

## Rescale comparisons

August 19, 2022

### 1 Introduction

This document studies whether setting `rescale = TRUE` helps with estimation using the same examples as described in the document titled *Matrix Decomposition Comparisons* (available [here](#)).

The `rescale` option was originally developed to address stability issues in the QCQP problem through rescaling the Gram matrix. Currently, `ivmte` only allows rescaling in the QCQP problems. I will allow rescaling in LP problems in the future.

To explain how I rescaled the QCQP problems, let  $x$  denote the unknown variables,  $A$  denote the design matrix,  $b$  denote the vector of observed outcomes, and  $L$  denote the linear constraint matrix. The least squares criterion is  $x' A' A x - 2x' A' b + b' b$ .

1. `qp0` (no decomposition): Let  $K$  denote a diagonal matrix whose  $i^{\text{th}}$  diagonal is equal to the  $\ell_2$  norm of the  $i^{\text{th}}$  column of  $A$ , i.e.,  $K_{i,i} \equiv \|A_{\cdot,i}\|$ . The QCQP model is defined using the rescaled design matrix  $\tilde{A} \equiv AK^{-1}$  and the rescaled linear constraint matrix  $\tilde{L} \equiv LK^{-1}$ .
2. `qp1` (QR decomposition, no substitutions): By the QR decomposition,  $A = QR$ , where  $Q$  is an orthogonal matrix. The Gram matrix  $A' A$  defining the quadratic component of the model may then be written as

$$A' A = (R' Q') Q R = R' R.$$

Let  $K$  denote a diagonal matrix whose  $i^{\text{th}}$  diagonal is equal to the  $\ell_2$  norm of the  $i^{\text{th}}$  column of  $R$ , i.e.,  $K_{i,i} \equiv \|R_{\cdot,i}\|$ . The QCQP problem is defined using  $\tilde{A} \equiv AK^{-1}$  and  $\tilde{L} = LK^{-1}$ .

3. `qp3` (QR decomposition,  $y = Rx$ ): By the QR decomposition and substitution  $y = Rx$ , the least squares criterion may be written as

$$\begin{aligned} x' A' A x - 2x' A' b + b' b &= x' R' R x - 2x' R' Q' b + b' b \\ &= y' y - 2y' Q' b + b' b. \end{aligned}$$

Since small entries in the constraint matrix  $L$  are due to small entries in  $R$ , I define

the diagonal matrix  $K$  such that  $K_{i,i} \equiv \|R_{:,i}\|$ . The QCQP model is defined using the rescaled constraint matrix  $\tilde{L} \equiv LK^{-1}$ . The substitution  $y = Rx$  then becomes  $y = \tilde{R}x$ , where  $\tilde{R} \equiv RK^{-1}$ .

4. qp4 (Cholesky decomposition,  $y = Cx$ ): By the Cholesky decomposition,  $A'A = C'C$ . Using the substitution  $y = Cx$ , the least squares criterion may be written as

$$\begin{aligned} x'A'Ax - 2x'A'b + b'b &= x'C'Cx - 2x'A'b + b'b \\ &= y'y - 2x'A'b + b'b. \end{aligned}$$

Since small entries in the constraint matrix  $L$  are due to small entries in  $C$ , I define the diagonal matrix  $K$  such that  $K_{i,i} \equiv \|C_{:,i}\|$ . The QCQP model is defined using the rescaled constraint matrix  $\tilde{L} \equiv LK^{-1}$ . The substitution  $y = Cx$  then becomes  $y = \tilde{C}x$ , where  $\tilde{C} \equiv CK^{-1}$ .

5. qp5 (Row/column rescaling): This approach first rescales the rows of  $A$  so that the minimum order of magnitude is  $-3$ . The columns of  $A$  are then rescaled so the minimum order of magnitude is also  $-3$ .

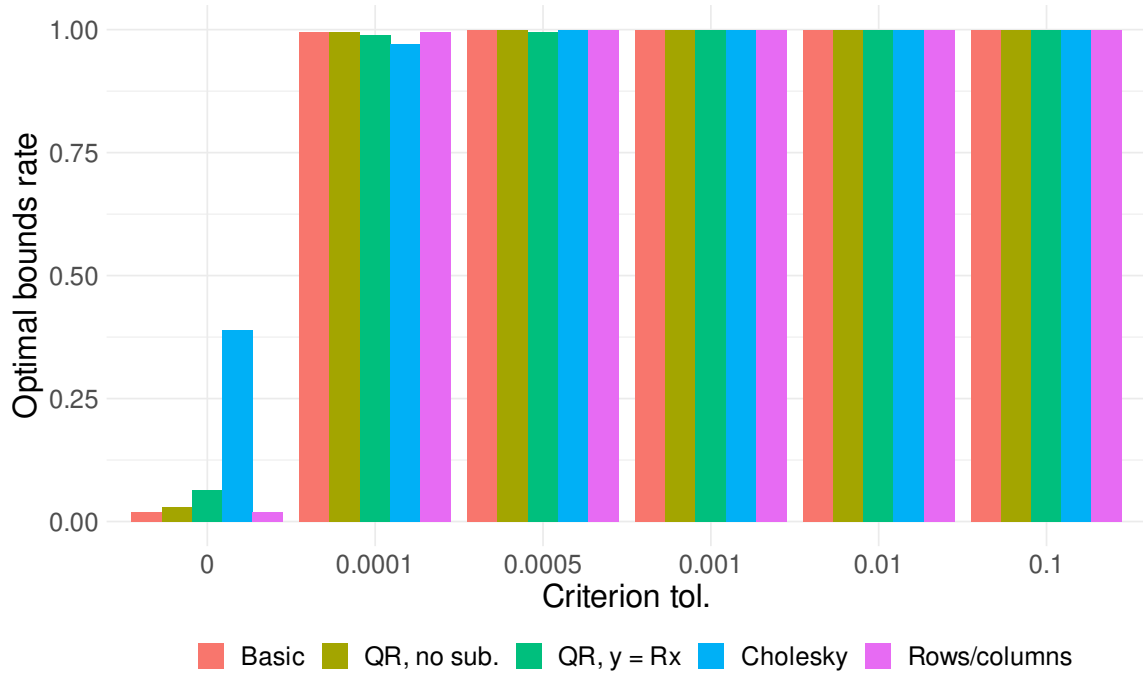
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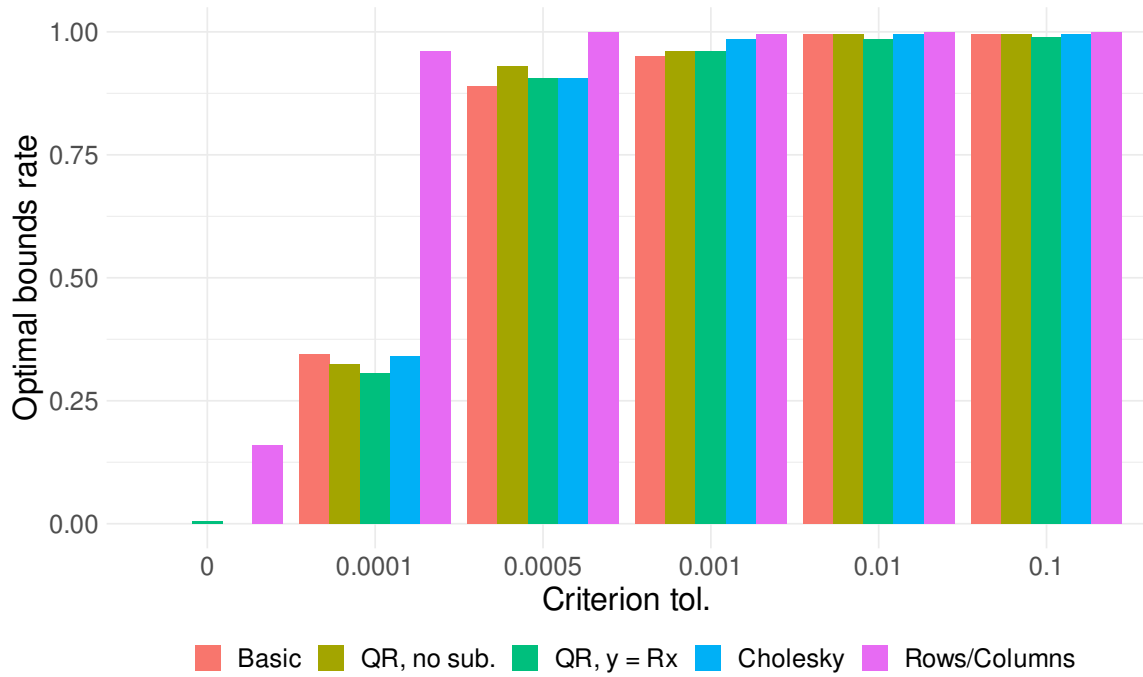
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## 2 Optimal upper and lower bounds

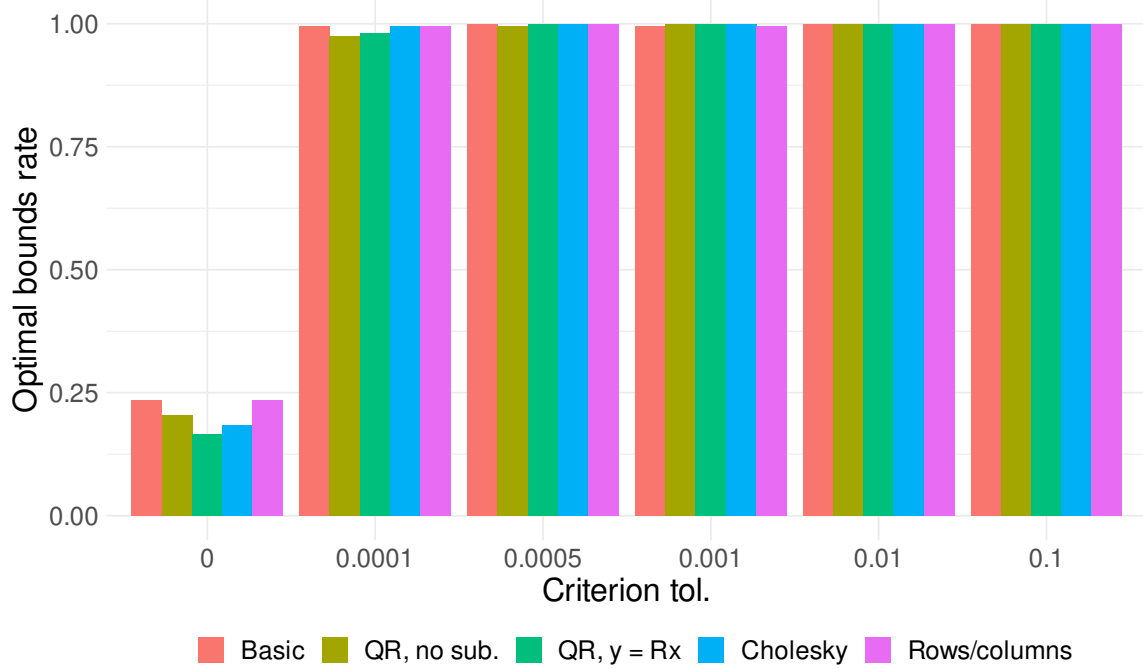
### 2.1 Case 1, QCQP, unscaled



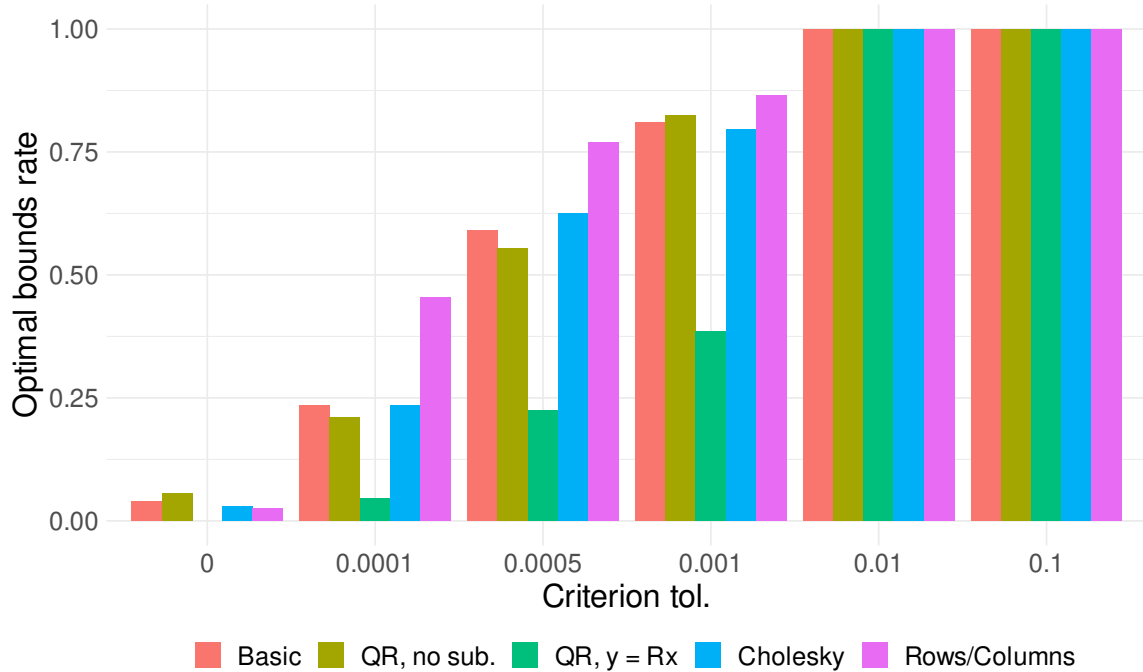
### 2.2 Case 1, QCQP, rescaled



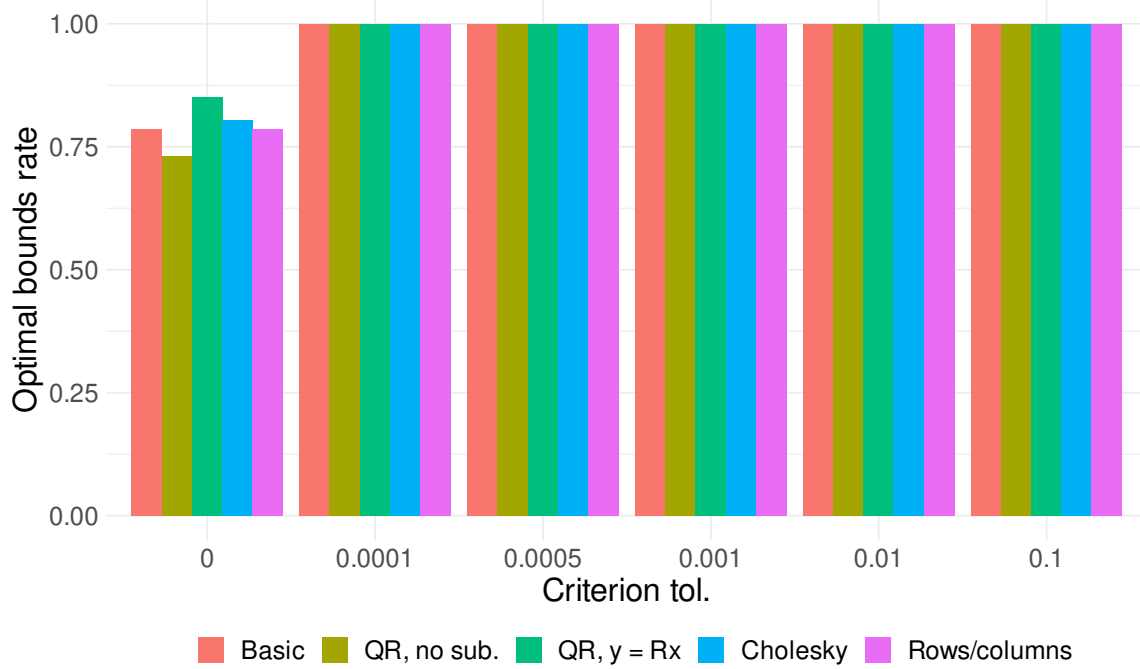
### 2.3 Case 2, QCQP, unscaled



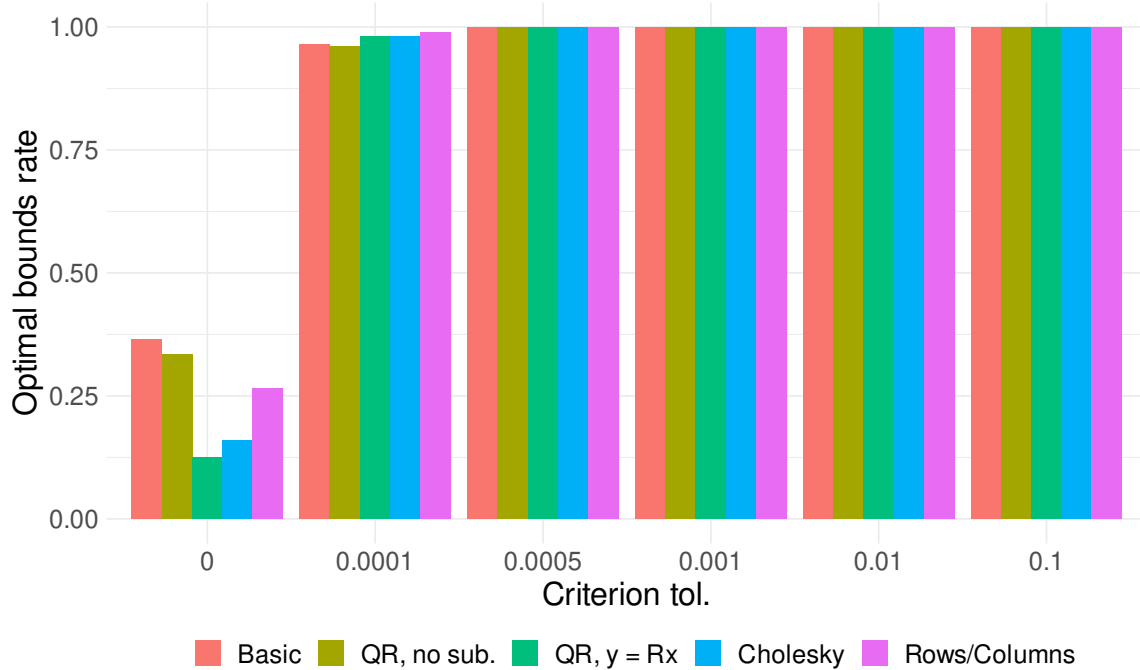
### 2.4 Case 2, QCQP, rescaled



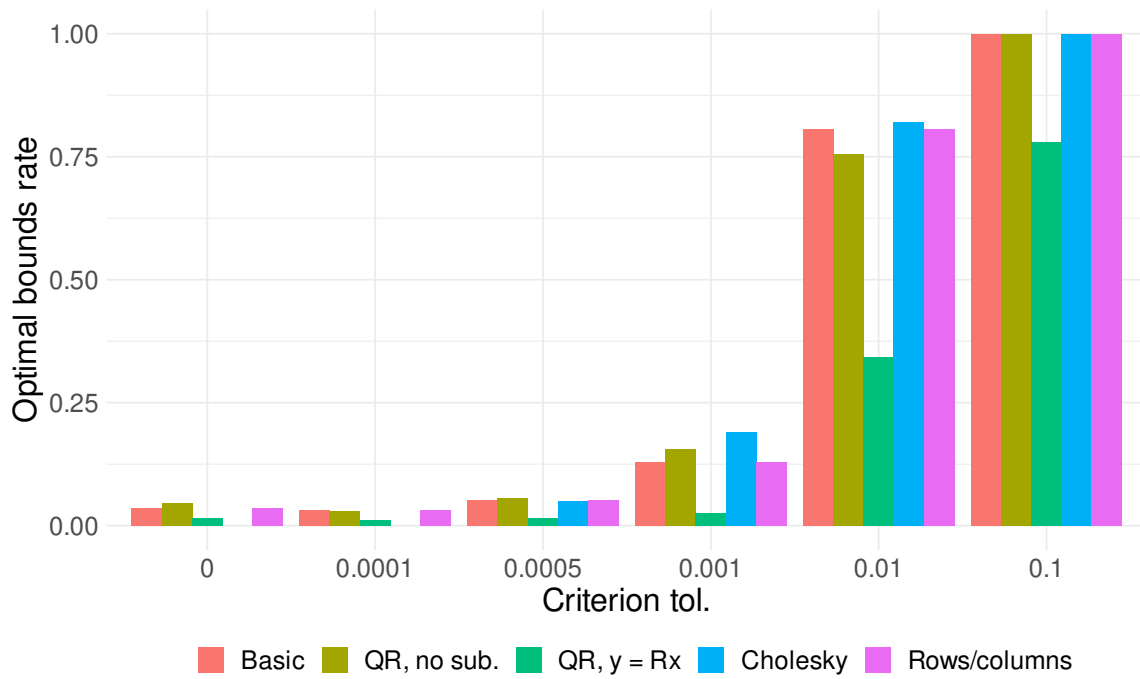
### 2.5 Case 3, QCQP, unscaled



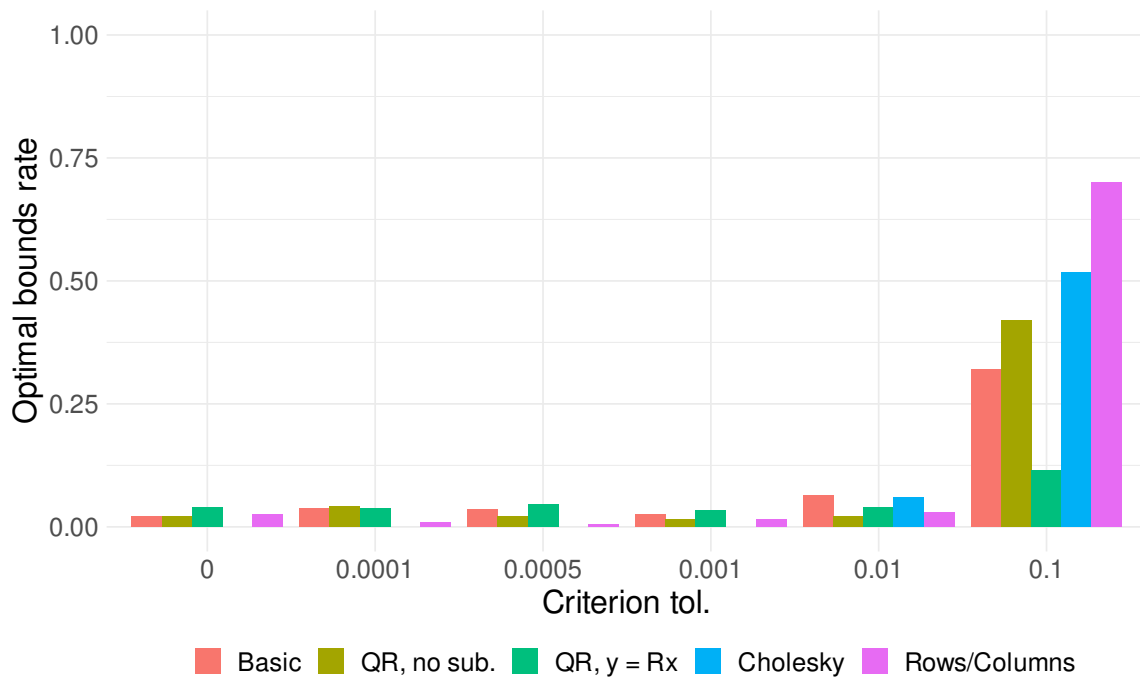
### 2.6 Case 3, QCQP, rescaled



### 2.7 Case 4, QCQP, unscaled



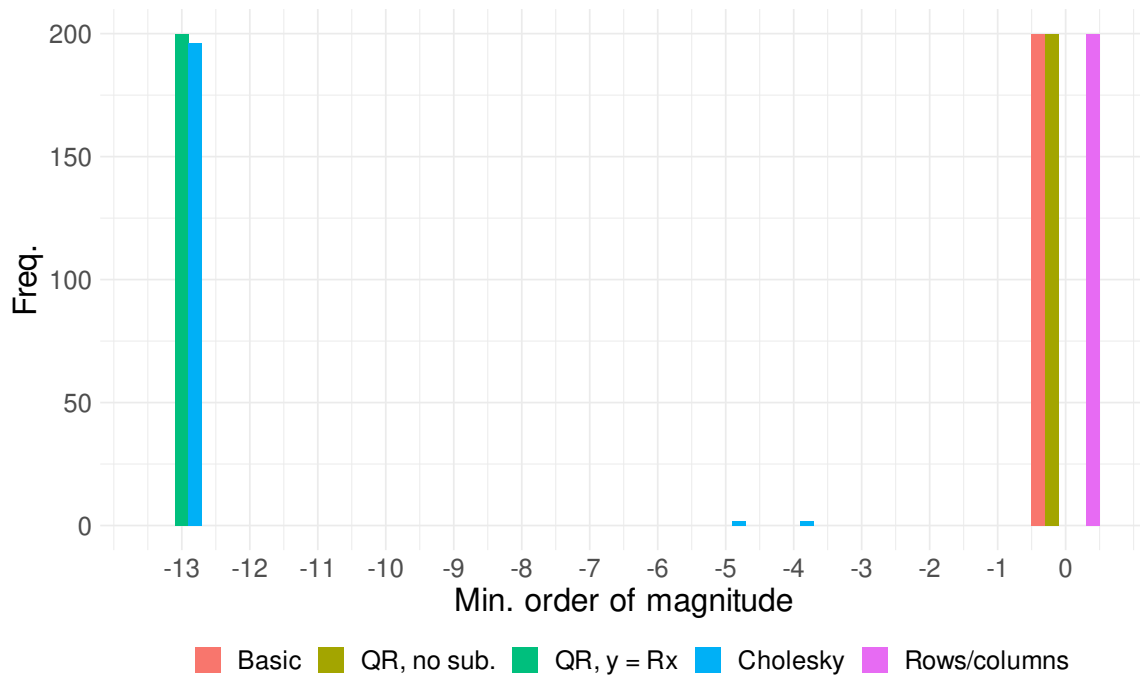
### 2.8 Case 4, QCQP, rescaled



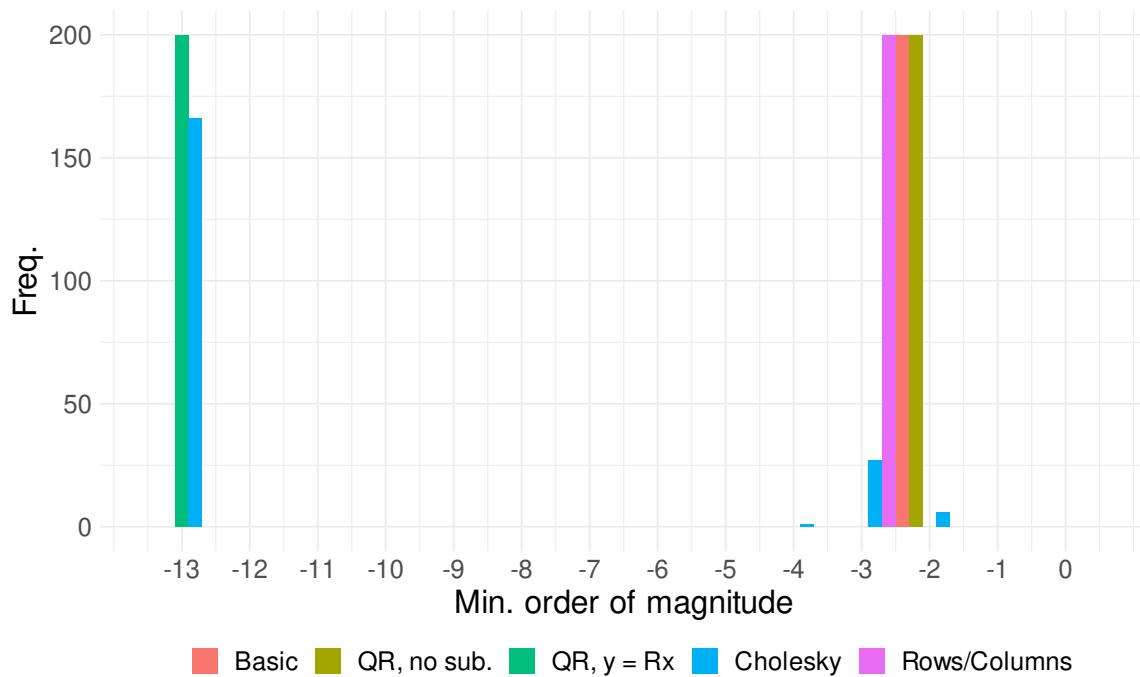


### 3 Min. order of mag. in linear constraint matrix

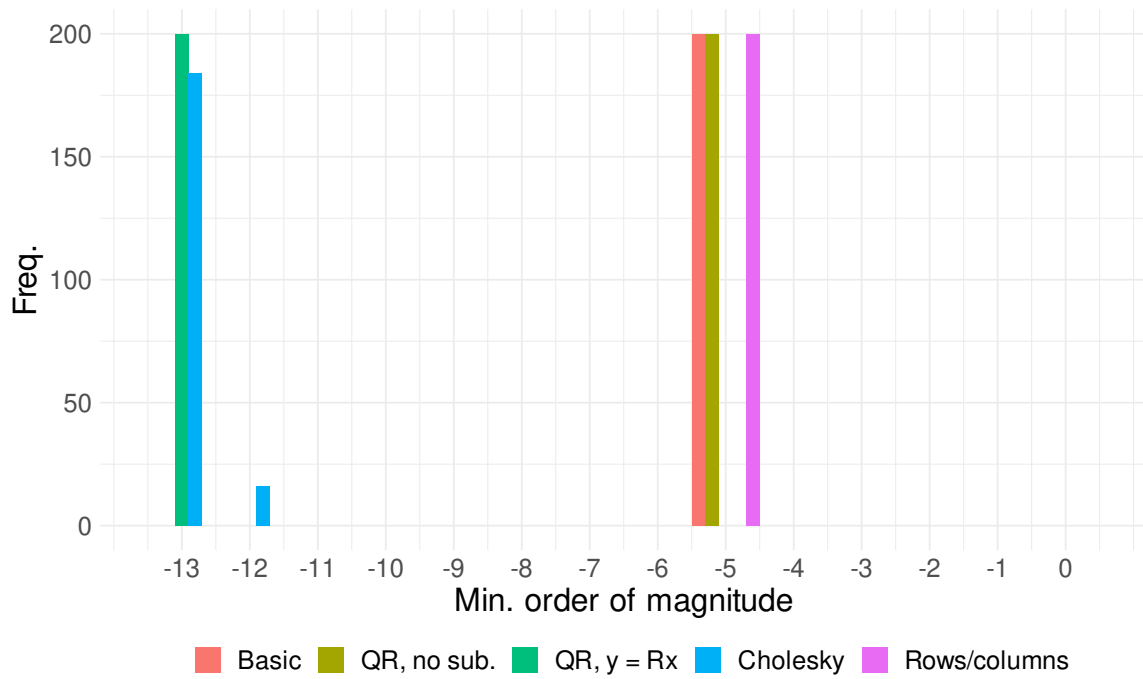
#### 3.1 Case 1, QCQP, unscaled



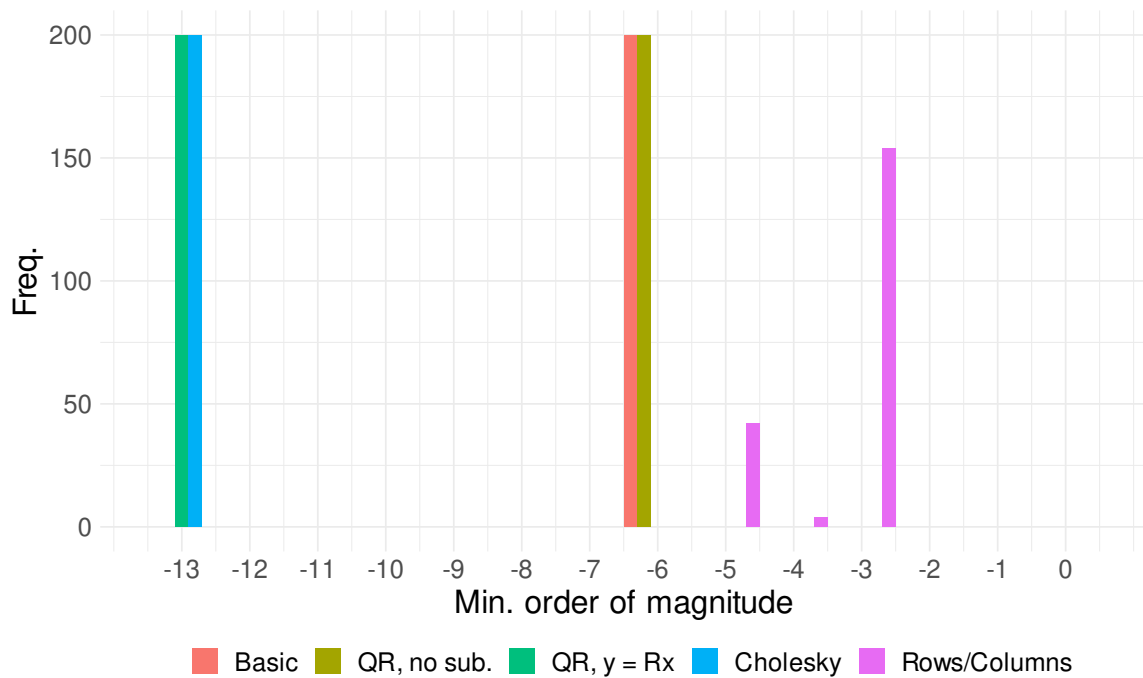
#### 3.2 Case 1, QCQP, rescaled



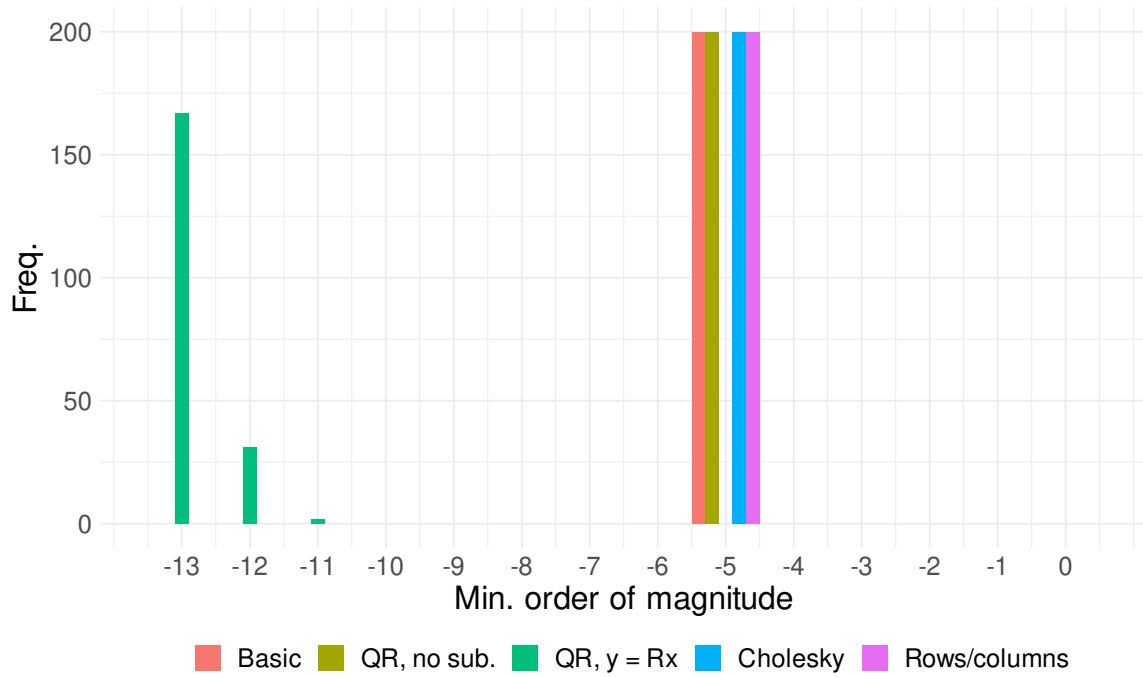
### 3.3 Case 2, QCQP, unscaled



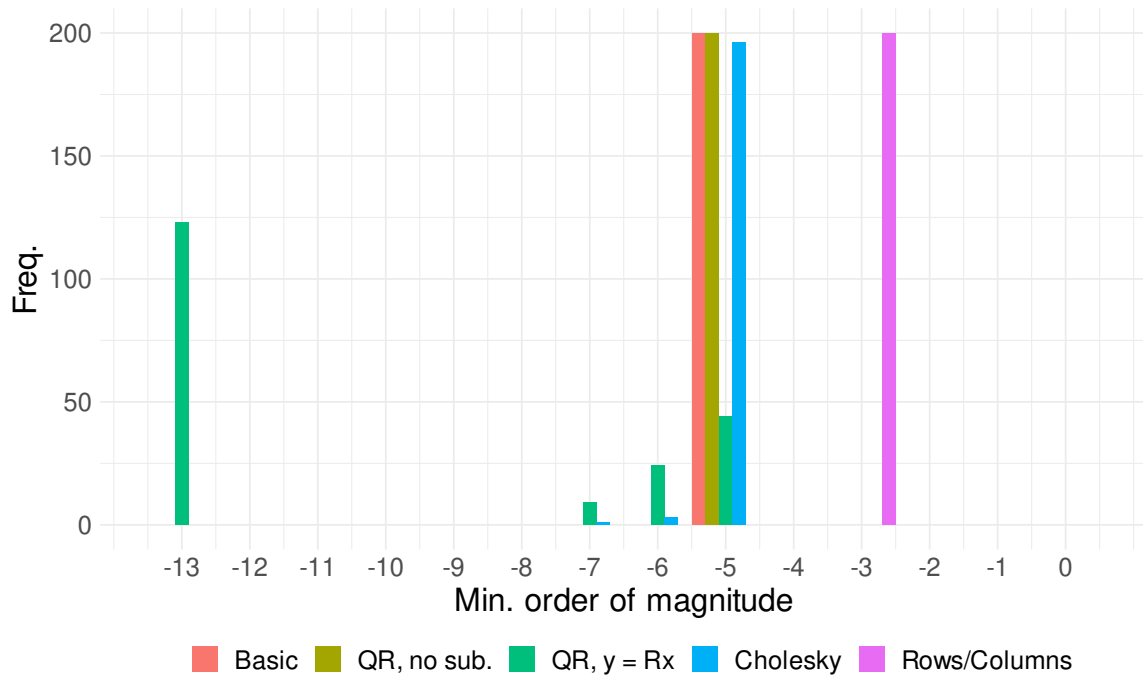
### 3.4 Case 2, QCQP, rescaled



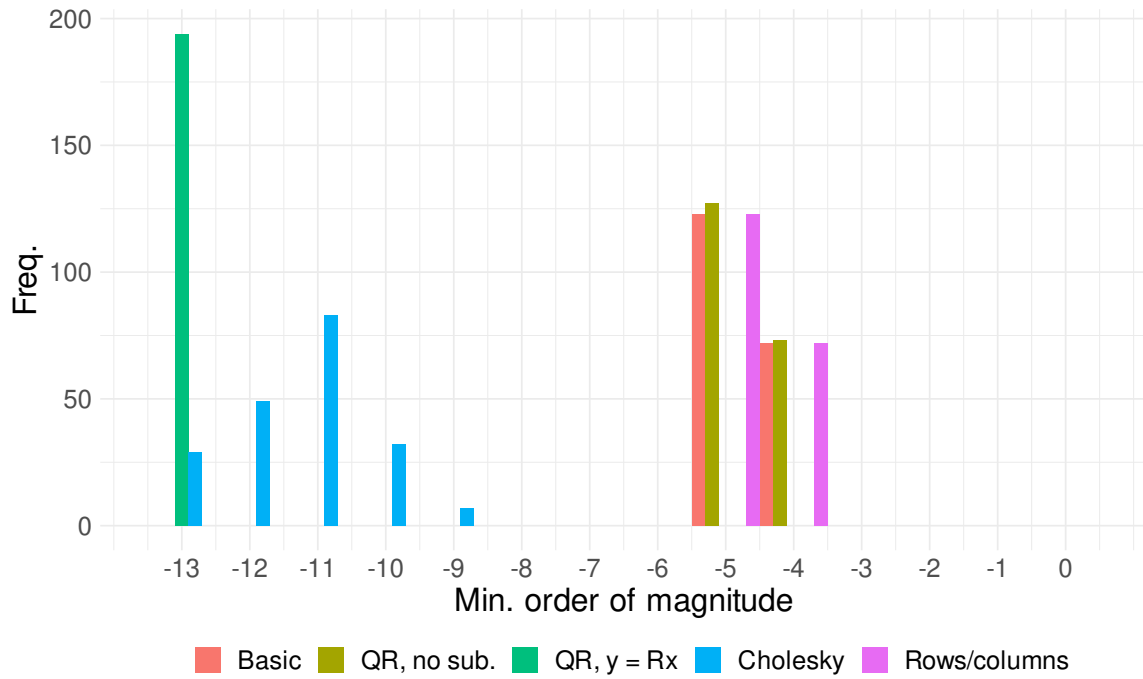
### 3.5 Case 3, QCQP, unscaled



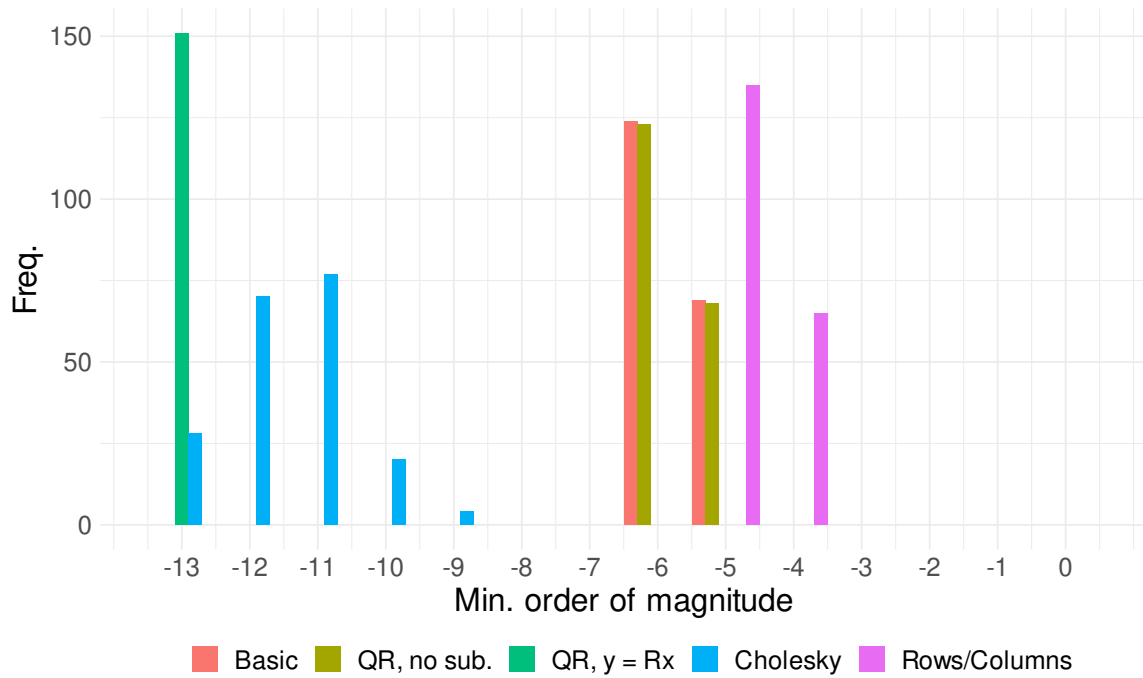
### 3.6 Case 3, QCQP, rescaled



### 3.7 Case 4, QCQP, unscaled

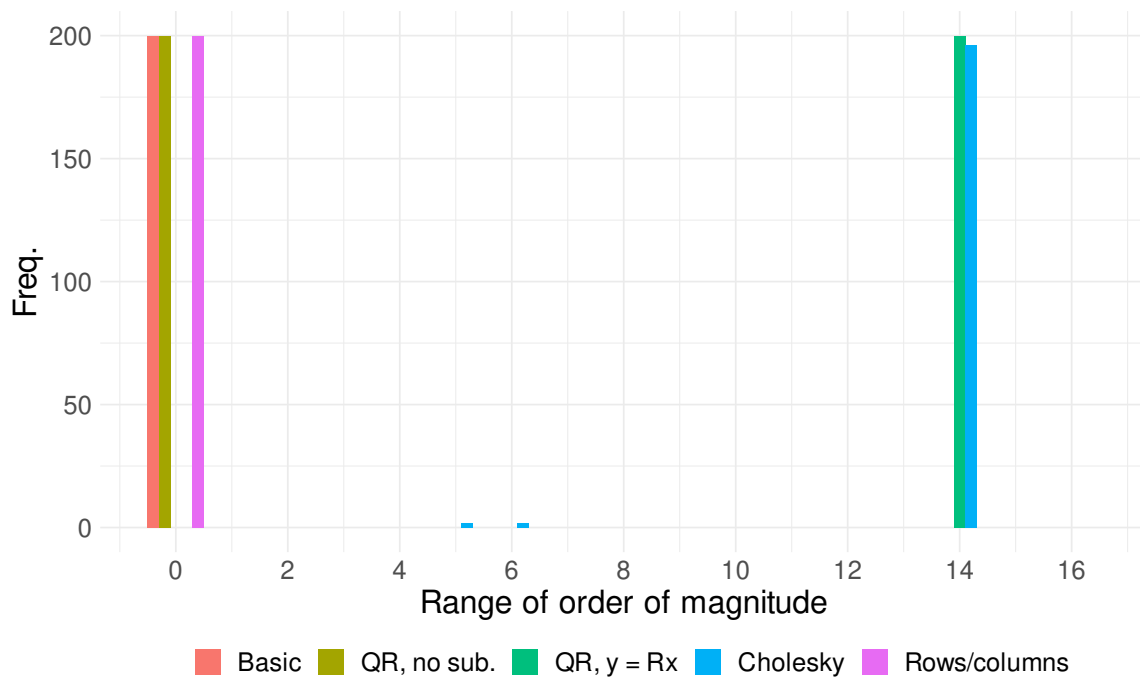


### 3.8 Case 4, QCQP, rescaled

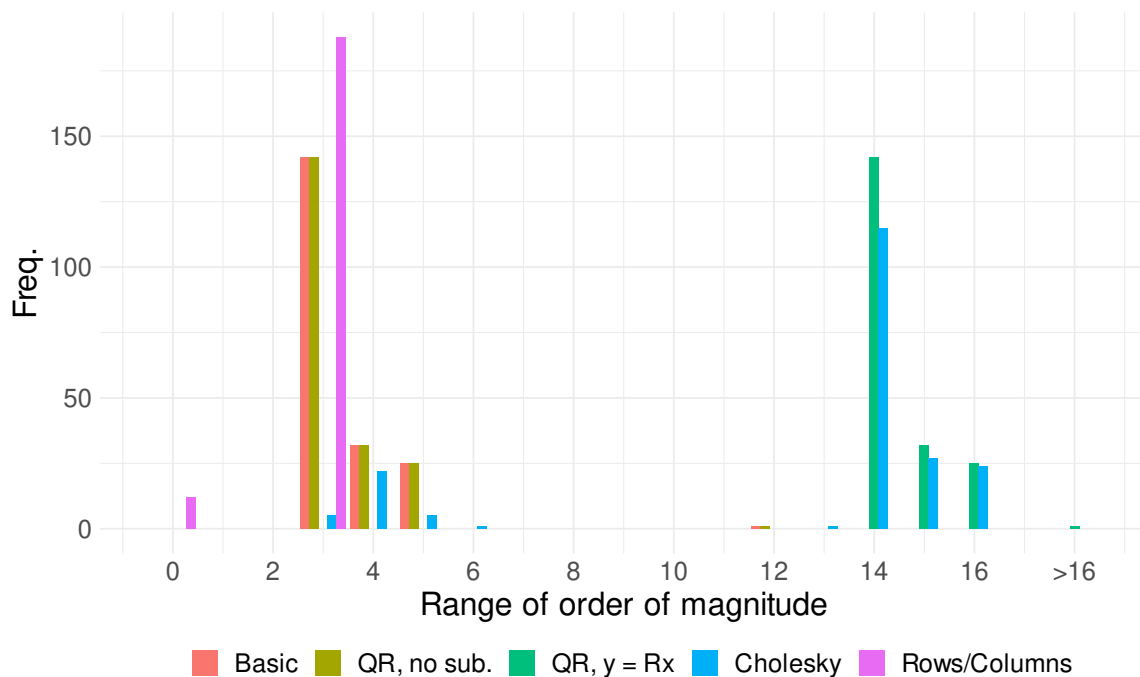


## 4 Range of order of mag. in linear constraint matrix

### 4.1 Case 1, QCQP, unscaled

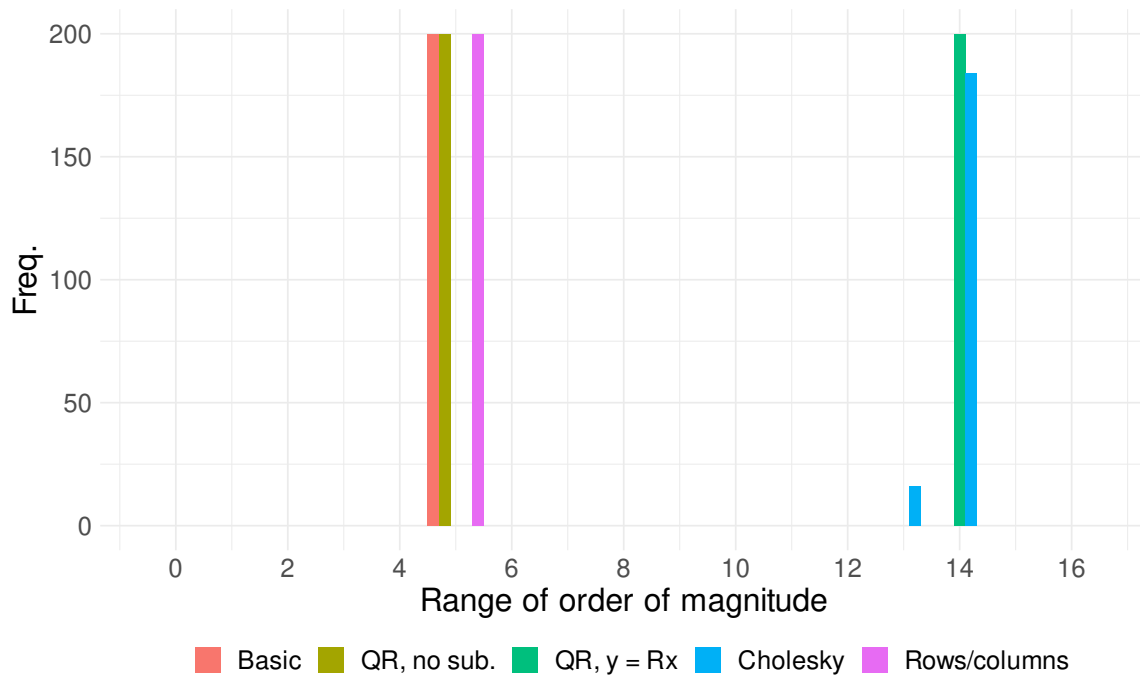


### 4.2 Case 1, QCQP, rescaled

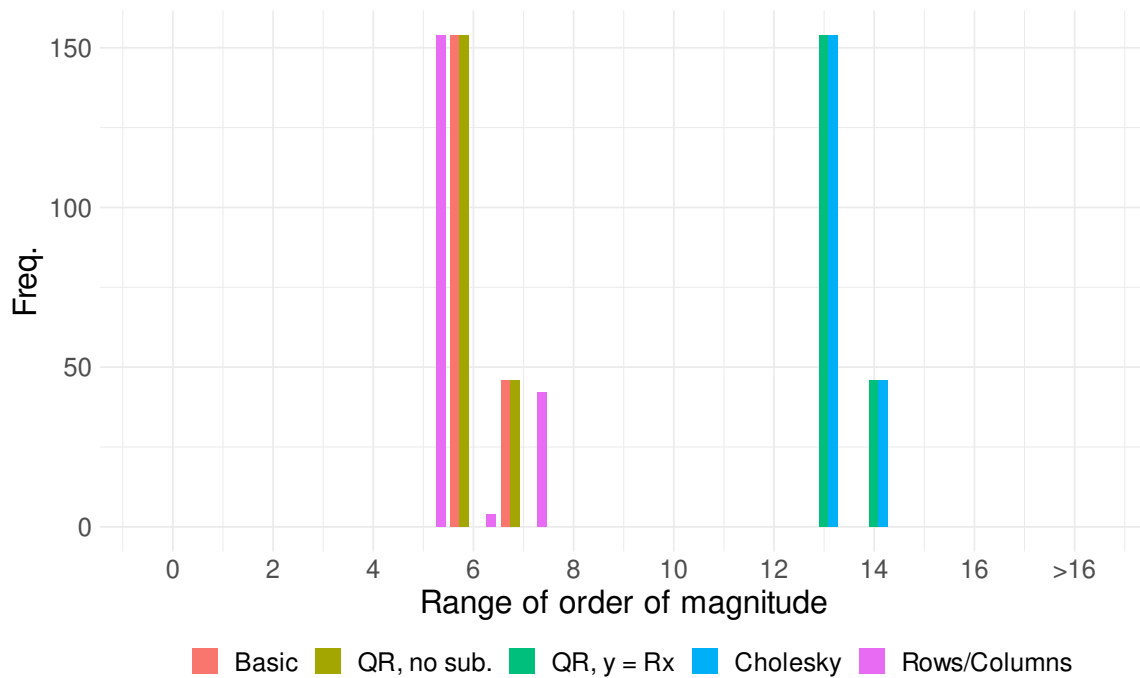


**Note:** When `rescale = TRUE`, ranges go up to as high as 30 orders of magnitude.

### 4.3 Case 2, QCQP, unscaled

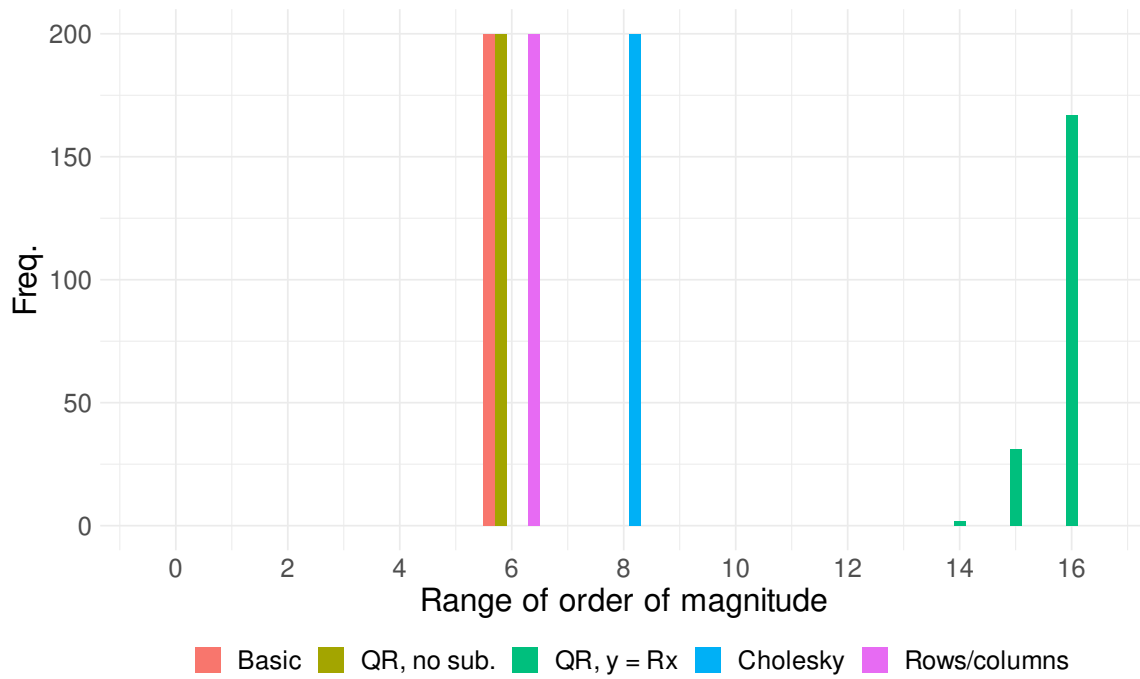


### 4.4 Case 2, QCQP, rescaled

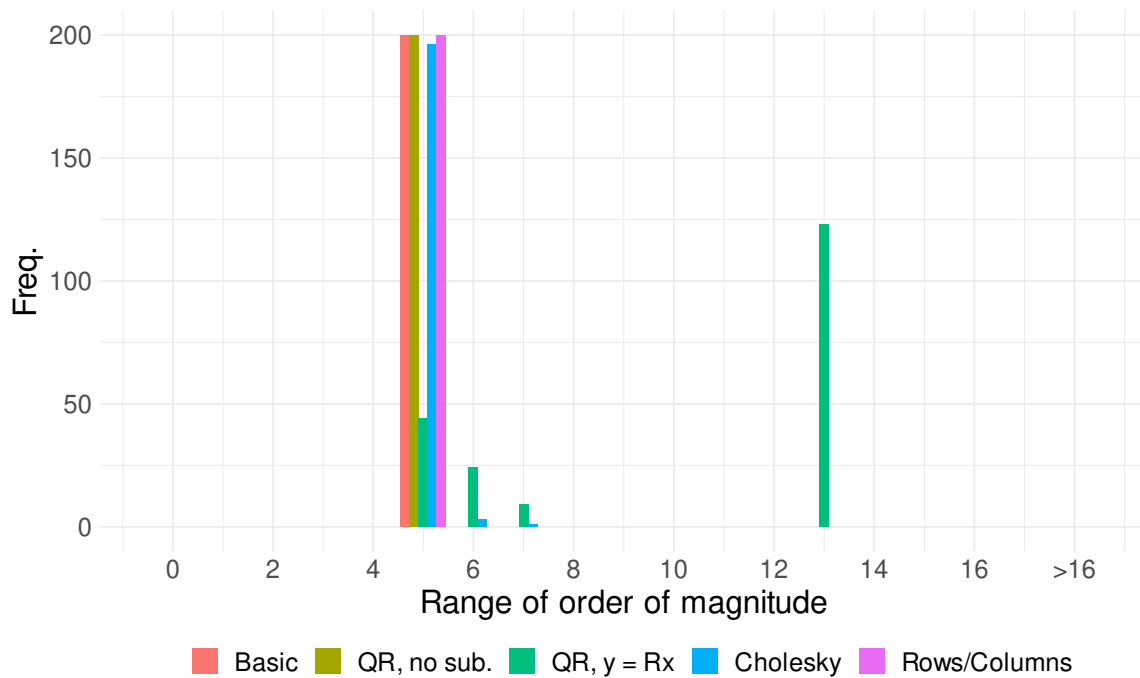


**Note:** When `rescale = TRUE`, ranges go up to as high as 30 orders of magnitude.

### 4.5 Case 3, QCQP, unscaled

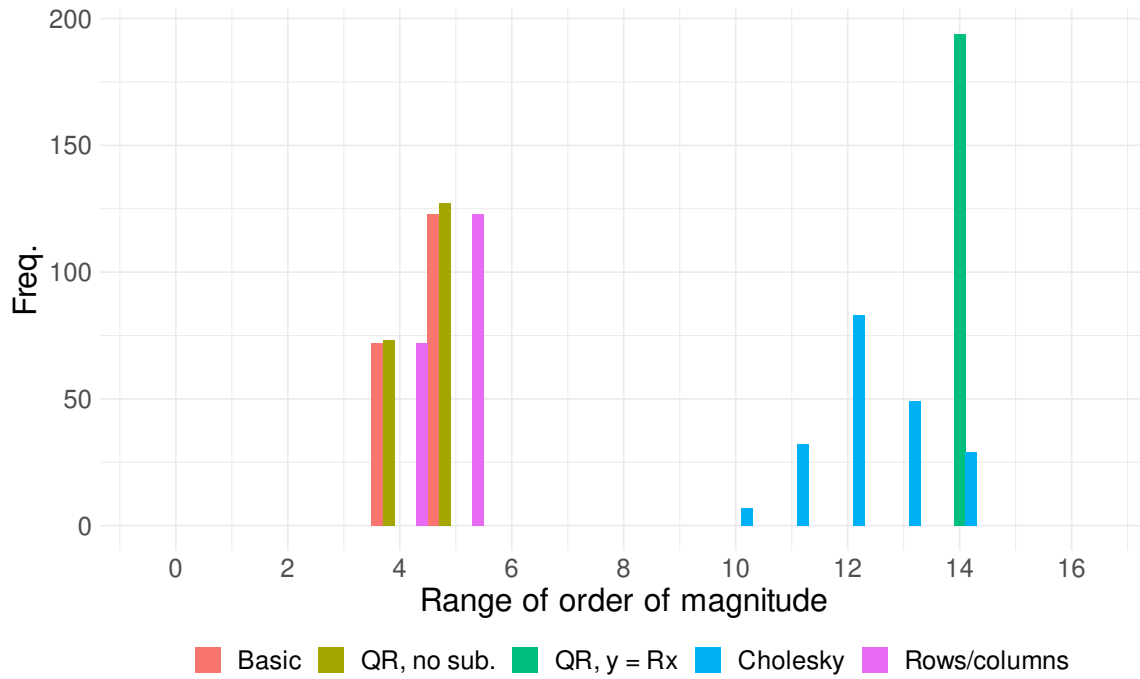


### 4.6 Case 3, QCQP, rescaled

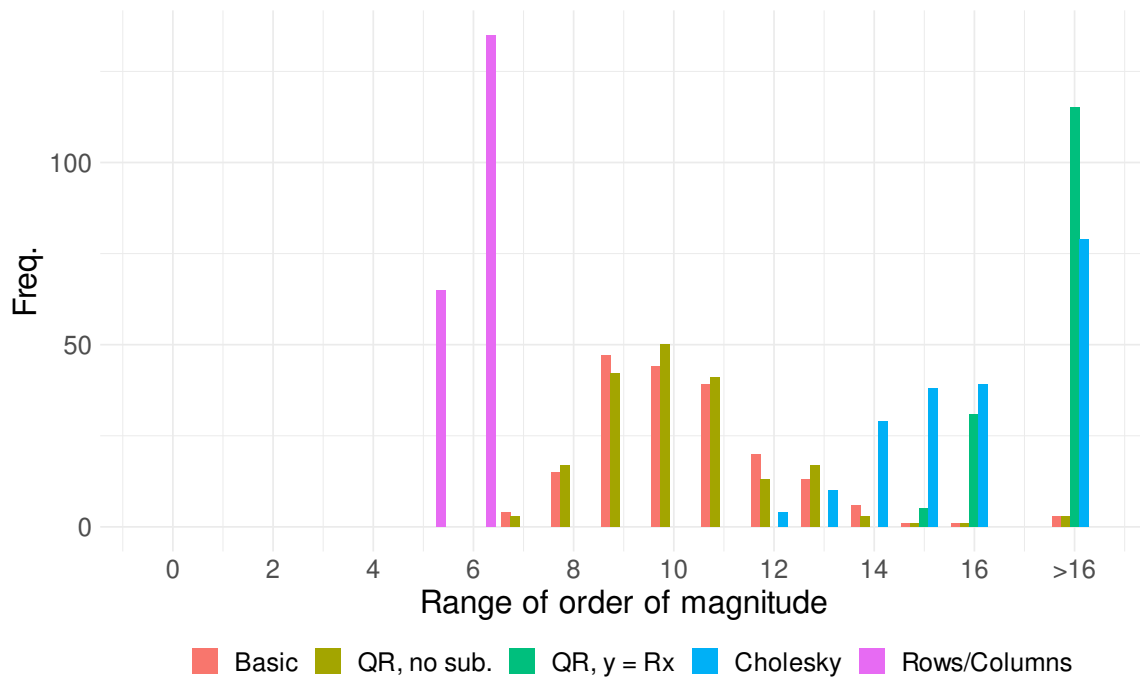


**Note:** When `rescale = TRUE`, ranges go up to as high as 30 orders of magnitude.

### 4.7 Case 4, QCQP, unscaled



### 4.8 Case 4, QCQP, rescaled

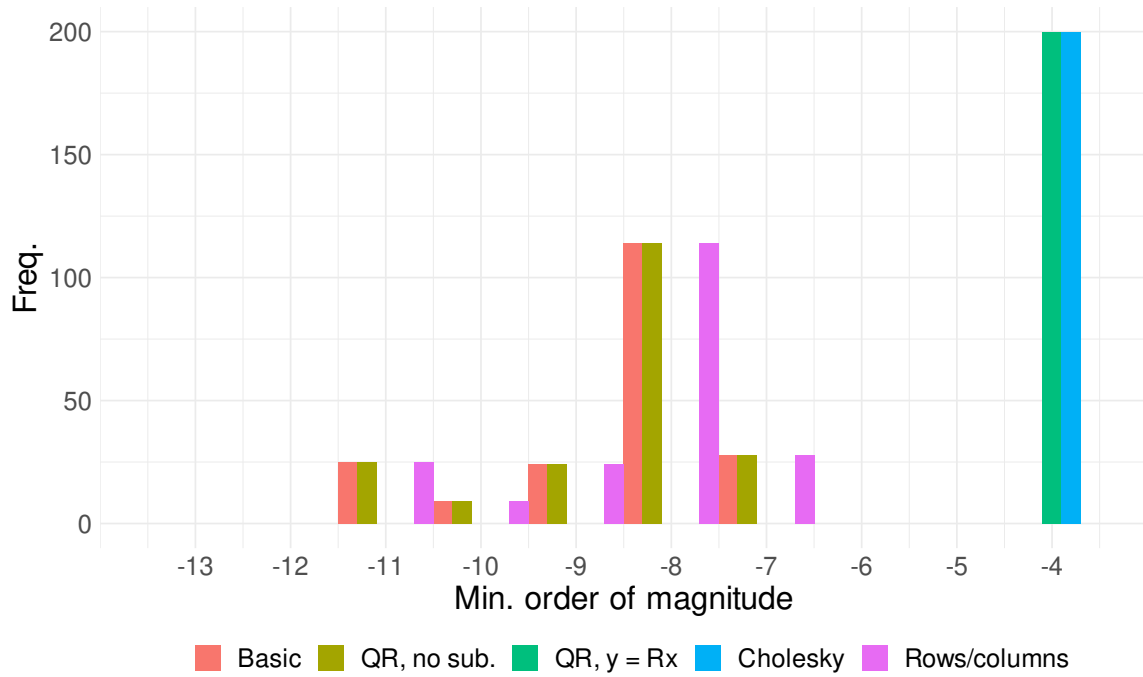


**Note:** When `rescale = TRUE`, ranges go up to as high as 30 orders of magnitude.

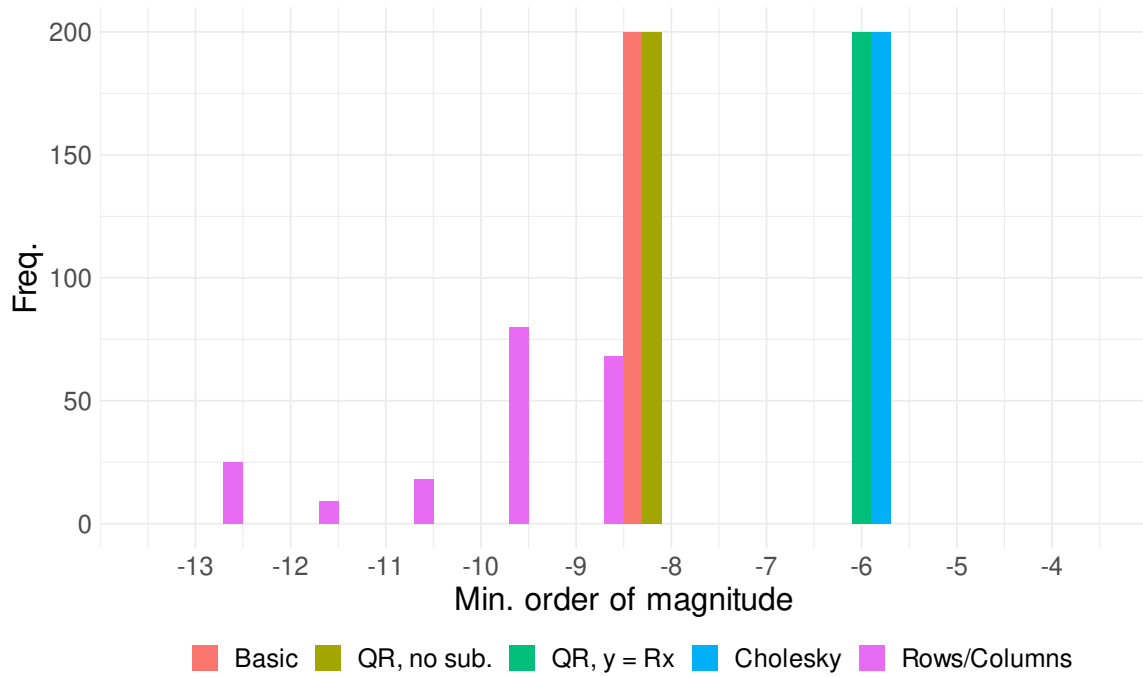


## 5 Min. order of mag. in quadratic constraint matrix

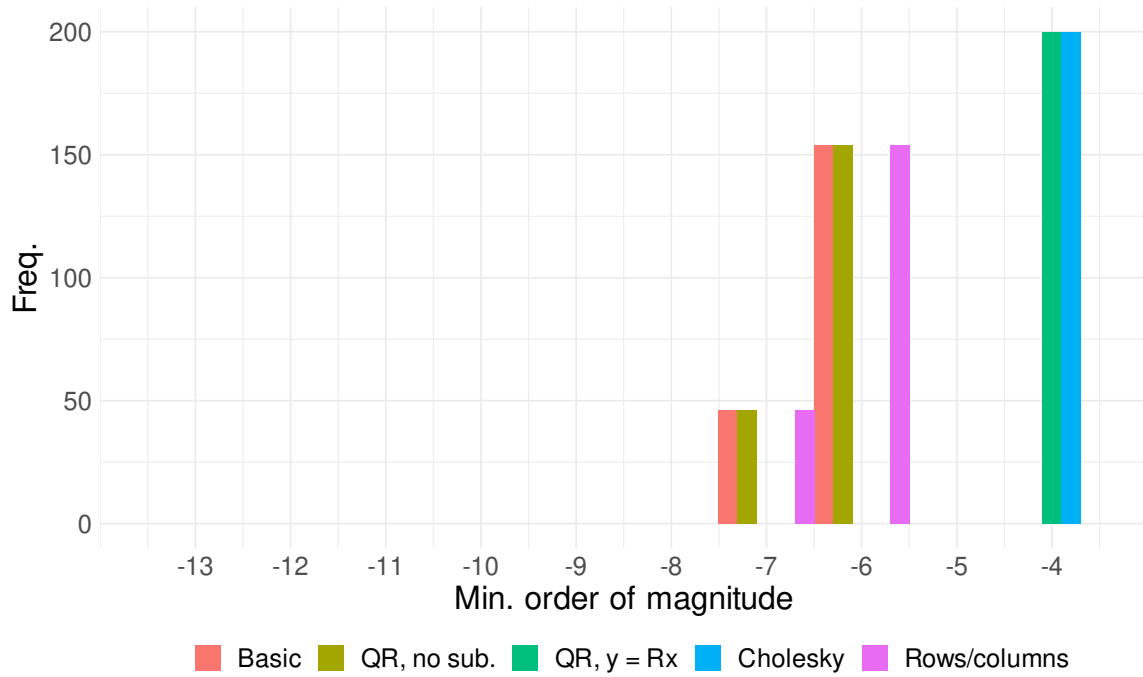
### 5.1 Case 1, QCQP, unscaled



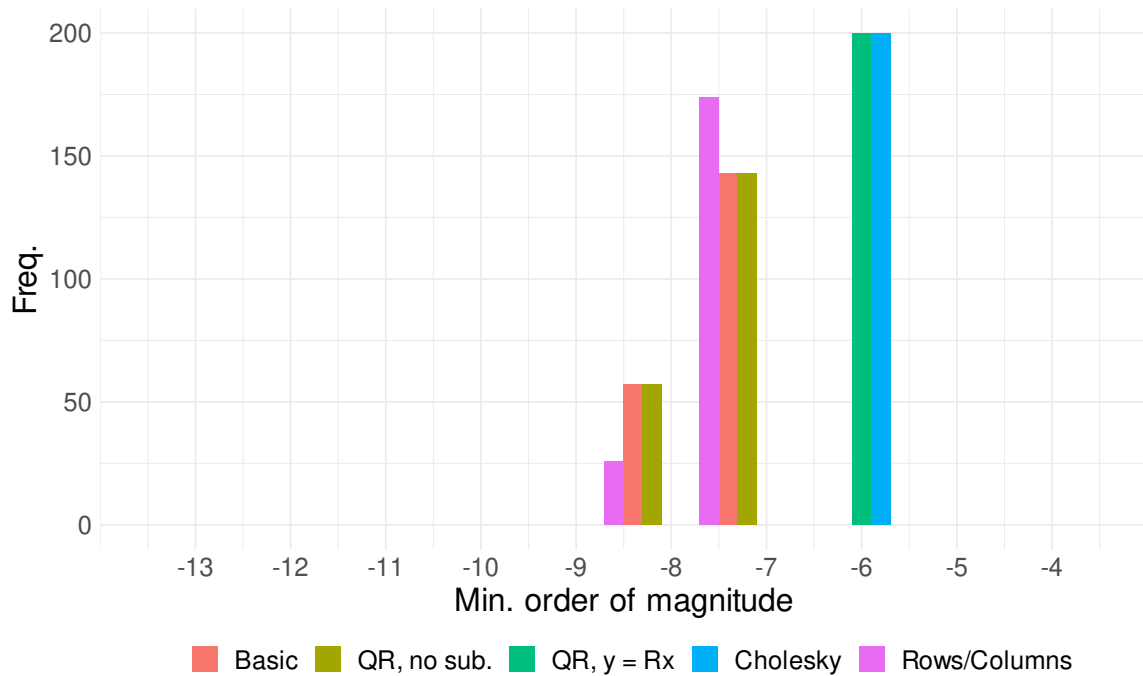
### 5.2 Case 1, QCQP, rescaled



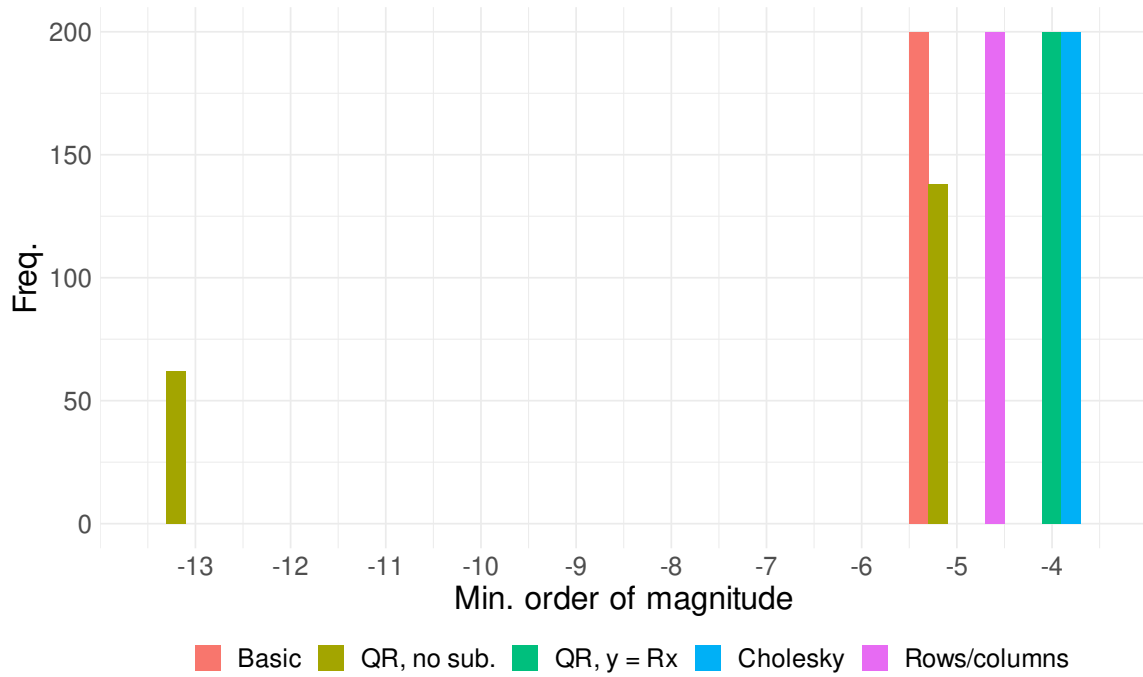
### 5.3 Case 2, QCQP, unscaled



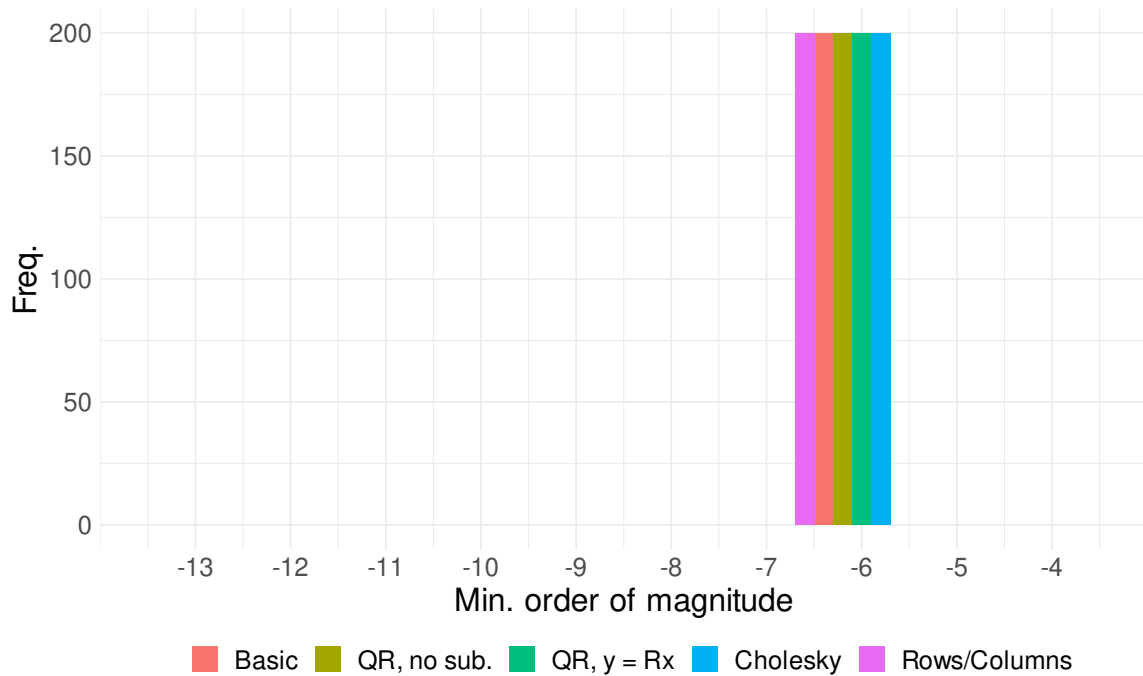
### 5.4 Case 2, QCQP, rescaled



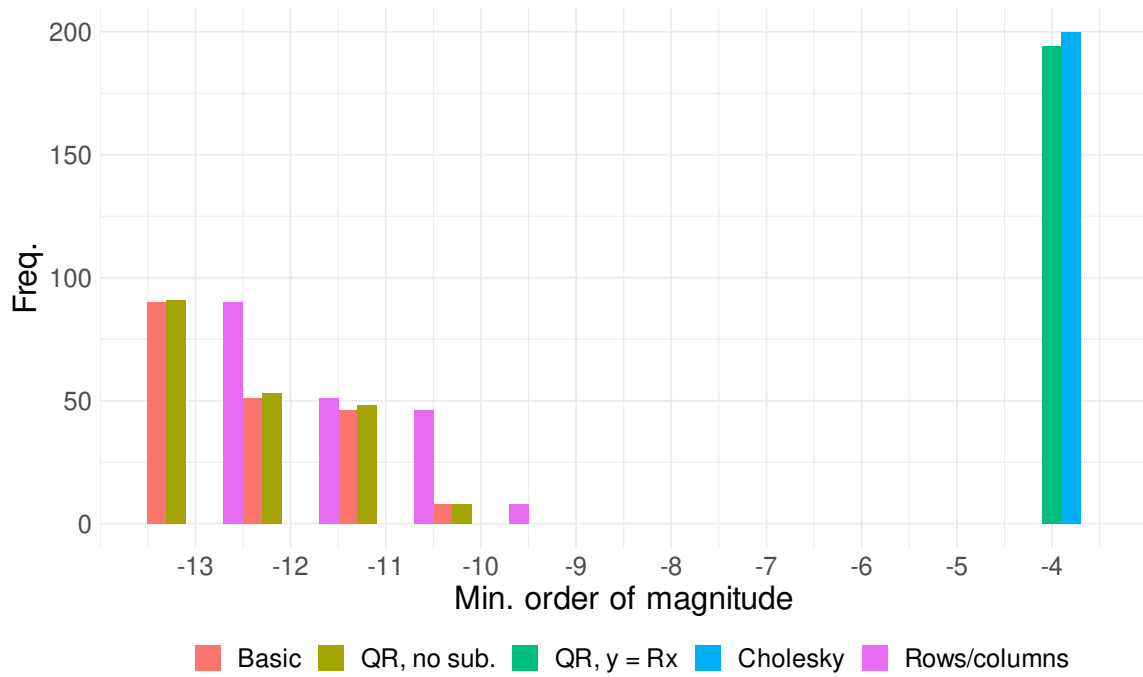
### 5.5 Case 3, QCQP, unscaled



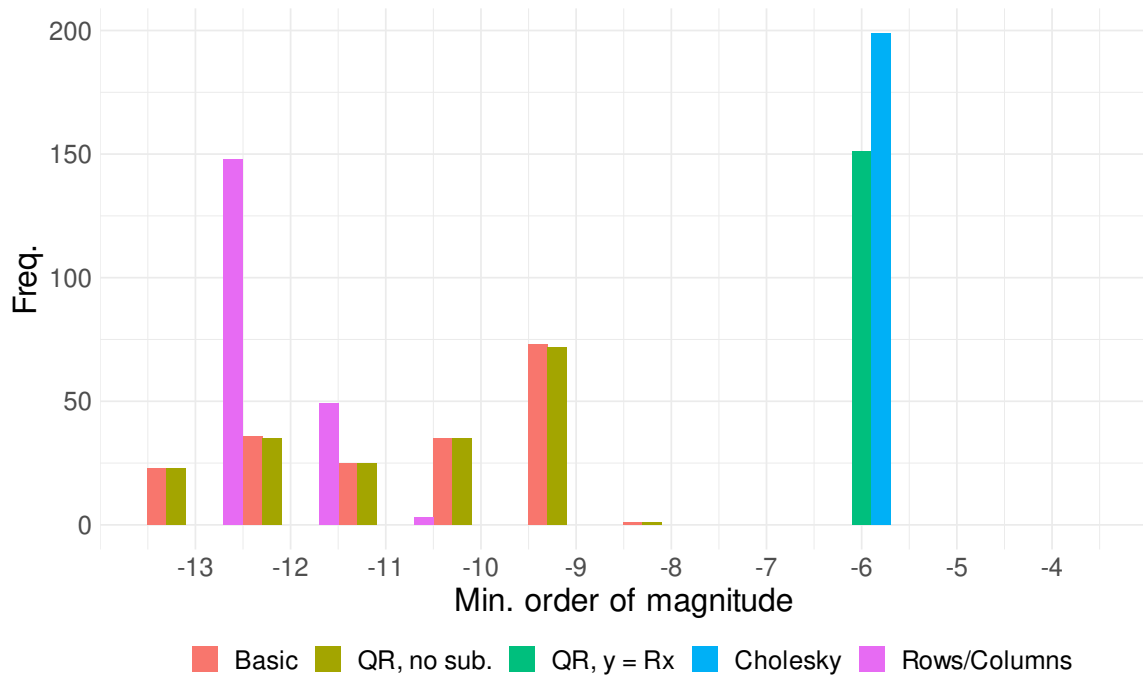
### 5.6 Case 3, QCQP, rescaled



### 5.7 Case 4, QCQP, unscaled



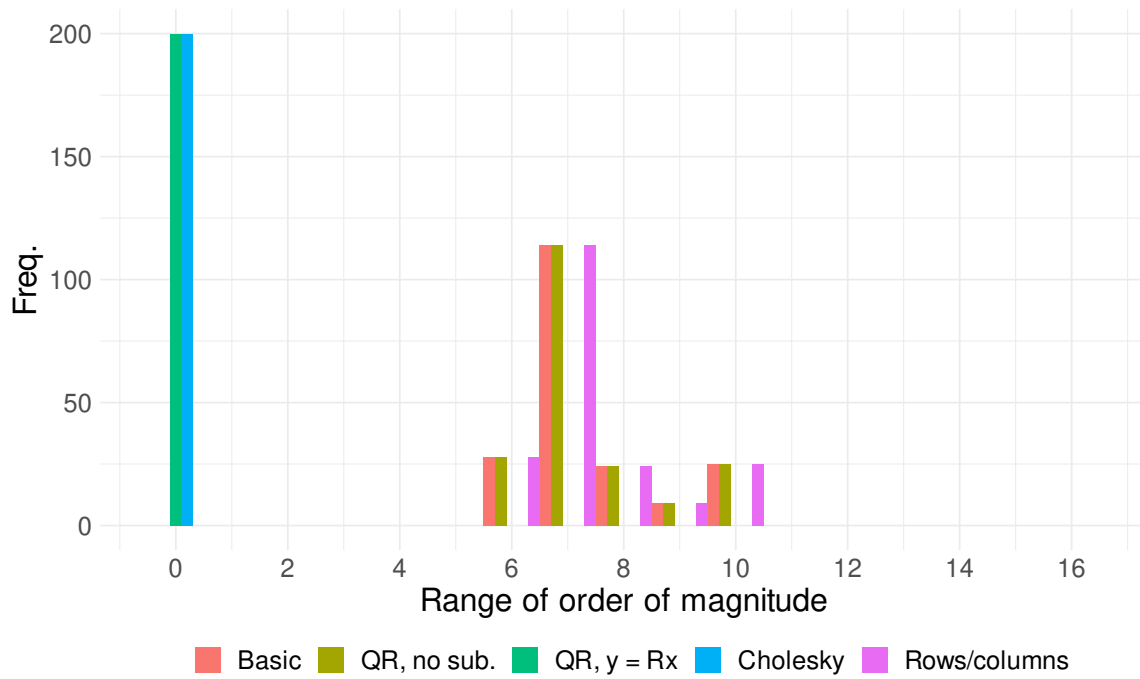
### 5.8 Case 4, QCQP, rescaled



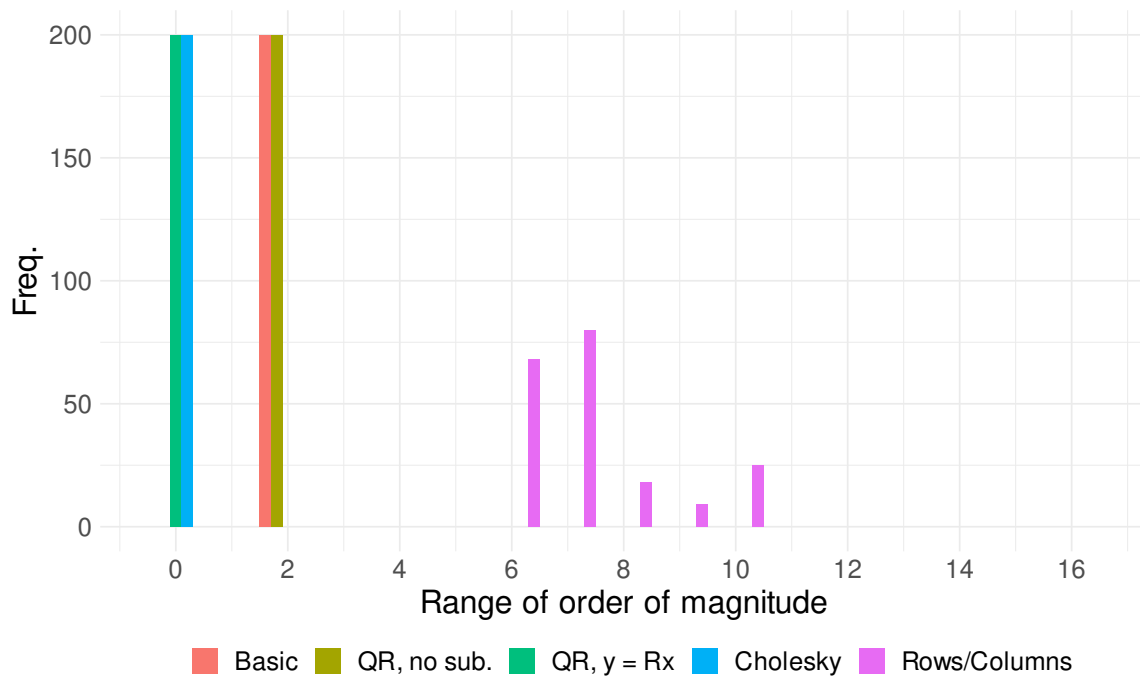
**Note:** Blue bar does not sum to 200 (total number of simulations) because some simulations returned errors and the QCQP model could not be saved.

## 6 Range of order of mag. in quadratic constraint matrix

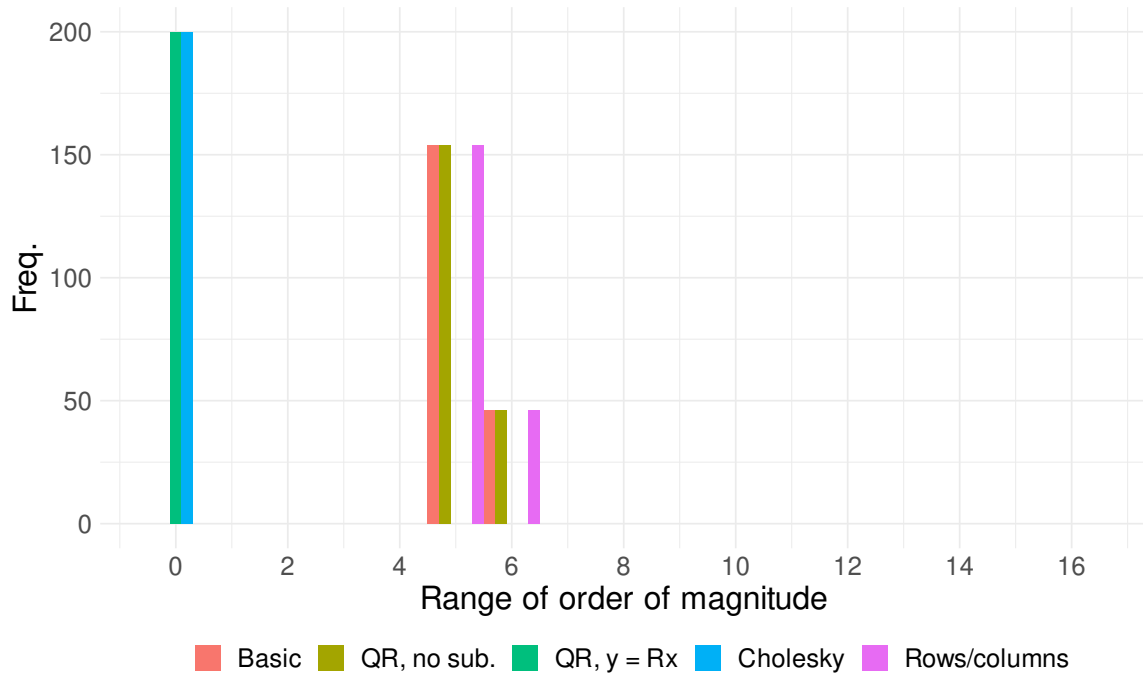
### 6.1 Case 1, QCQP, unscaled



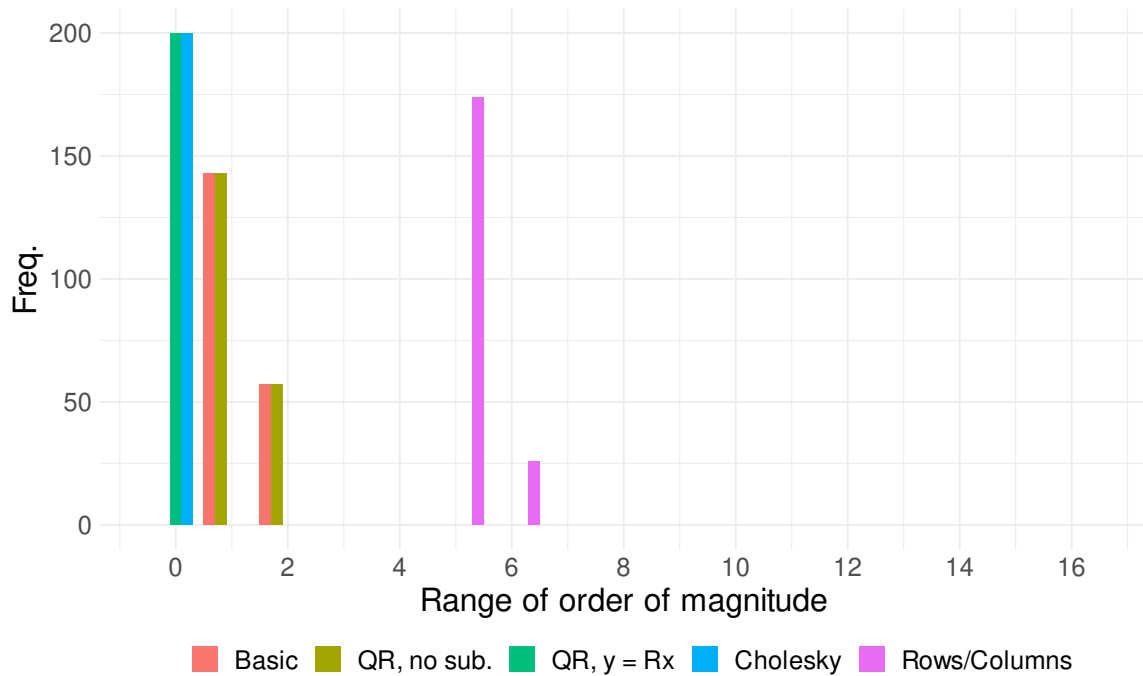
### 6.2 Case 1, QCQP, rescaled



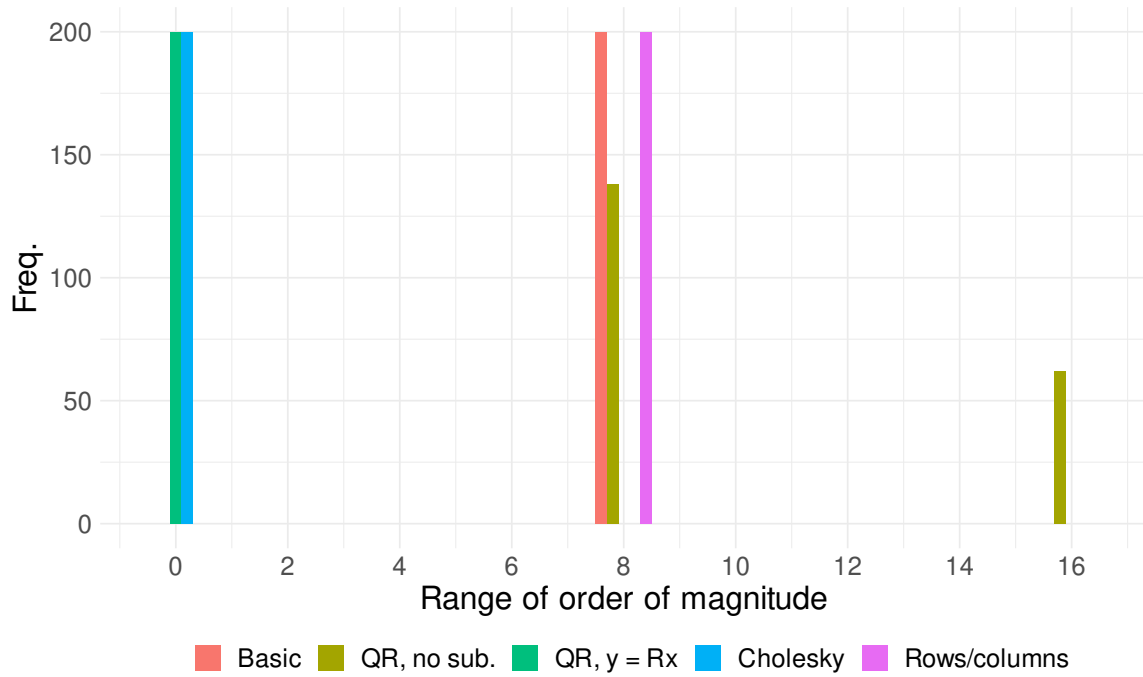
### 6.3 Case 2, QCQP, unscaled



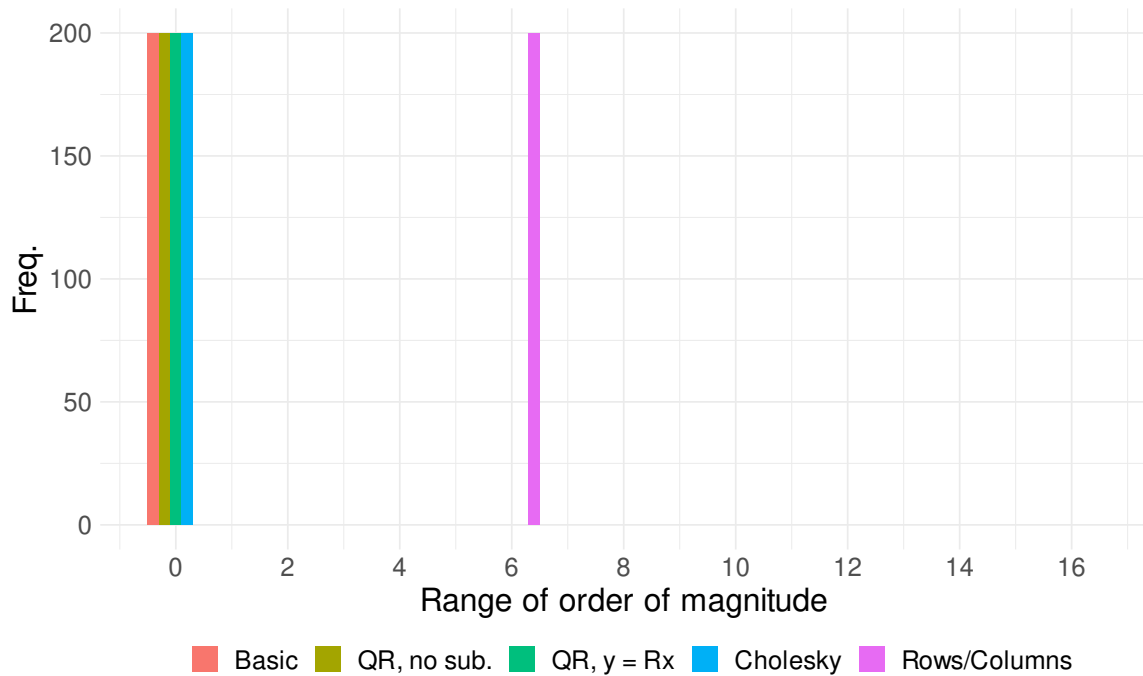
### 6.4 Case 2, QCQP, rescaled



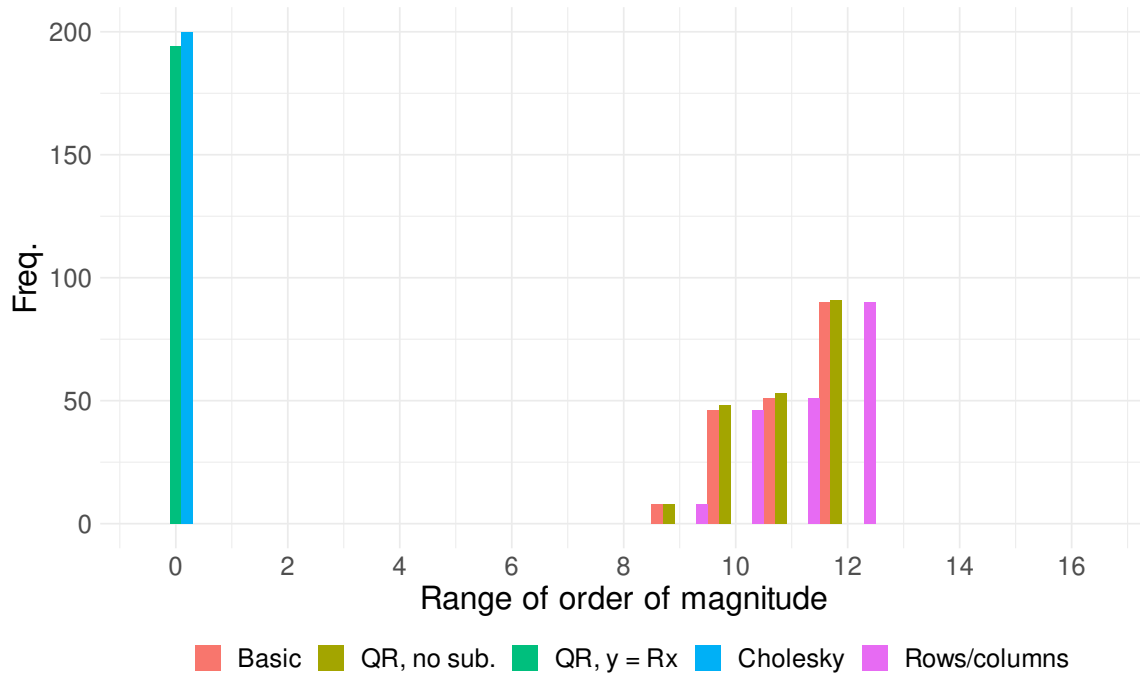
### 6.5 Case 3, QCQP, unscaled



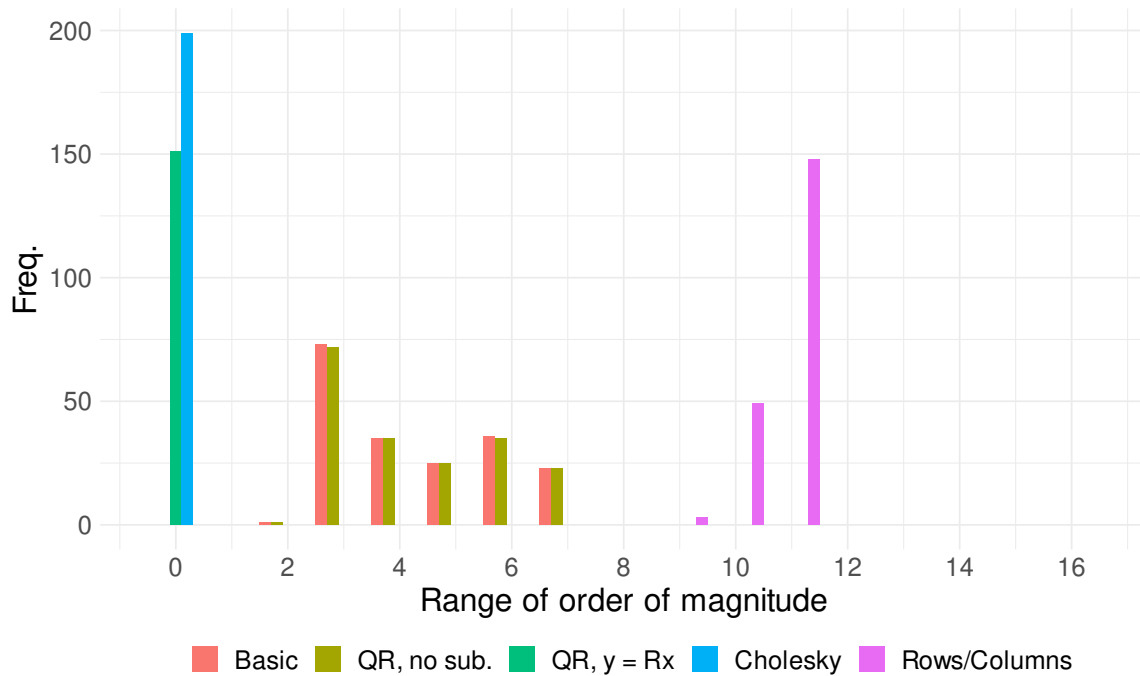
### 6.6 Case 3, QCQP, rescaled



### 6.7 Case 4, QCQP, unscaled



### 6.8 Case 4, QCQP, rescaled

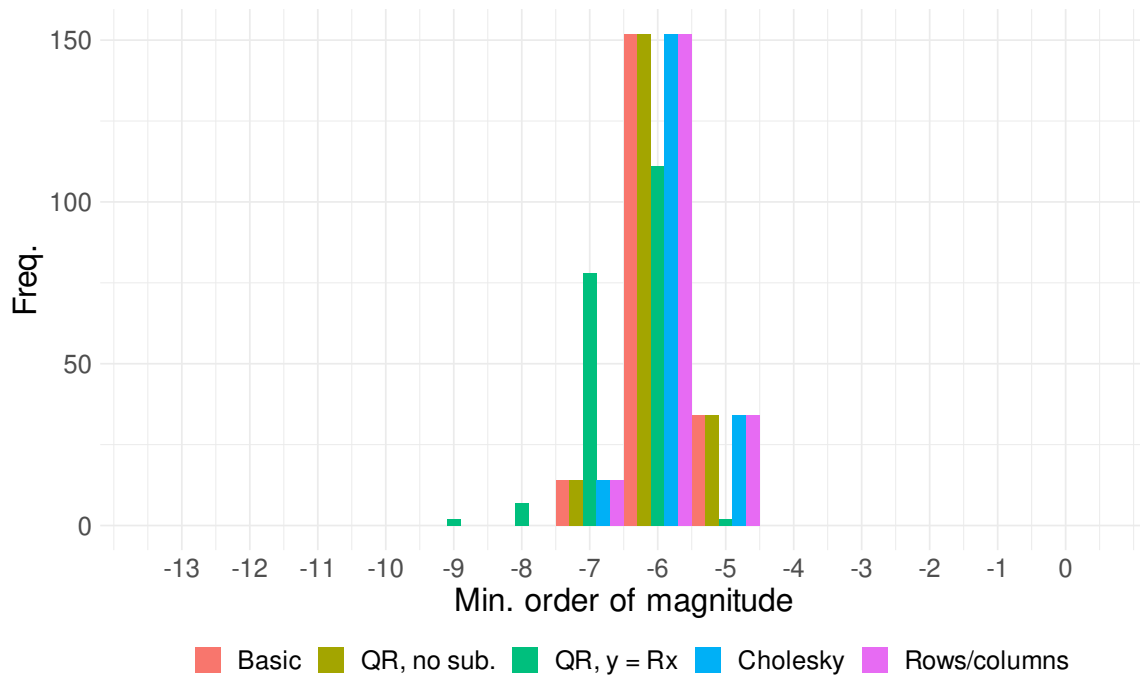


**Note:** Blue bar does not sum to 200 (total number of simulations) because some simulations returned errors and the QCQP model could not be saved.

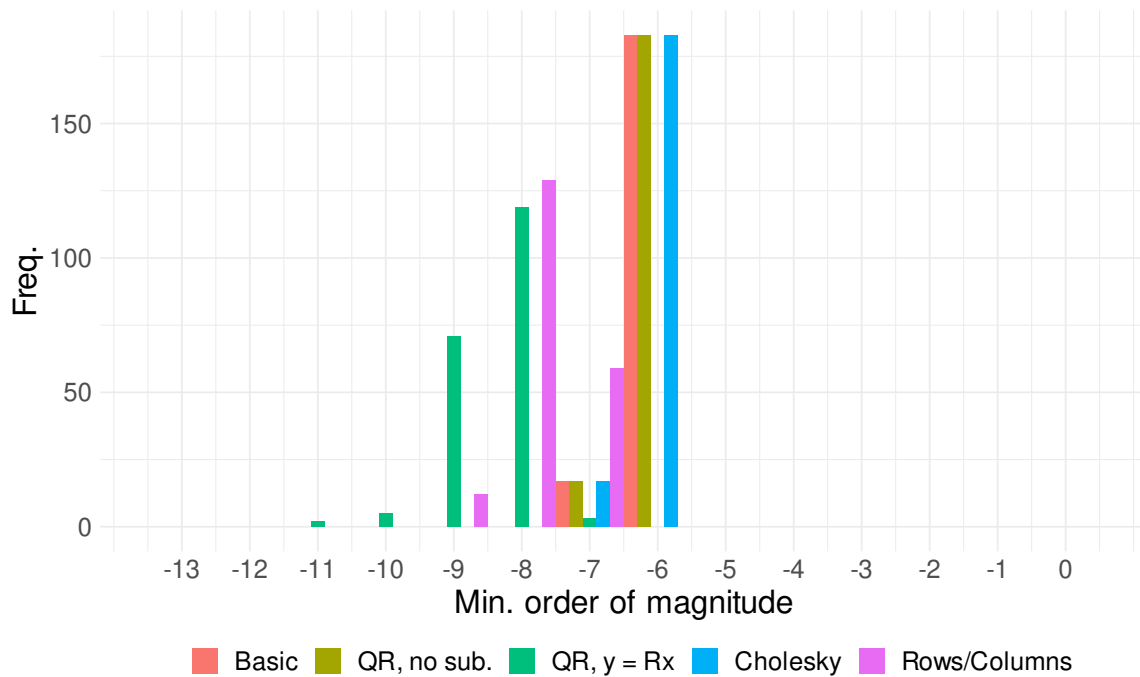


## 7 Min. order of mag. in quadratic constraint vector

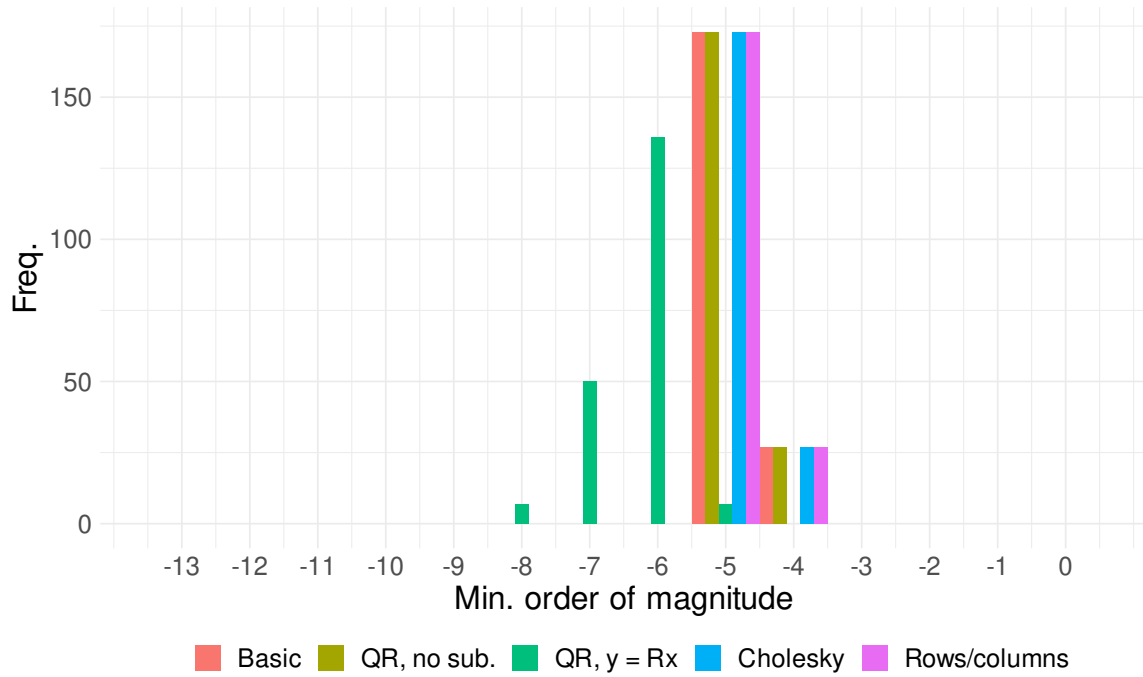
### 7.1 Case 1, QCQP, unscaled



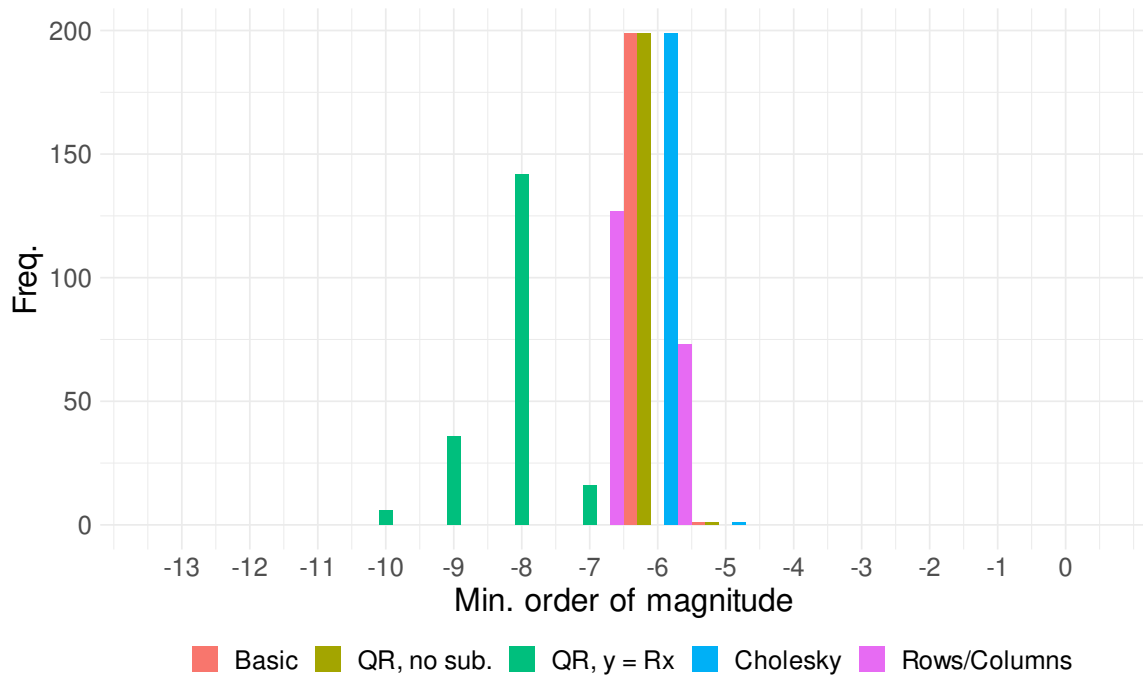
### 7.2 Case 1, QCQP, rescaled



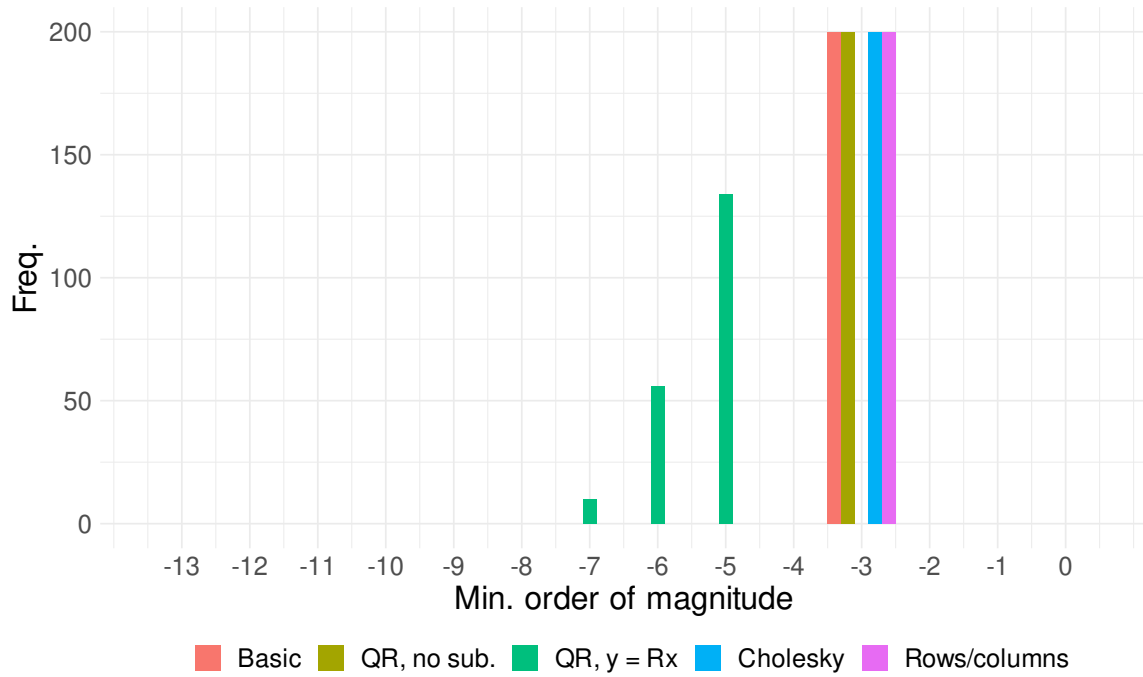
### 7.3 Case 2, QCQP, unscaled



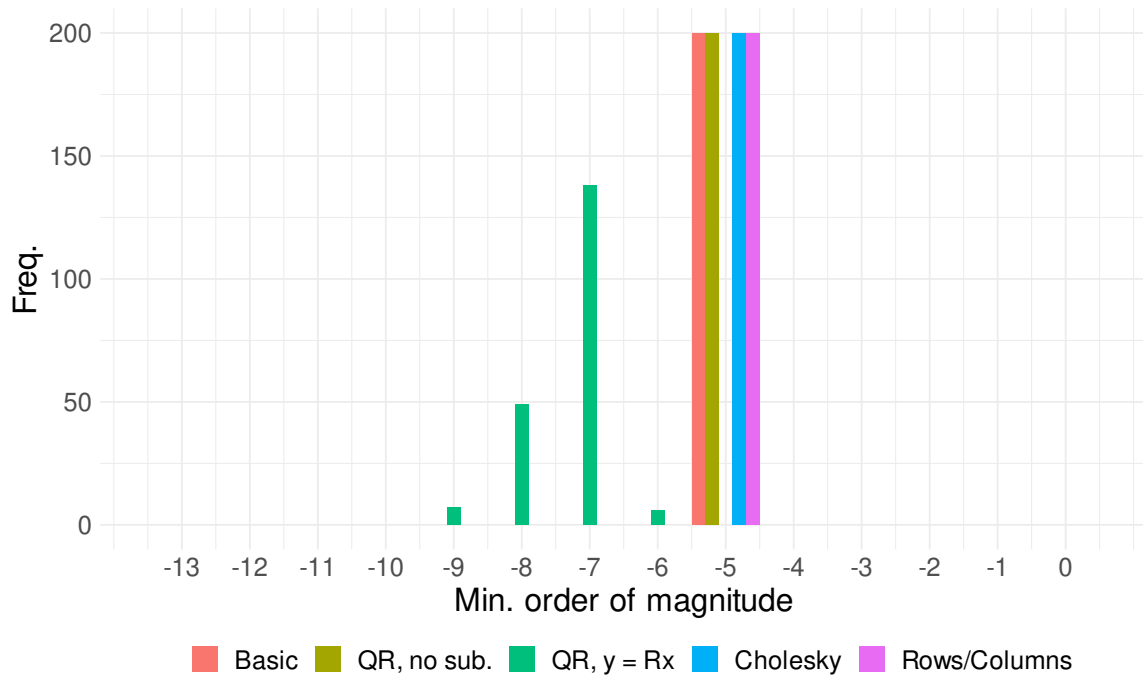
### 7.4 Case 2, QCQP, rescaled



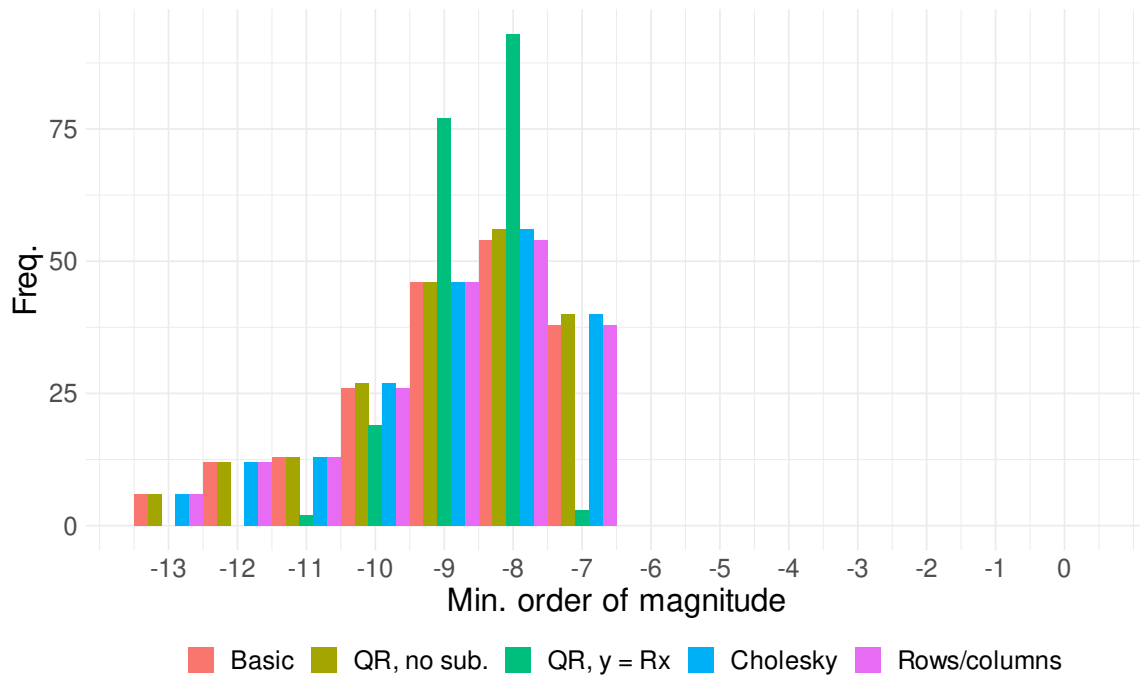
### 7.5 Case 3, QCQP, unscaled



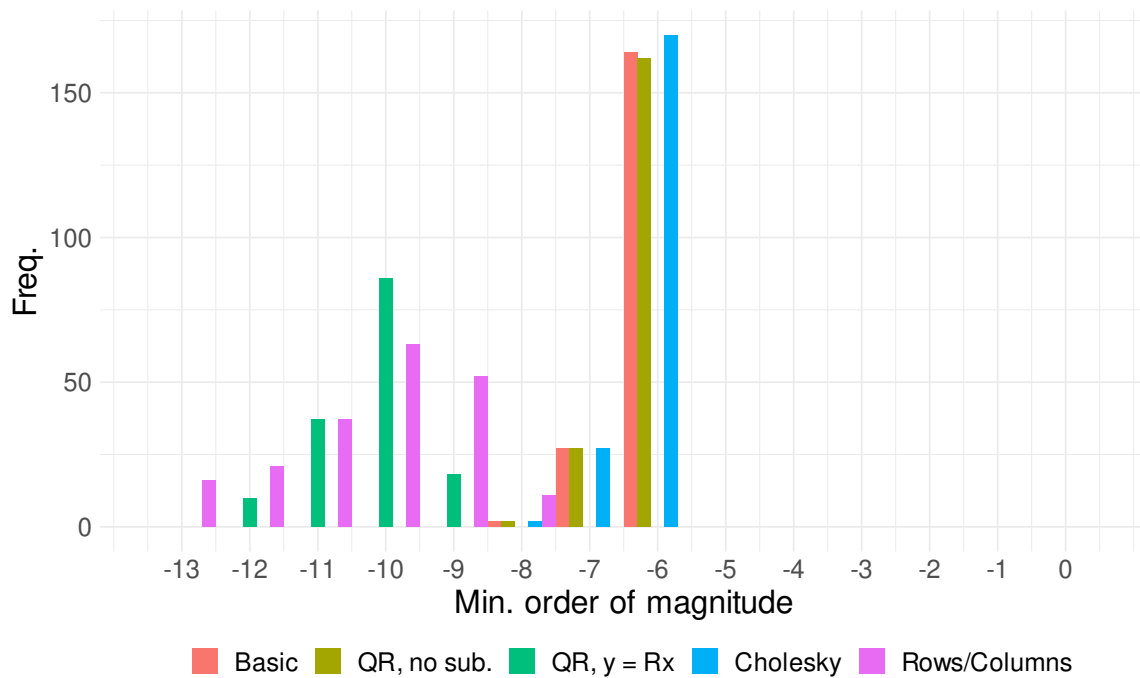
### 7.6 Case 3, QCQP, rescaled



### 7.7 Case 4, QCQP, unscaled



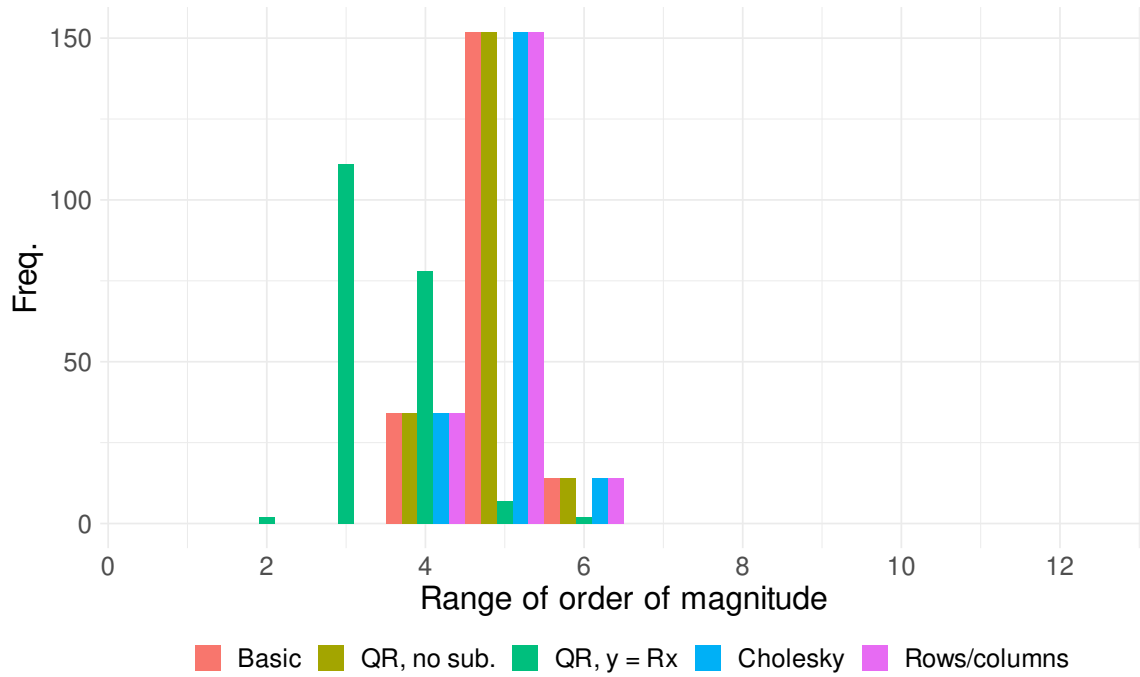
### 7.8 Case 4, QCQP, rescaled



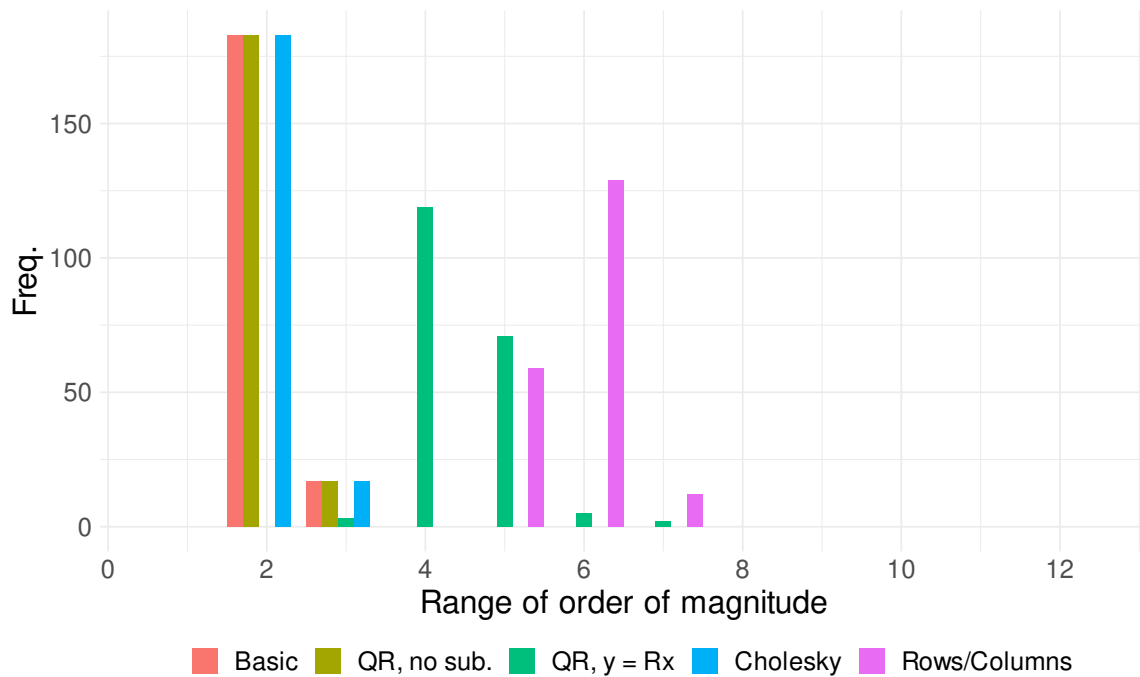
**Note:** Blue bar does not sum to 200 (total number of simulations) because some simulations returned errors and the QCQP model could not be saved.

## 8 Range of order of mag. in quadratic constraint vector

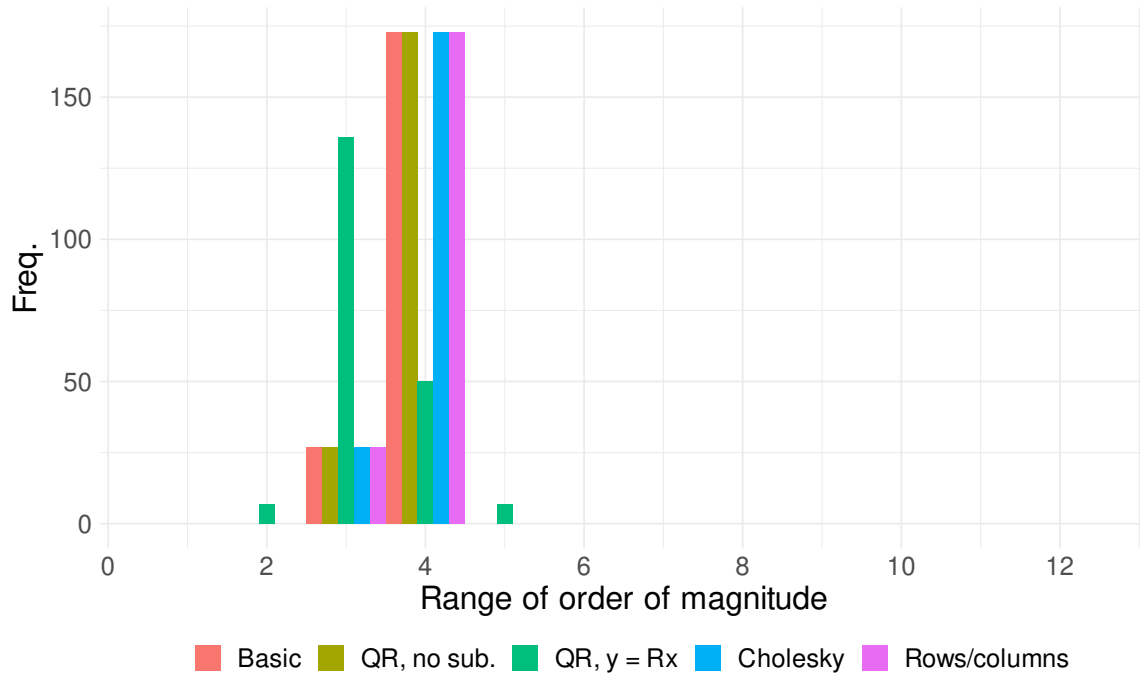
### 8.1 Case 1, QCQP, unscaled



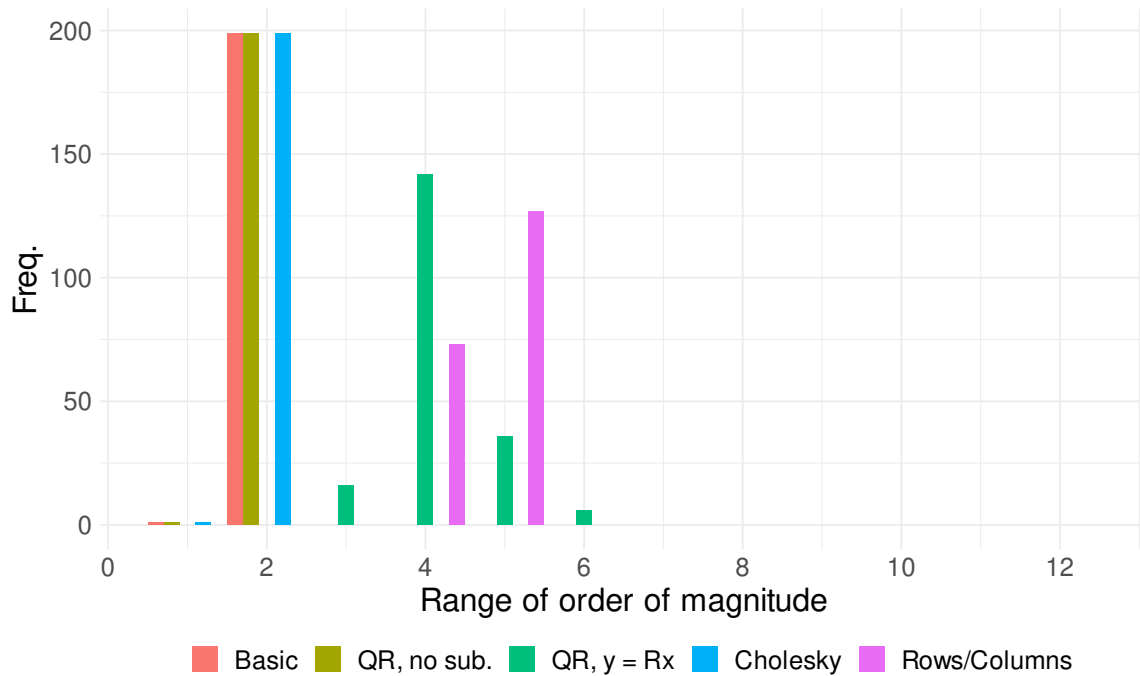
### 8.2 Case 1, QCQP, rescaled



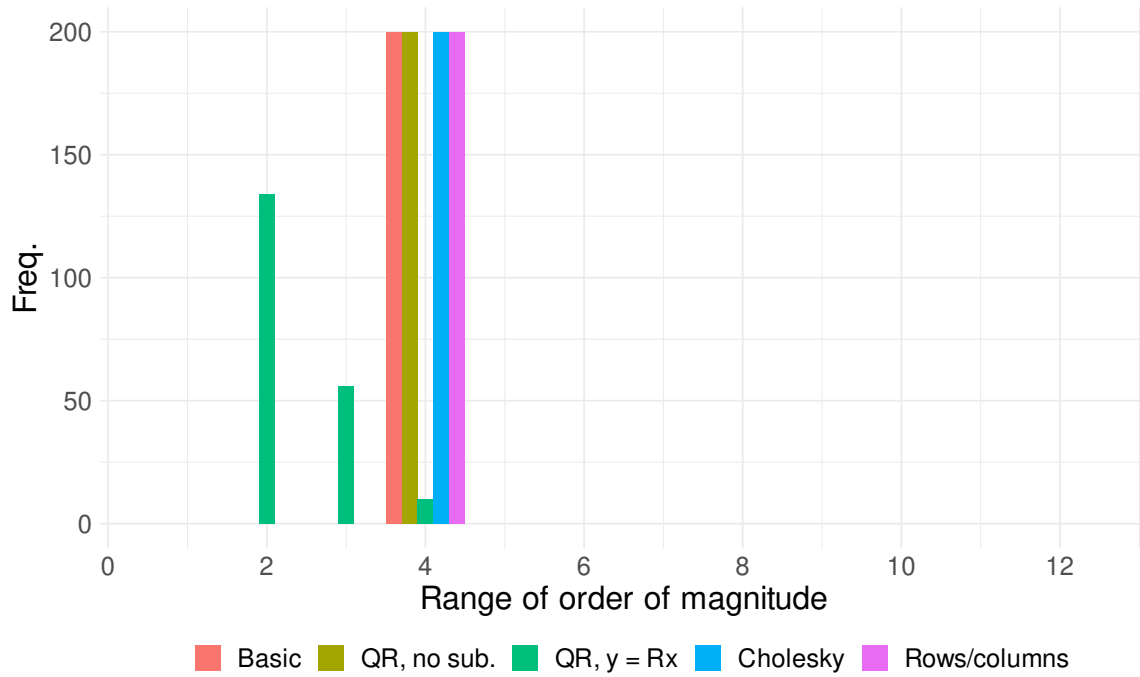
### 8.3 Case 2, QCQP, unscaled



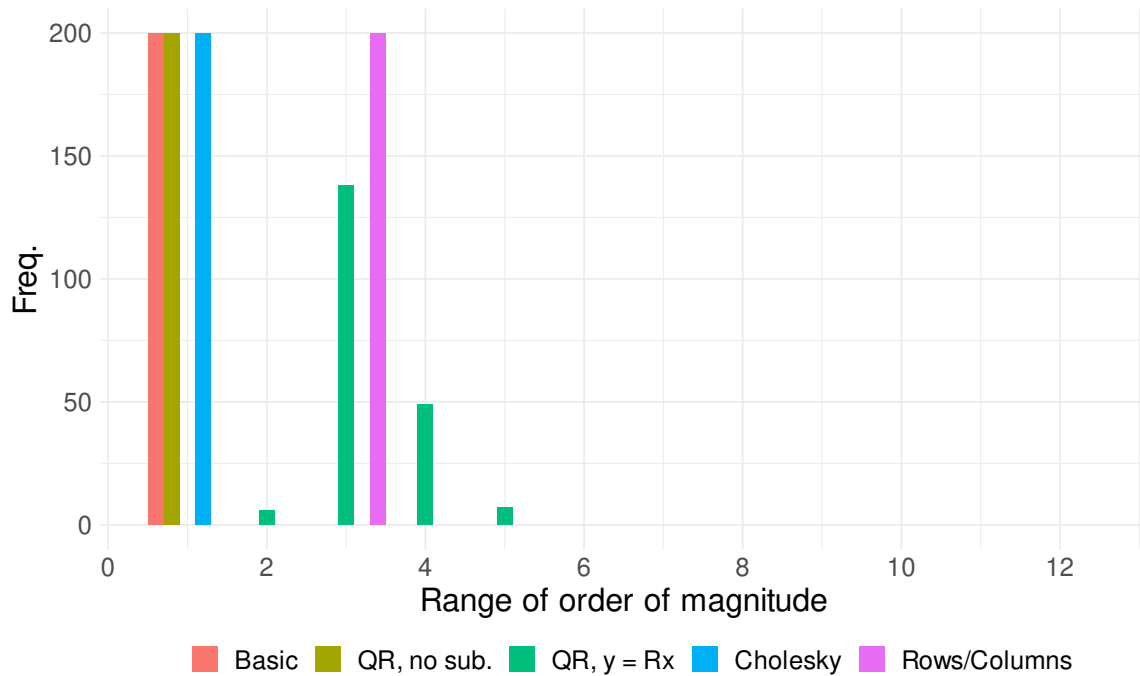
### 8.4 Case 2, QCQP, rescaled



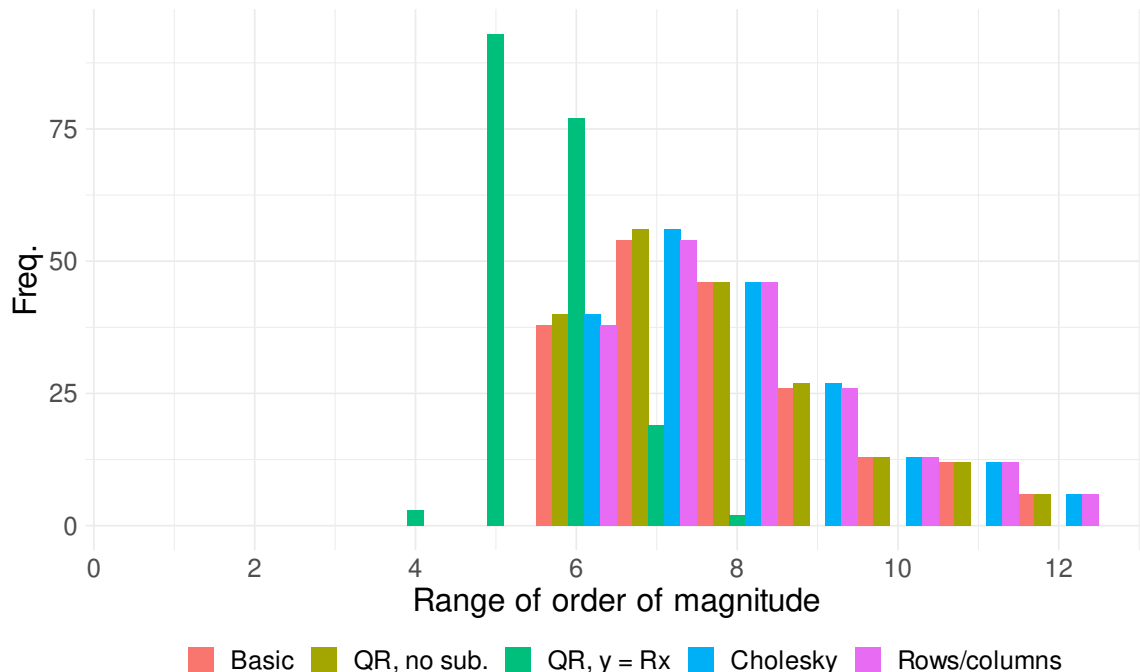
### 8.5 Case 3, QCQP, unscaled



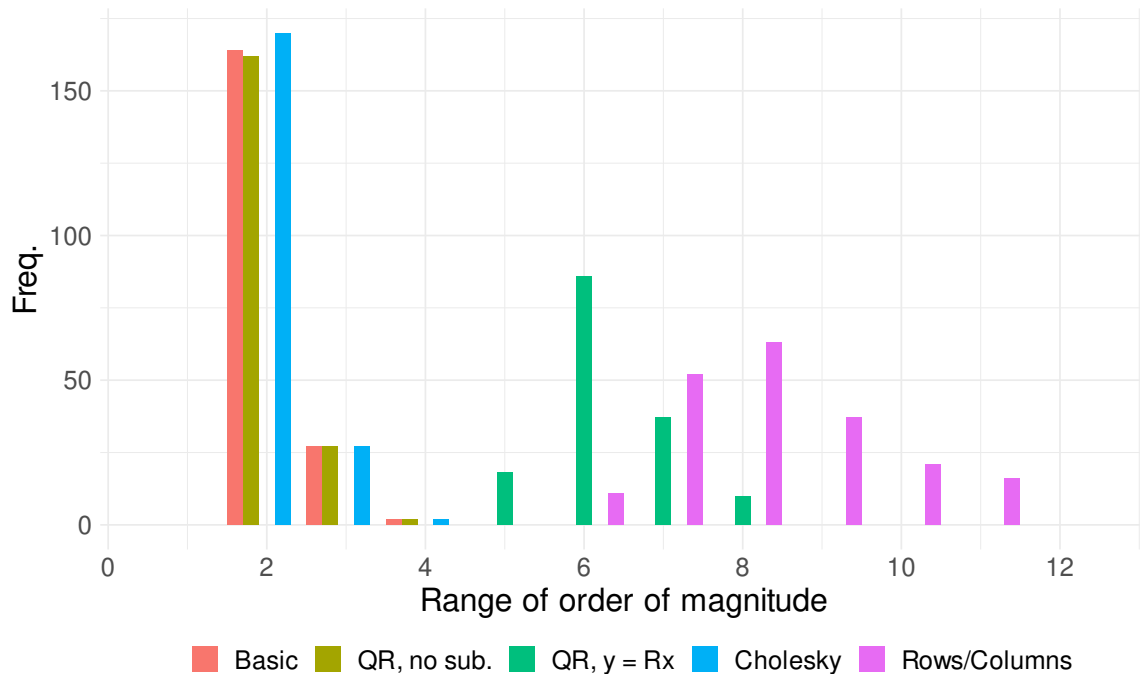
### 8.6 Case 3, QCQP, rescaled



### 8.7 Case 4, QCQP, unscaled



### 8.8 Case 4, QCQP, rescaled



**Note:** Blue bar does not sum to 200 (total number of simulations) because some simulations returned errors and the QCQP model could not be saved.