Rescale comparisons

August 19, 2022

1 Introduction

This document studies whether setting `rescale = TRUE` helps with estimation using the same examples as described in the document titled *Matrix Decomposition Comparisons* (available here).

The `rescale` option was originally developed to address stability issues in the QCQP problem through rescaling the Gram matrix. Currently, `ivmte` only allows rescaling in the QCQP problems. I will allow rescaling in LP problems in the future.

To explain how I rescaled the QCQP problems, let x denote the unknown variables, A denote the design matrix, b denote the vector of observed outcomes, and L denote the linear constraint matrix. The least squares criterion is $x'A'Ax - 2x'A'b + b'b$.

1. `qp0` (no decomposition): Let K denote a diagonal matrix whose i^{th} diagonal is equal to the ℓ_2 norm of the i^{th} column of A, i.e., $K_{i,i} \equiv \|A_{\cdot,i}\|$. The QCQP model is defined using the rescaled design matrix $\widetilde{A} \equiv AK^{-1}$ and the rescaled linear constraint matrix $\widetilde{L} \equiv LK^{-1}$.

2. `qp1` (QR decomposition, no substitutions): By the QR decomposition, $A = QR$, where Q is an orthogonal matrix. The Gram matrix $A'A$ defining the quadratic component of the model may then be written as

$$A'A = (R'Q')QR = R'R.$$

   Let K denote a diagonal matrix whose i^{th} diagonal is equal to the ℓ_2 norm of the i^{th} column of R, i.e., $K_{i,i} \equiv \|R_{\cdot,i}\|$. The QCQP problem is defined using $\widetilde{A} \equiv AK^{-1}$ and $\widetilde{L} = LK^{-1}$.

3. `qp3` (QR decomposition, $y = Rx$): By the QR decomposition and substitution $y = Rx$, the least squares criterion may be written as

$$\begin{aligned} x'A'Ax - 2x'A'b + b'b &= x'R'Rx - 2x'R'Q'b + b'b \\ &= y'y - 2y'Q'b + b'b. \end{aligned}$$

   Since small entries in the constraint matrix L are due to small entries in R, I define

the diagonal matrix K such that $K_{i,i} \equiv \|R_{\cdot,i}\|$. The QCQP model is defined using the rescaled constraint matrix $\widetilde{L} \equiv LK^{-1}$. The substitution $y = Rx$ then becomes $y = \widetilde{R}x$, where $\widetilde{R} \equiv RK^{-1}$.

4. `qp4` (Cholesky decomposition, $y = Cx$): By the Cholesky decomposition, $A'A = C'C$. Using the substitution $y = Cx$, the least squares criterion may be written as

$$\begin{aligned} x'A'Ax - 2x'A'b + b'b &= x'C'Cx - 2x'A'b + b'b \\ &= y'y - 2x'A'b + b'b. \end{aligned}$$

Since small entries in the constraint matrix L are due to small entries in C, I define the diagonal matrix K such that $K_{i,i} \equiv \|C_{\cdot,i}\|$. The QCQP model is defined using the rescaled constraint matrix $\widetilde{L} \equiv LK^{-1}$. The substitution $y = Cx$ then becomes $y = \widetilde{C}x$, where $\widetilde{C} \equiv CK^{-1}$.

5. `qp5` (Row/column rescaling): This approach first rescales the columns of A so that the minimum order of magnitude is -3. The rows of A are then rescaled so the minimum order of magnitude is also -3.

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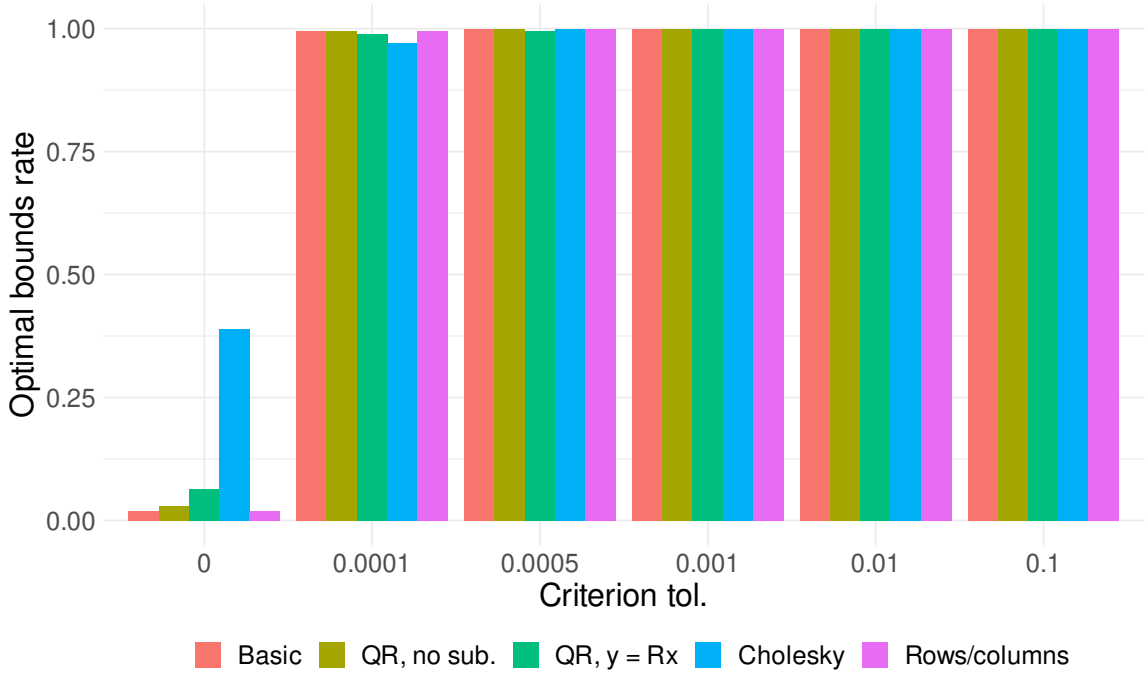
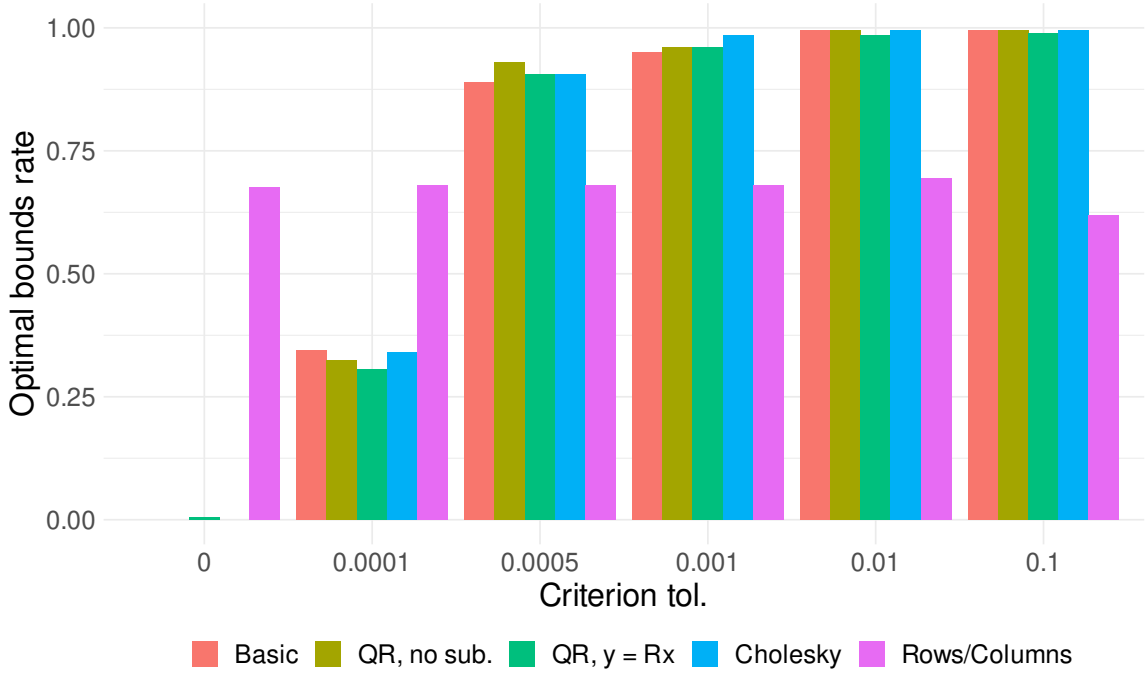
2 Optimal upper and lower bounds

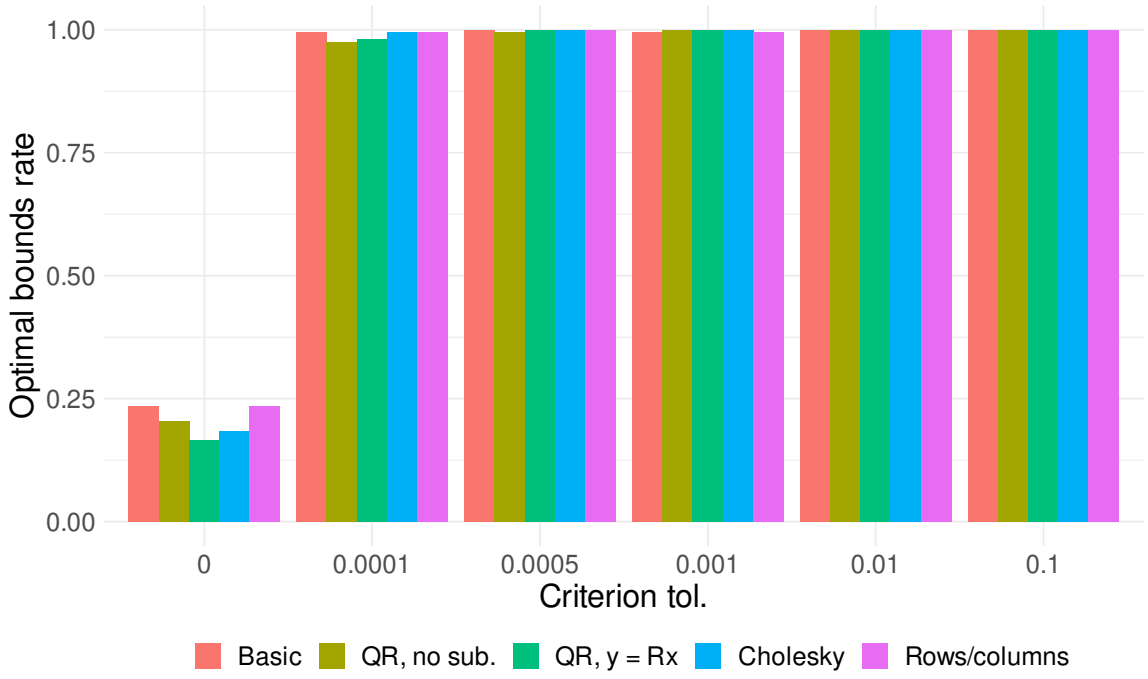
2.1 Case 1, QCQP, unscaled



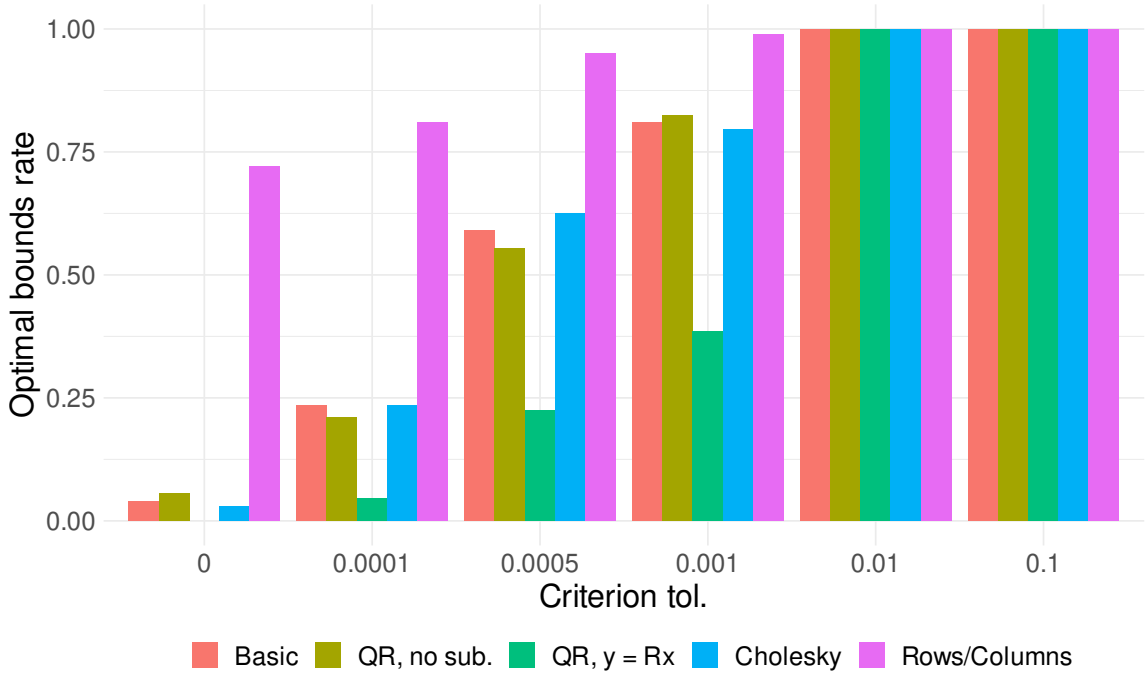
2.2 Case 1, QCQP, rescaled



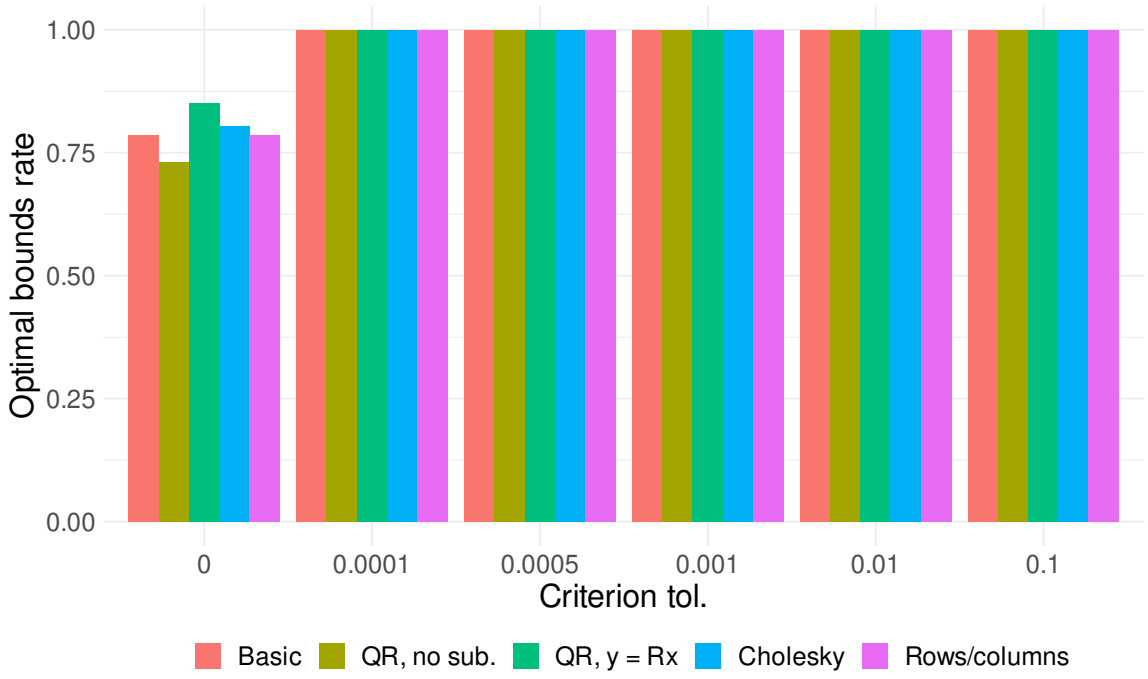
2.3 Case 2, QCQP, unscaled



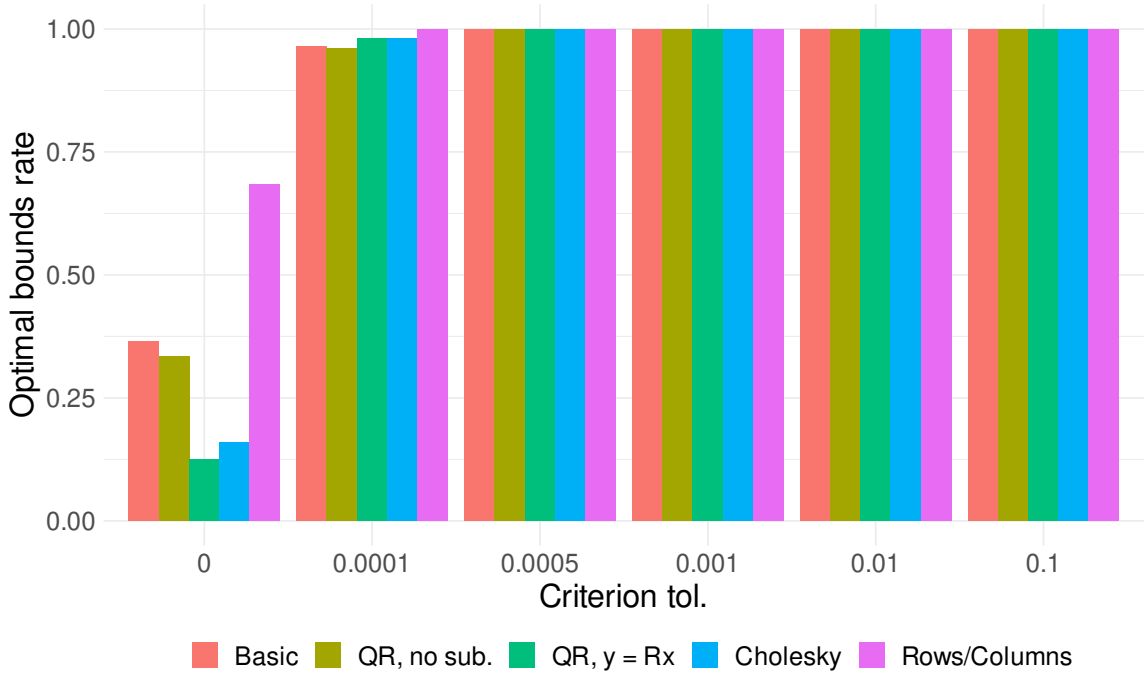
2.4 Case 2, QCQP, rescaled



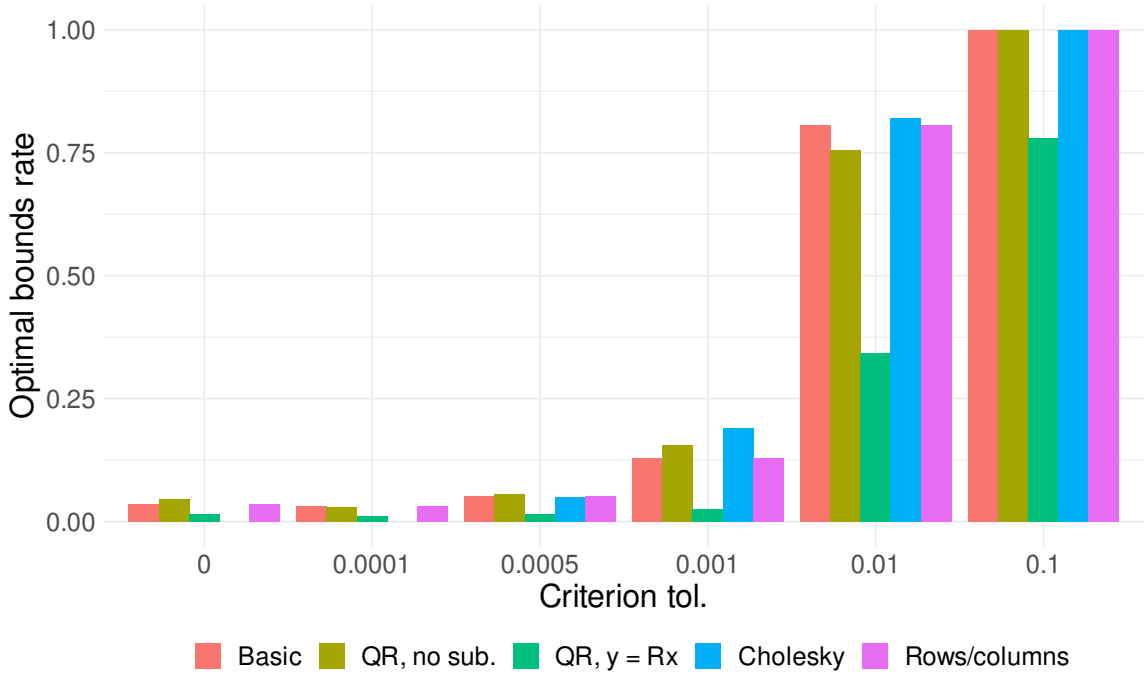
2.5 Case 3, QCQP, unscaled



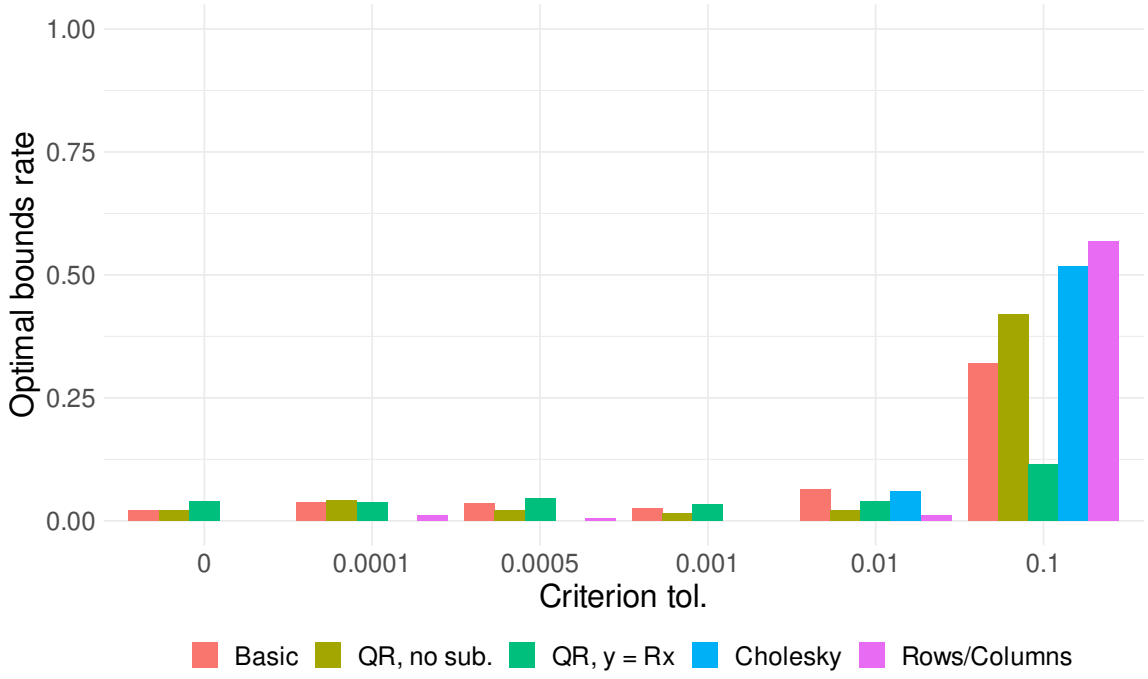
2.6 Case 3, QCQP, rescaled



2.7 Case 4, QCQP, unscaled

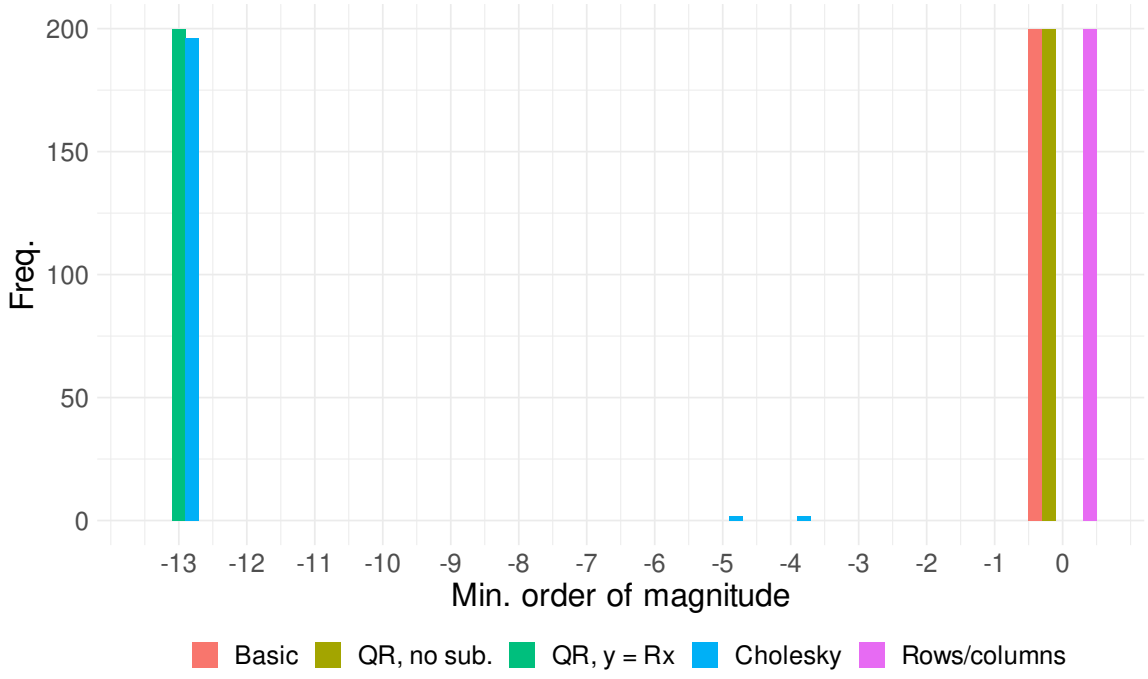


2.8 Case 4, QCQP, rescaled

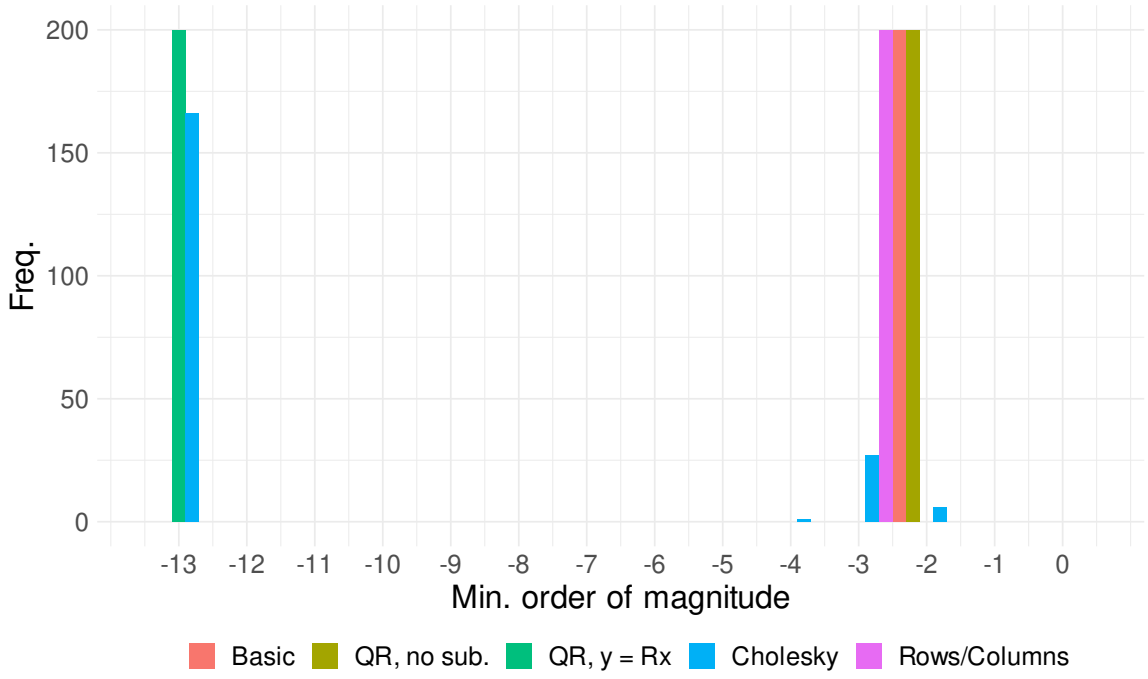


3 Min. order of mag. in linear constraint matrix

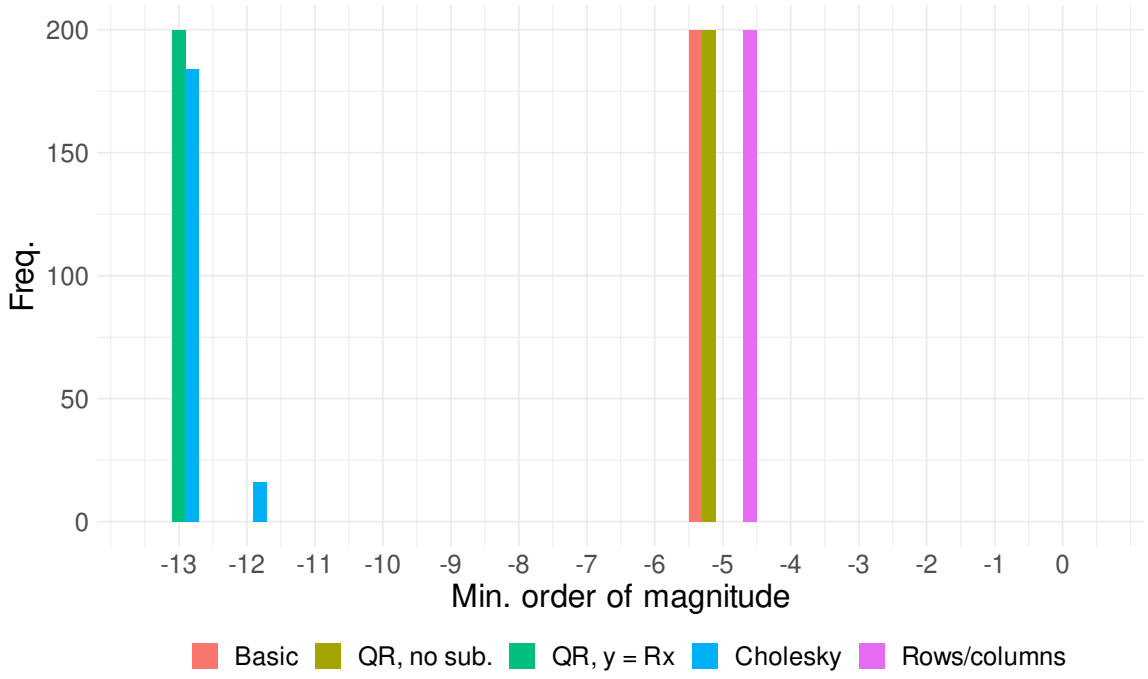
3.1 Case 1, QCQP, unscaled



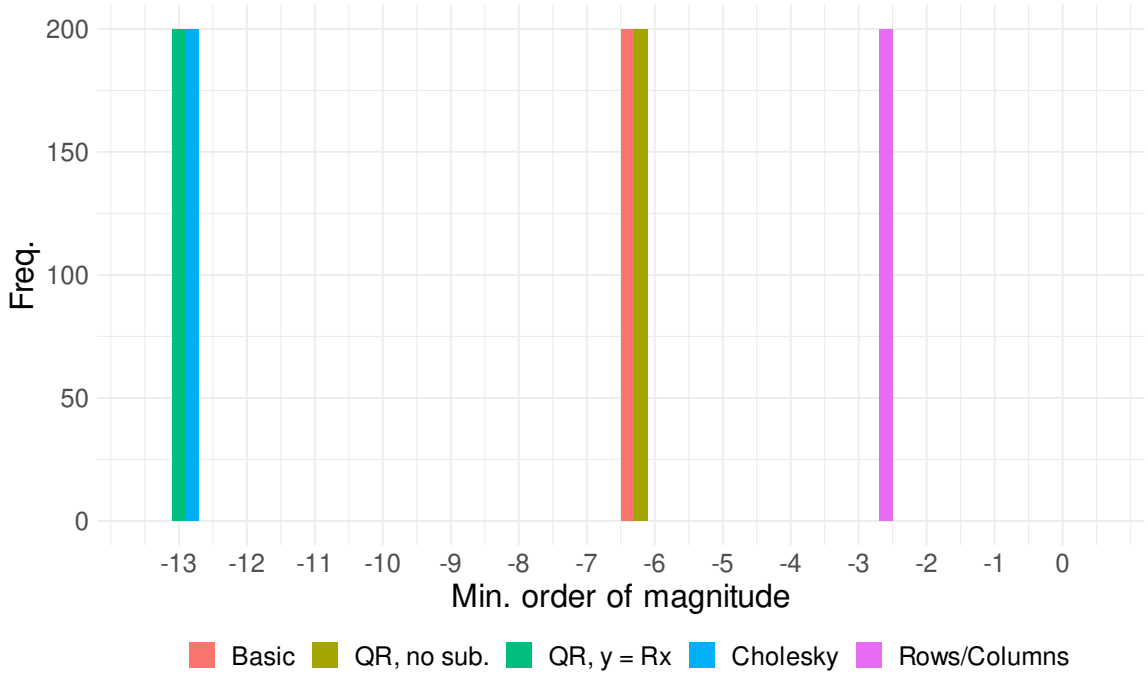
3.2 Case 1, QCQP, rescaled



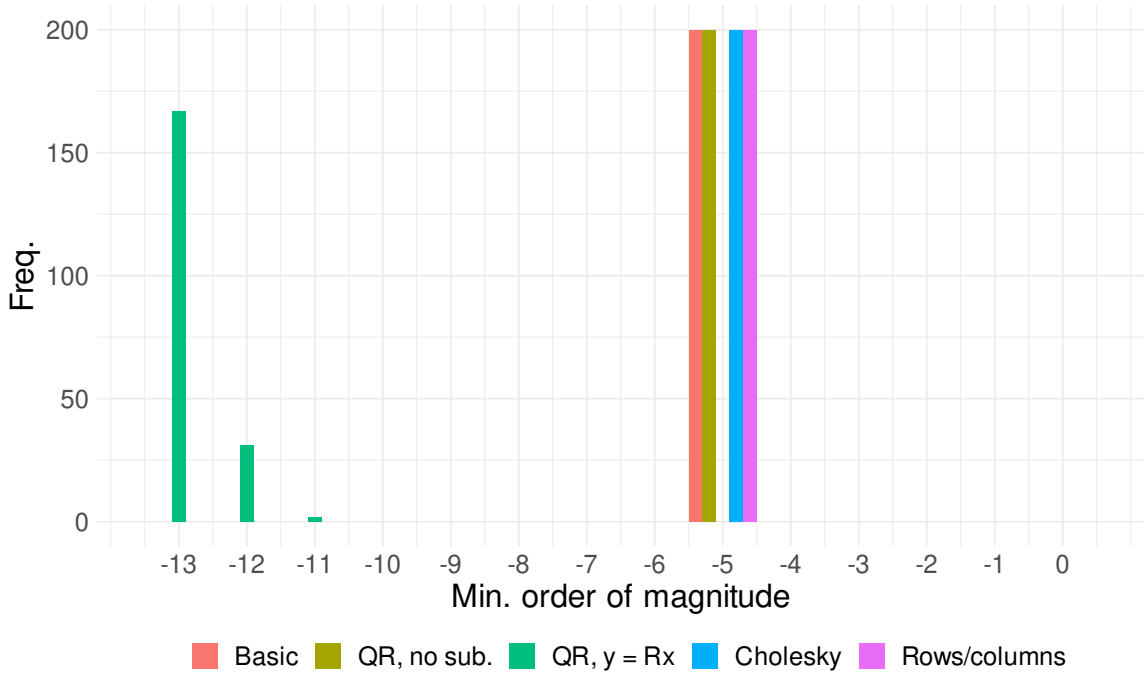
3.3 Case 2, QCQP, unscaled



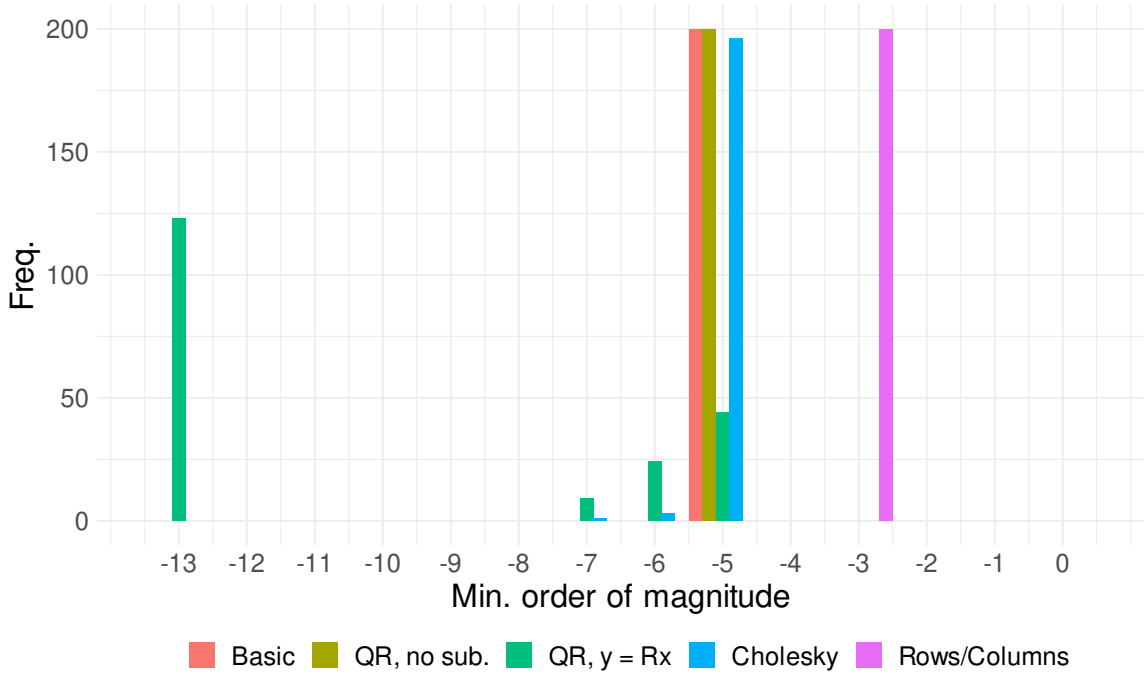
3.4 Case 2, QCQP, rescaled



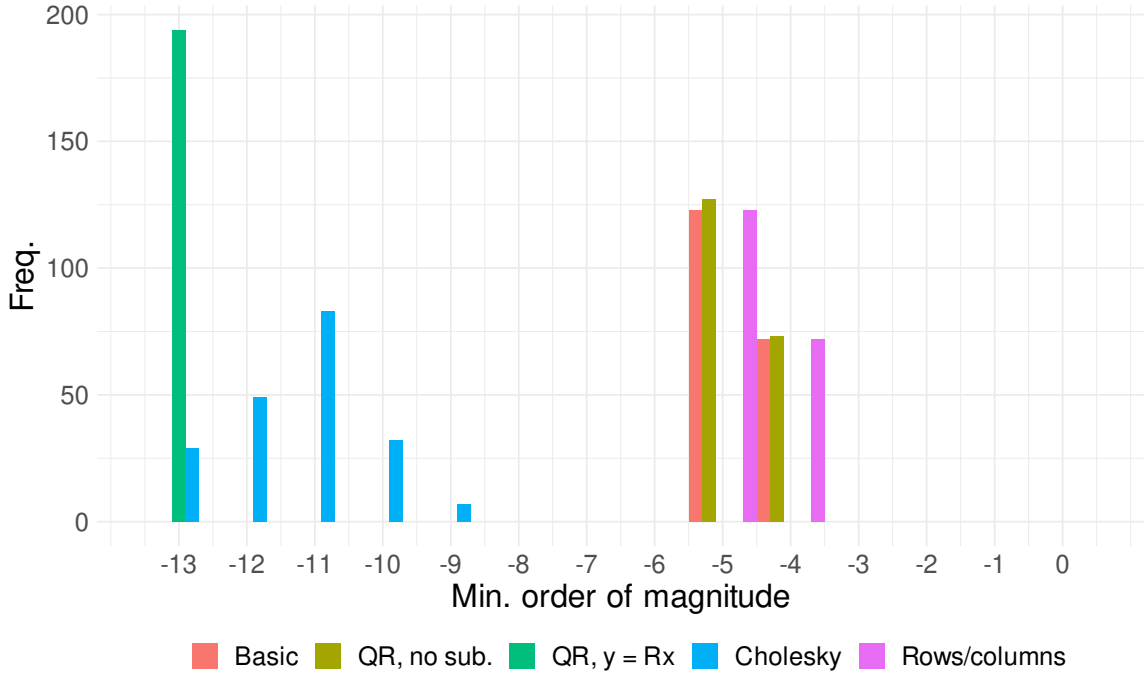
3.5 Case 3, QCQP, unscaled



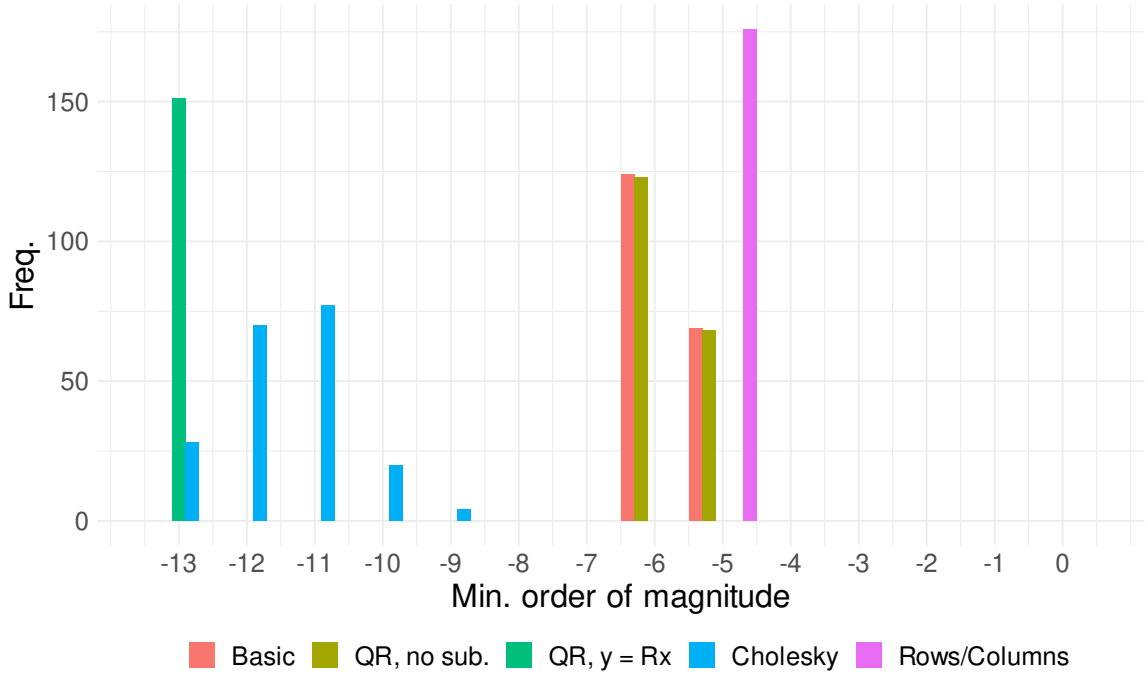
3.6 Case 3, QCQP, rescaled



3.7 Case 4, QCQP, unscaled

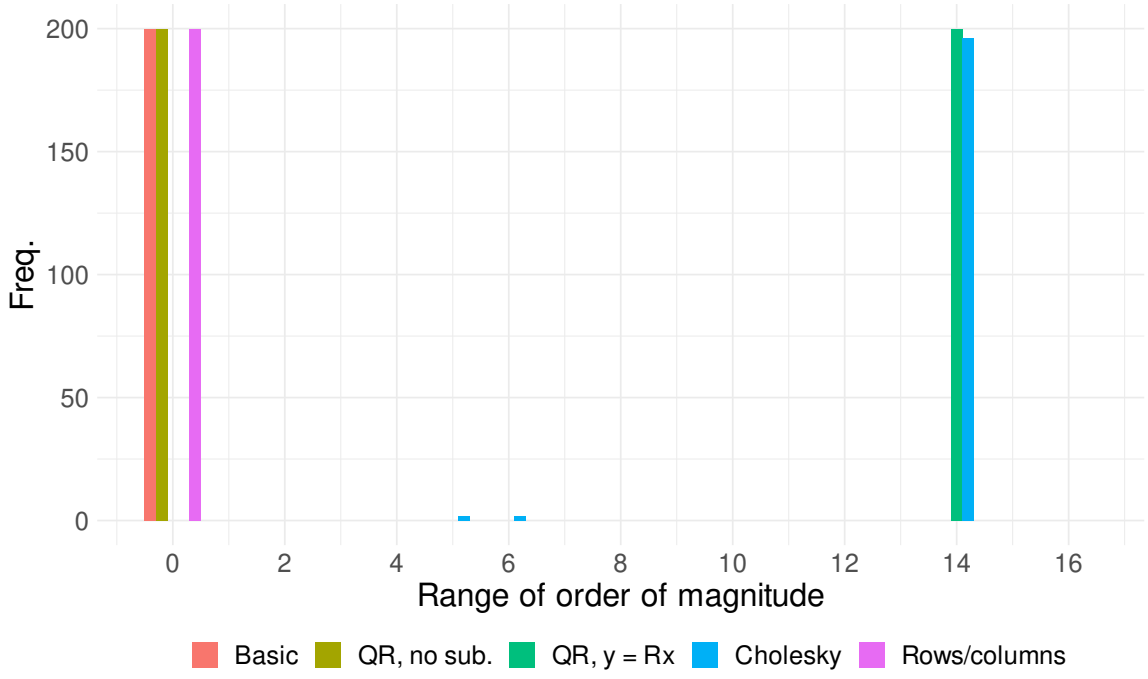


3.8 Case 4, QCQP, rescaled

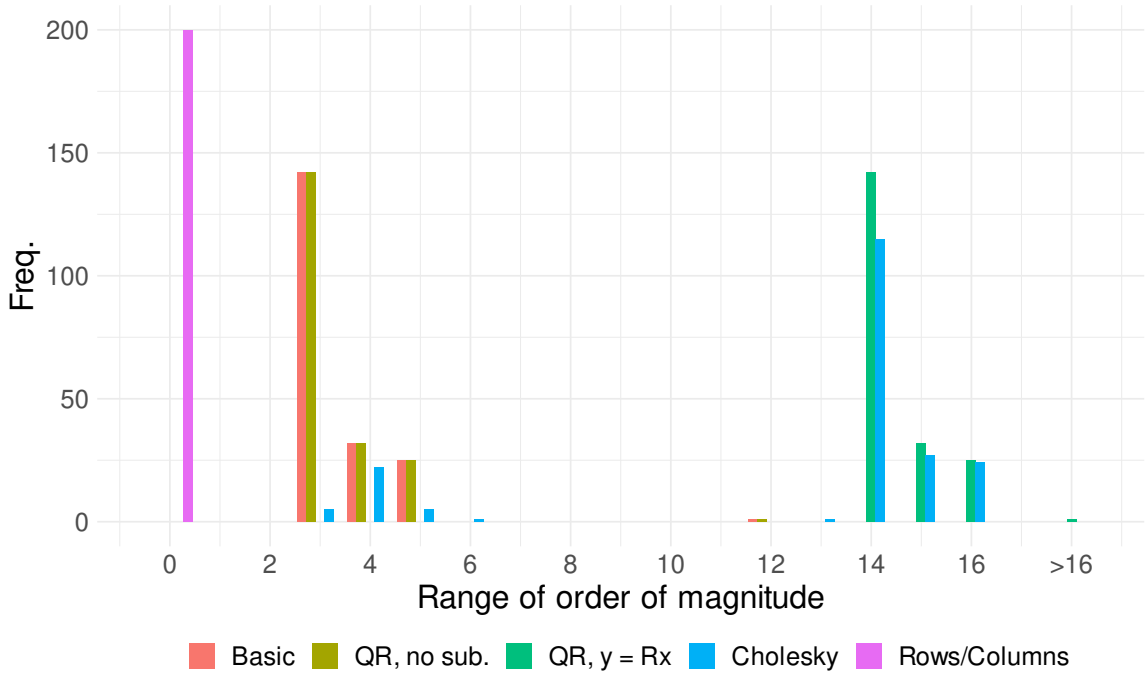


4 Range of order of mag. in linear constraint matrix

4.1 Case 1, QCQP, unscaled

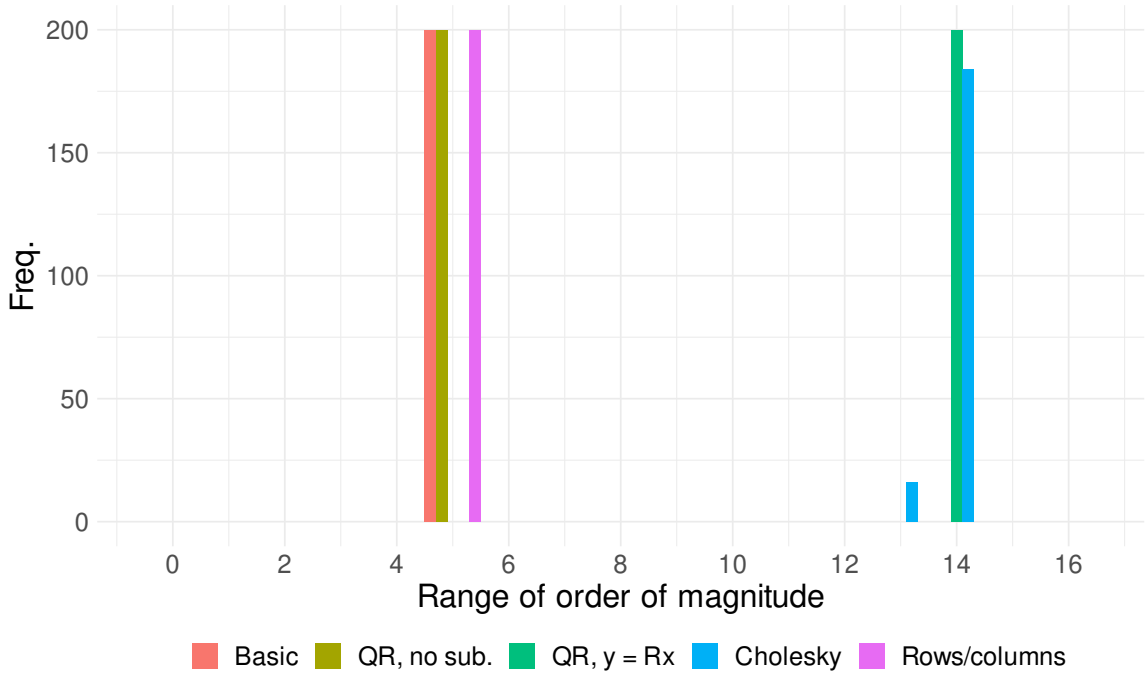


4.2 Case 1, QCQP, rescaled

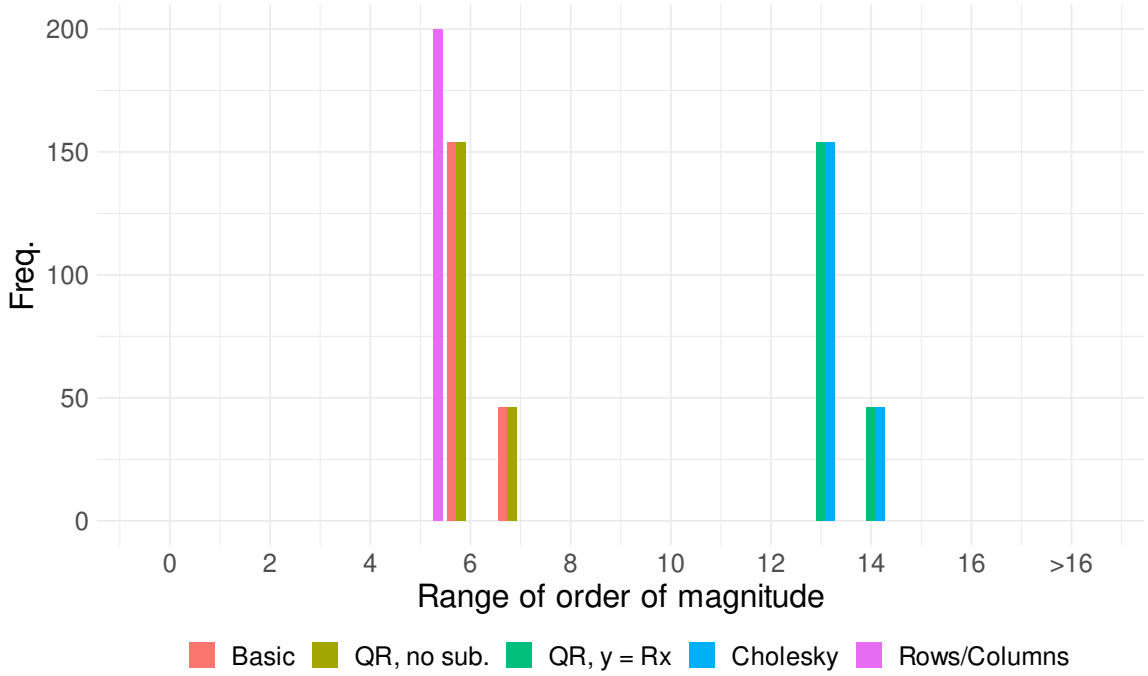


Note: When `rescale = TRUE`, ranges go up to as high as 30 orders of magnitude.

4.3 Case 2, QCQP, unscaled

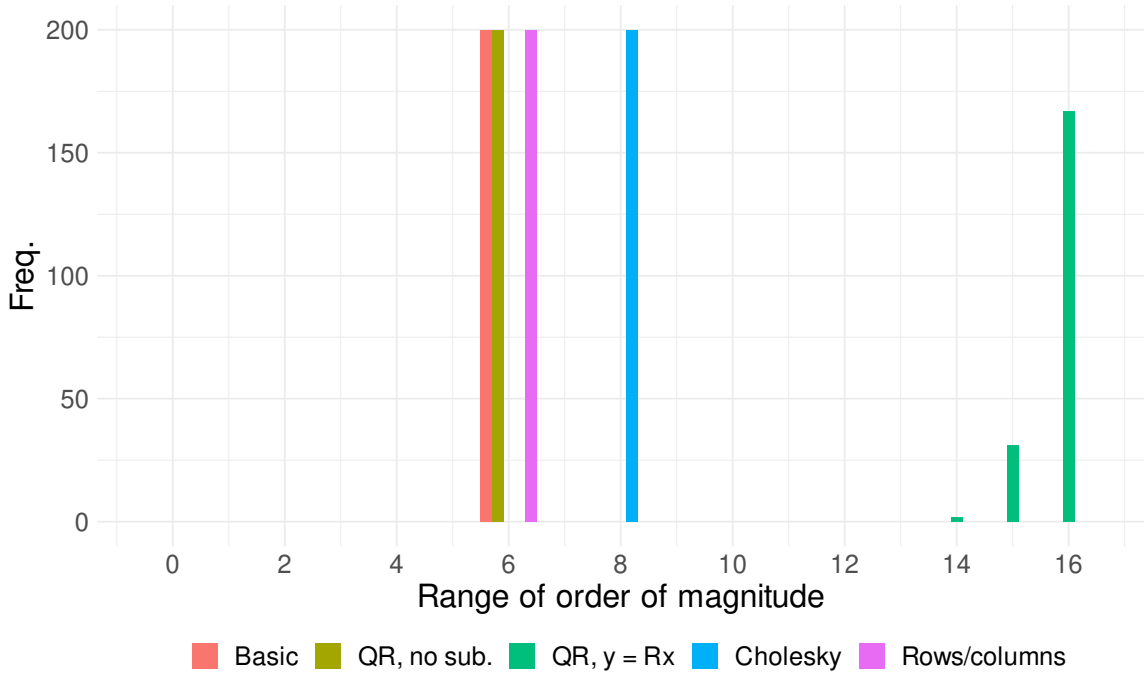


4.4 Case 2, QCQP, rescaled

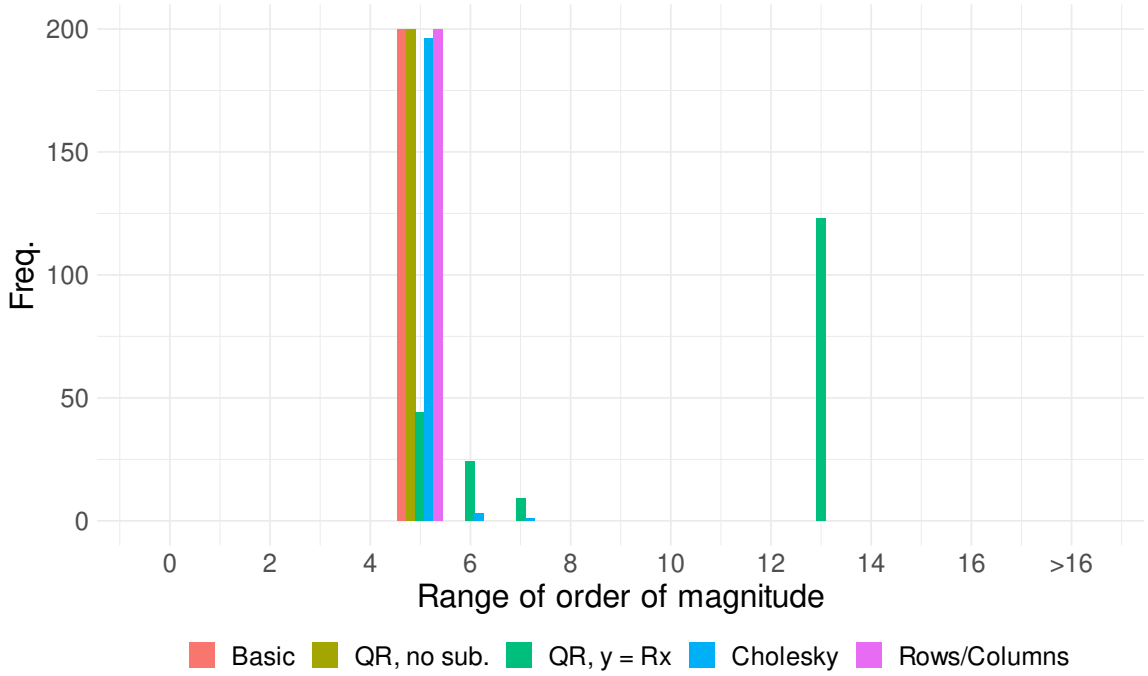


Note: When `rescale = TRUE`, ranges go up to as high as 30 orders of magnitude.

4.5 Case 3, QCQP, unscaled

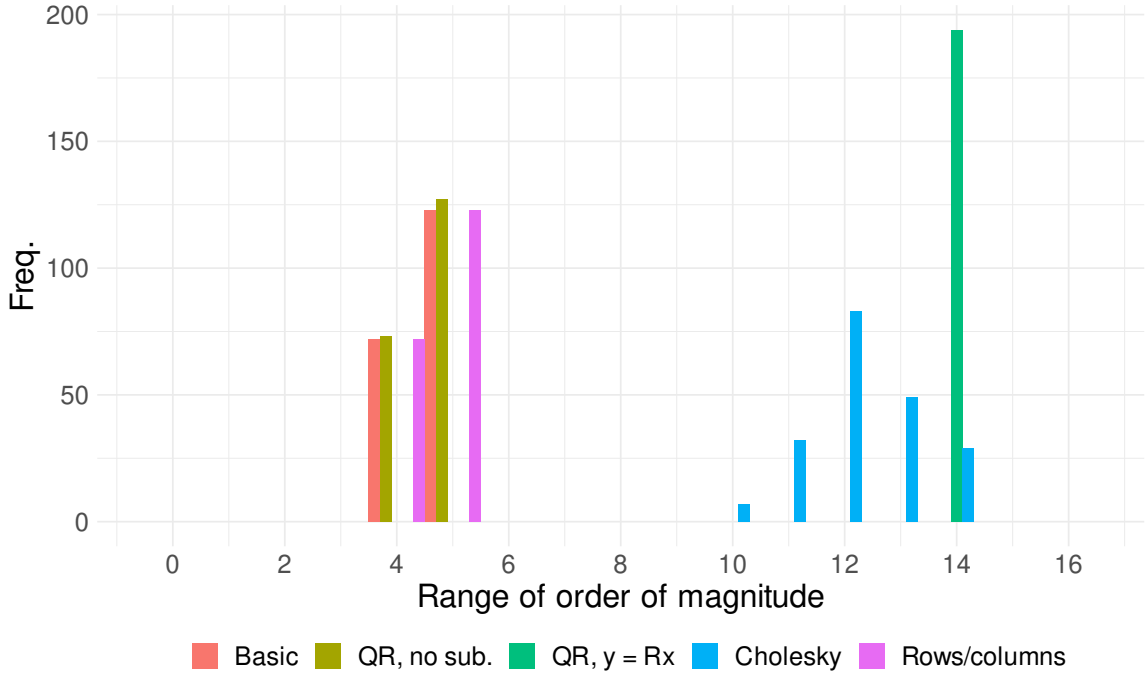


4.6 Case 3, QCQP, rescaled

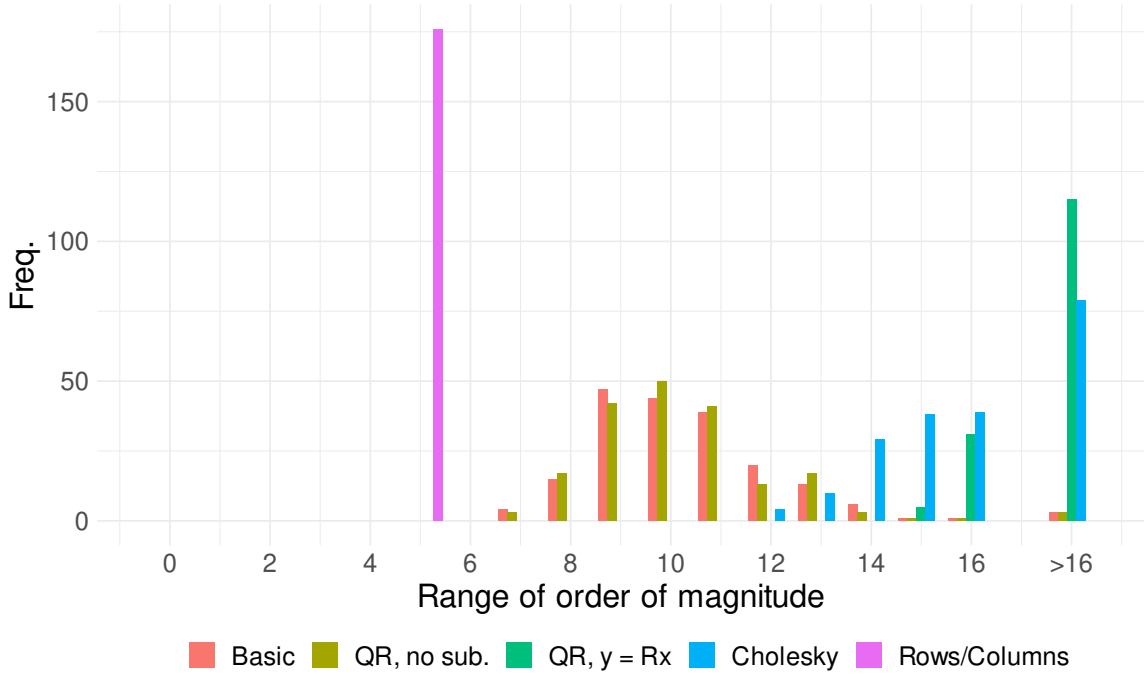


Note: When `rescale = TRUE`, ranges go up to as high as 30 orders of magnitude.

4.7 Case 4, QCQP, unscaled



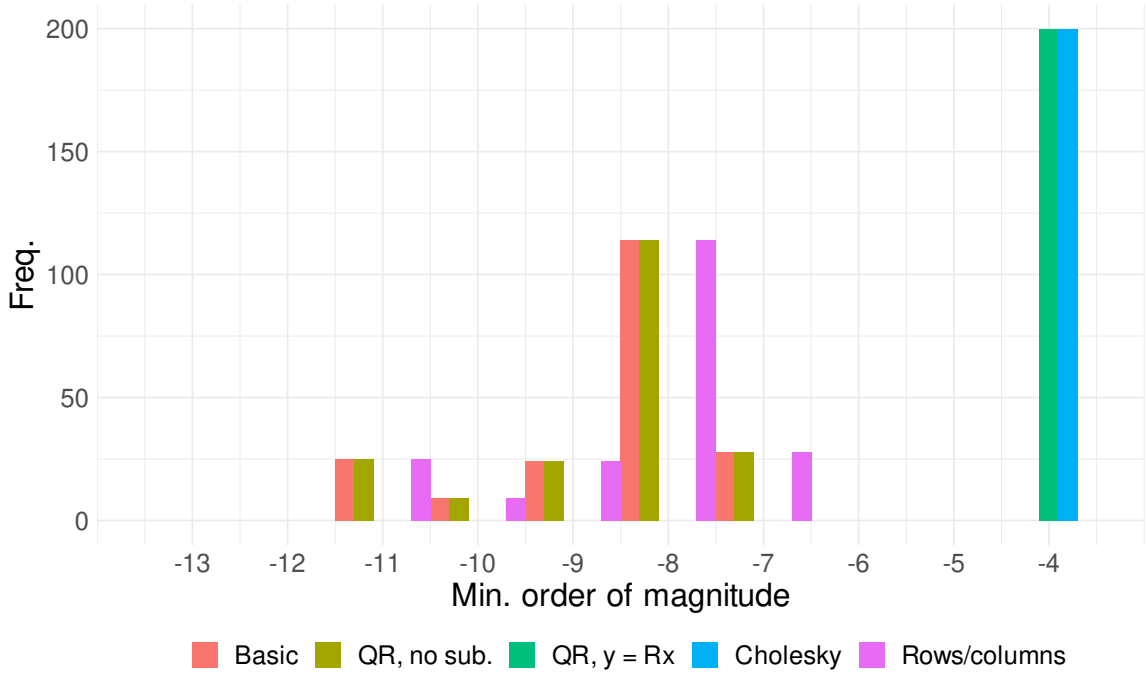
4.8 Case 4, QCQP, rescaled



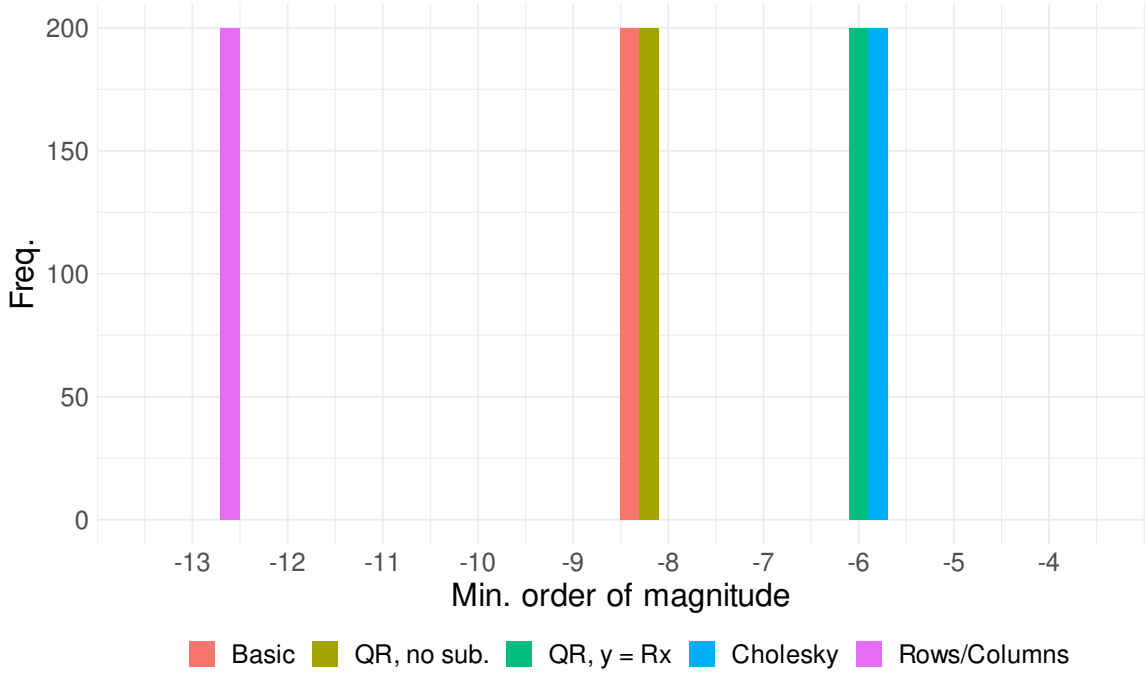
Note: When `rescale = TRUE`, ranges go up to as high as 30 orders of magnitude.

5 Min. order of mag. in quadratic constraint matrix

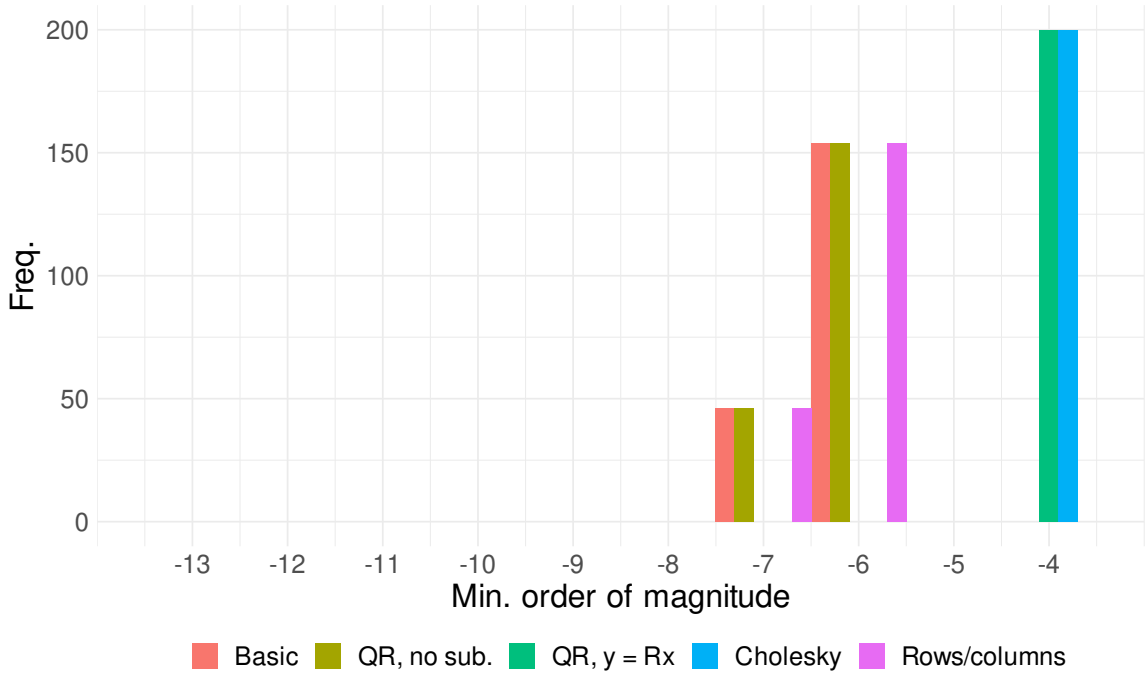
5.1 Case 1, QCQP, unscaled



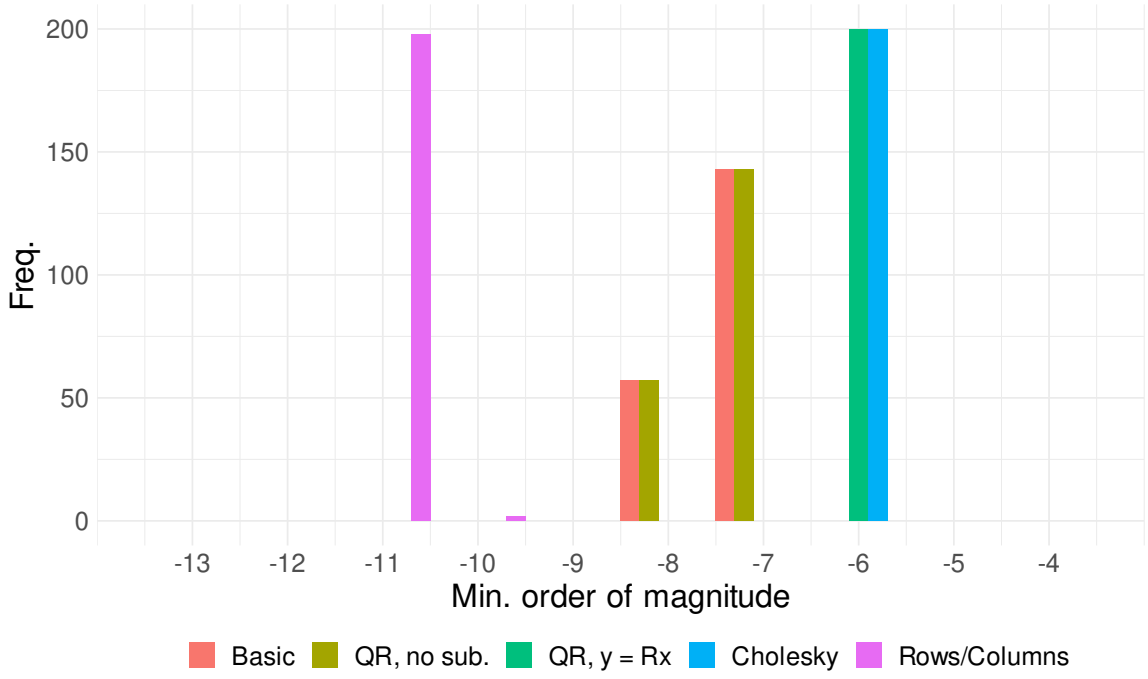
5.2 Case 1, QCQP, rescaled



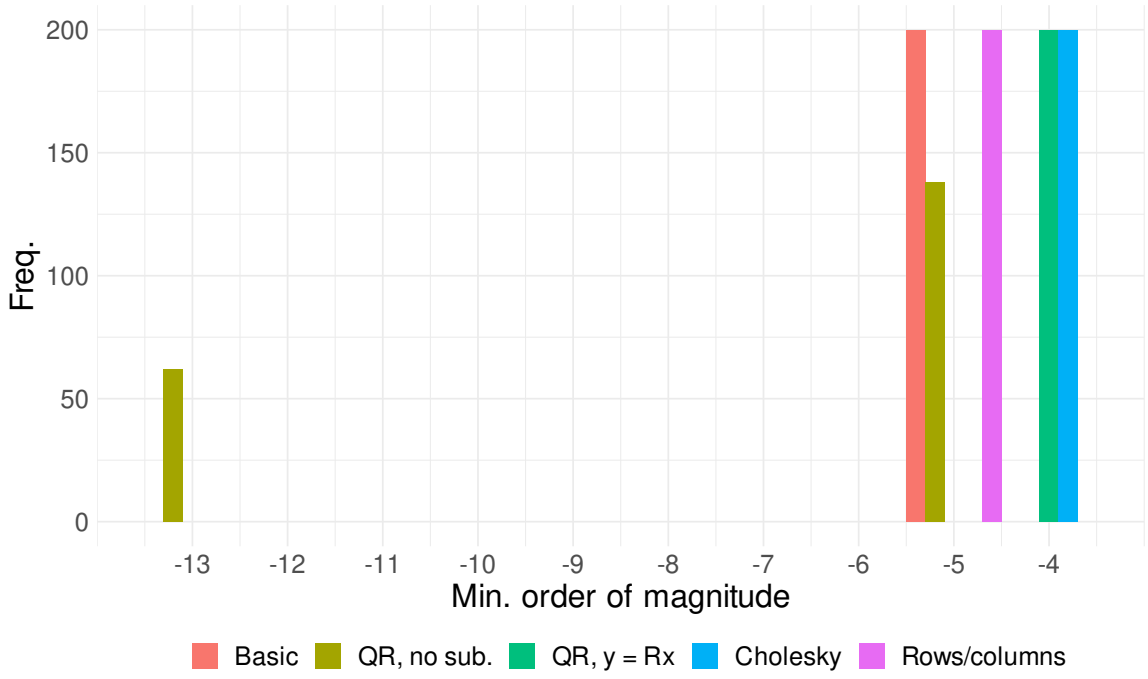
5.3 Case 2, QCQP, unscaled



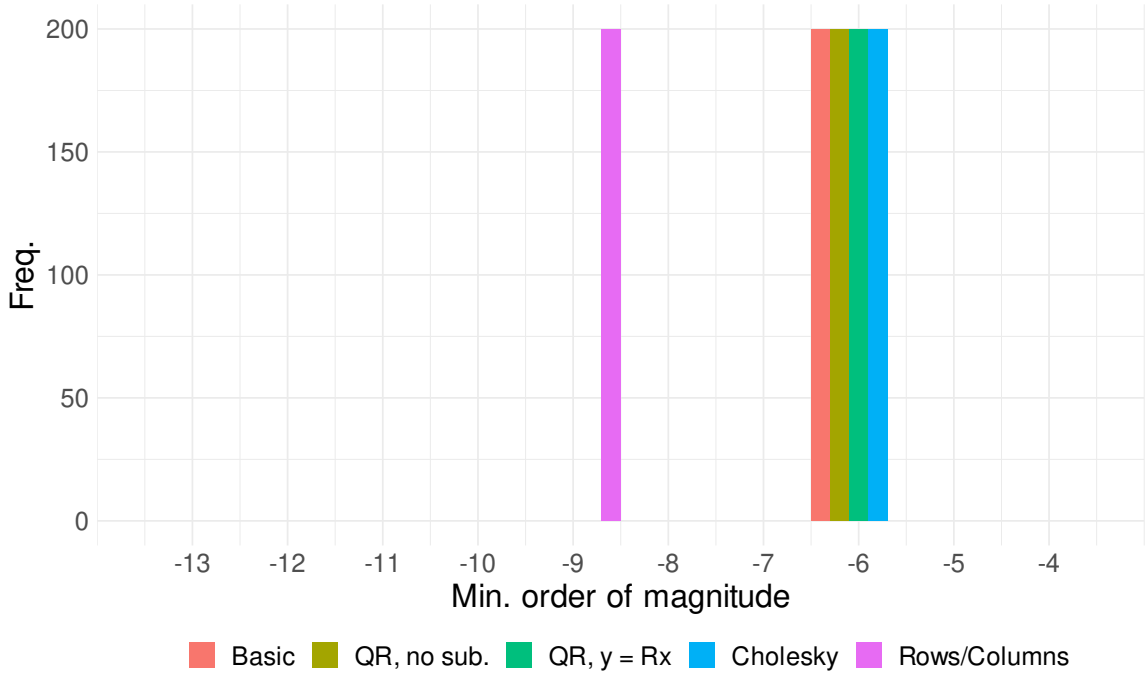
5.4 Case 2, QCQP, rescaled



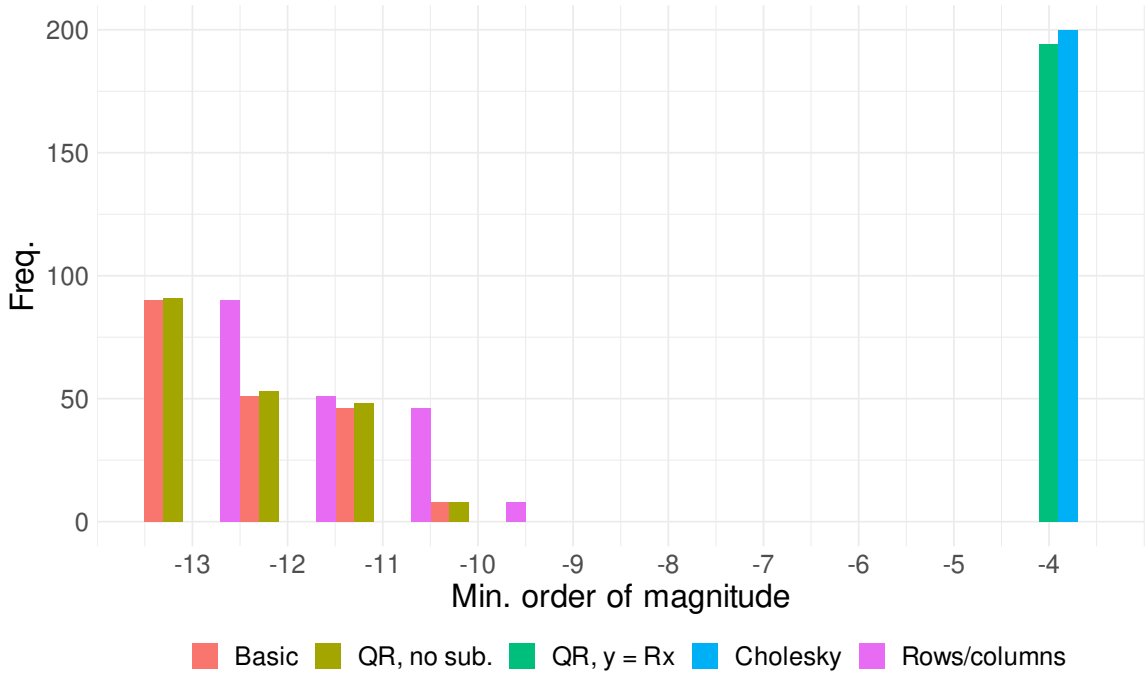
5.5 Case 3, QCQP, unscaled



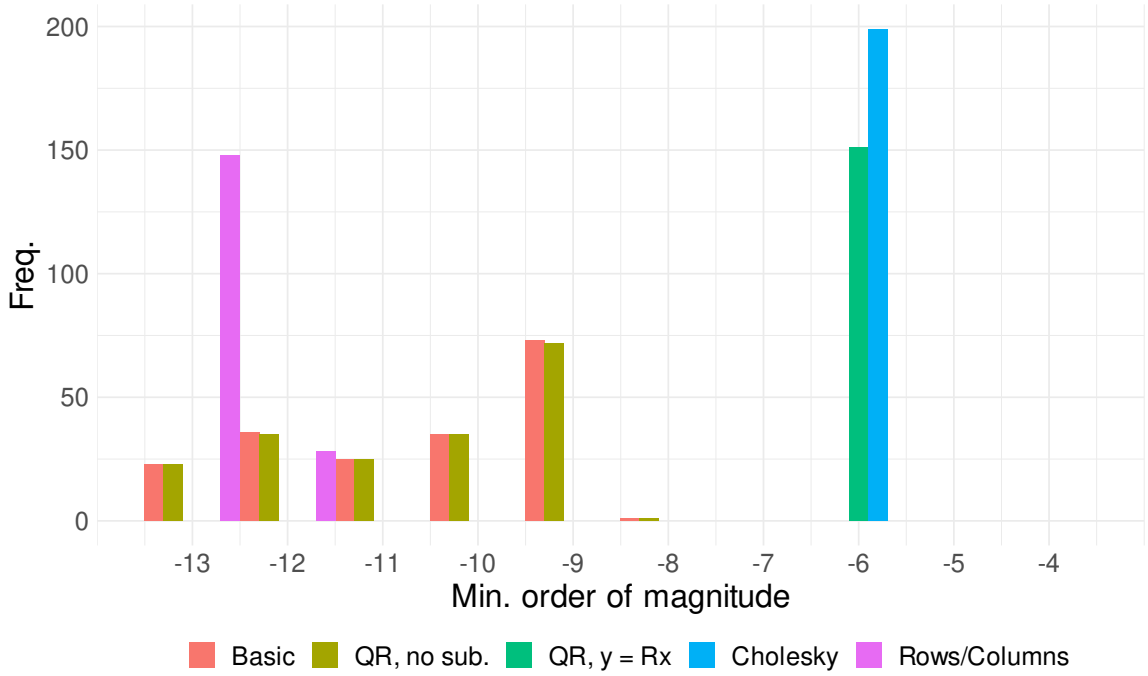
5.6 Case 3, QCQP, rescaled



5.7 Case 4, QCQP, unscaled



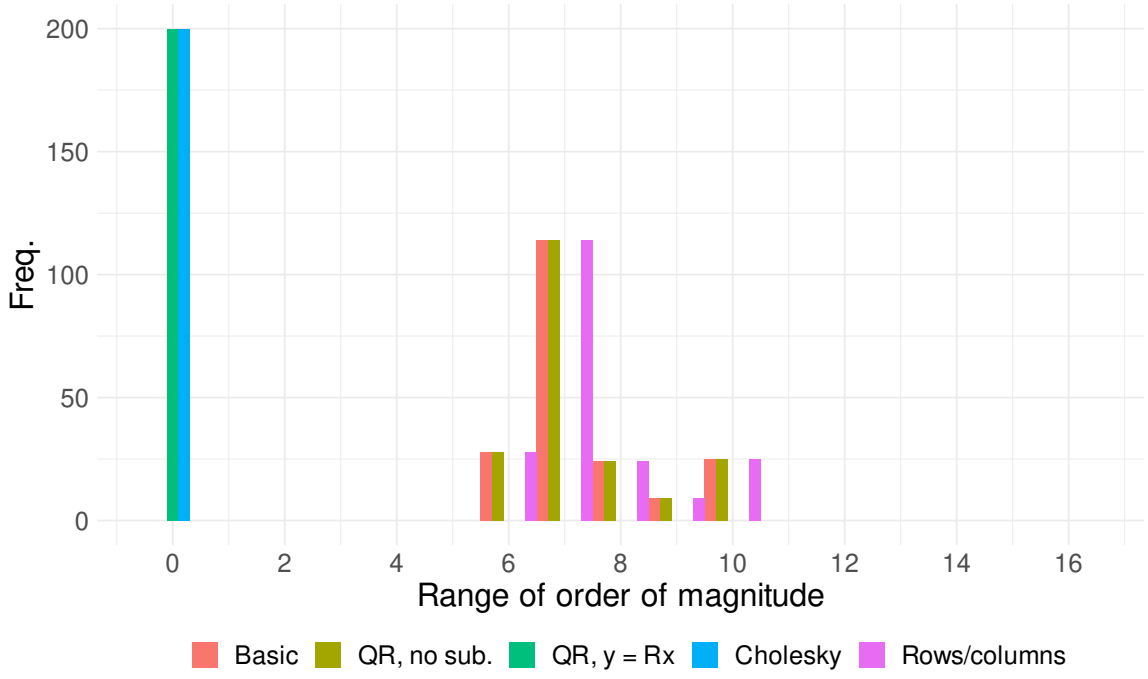
5.8 Case 4, QCQP, rescaled



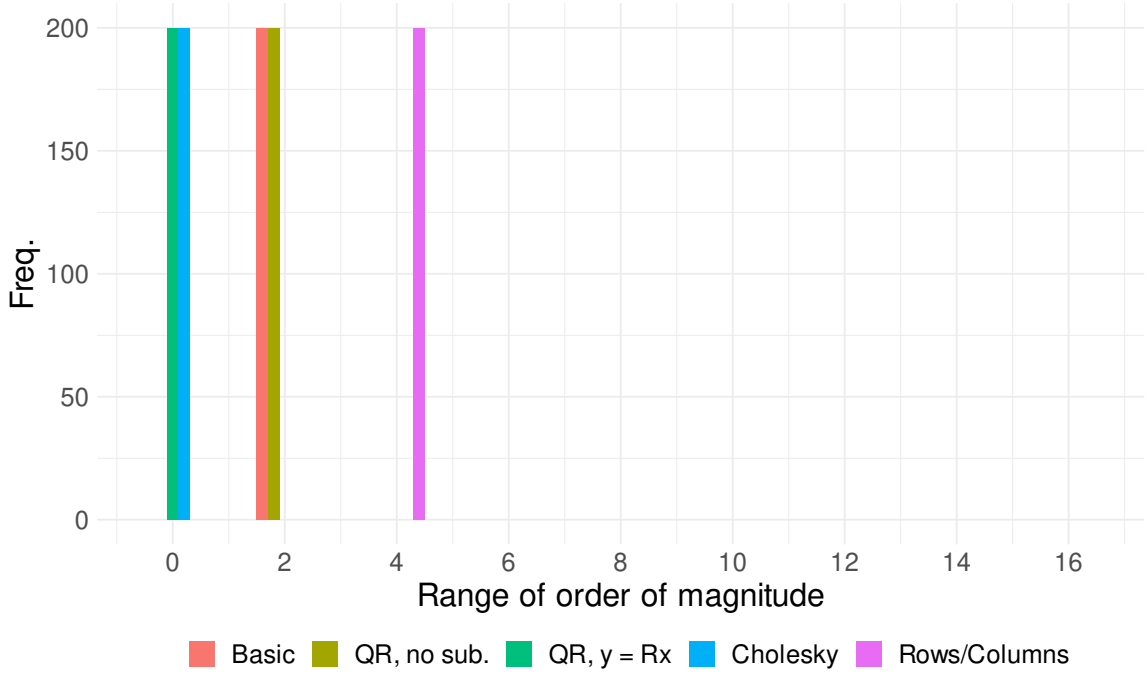
Note: Blue bar does not sum to 200 (total number of simulations) because some simulations returned errors and the QCQP model could not be saved.

6 Range of order of mag. in quadratic constraint matrix

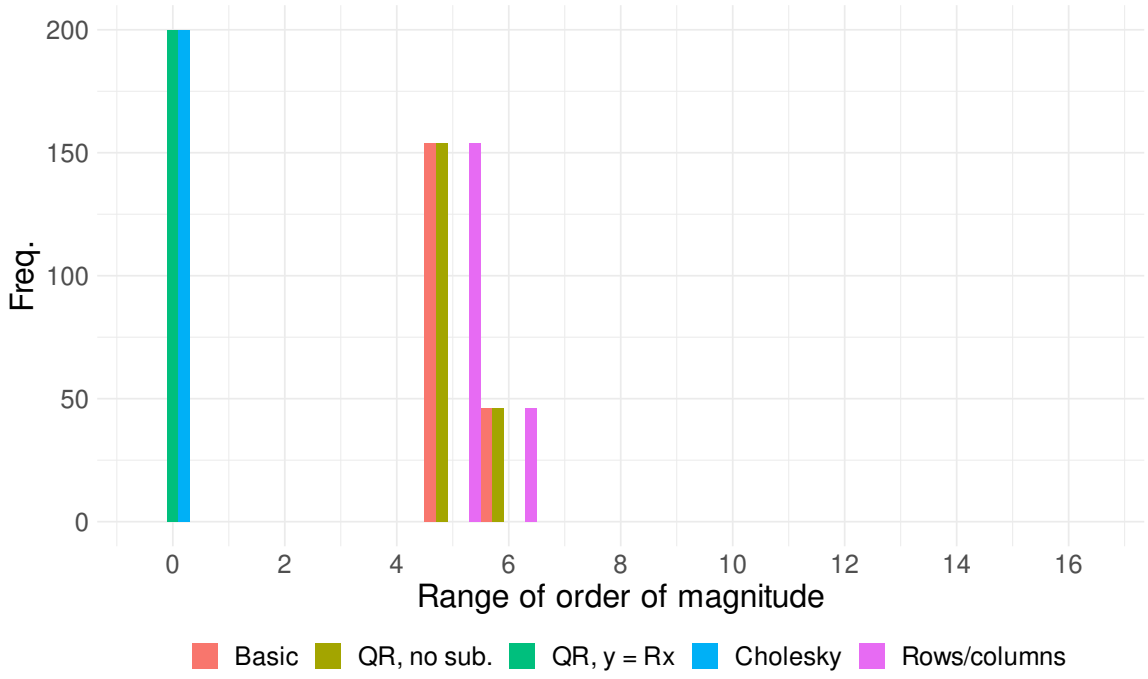
6.1 Case 1, QCQP, unscaled



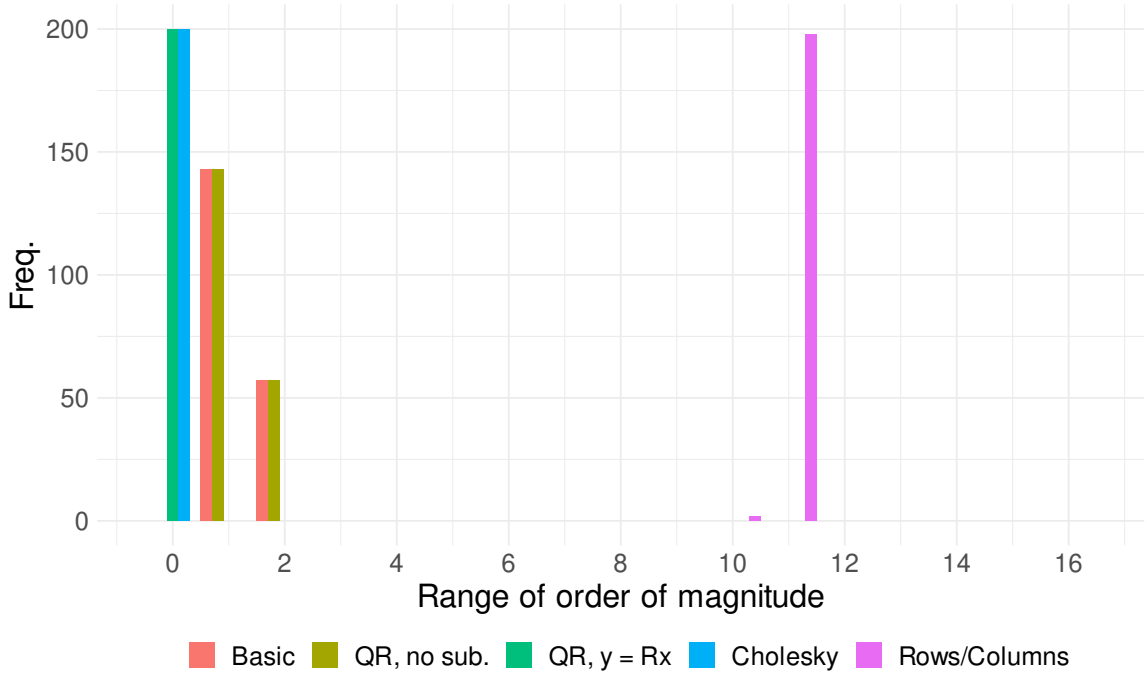
6.2 Case 1, QCQP, rescaled



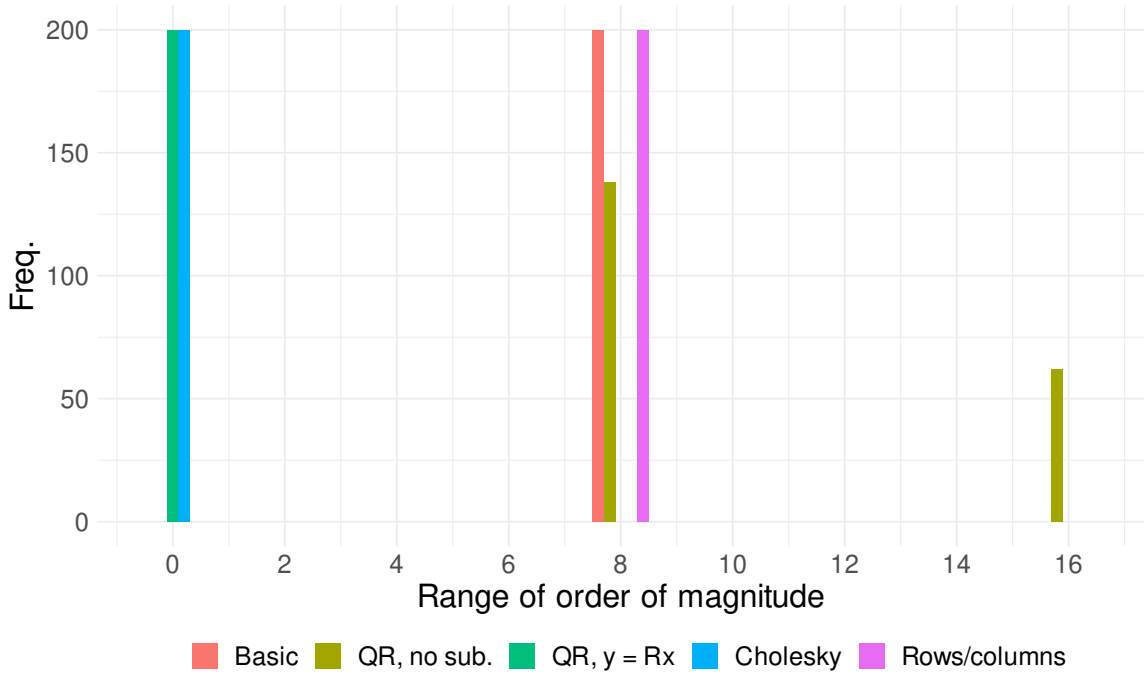
6.3 Case 2, QCQP, unscaled



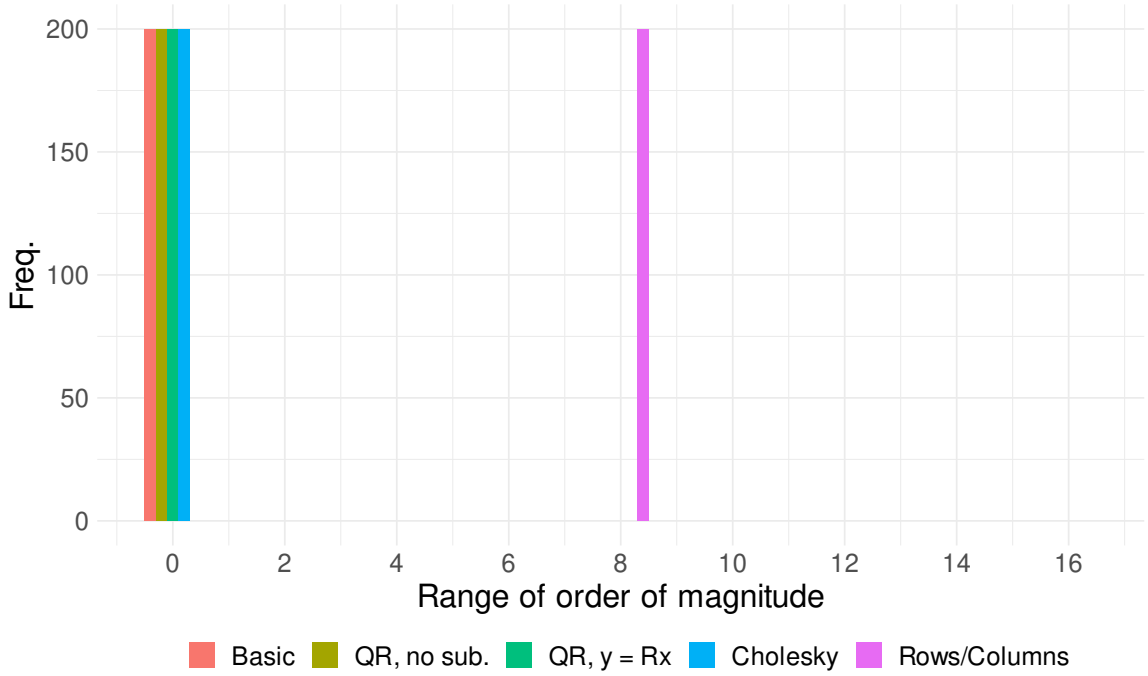
6.4 Case 2, QCQP, rescaled



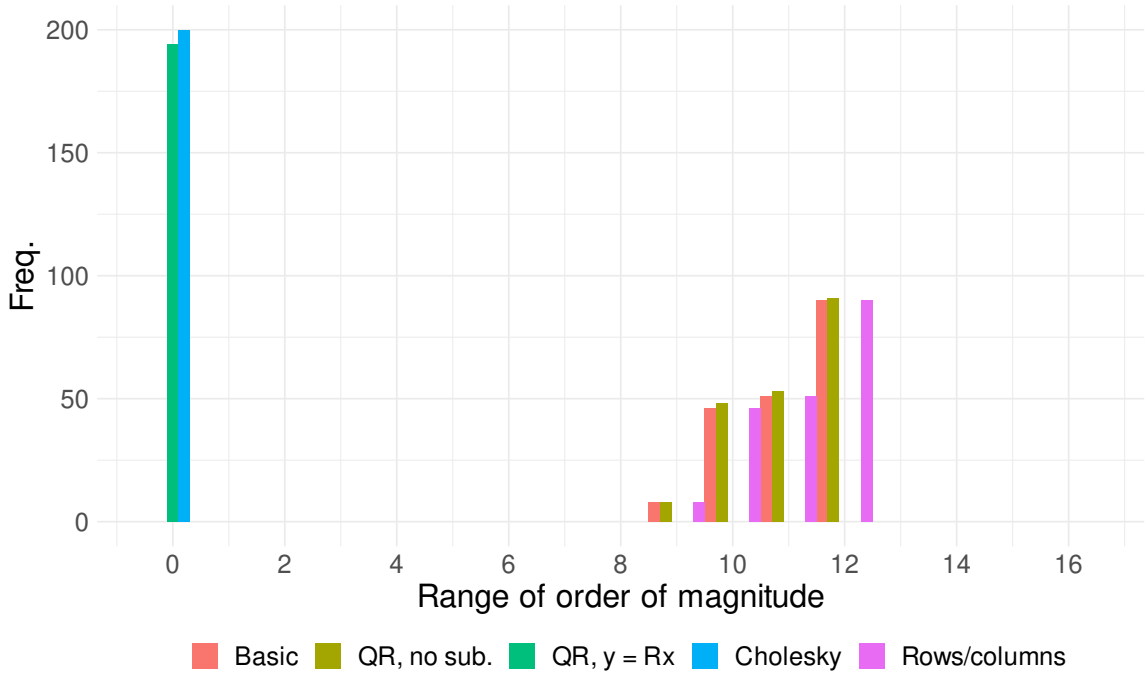
6.5 Case 3, QCQP, unscaled



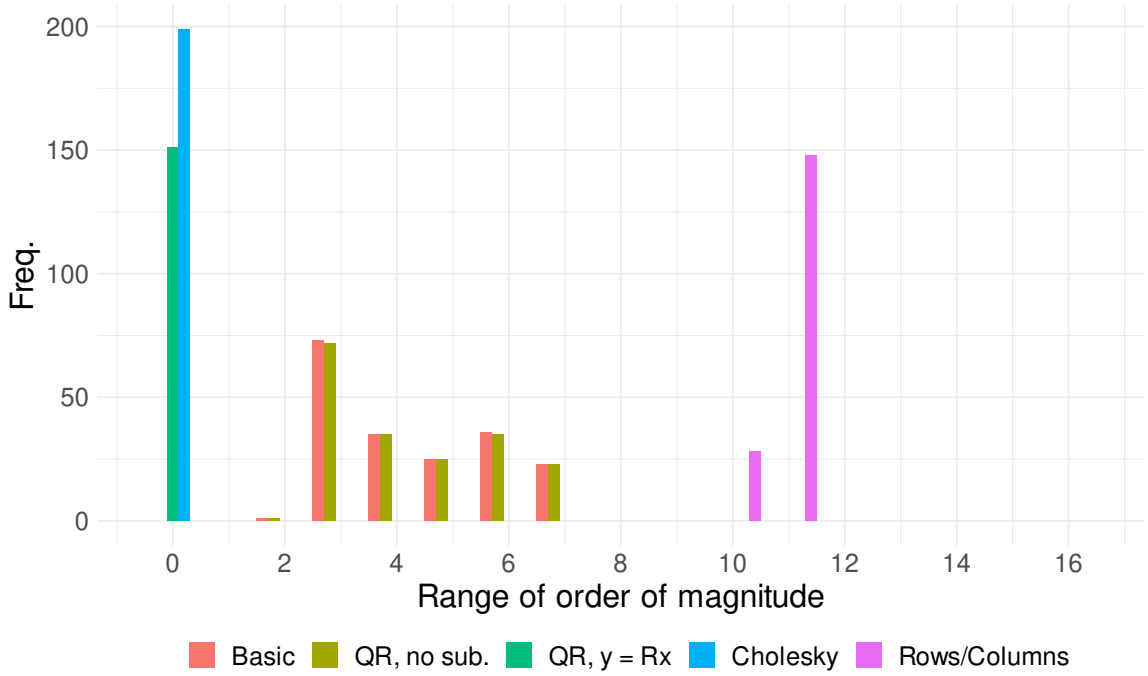
6.6 Case 3, QCQP, rescaled



6.7 Case 4, QCQP, unscaled



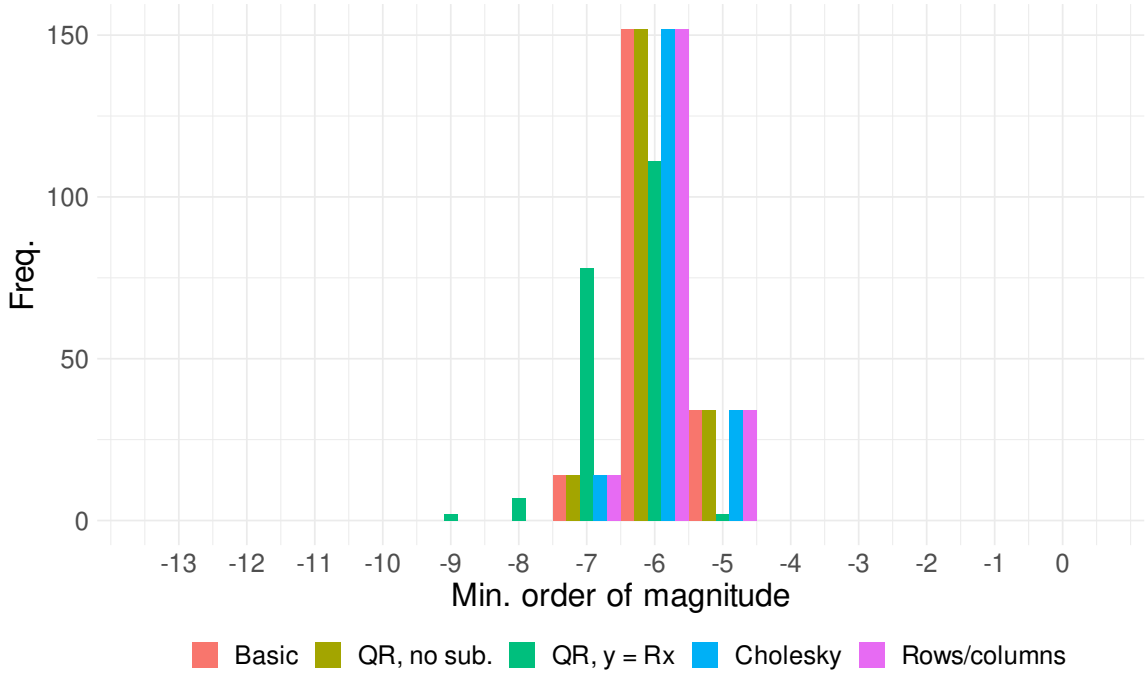
6.8 Case 4, QCQP, rescaled



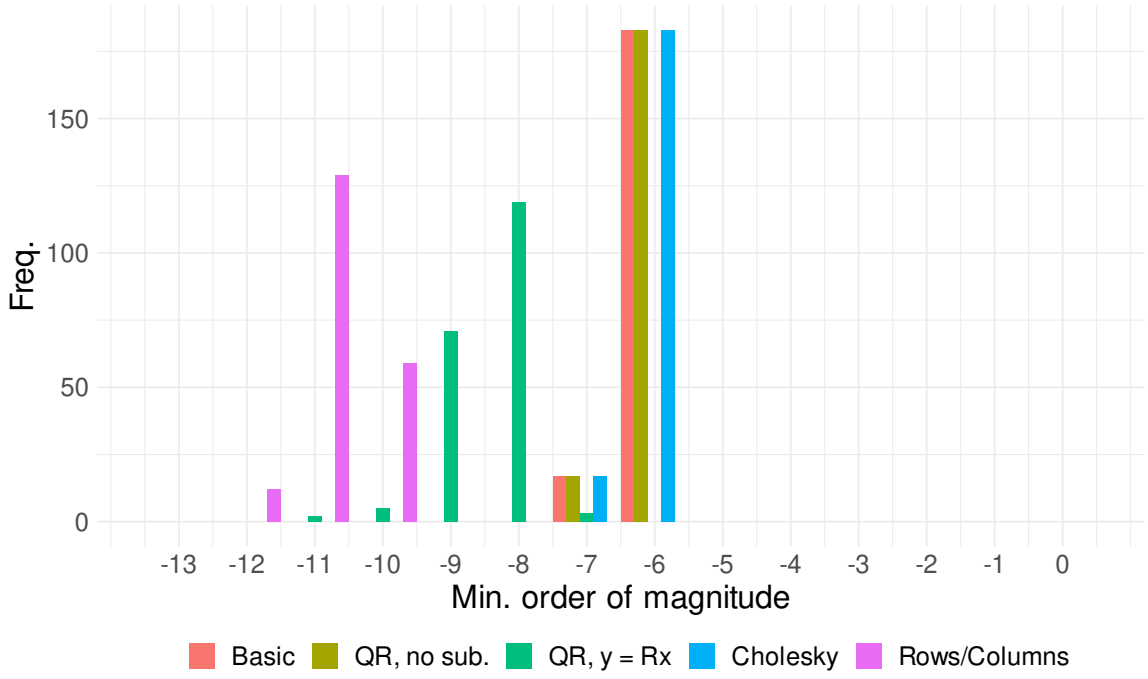
Note: Blue bar does not sum to 200 (total number of simulations) because some simulations returned errors and the QCQP model could not be saved.

7 Min. order of mag. in quadratic constraint vector

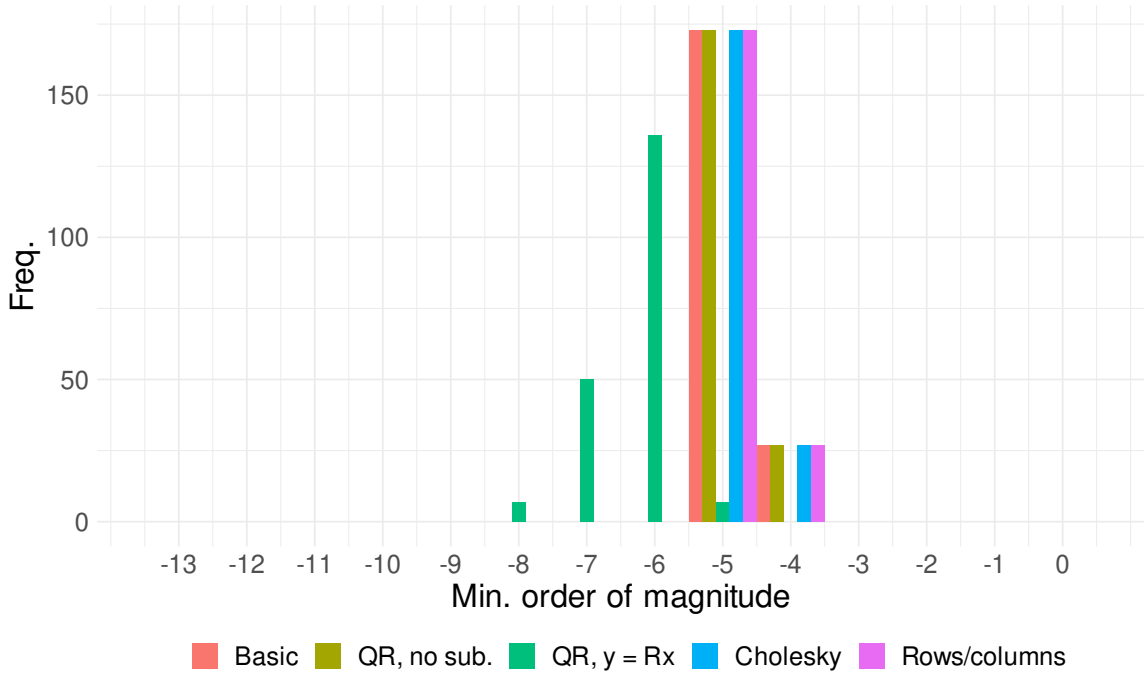
7.1 Case 1, QCQP, unscaled



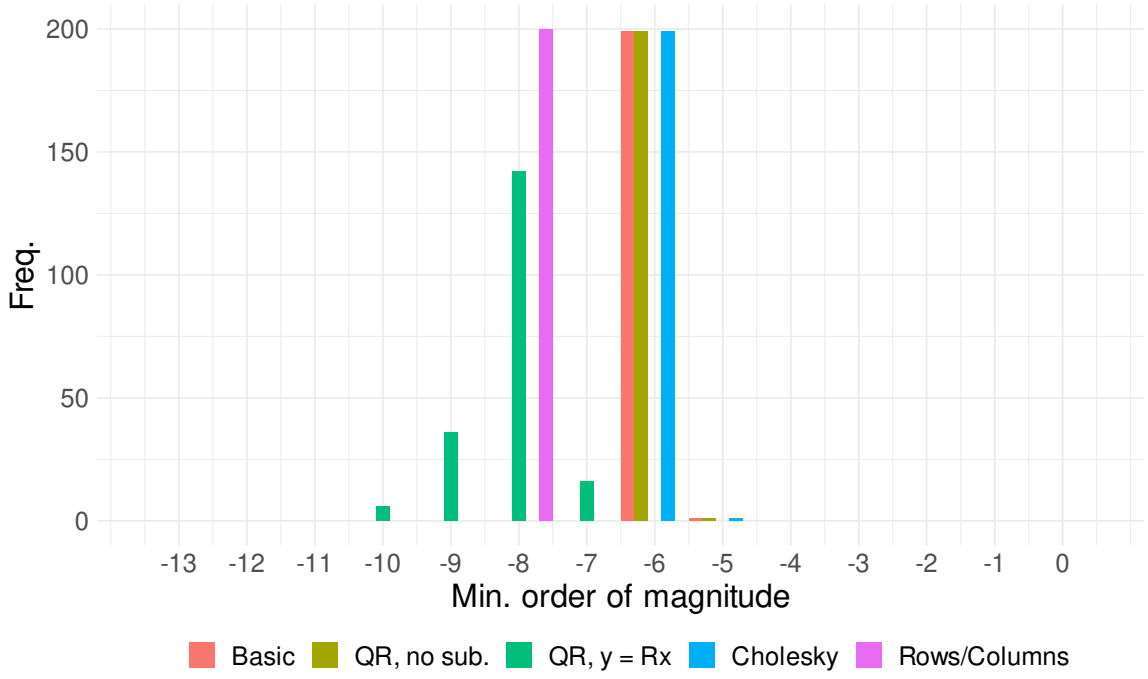
7.2 Case 1, QCQP, rescaled



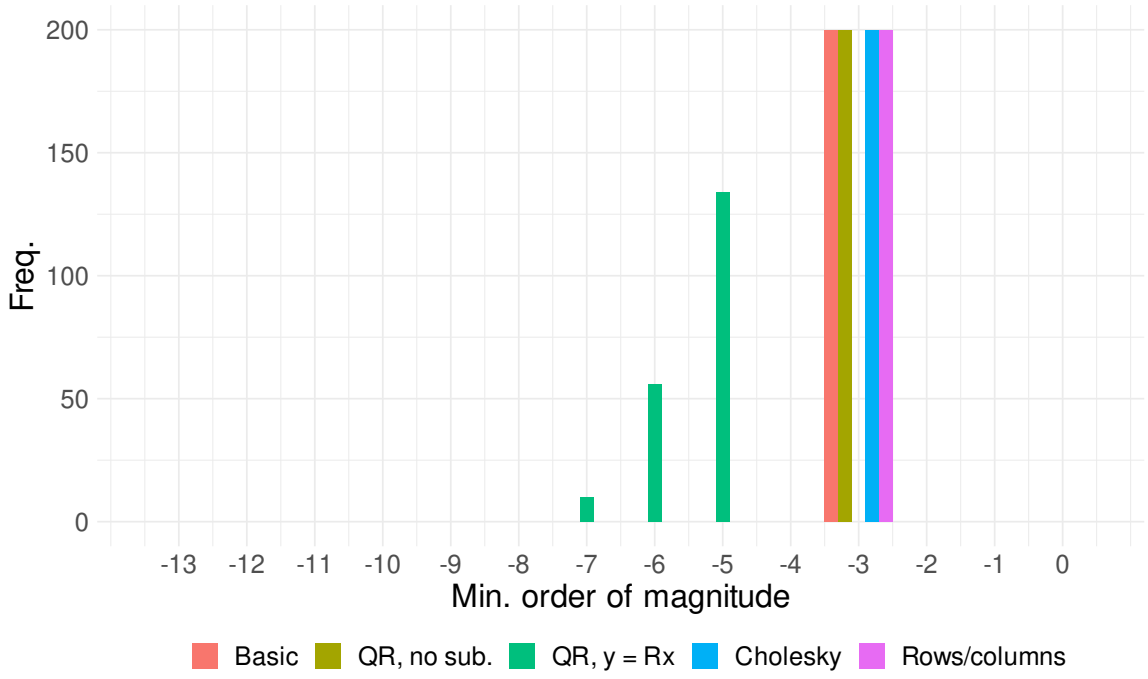
7.3 Case 2, QCQP, unscaled



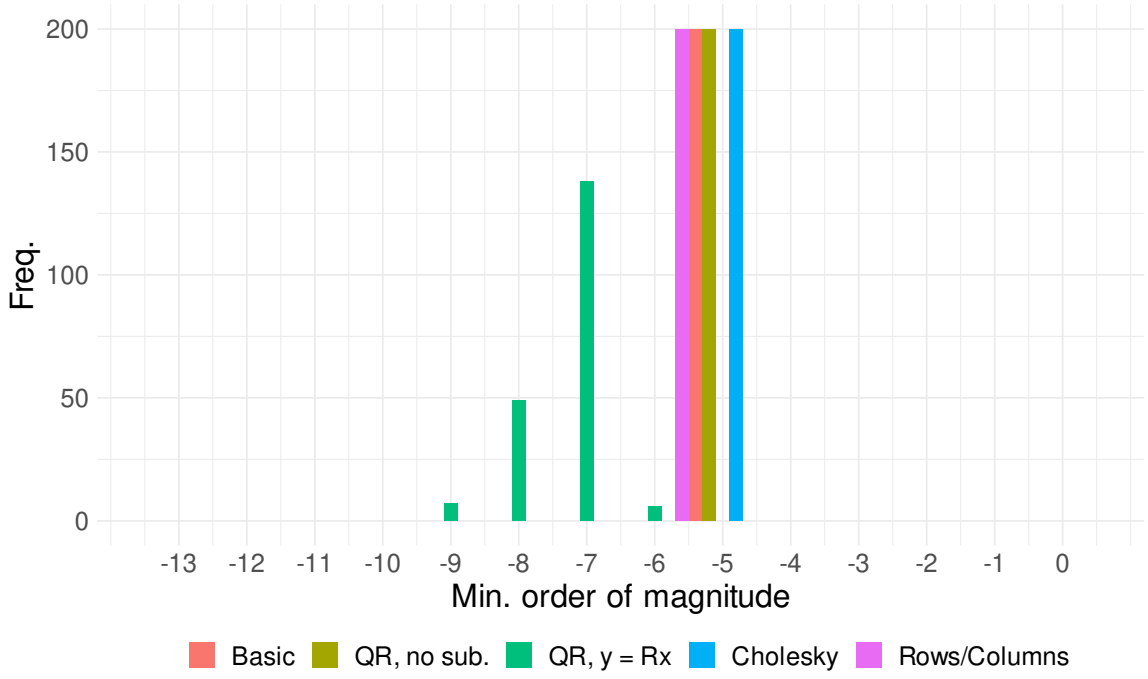
7.4 Case 2, QCQP, rescaled



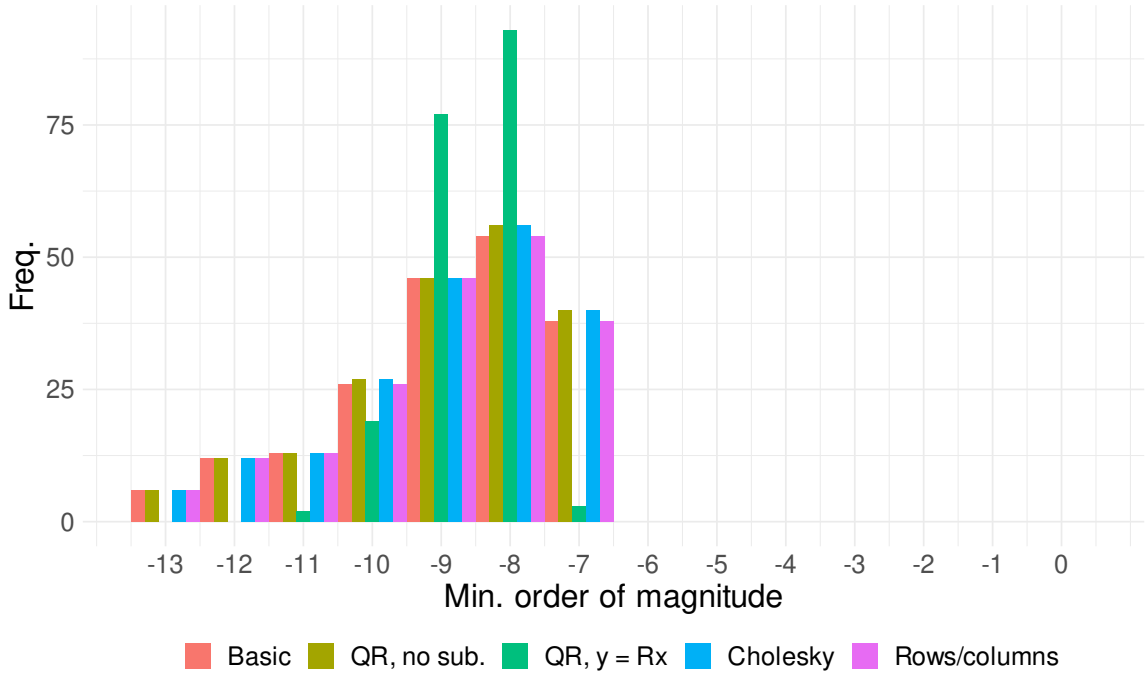
7.5 Case 3, QCQP, unscaled



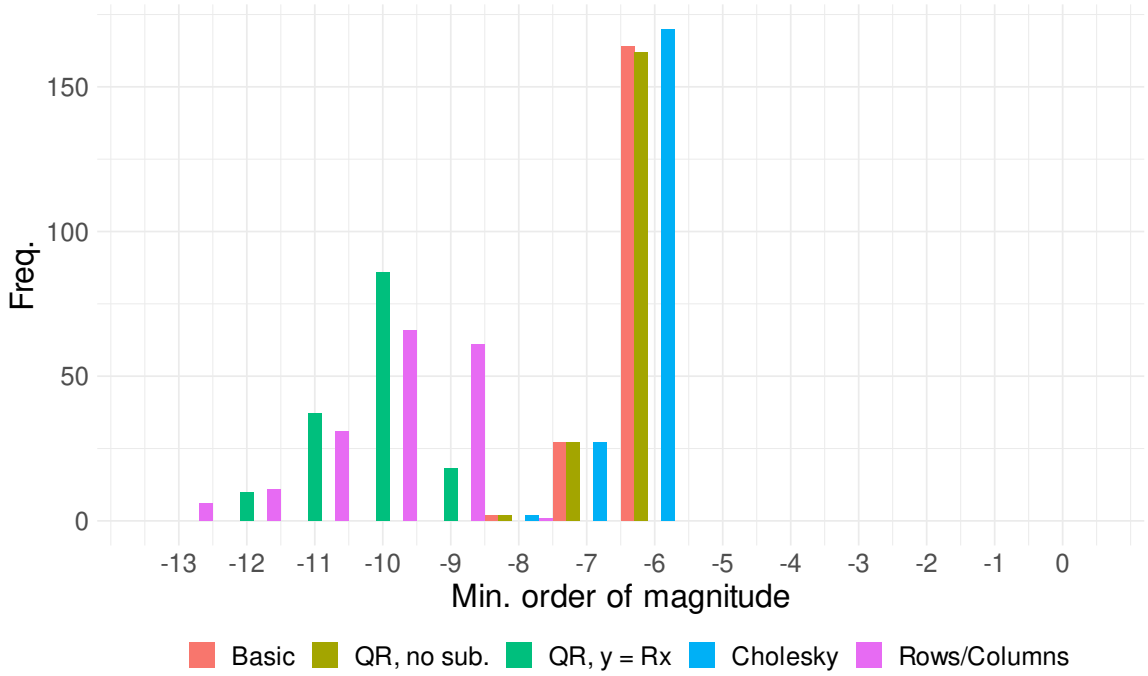
7.6 Case 3, QCQP, rescaled



7.7 Case 4, QCQP, unscaled



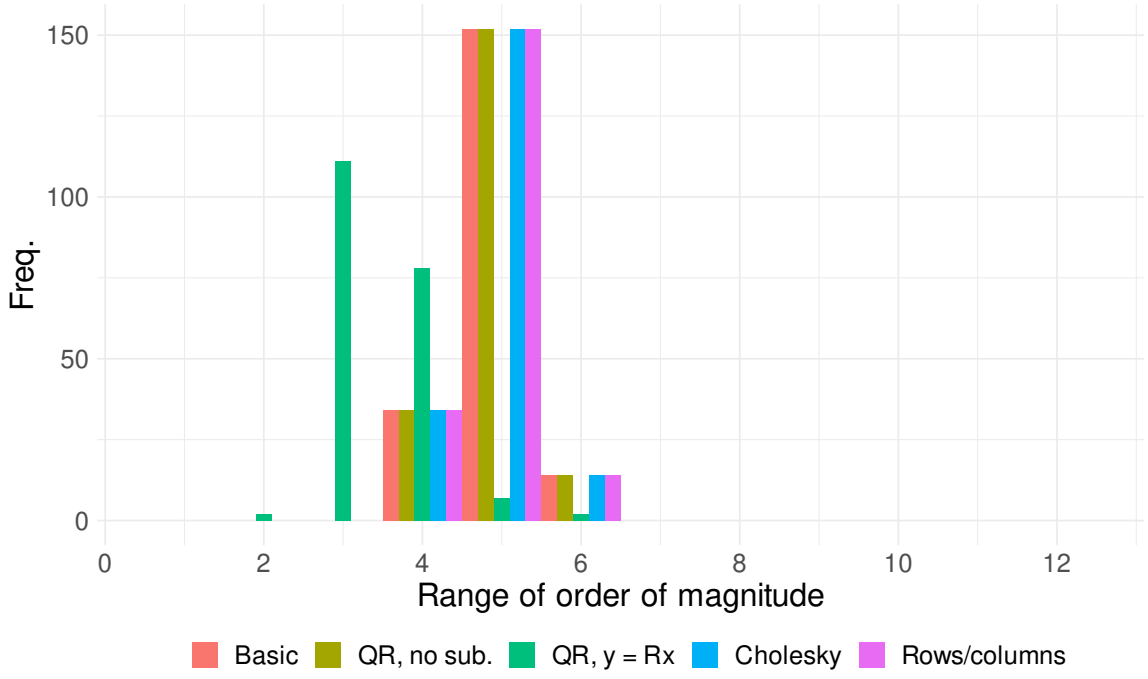
7.8 Case 4, QCQP, rescaled



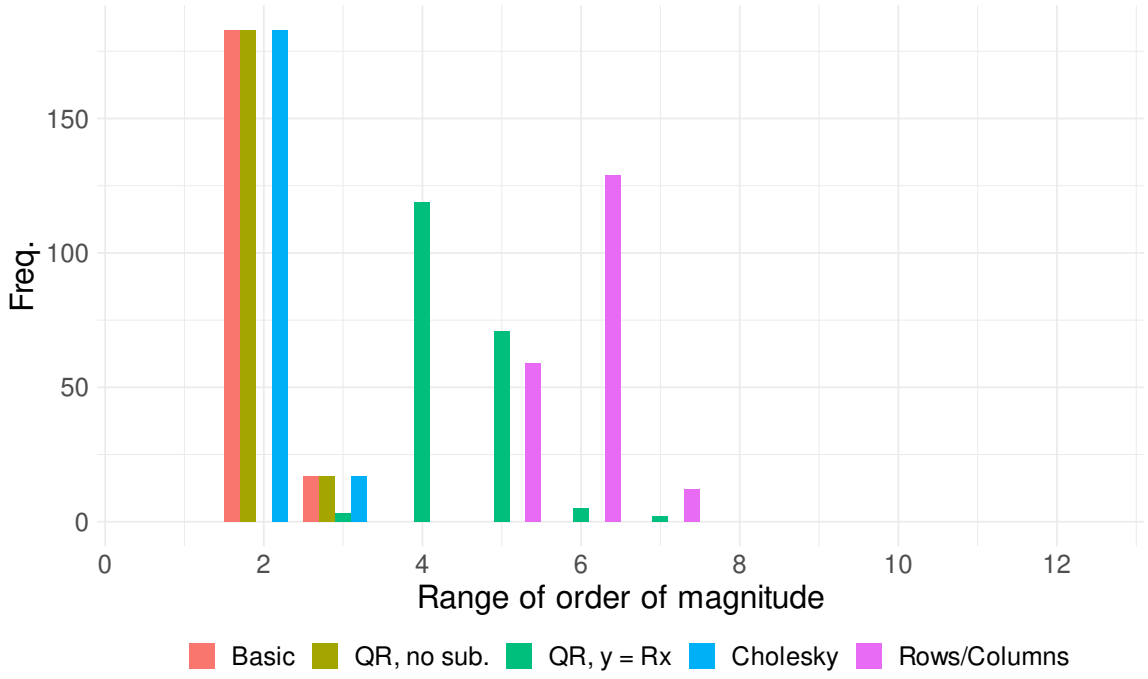
Note: Blue bar does not sum to 200 (total number of simulations) because some simulations returned errors and the QCQP model could not be saved.

8 Range of order of mag. in quadratic constraint vector

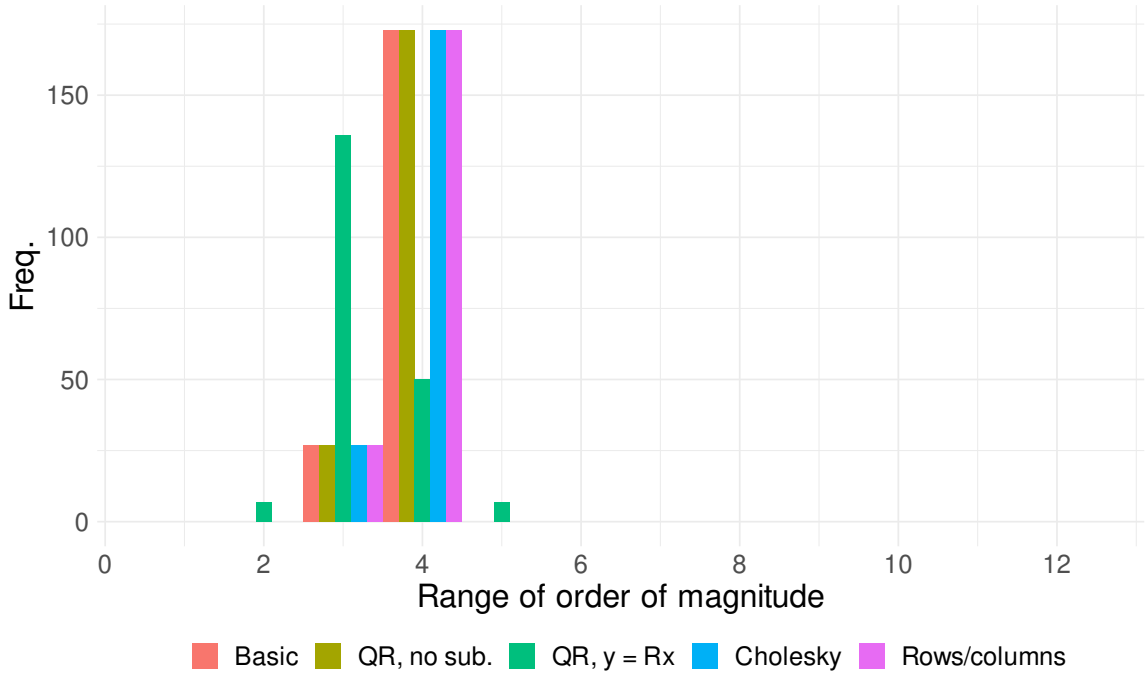
8.1 Case 1, QCQP, unscaled



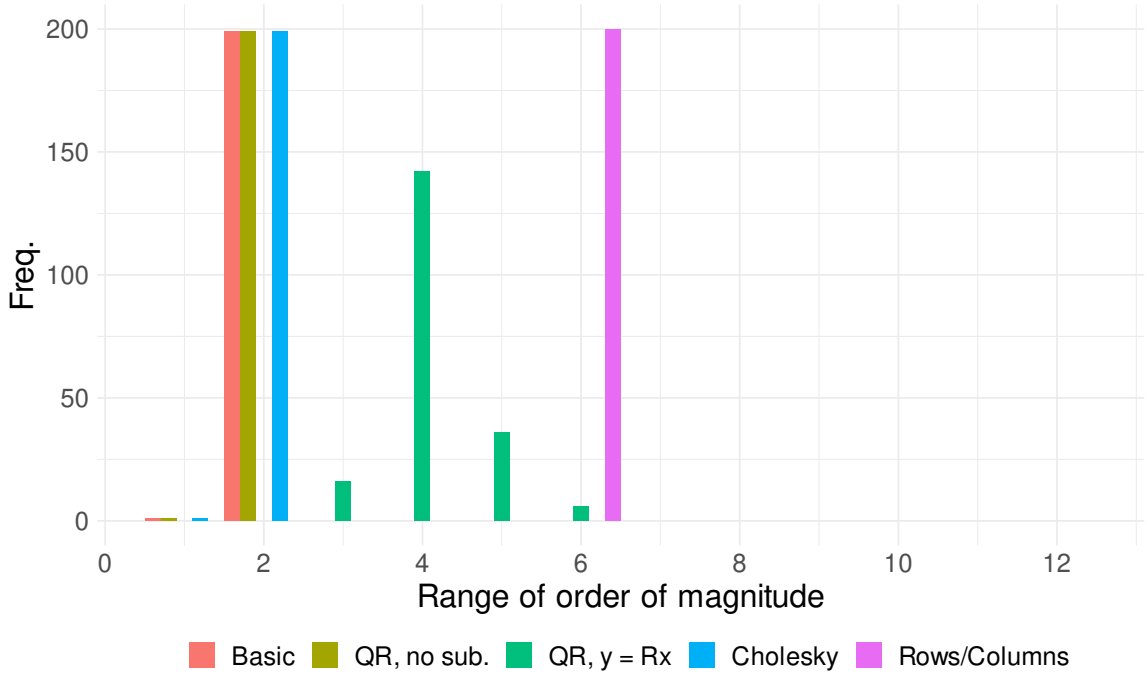
8.2 Case 1, QCQP, rescaled



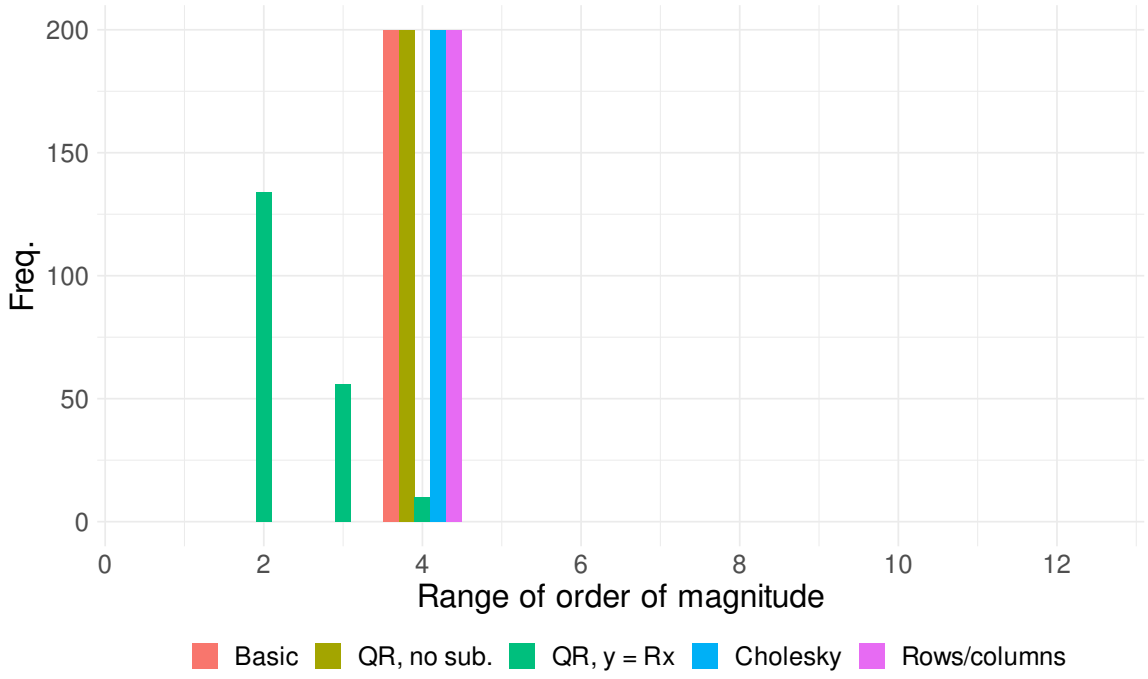
8.3 Case 2, QCQP, unscaled



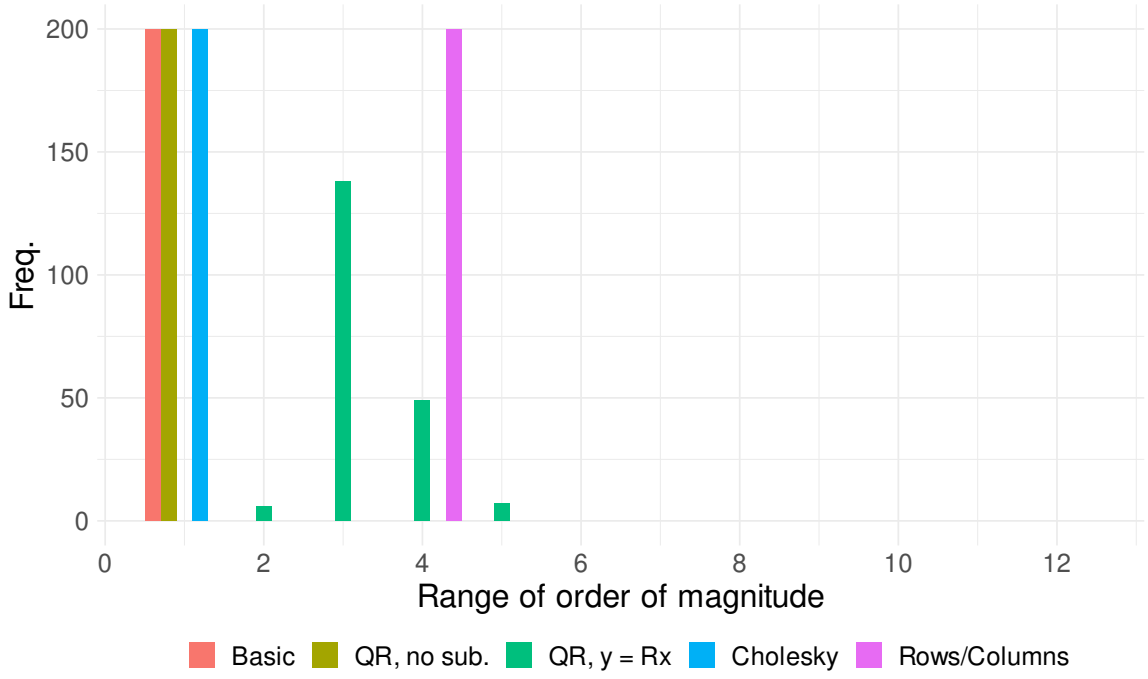
8.4 Case 2, QCQP, rescaled



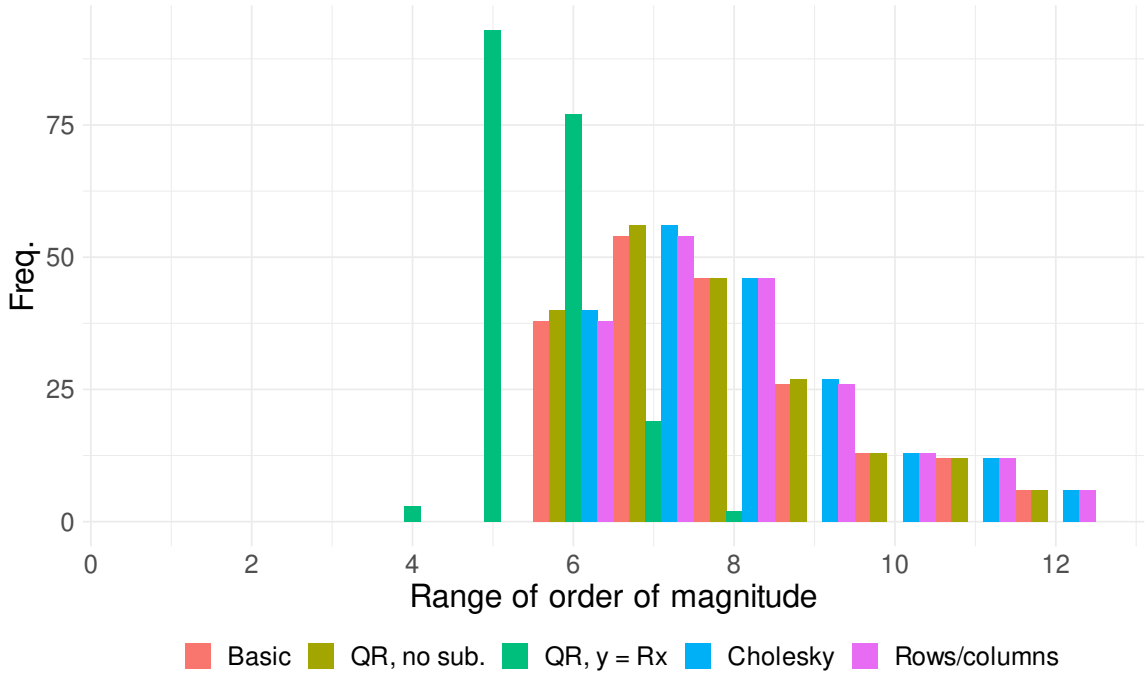
8.5 Case 3, QCQP, unscaled



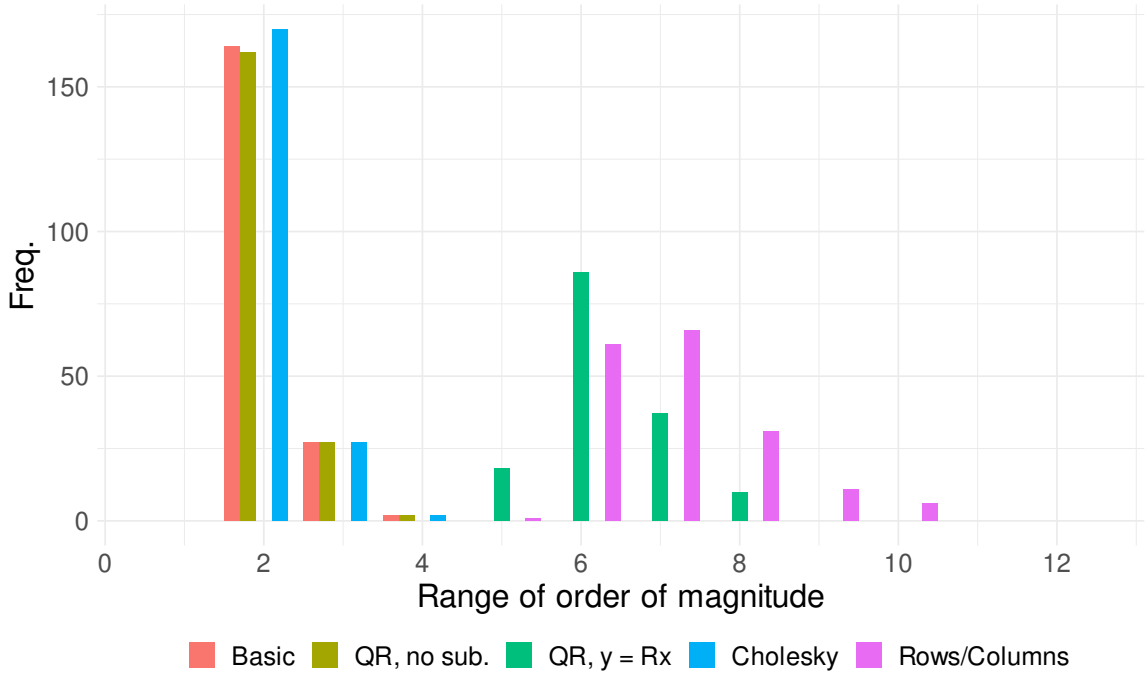
8.6 Case 3, QCQP, rescaled



8.7 Case 4, QCQP, unscaled



8.8 Case 4, QCQP, rescaled



Note: Blue bar does not sum to 200 (total number of simulations) because some simulations returned errors and the QCQP model could not be saved.