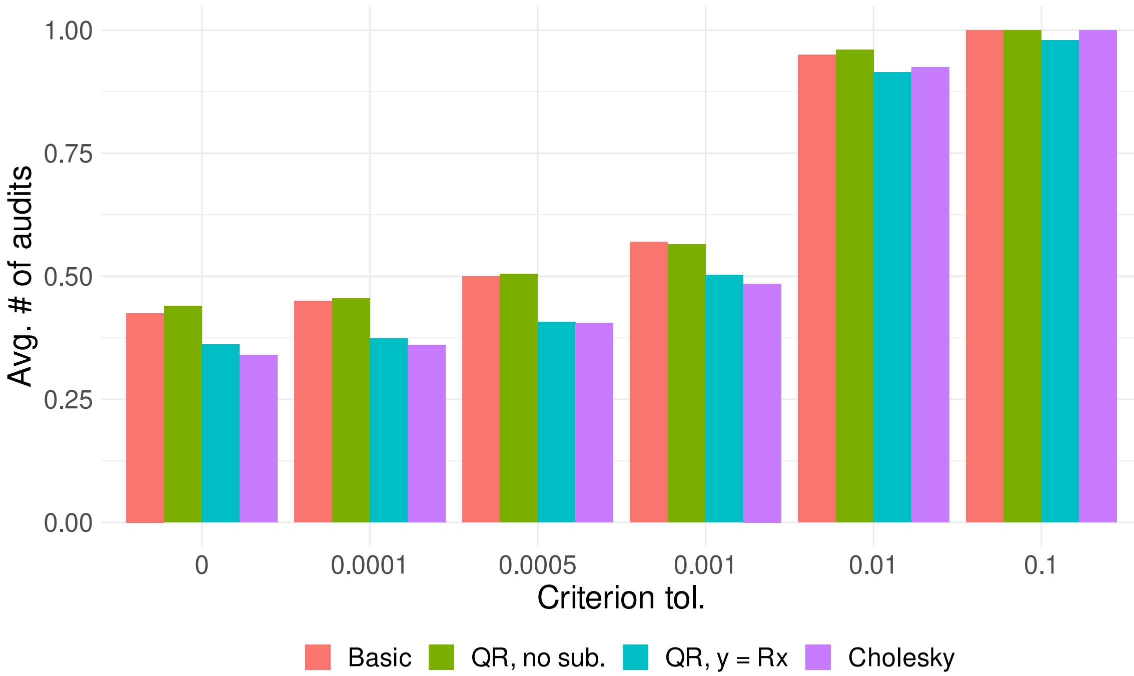


3.8 Case 4, QCQP



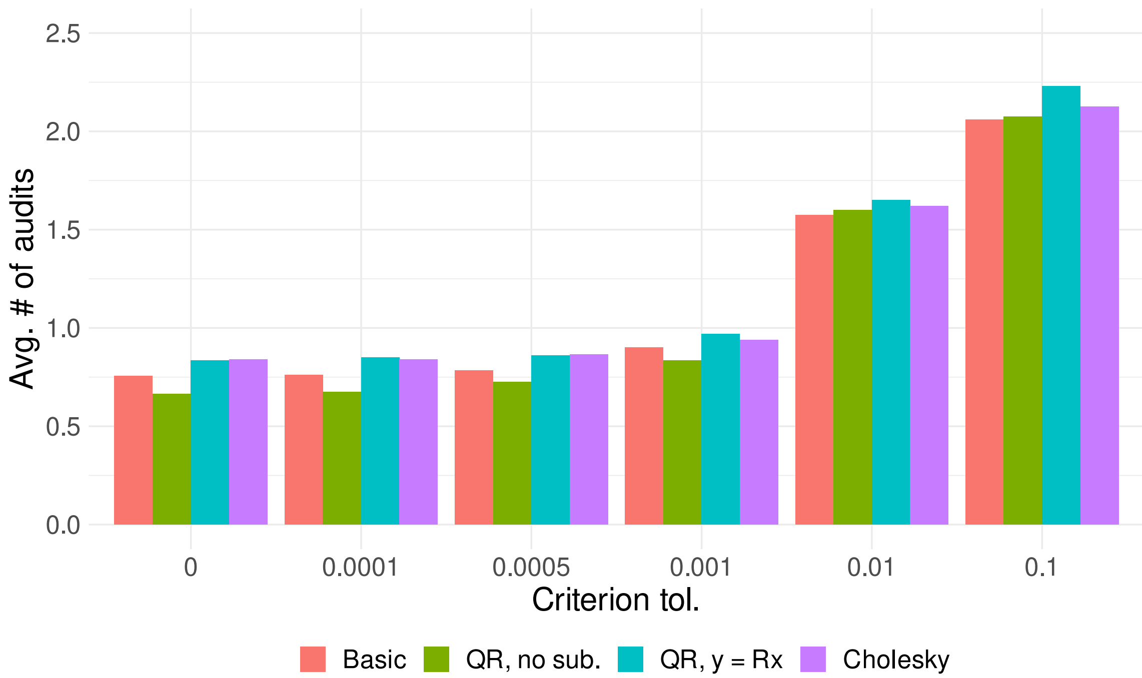
4 Average number of successful audits before termination

4.1 Case 1, LP



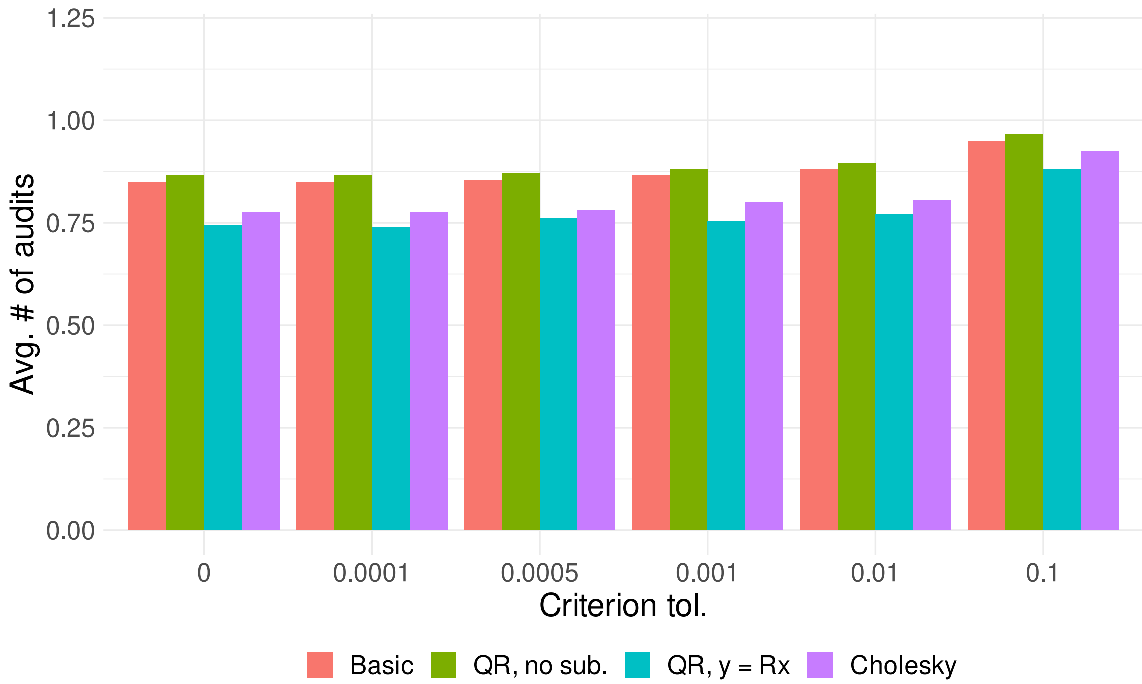
Note: There are 0 successful audits if the `ivmte` is runs into an error (infeasibility, unboundedness, numerical issues) on the very first audit.

4.2 Case 2, LP



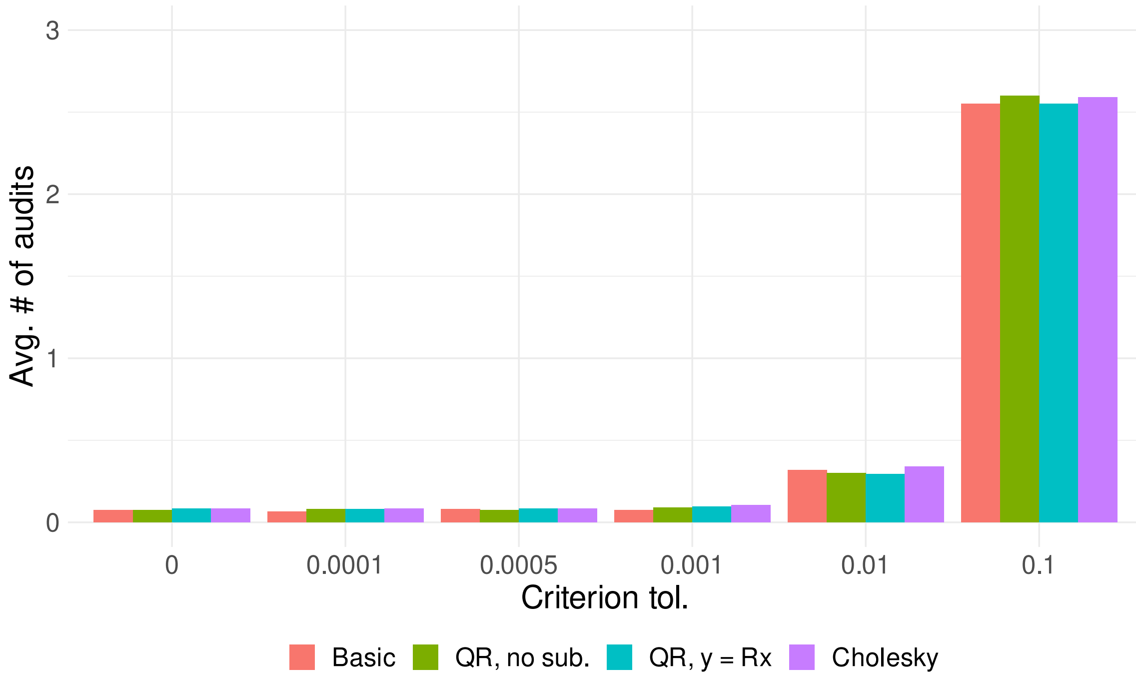
Note: There are 0 successful audits if the `ivmte` is runs into an error (infeasibility, unboundedness, numerical issues) on the very first audit.

4.3 Case 3, LP



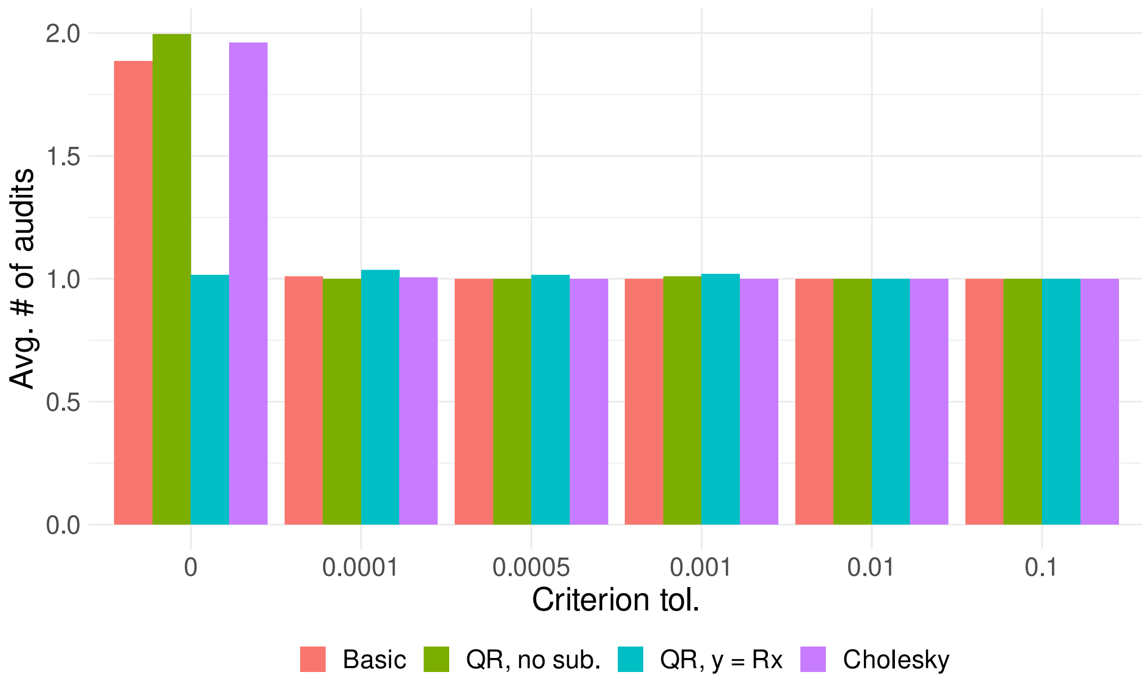
Note: There are 0 successful audits if the `ivmte` is runs into an error (infeasibility, unboundedness, numerical issues) on the very first audit.

4.4 Case 4, LP



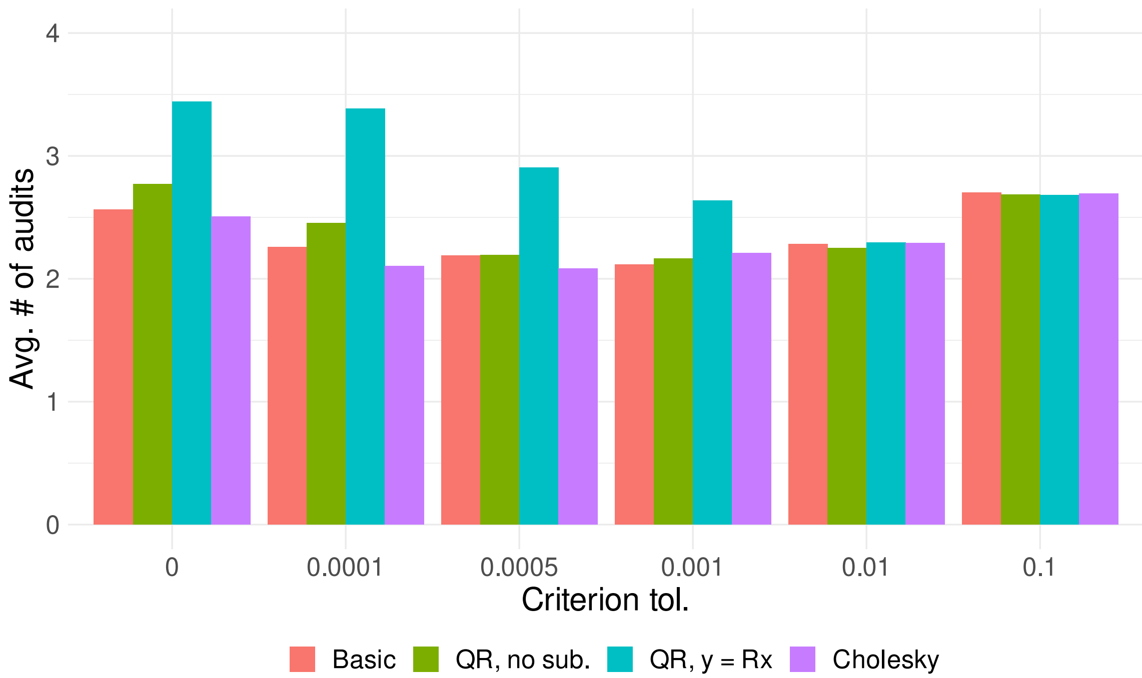
Note: There are 0 successful audits if the `ivmte` is runs into an error (infeasibility, unboundedness, numerical issues) on the very first audit.

4.5 Case 1, QCQP



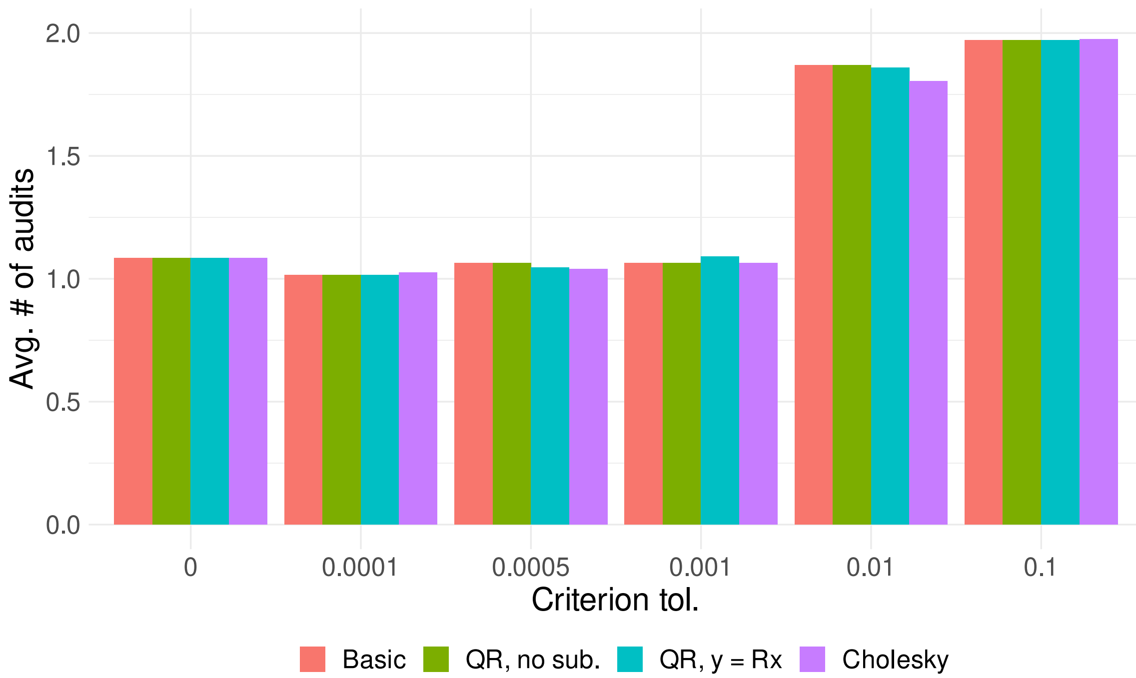
Note: There are 0 successful audits if the `ivmte` is runs into an error (infeasibility, unboundedness, numerical issues) on the very first audit.

4.6 Case 2, QCQP



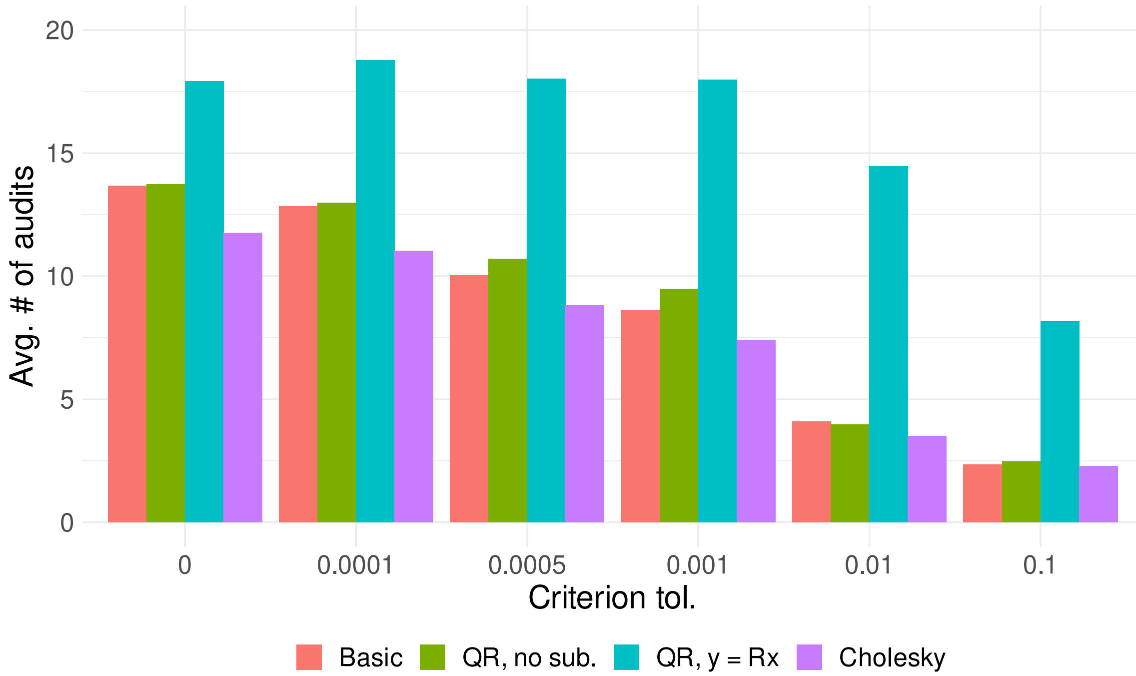
Note: There are 0 successful audits if the `ivmte` is runs into an error (infeasibility, unboundedness, numerical issues) on the very first audit.

4.7 Case 3, QCQP



Note: There are 0 successful audits if the `ivmte` is runs into an error (infeasibility, unboundedness, numerical issues) on the very first audit.

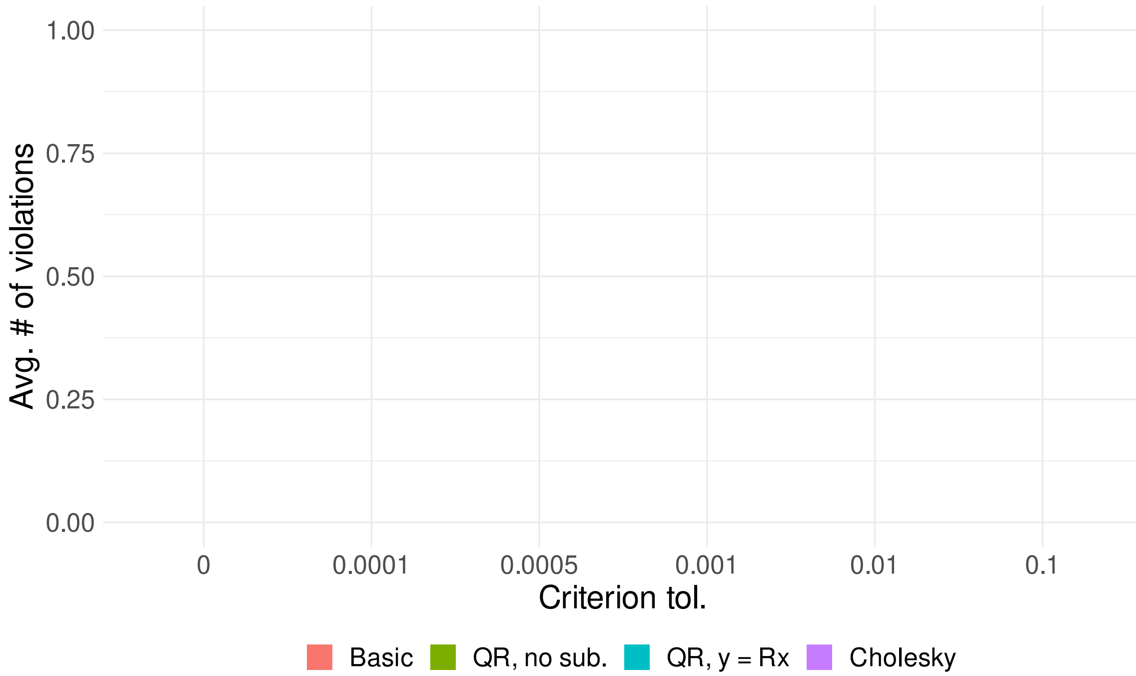
4.8 Case 4, QCQP



Note: There are 0 successful audits if the `ivmte` is runs into an error (infeasibility, unboundedness, numerical issues) on the very first audit.

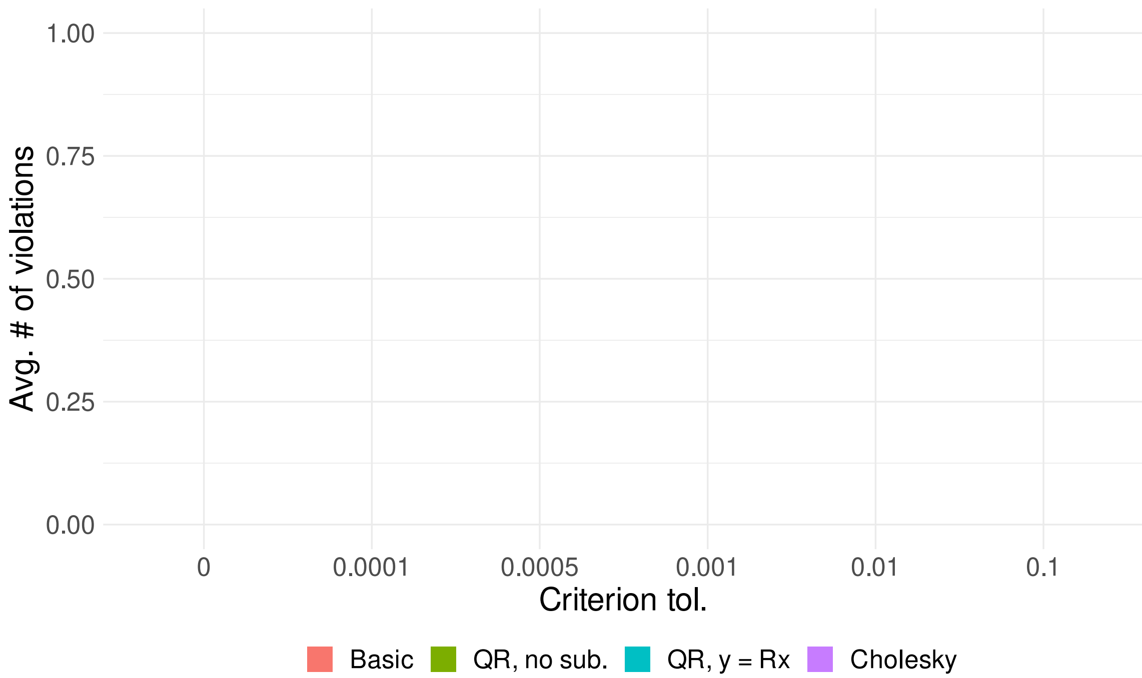
5 Average number remaining violations

5.1 Case 1, LP



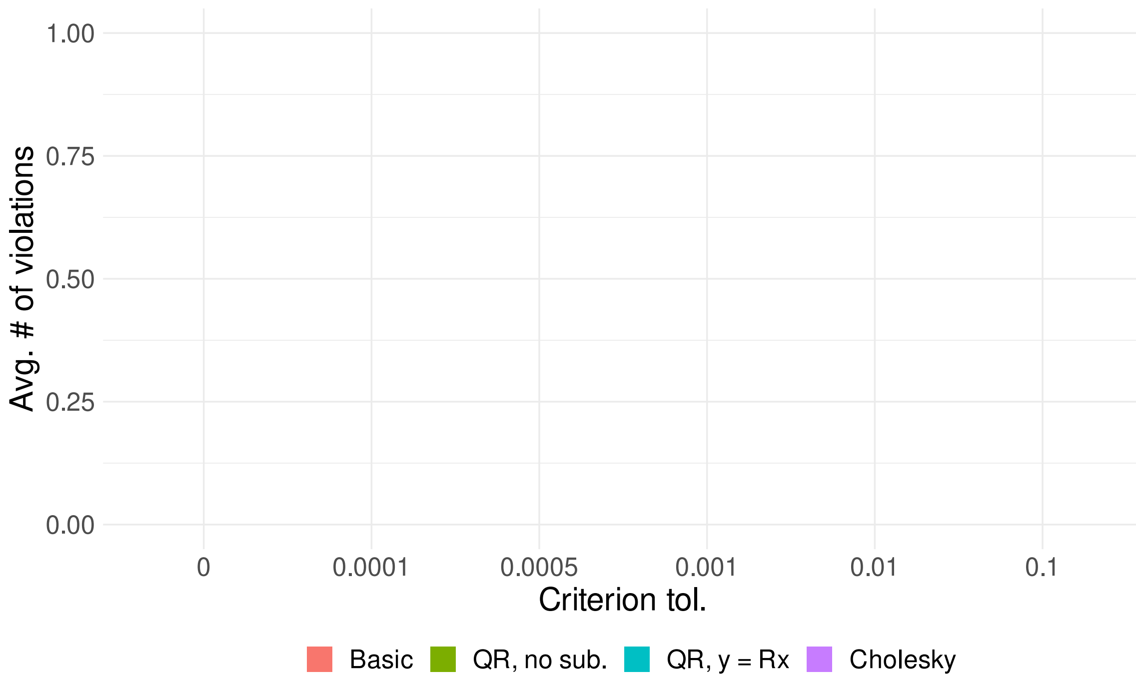
Note: If a set of violations cannot be resolved after three audits, `ivmte` terminates the estimation procedure and returns the estimated bounds.

5.2 Case 2, LP



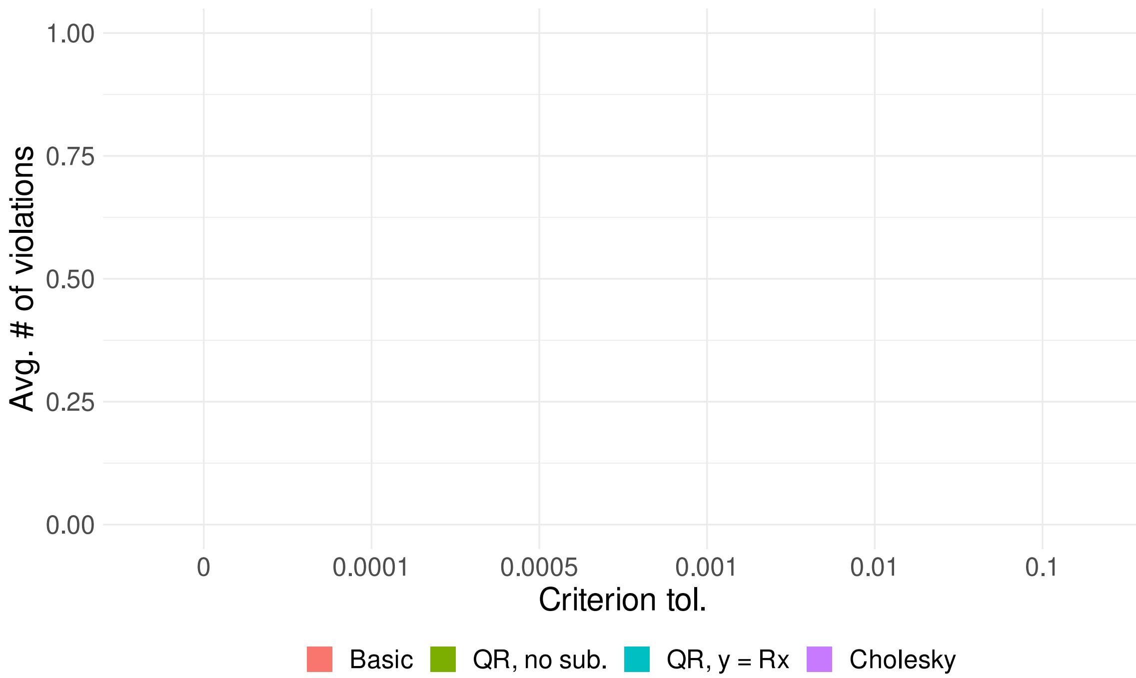
Note: If a set of violations cannot be resolved after three audits, `ivmte` terminates the estimation procedure and returns the estimated bounds.

5.3 Case 3, LP



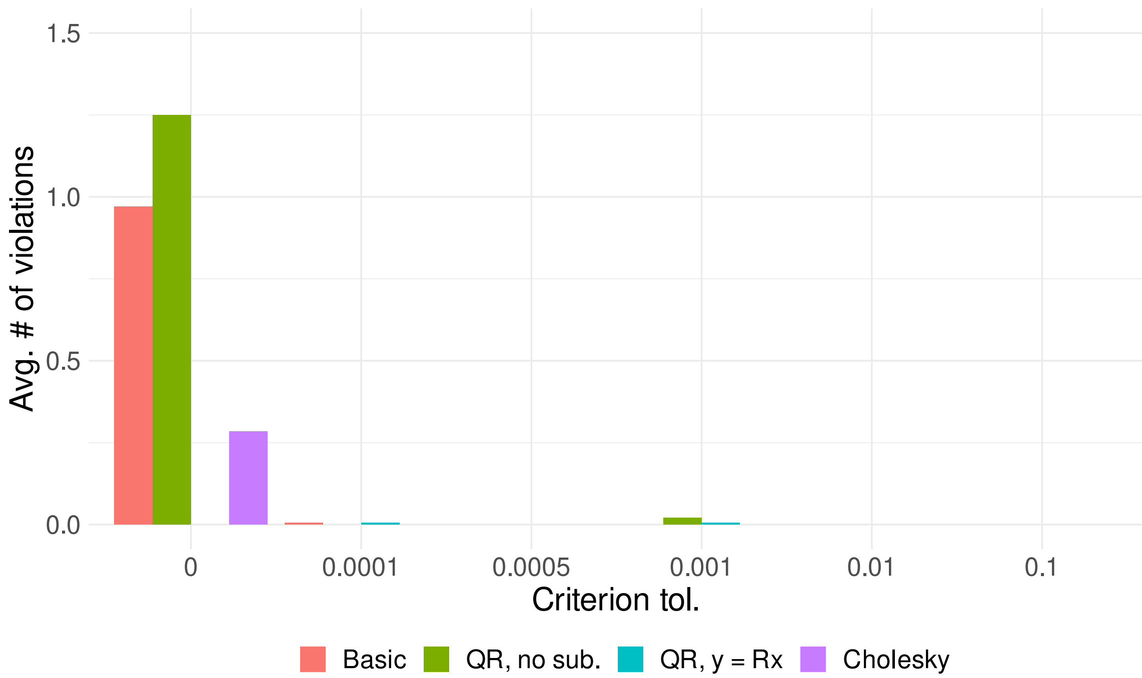
Note: If a set of violations cannot be resolved after three audits, `ivmte` terminates the estimation procedure and returns the estimated bounds.

5.4 Case 4, LP



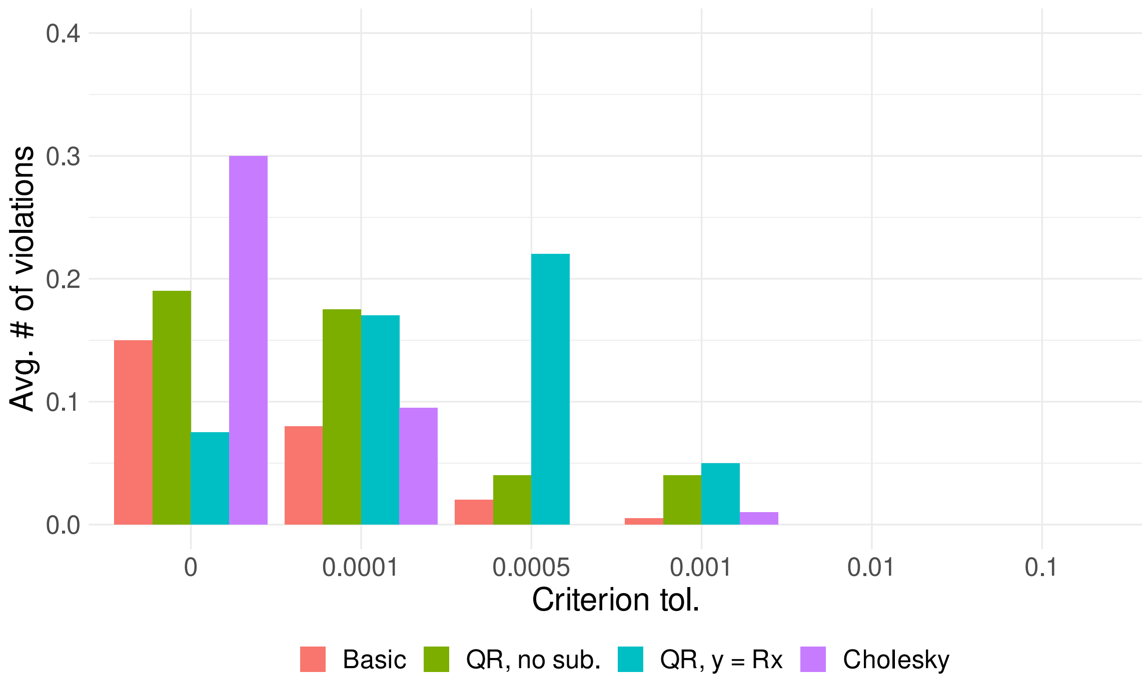
Note: If a set of violations cannot be resolved after three audits, `ivmte` terminates the estimation procedure and returns the estimated bounds.

5.5 Case 1, QCQP



Note: If a set of violations cannot be resolved after three audits, `ivmte` terminates the estimation procedure and returns the estimated bounds.

5.6 Case 2, QCQP



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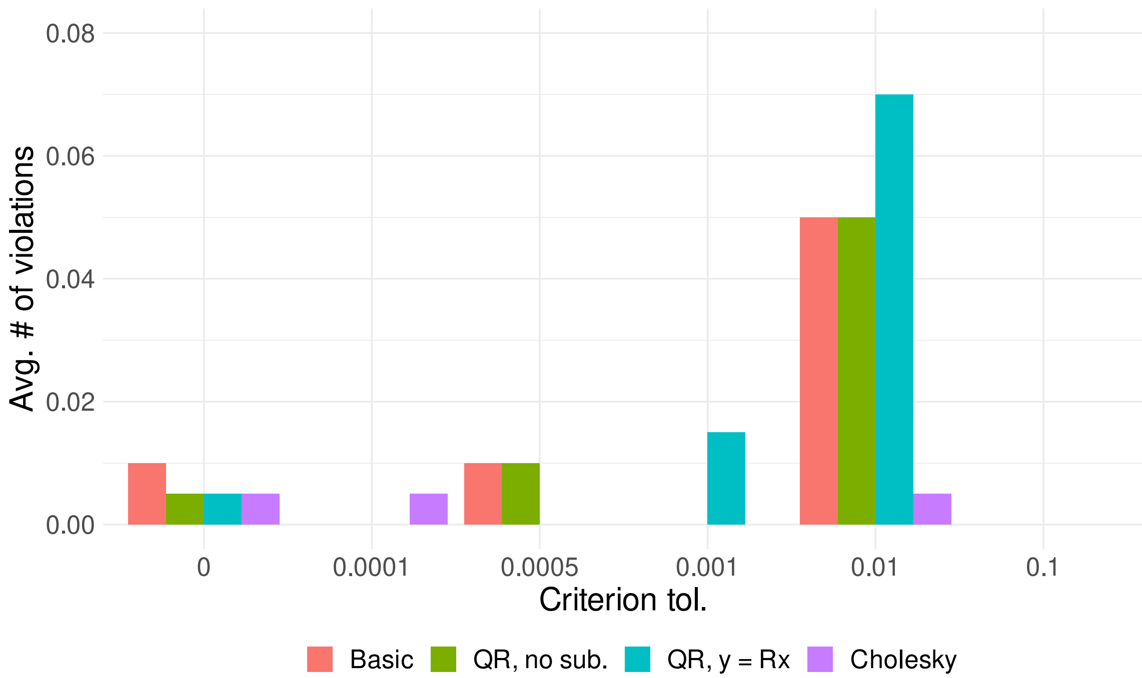
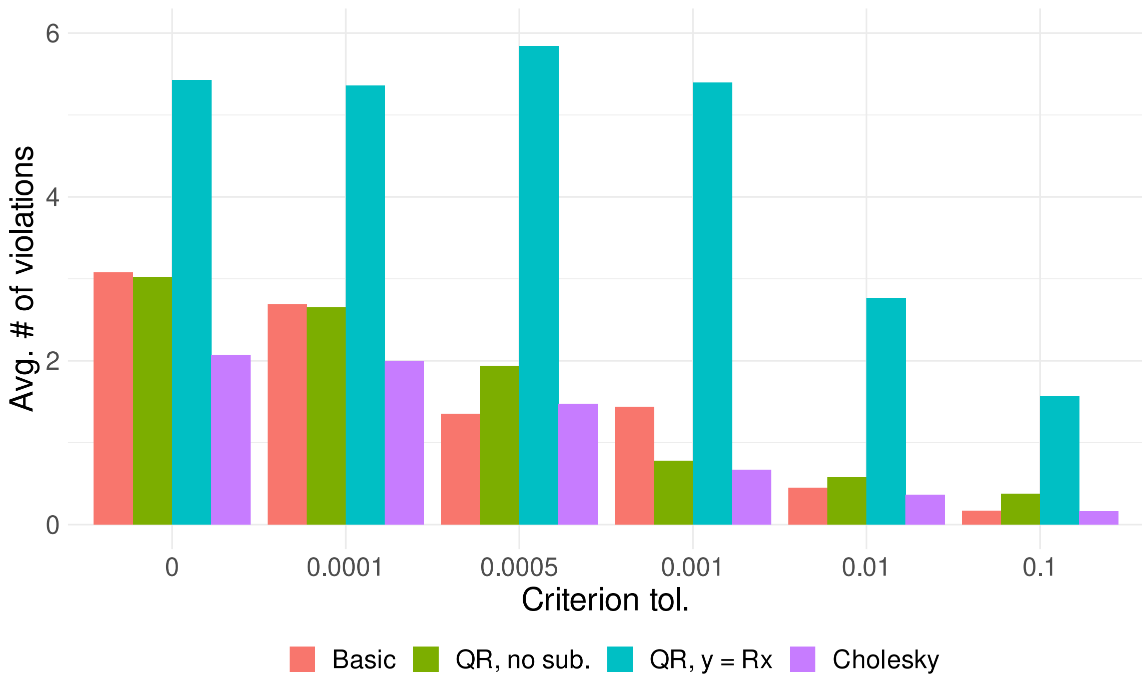
5.7 Case 3, QCQP



Note: If a set of violations cannot be resolved after three audits, `ivmte` terminates the estimation procedure and returns the estimated bounds.

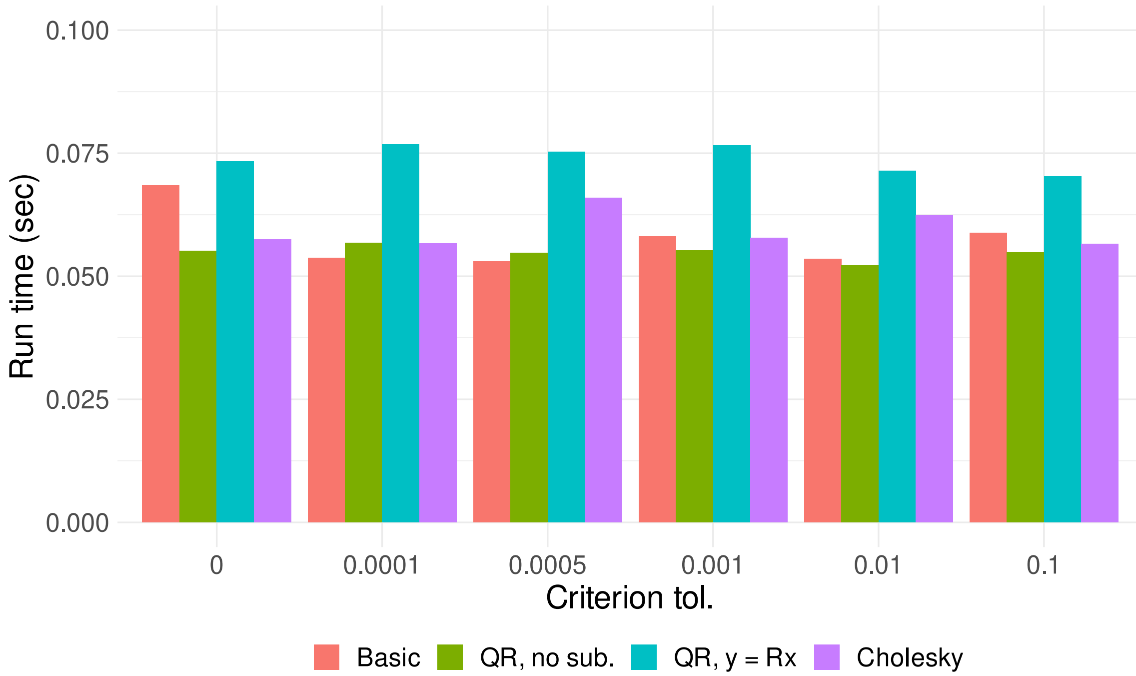
5.8 Case 4, QCQP



Note: If a set of violations cannot be resolved after three audits, ivmte terminates the estimation procedure and returns the estimated bounds.

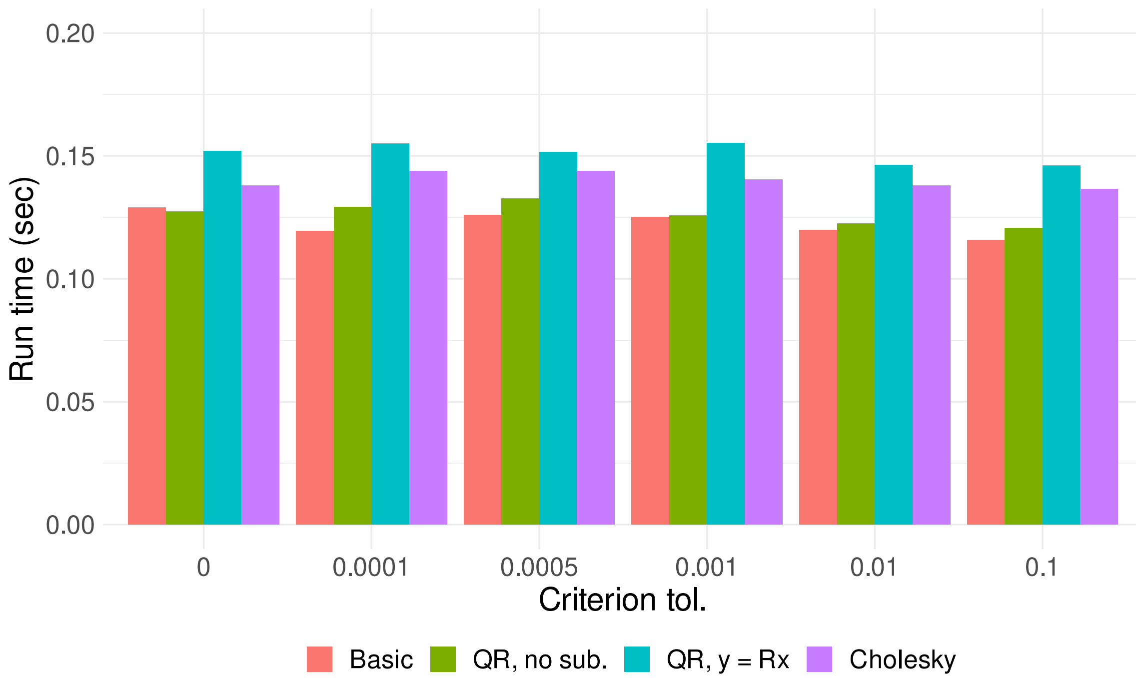
6 Average Gurobi run time

6.1 Case 1, LP



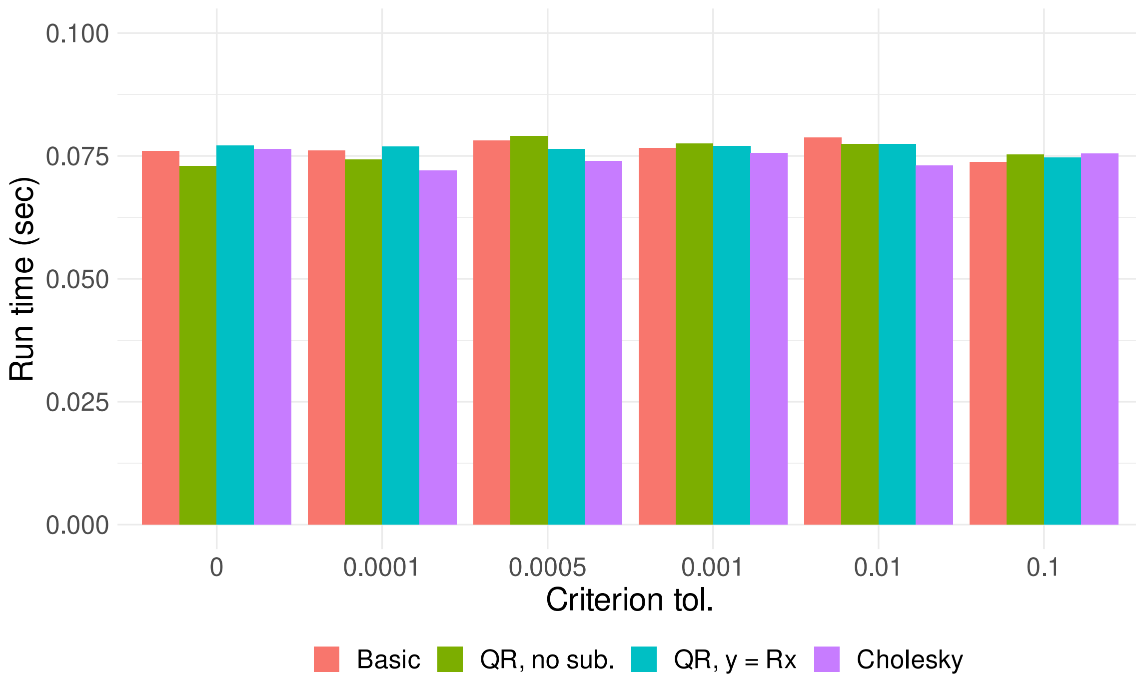
Note: Gurobi run time includes the time taken to minimize the criterion, and obtain the lower and upper bound estimates.

6.2 Case 2, LP



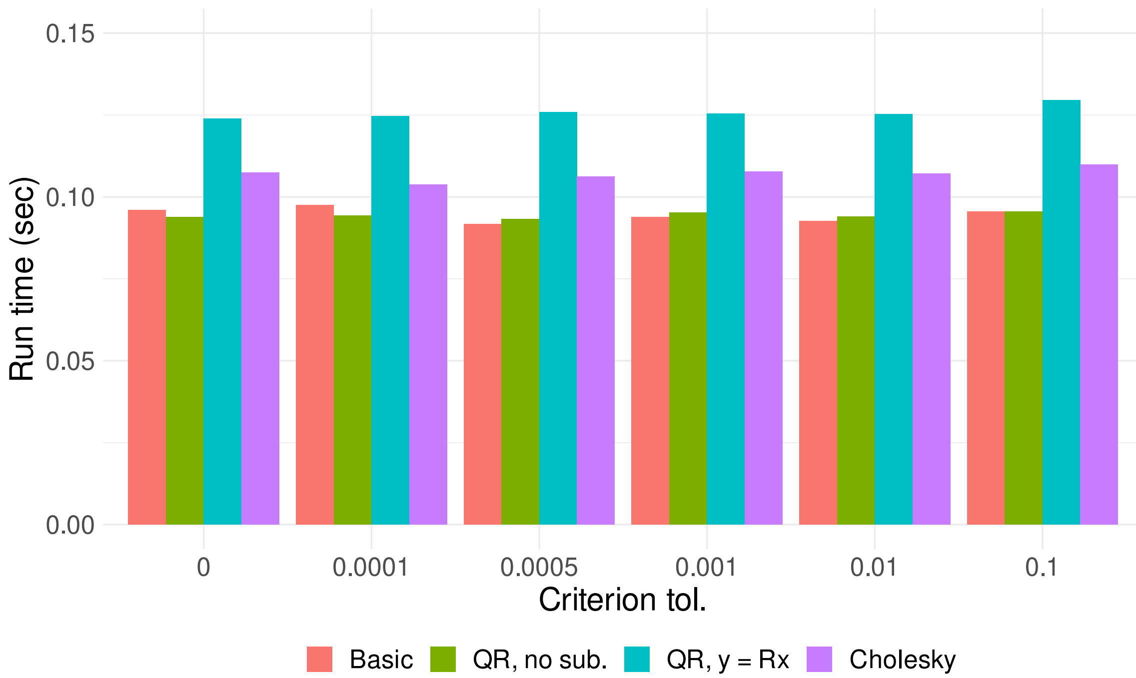
Note: Gurobi run time includes the time taken to minimize the criterion, and obtain the lower and upper bound estimates.

6.3 Case 3, LP



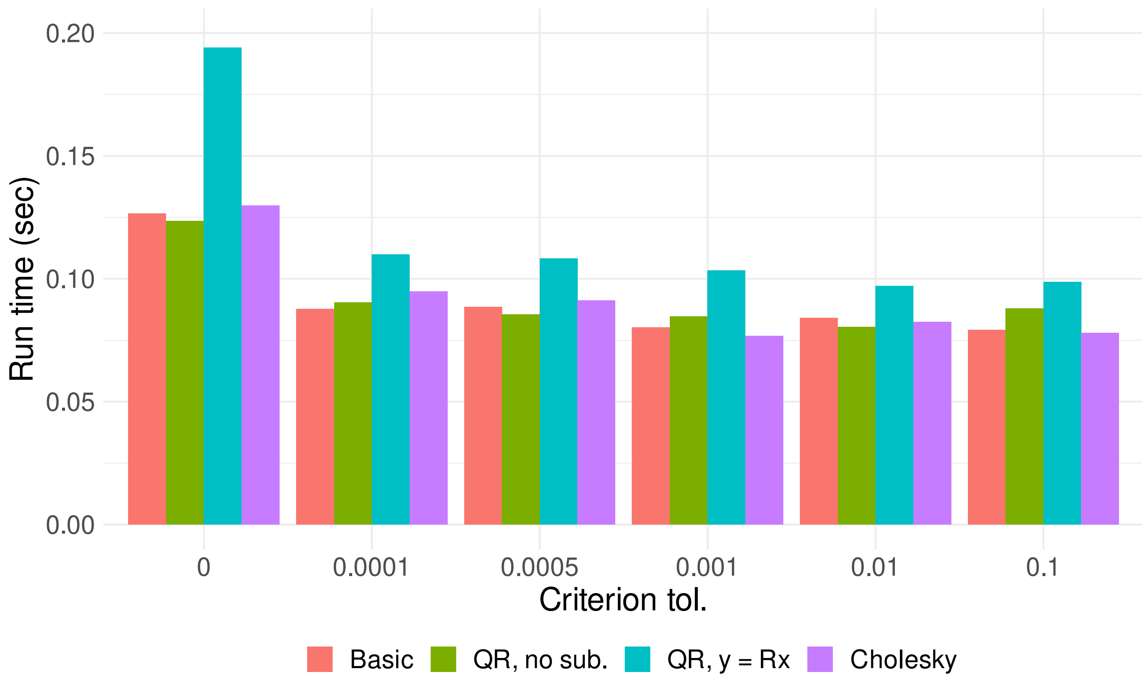
Note: Gurobi run time includes the time taken to minimize the criterion, and obtain the lower and upper bound estimates.

6.4 Case 4, LP



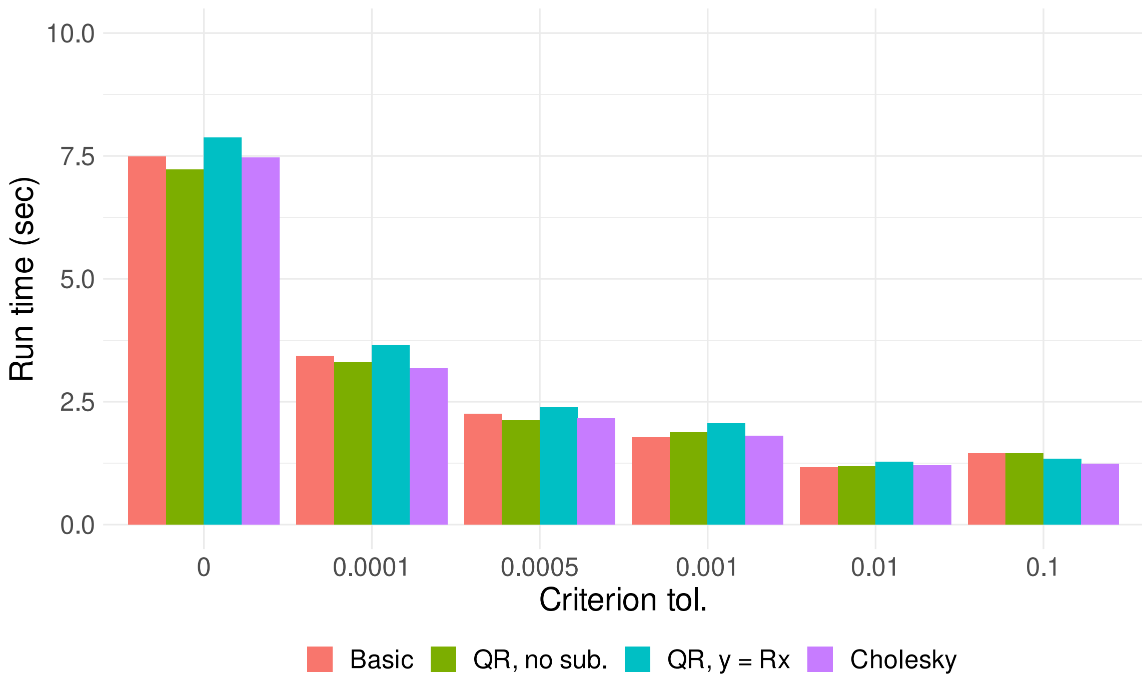
Note: Gurobi run time includes the time taken to minimize the criterion, and obtain the lower and upper bound estimates.

6.5 Case 1, QCQP



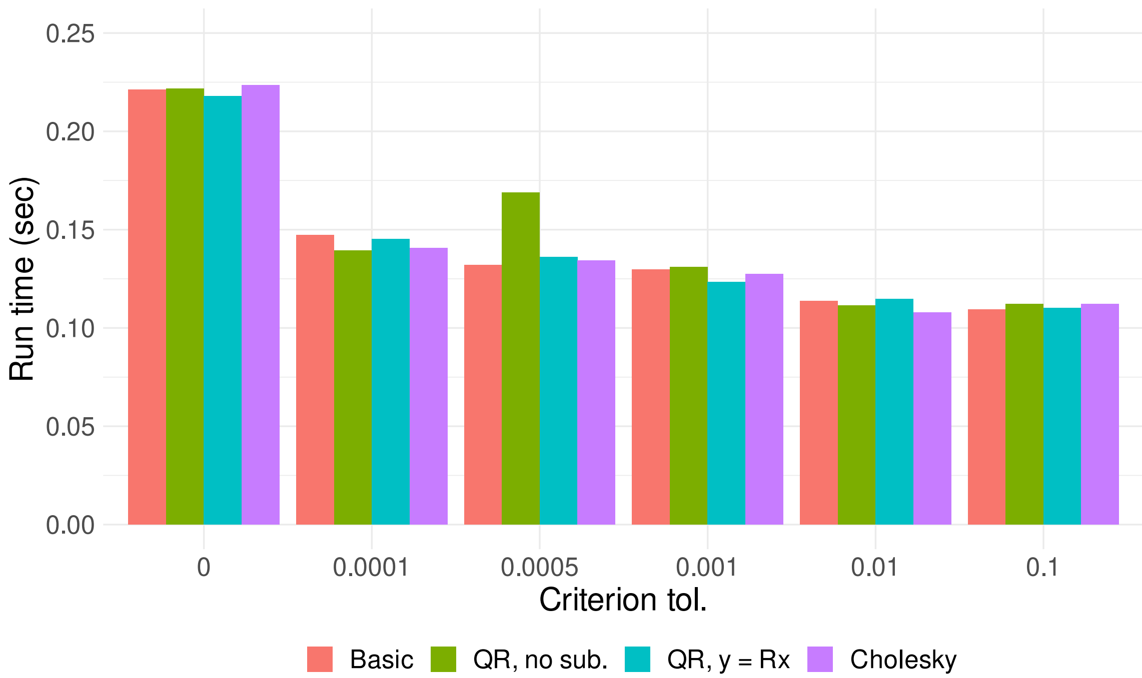
Note: Gurobi run time includes the time taken to minimize the criterion, and obtain the lower and upper bound estimates.

6.6 Case 2, QCQP



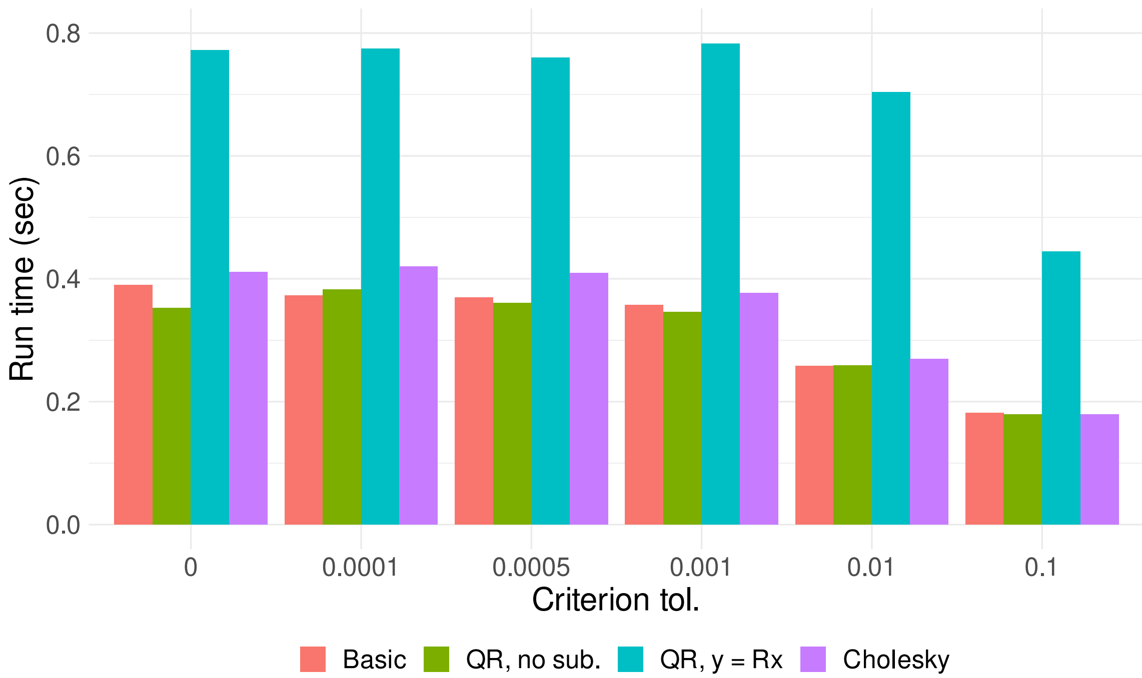
Note: Gurobi run time includes the time taken to minimize the criterion, and obtain the lower and upper bound estimates.

6.7 Case 3, QCQP



Note: Gurobi run time includes the time taken to minimize the criterion, and obtain the lower and upper bound estimates.

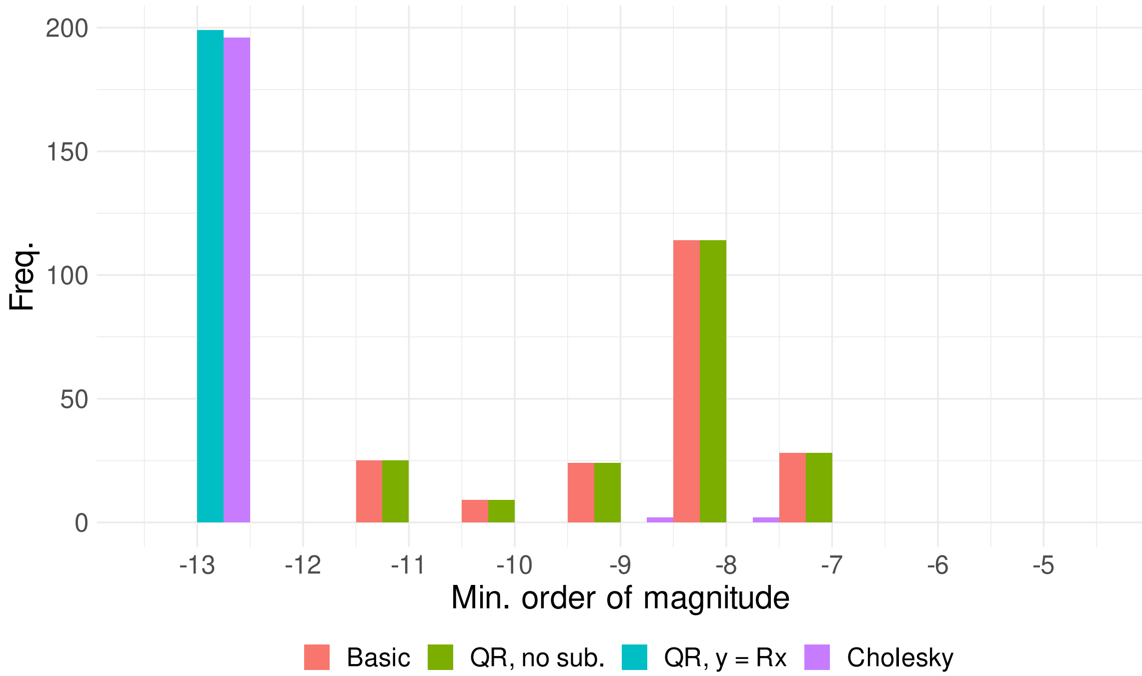
6.8 Case 4, QCQP



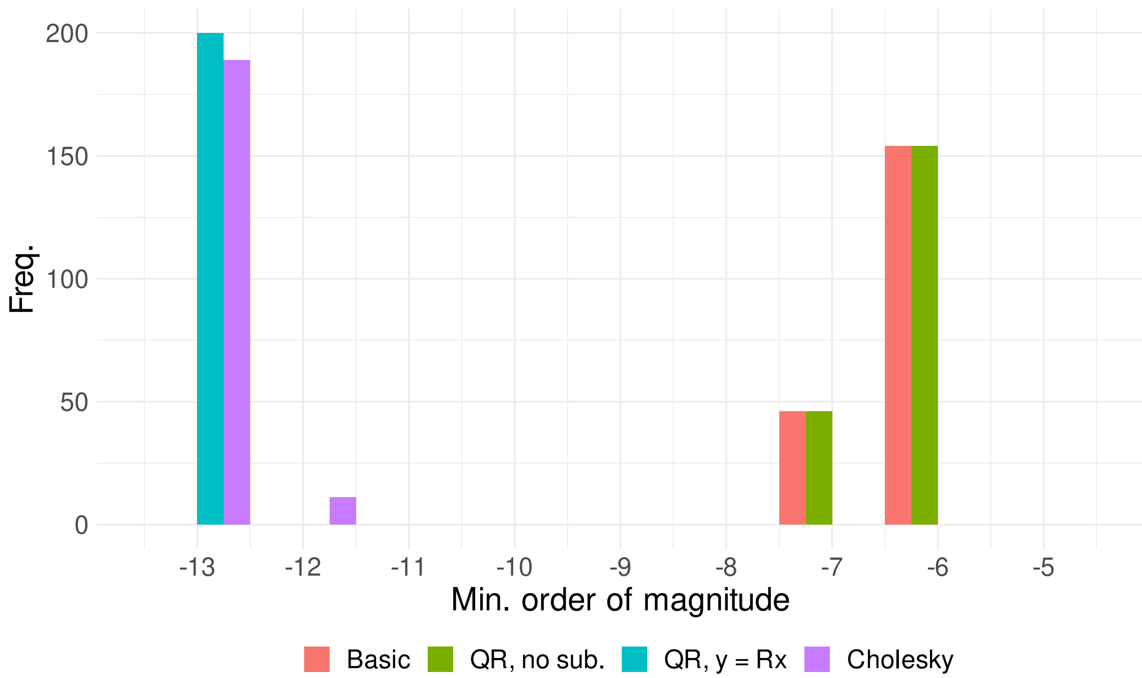
Note: Gurobi run time includes the time taken to minimize the criterion, and obtain the lower and upper bound estimates.

7 Minimum order of magnitude

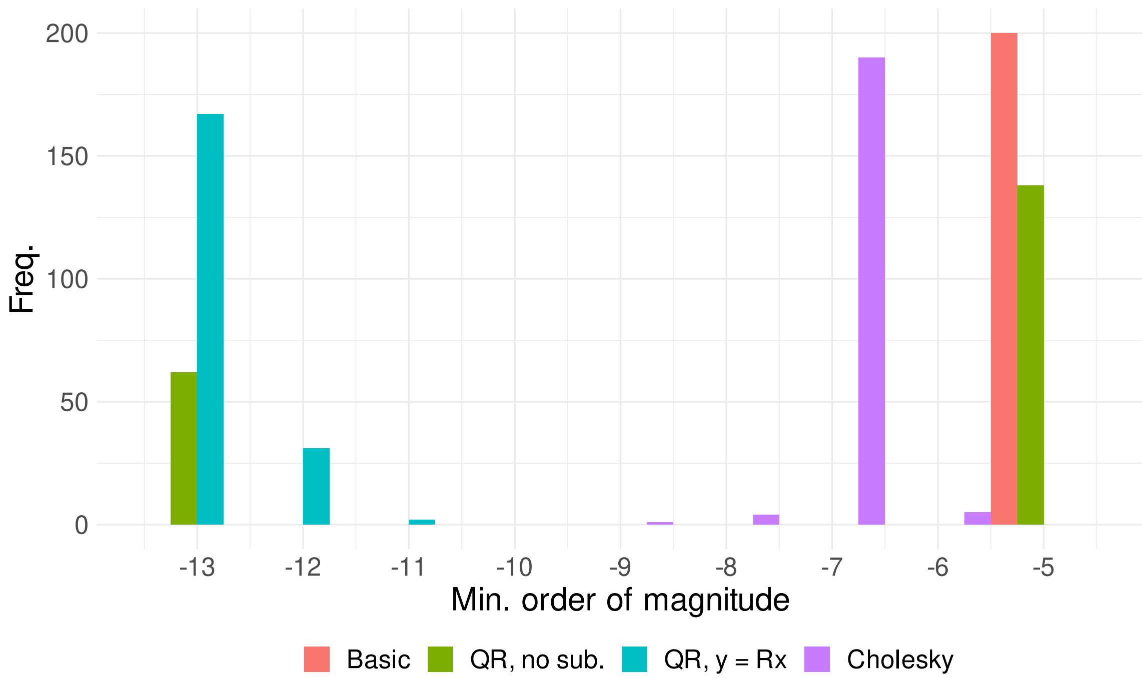
7.1 Case 1, LP



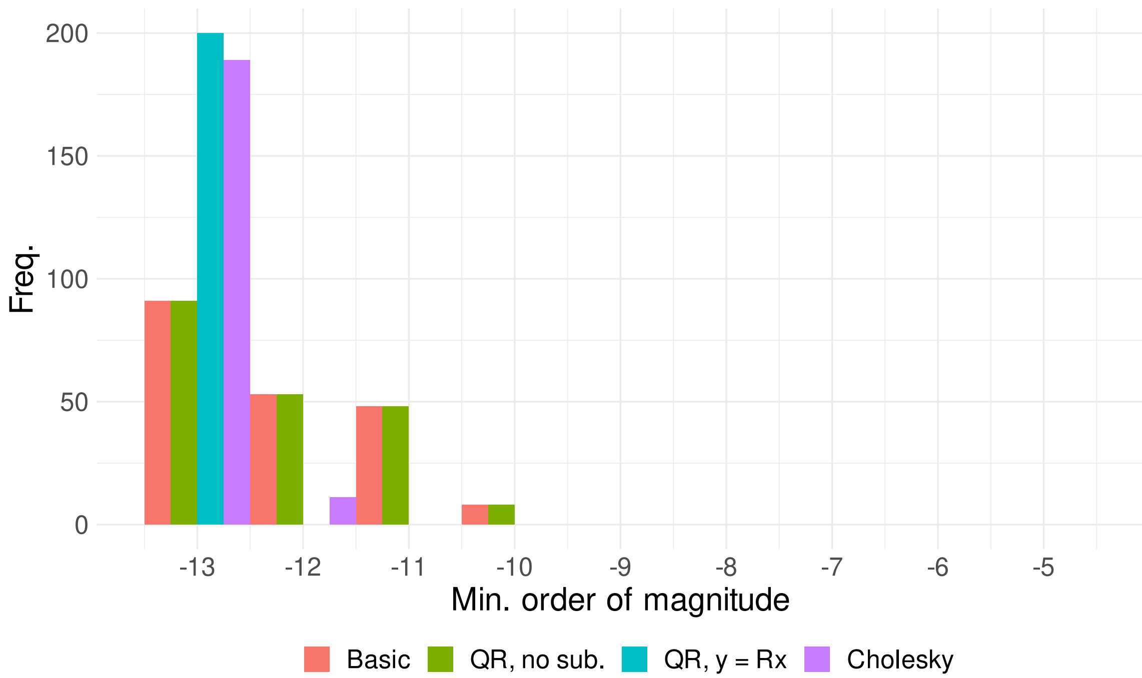
7.2 Case 2, LP


200
150
100
50
0
Freq.
-13
-12
-11
-10
-9
-8
-7
-6
-5
Min. order of magnitude
Basic
QR, no sub.
QR, y = Rx
Cholesky

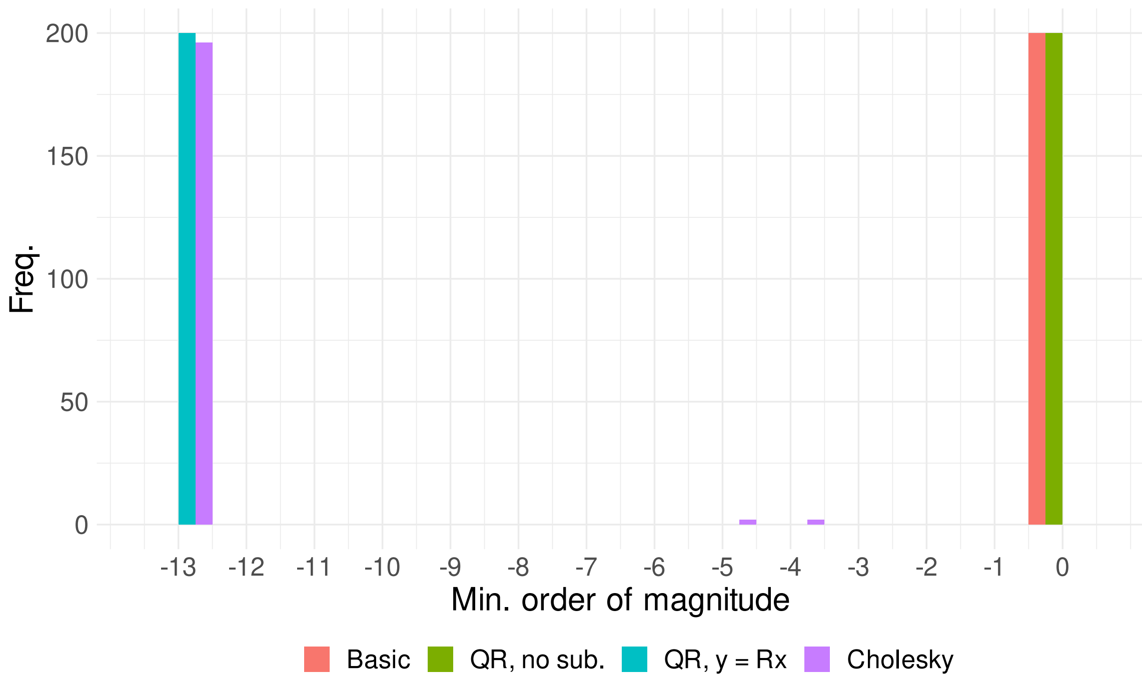
7.3 Case 3, LP


Freq.
200
150
100
50
0
-13
-12
-11
-10
-9
-8
-7
-6
-5
Min. order of magnitude
Basic
QR, no sub.
QR, y = Rx
Cholesky

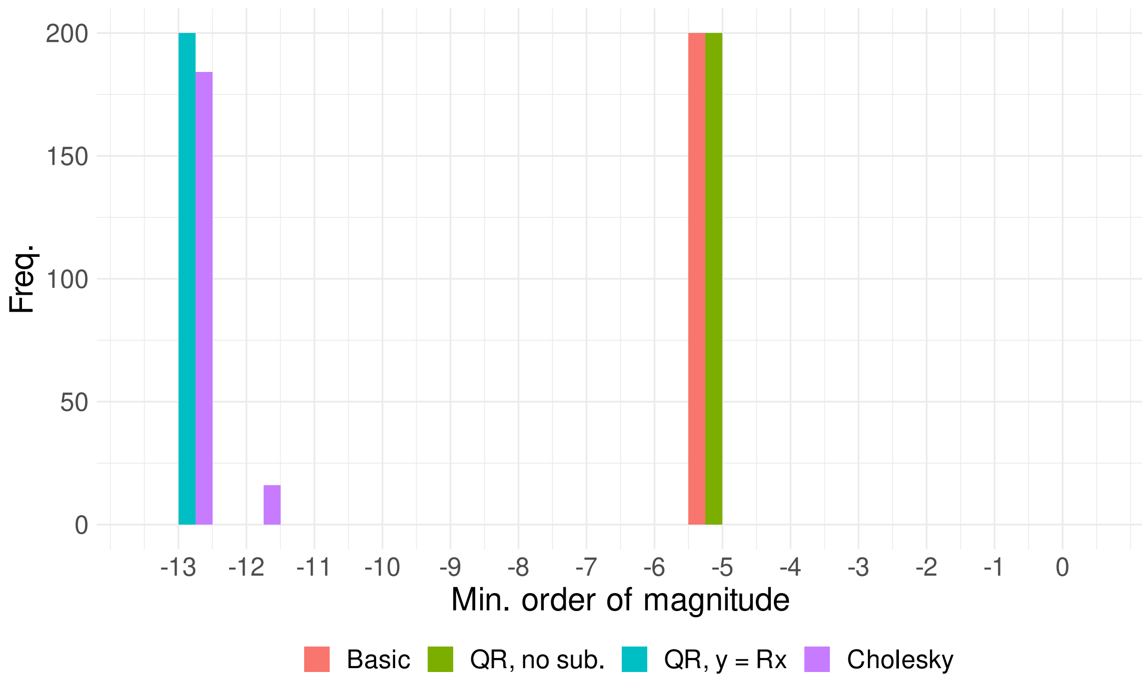
7.4 Case 4, LP


Freq.
200
150
100
50
0
-13
-12
-11
-10
-9
-8
-7
-6
-5
Min. order of magnitude
Basic
QR, no sub.
QR, y = Rx
Cholesky

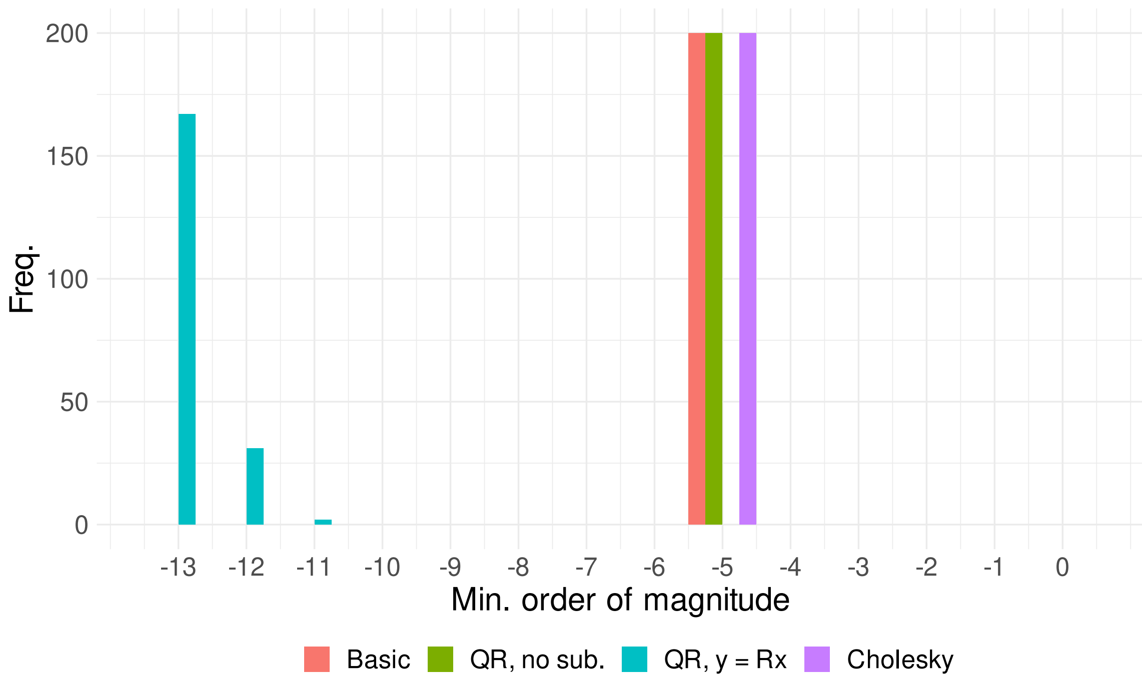
7.5 Case 1, QCQP



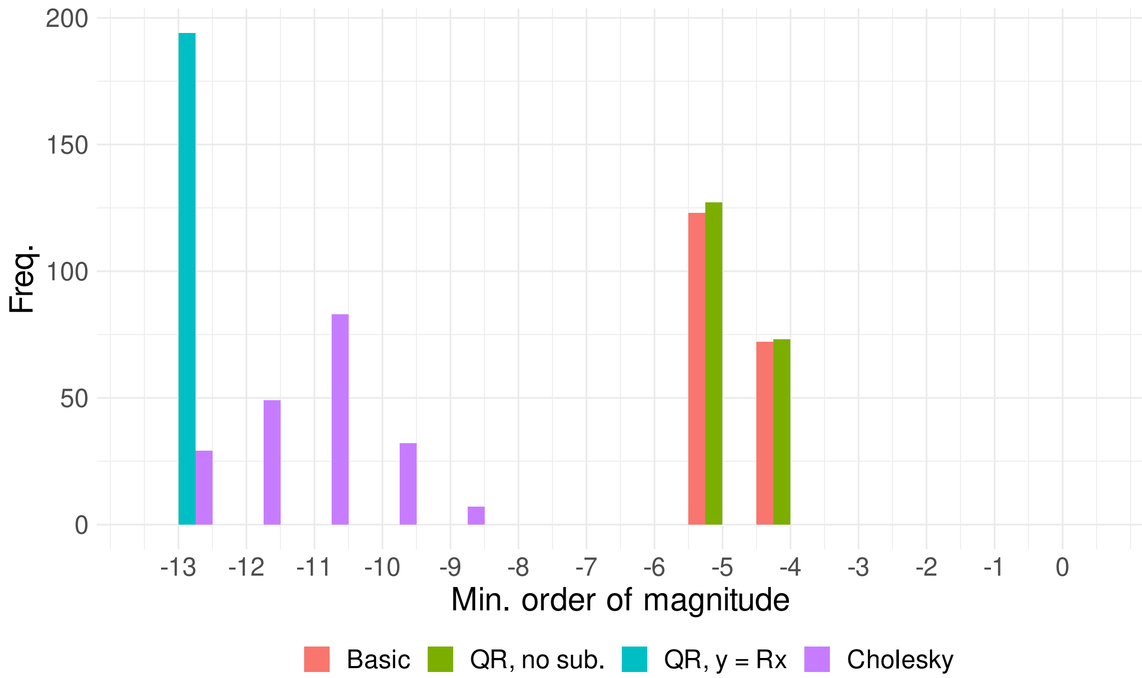
7.6 Case 2, QCQP



7.7 Case 3, QCQP

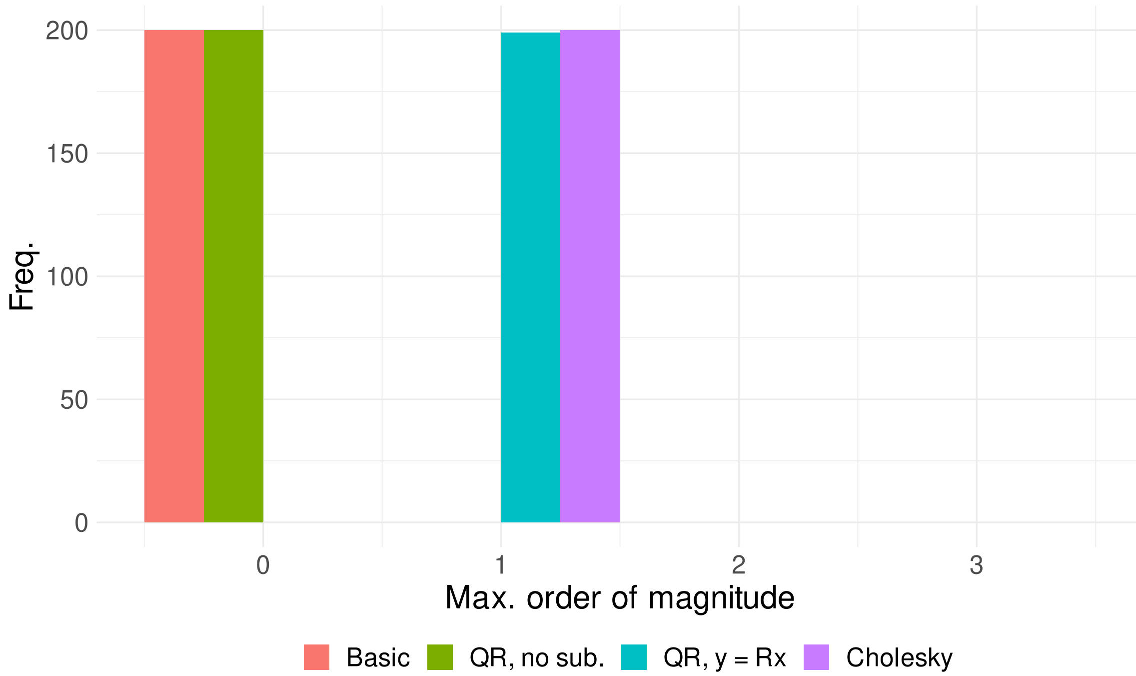


7.8 Case 4, QCQP

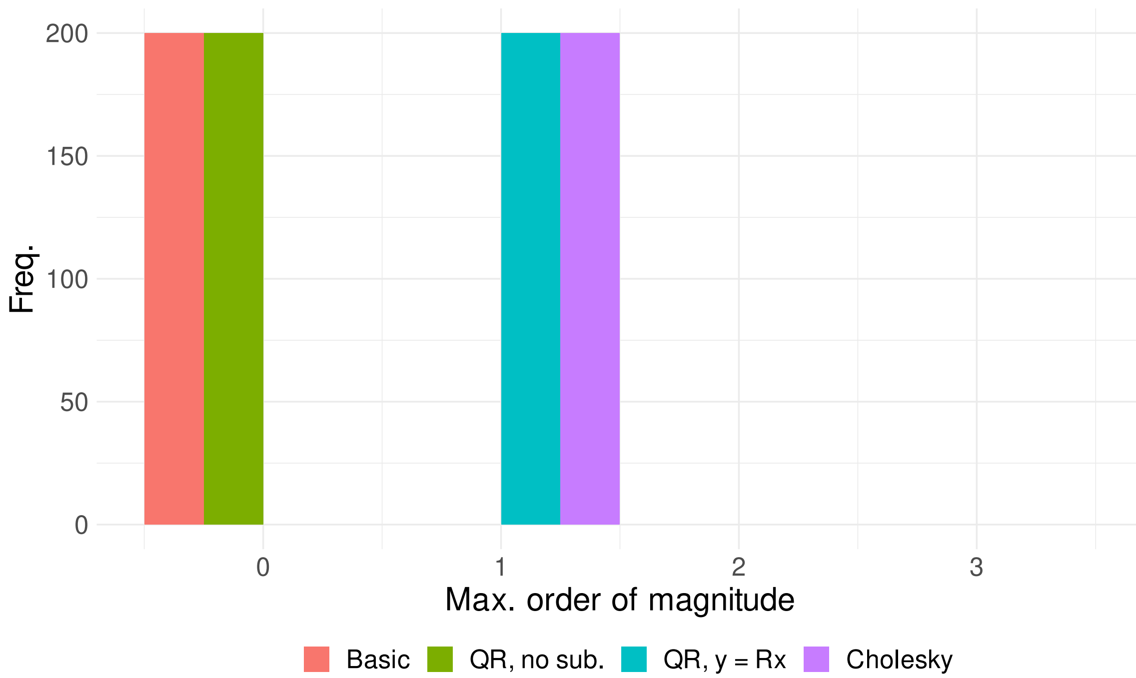


8 Maximum order of magnitude

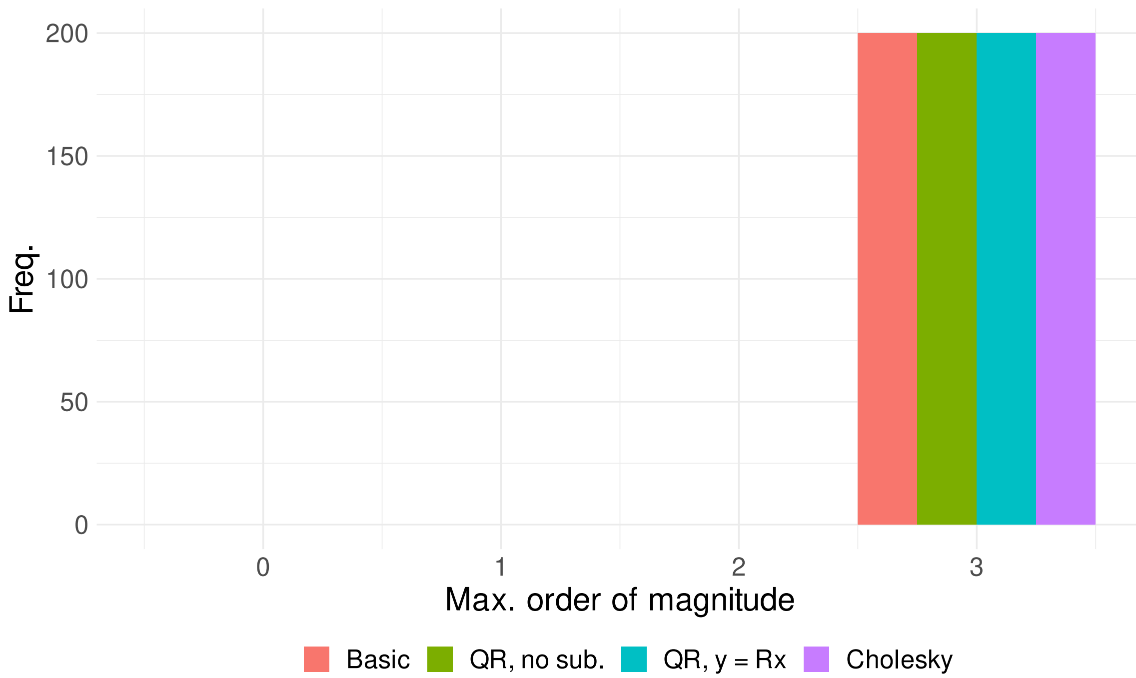
8.1 Case 1, LP



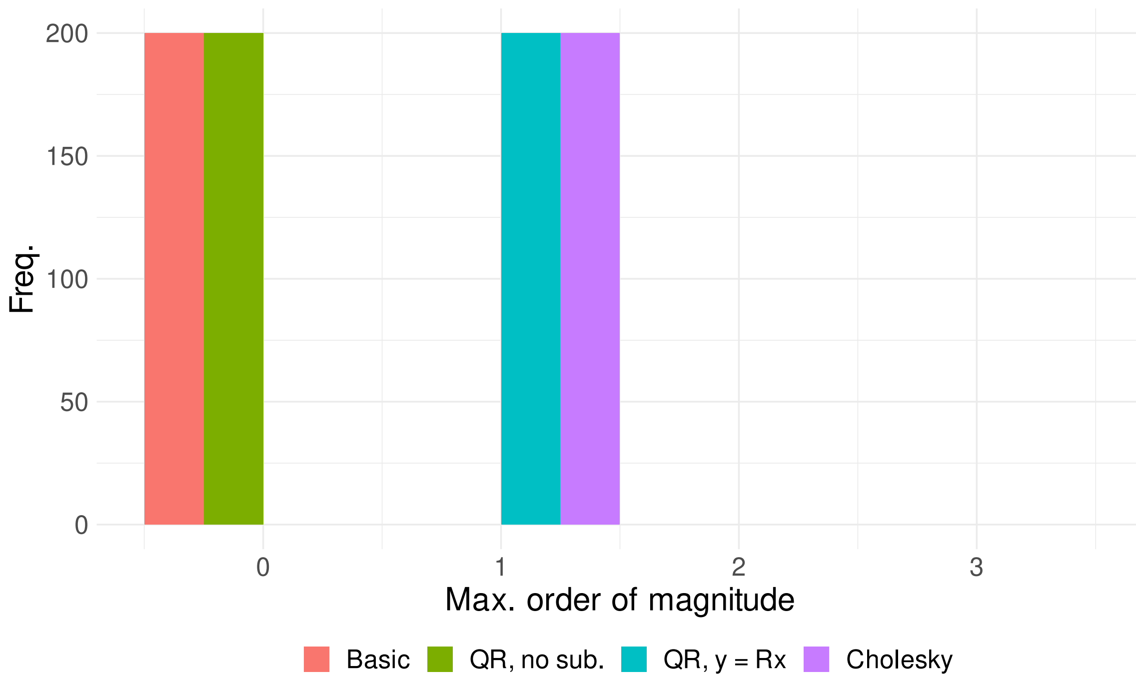
8.2 Case 2, LP


200
150
100
50
0
Freq.
0
1
2
3
Max. order of magnitude
Basic
QR, no sub.
QR, y = Rx
Cholesky

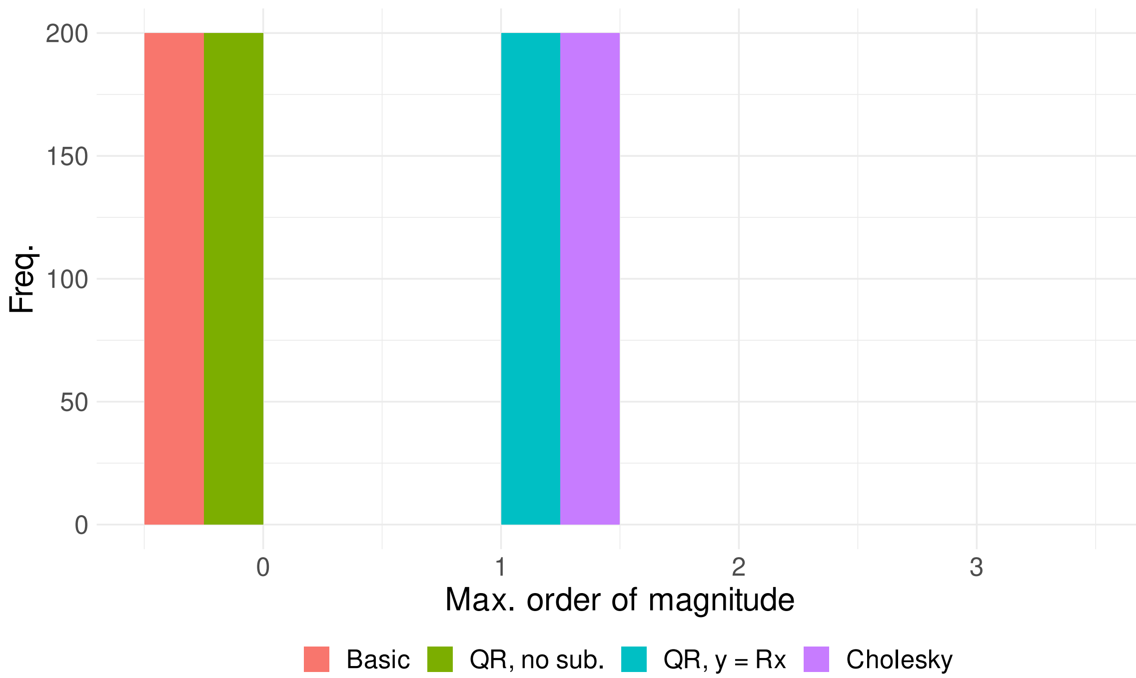
8.3 Case 3, LP


200
150
100
50
0
Freq.
0
1
2
3
Max. order of magnitude
Basic
QR, no sub.
QR, y = Rx
Cholesky

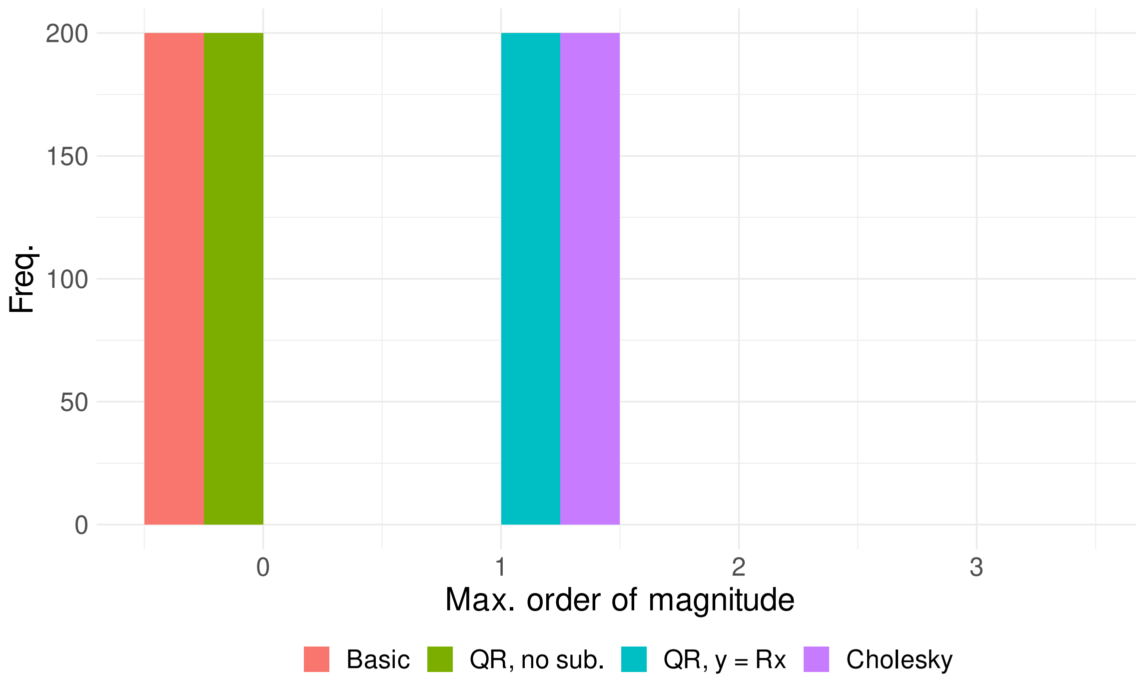
8.4 Case 4, LP


200
150
100
50
0
Freq.
0
1
2
3
Max. order of magnitude
Basic
QR, no sub.
QR, y = Rx
Cholesky

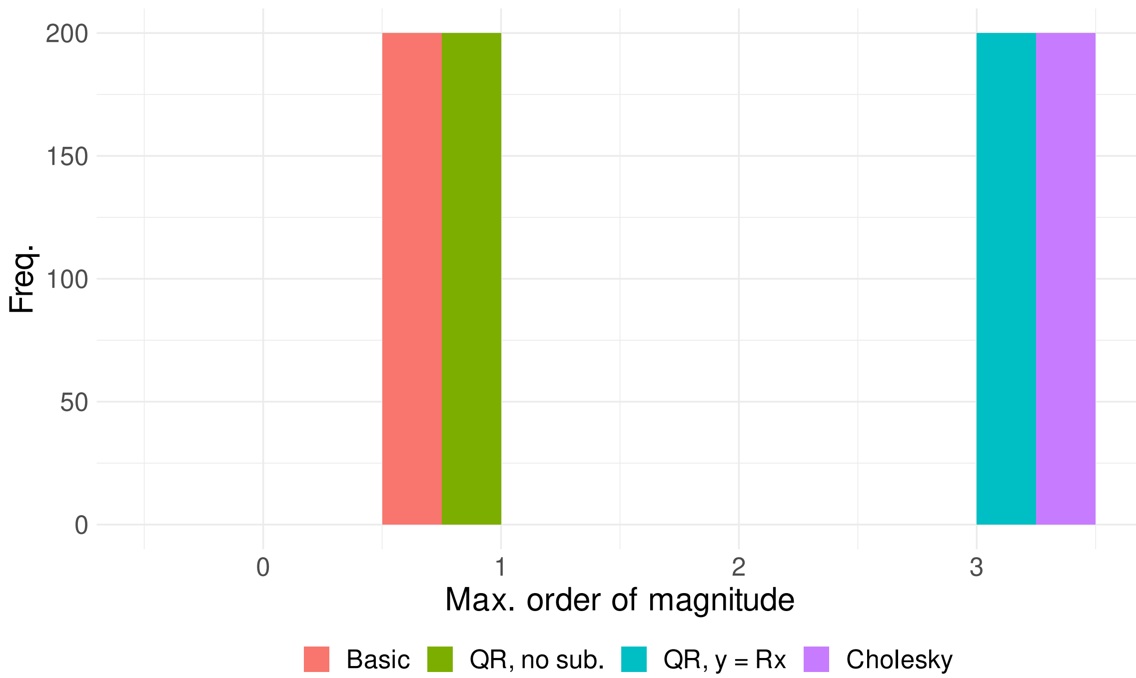
8.5 Case 1, QCQP



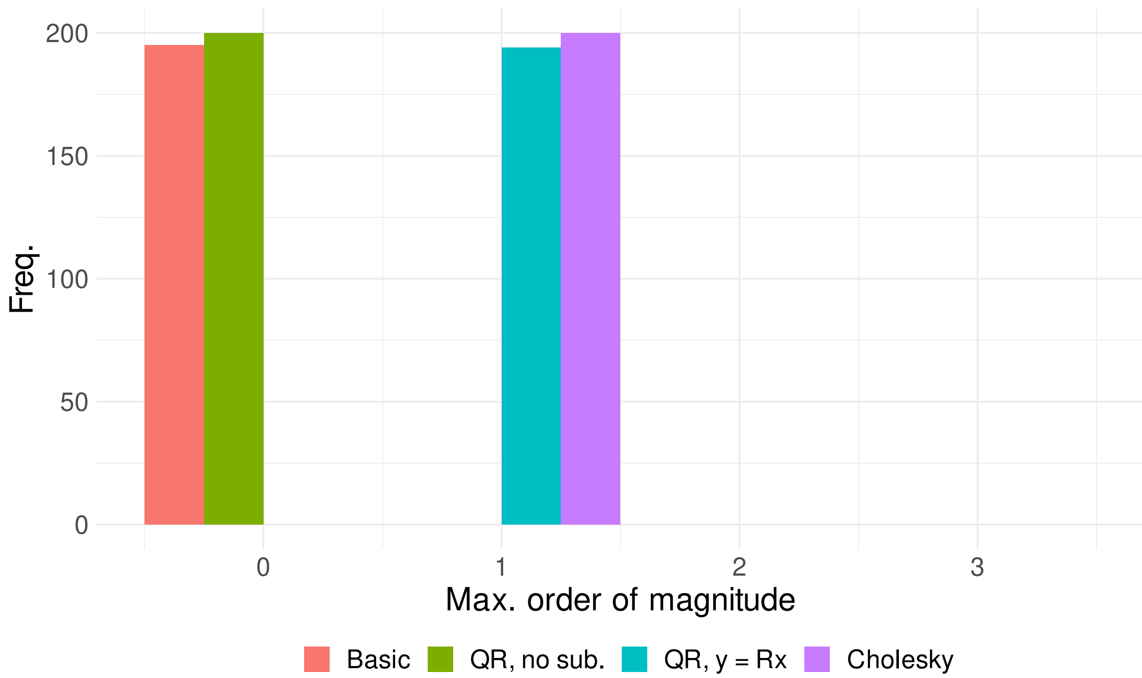
8.6 Case 2, QCQP



8.7 Case 3, QCQP

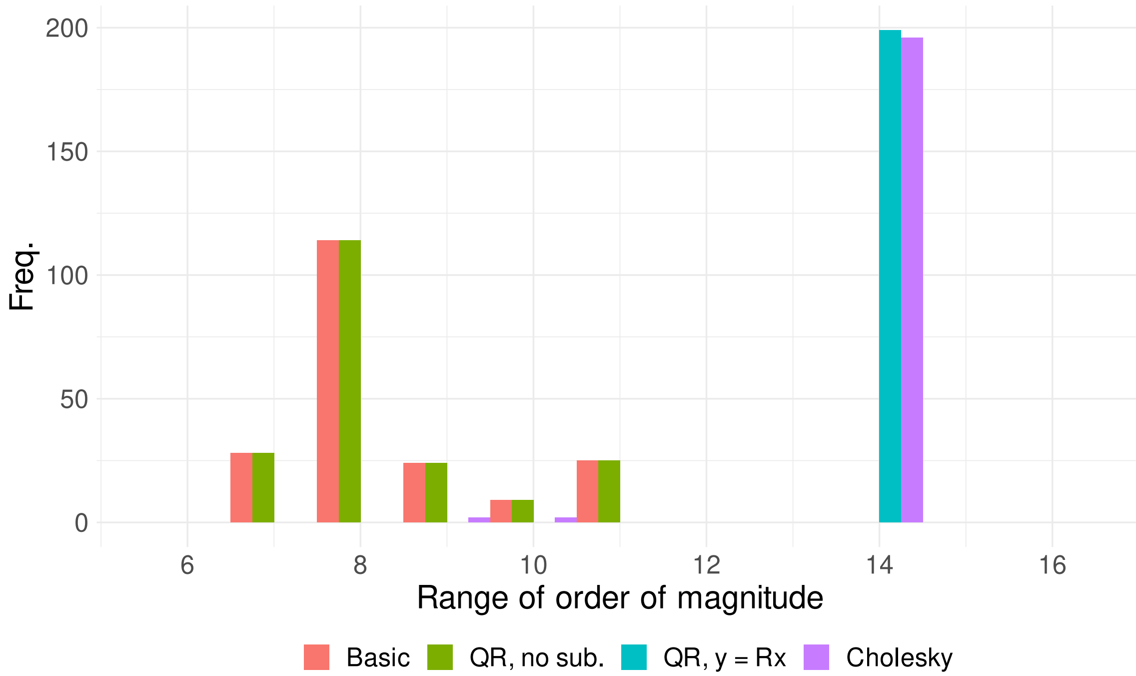


8.8 Case 4, QCQP



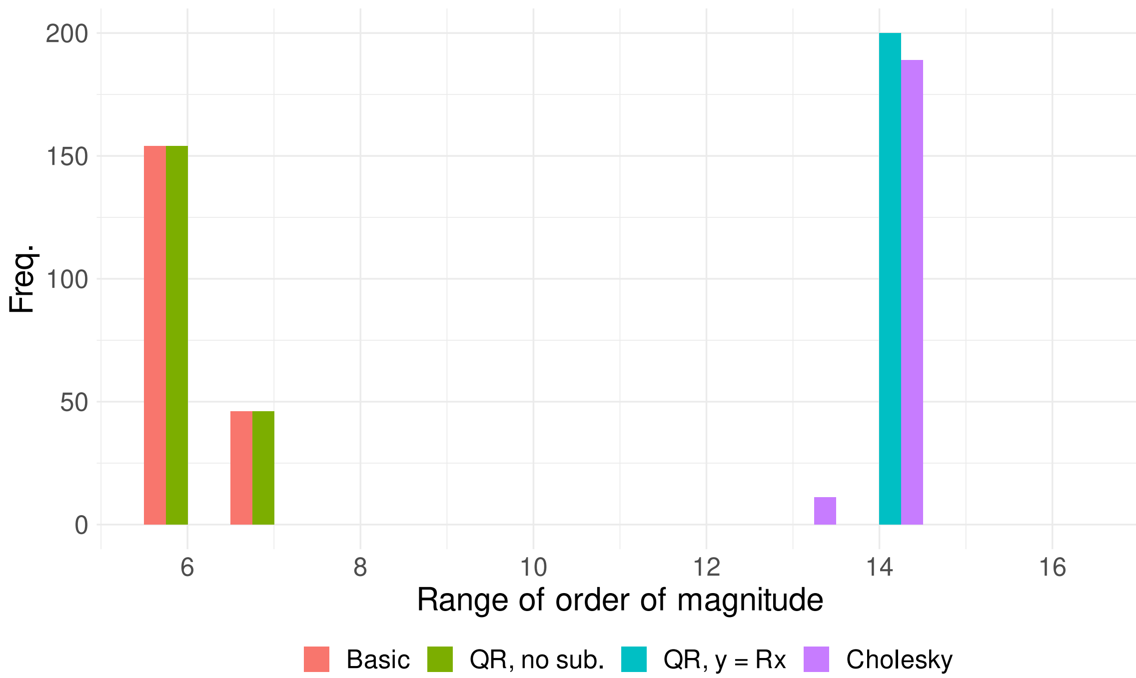
9 Range of order of magnitude

9.1 Case 1, LP



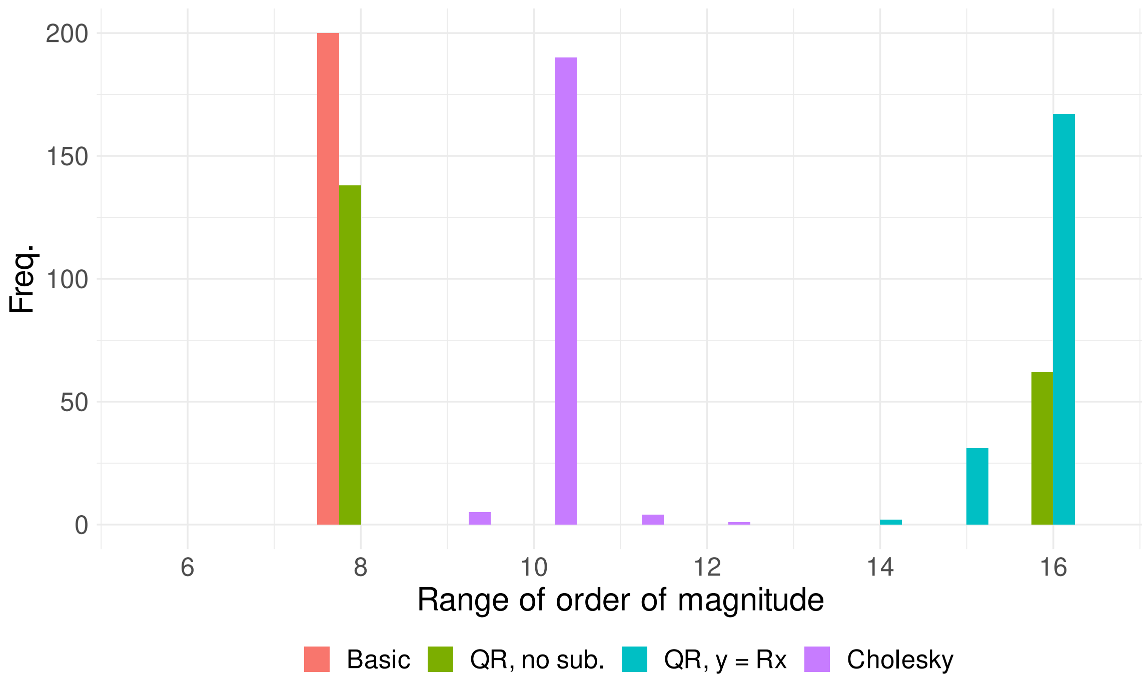
Note: The range is calculated as the difference between the minimum and maximum orders of magnitude.

9.2 Case 2, LP



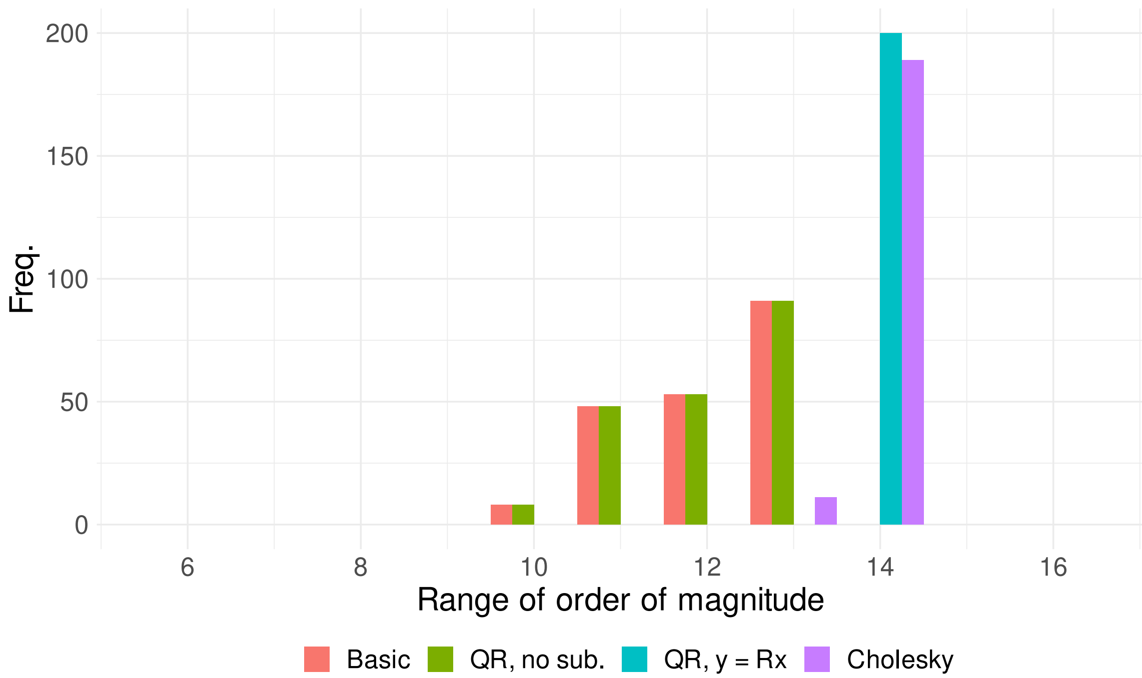
Note: The range is calculated as the difference between the minimum and maximum orders of magnitude.

9.3 Case 3, LP



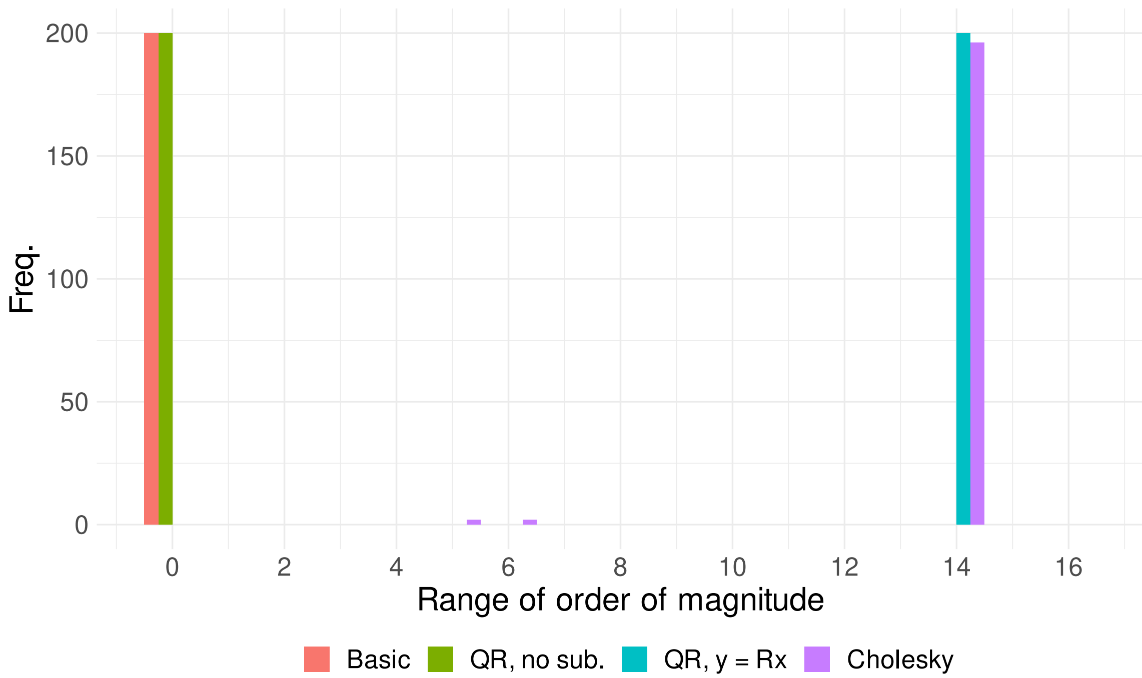
Note: The range is calculated as the difference between the minimum and maximum orders of magnitude.

9.4 Case 4, LP



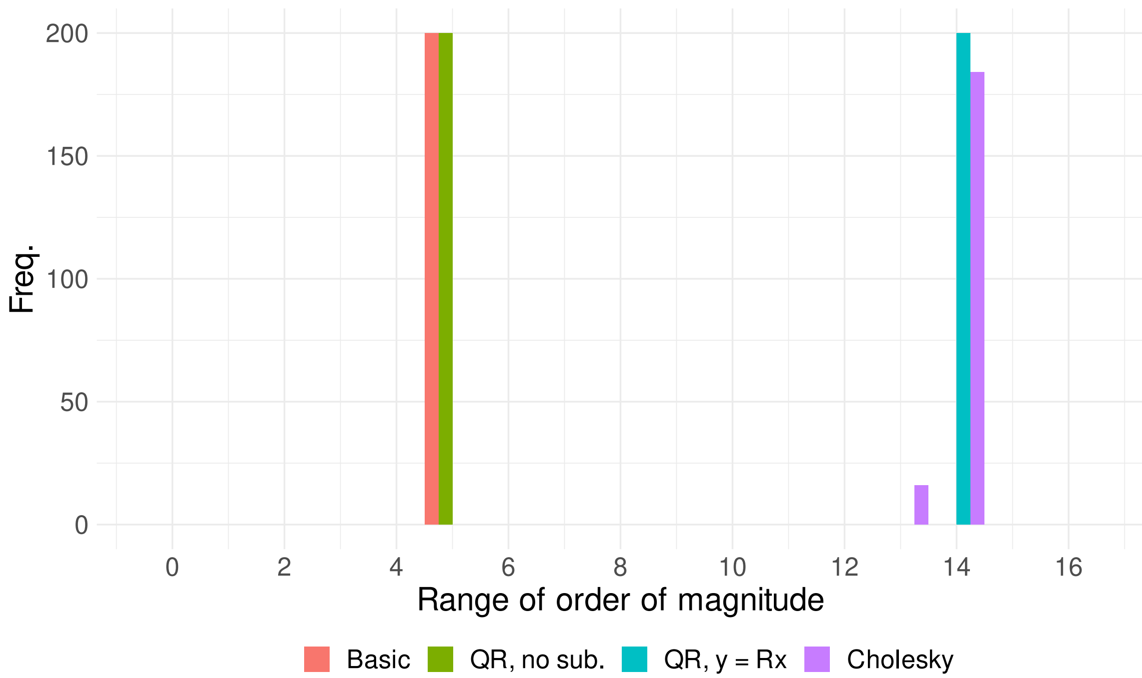
Note: The range is calculated as the difference between the minimum and maximum orders of magnitude.

9.5 Case 1, QCQP



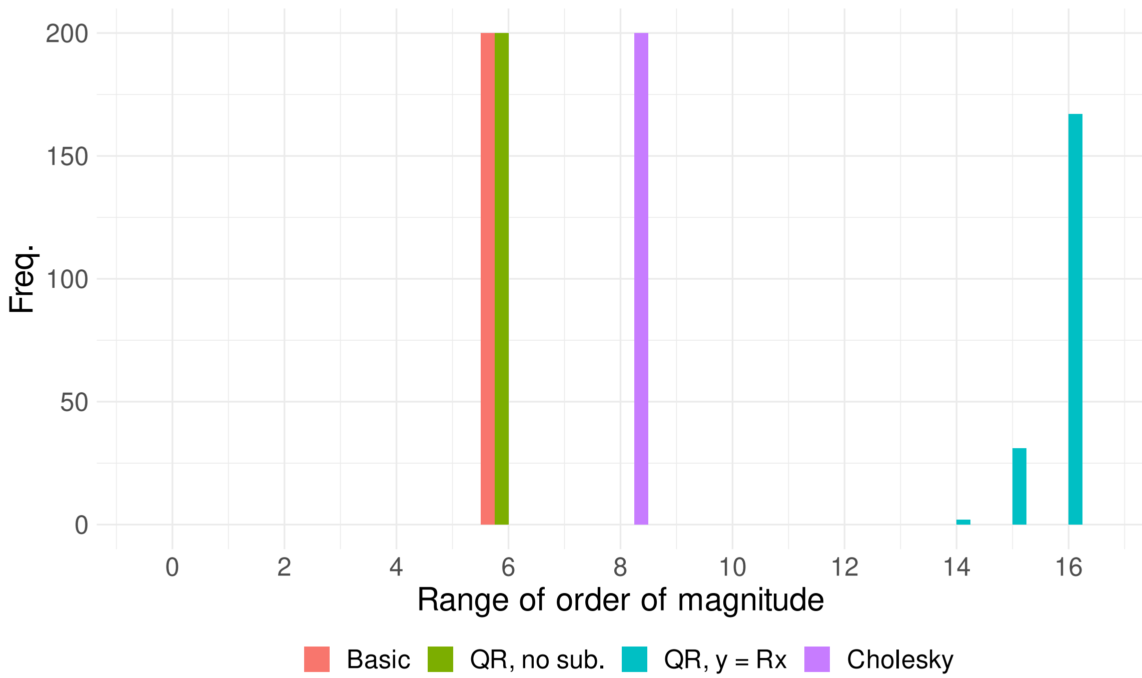
Note: The range is calculated as the difference between the minimum and maximum orders of magnitude.

9.6 Case 2, QCQP



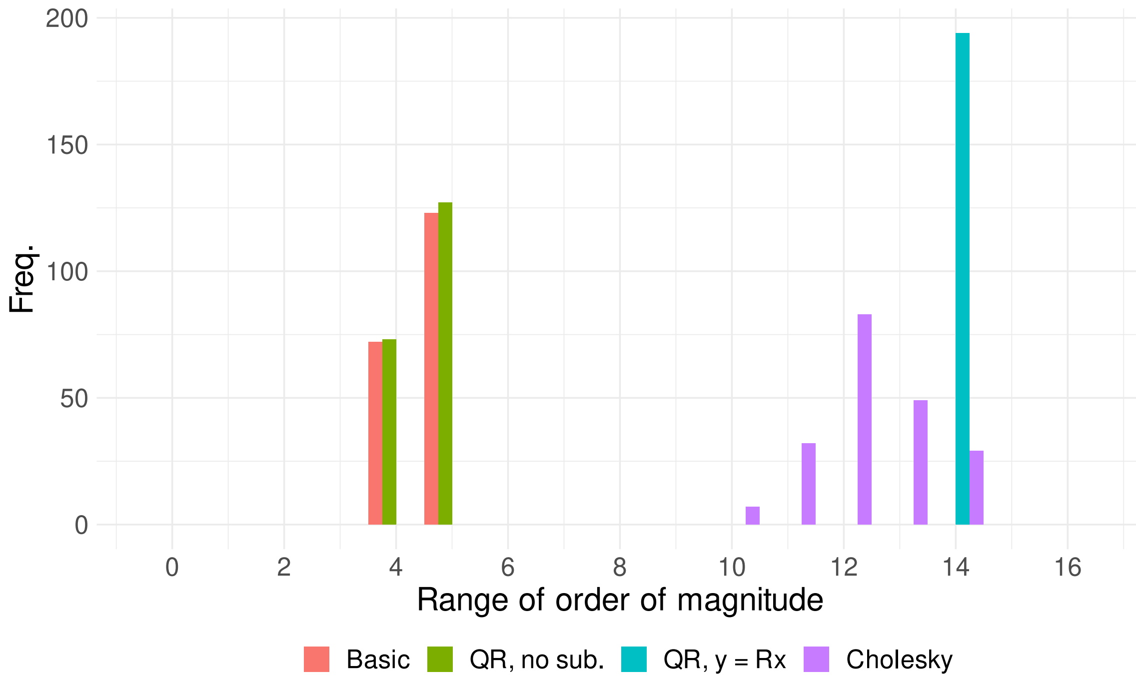
Note: The range is calculated as the difference between the minimum and maximum orders of magnitude.

9.7 Case 3, QCQP



Note: The range is calculated as the difference between the minimum and maximum orders of magnitude.

9.8 Case 4, QCQP



Note: The range is calculated as the difference between the minimum and maximum orders of magnitude.