Order of rescaling rows and columns can matter

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Suppose A is a 2×2 matrix and the order of magnitude of each entry is indicated by the following matrix,

$$\begin{bmatrix} -1 & -3 \\ 0 & 1 \end{bmatrix}.$$

Suppose we want the minimum magnitude in each column and row to be -3.

If we rescale the columns first and then the rows, then the magnitudes become

$$\begin{bmatrix} -1 & -3 \\ 0 & 1 \end{bmatrix} \xrightarrow{\text{Col.}} \begin{bmatrix} -3 & -3 \\ -2 & 1 \end{bmatrix} \xrightarrow{\text{Row}} \begin{bmatrix} -3 & -3 \\ -3 & 0 \end{bmatrix}.$$

The range of the orders of magnitude is 3.

If we rescale the rows first and then the columns, then the magnitudes become

$$\begin{bmatrix} -1 & -3 \\ 0 & 1 \end{bmatrix} \xrightarrow{\text{Row}} \begin{bmatrix} -1 & -3 \\ -3 & -2 \end{bmatrix} \xrightarrow{\text{Col.}} \begin{bmatrix} -1 & -3 \\ -3 & -2 \end{bmatrix}.$$

The range of the orders of magnitude is 2.

Other examples

For some matrices, the final magnitudes depend on whether the rows or columns are rescaled first, but the range of the magnitudes does not. Below is one such example, where the range is 1 regardless of whether the columns or rows are rescaled first.

$$\begin{bmatrix} -1 & -3 \\ 1 & 0 \end{bmatrix} \xrightarrow{\text{Col.}} \begin{bmatrix} -3 & -3 \\ -1 & 0 \end{bmatrix} \xrightarrow{\text{Row.}} \begin{bmatrix} -3 & -3 \\ -3 & -2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -3 \\ 1 & 0 \end{bmatrix} \xrightarrow{\text{Row}} \begin{bmatrix} -1 & -3 \\ -2 & -3 \end{bmatrix} \xrightarrow{\text{Col.}} \begin{bmatrix} -2 & -3 \\ -3 & -3 \end{bmatrix}$$

For some matrices, the final magnitudes are the same regardless of whether the

columns or rows are rescaled first. The range therefore also does not depend on whether the columns or rows are rescaled first.

$$\begin{bmatrix} -6 & -1 \\ -3 & 8 \end{bmatrix} \xrightarrow{\text{Col.}} \begin{bmatrix} -3 & -3 \\ 0 & 6 \end{bmatrix} \xrightarrow{\text{Row.}} \begin{bmatrix} -3 & -3 \\ -3 & 3 \end{bmatrix}$$

$$\begin{bmatrix} -6 & -1 \\ -3 & 8 \end{bmatrix} \xrightarrow{\text{Row}} \begin{bmatrix} -3 & 2 \\ -3 & 8 \end{bmatrix} \xrightarrow{\text{Col.}} \begin{bmatrix} -3 & -3 \\ -3 & 3 \end{bmatrix}$$