

# Where to from here?

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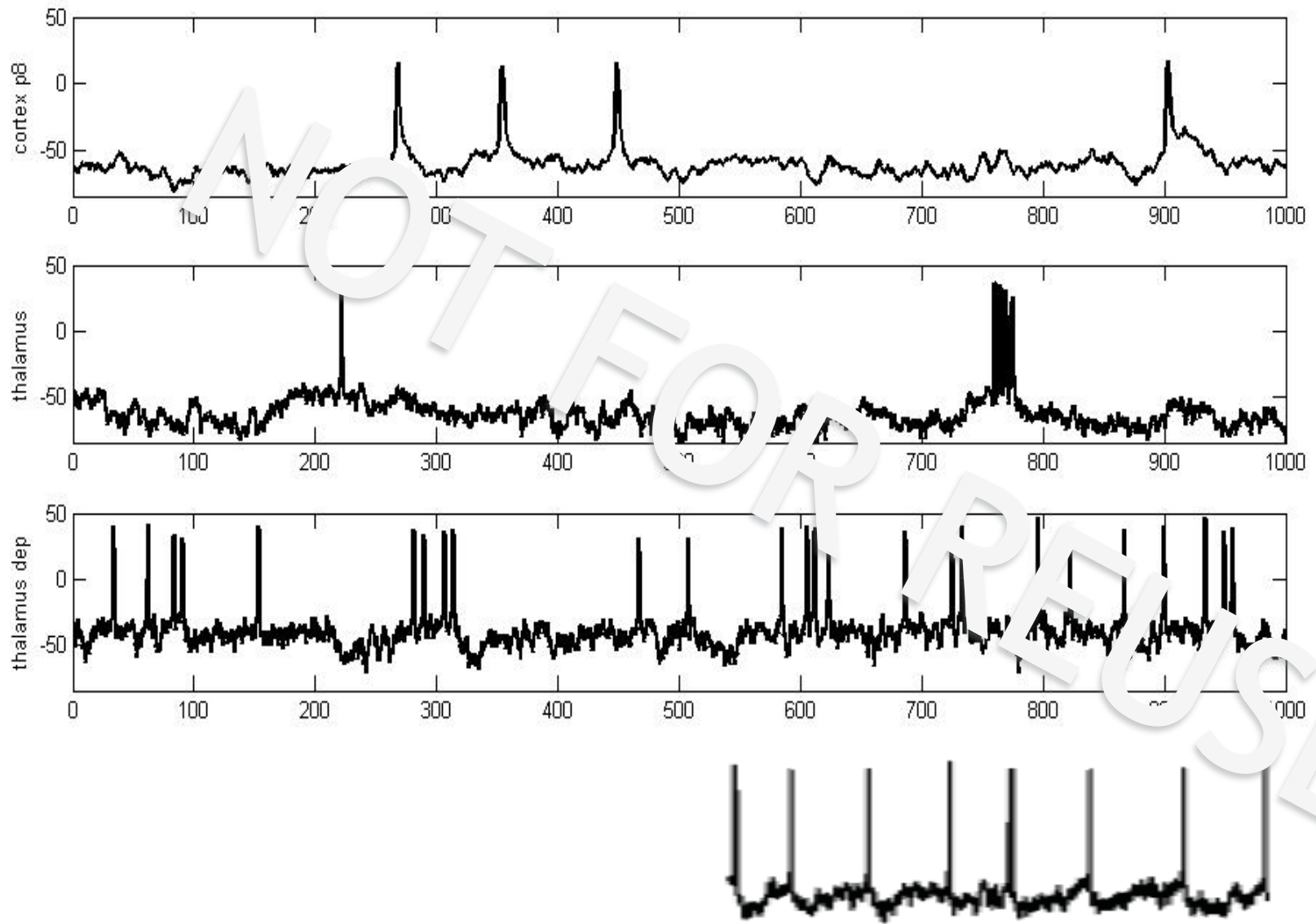
**Hodgkin-Huxley**

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graph TD; A([Hodgkin-Huxley]) --> B([Biophysical realism<br/>Ion channel physics<br/>Additional channels<br/>Geometry]); A --> C([Simplified models<br/>Fundamental dynamics<br/>Analytical tractability]);
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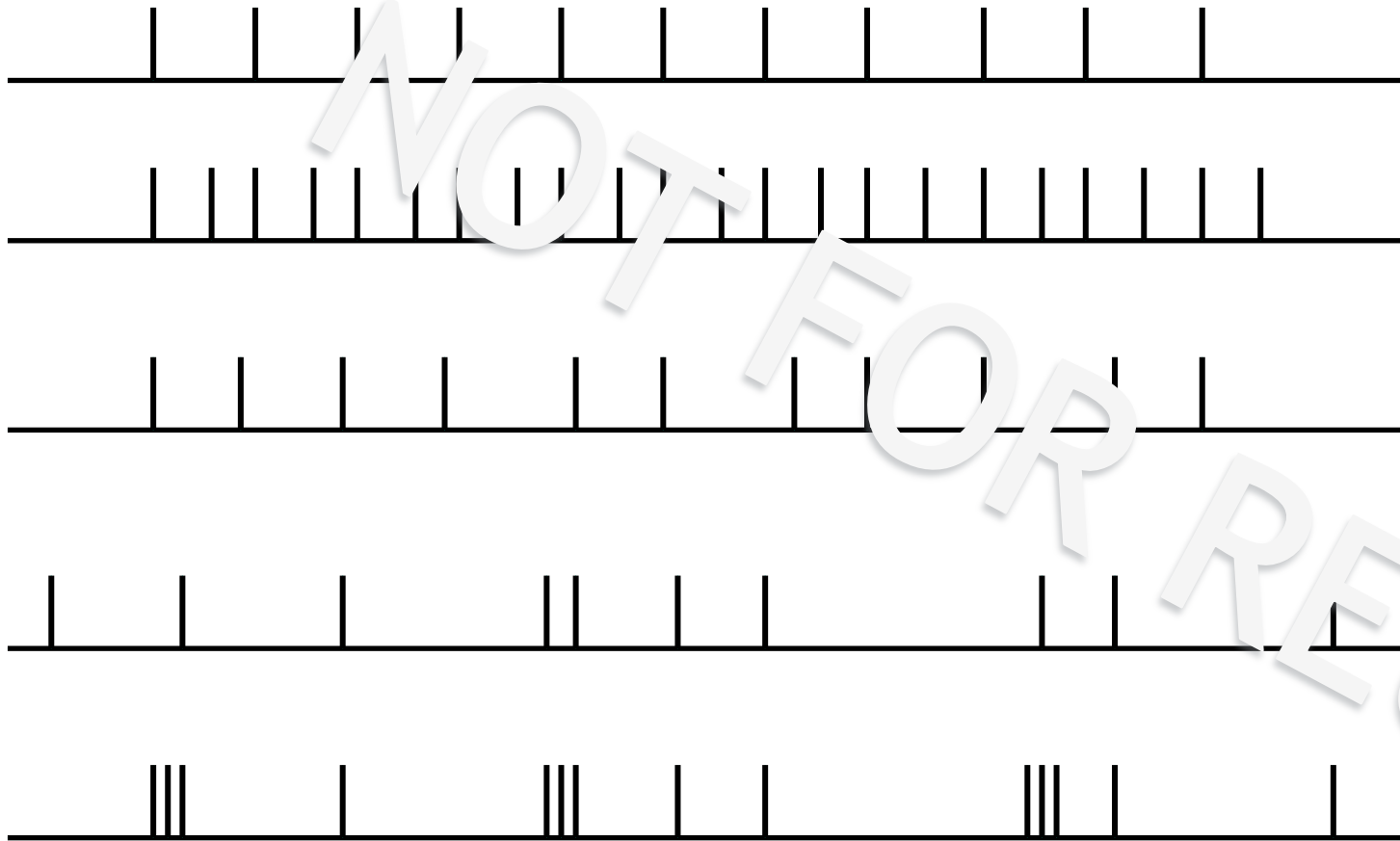
**Biophysical realism**  
Ion channel physics  
Additional channels  
Geometry

**Simplified models**  
Fundamental dynamics  
Analytical tractability

# The electric personalities of neurons



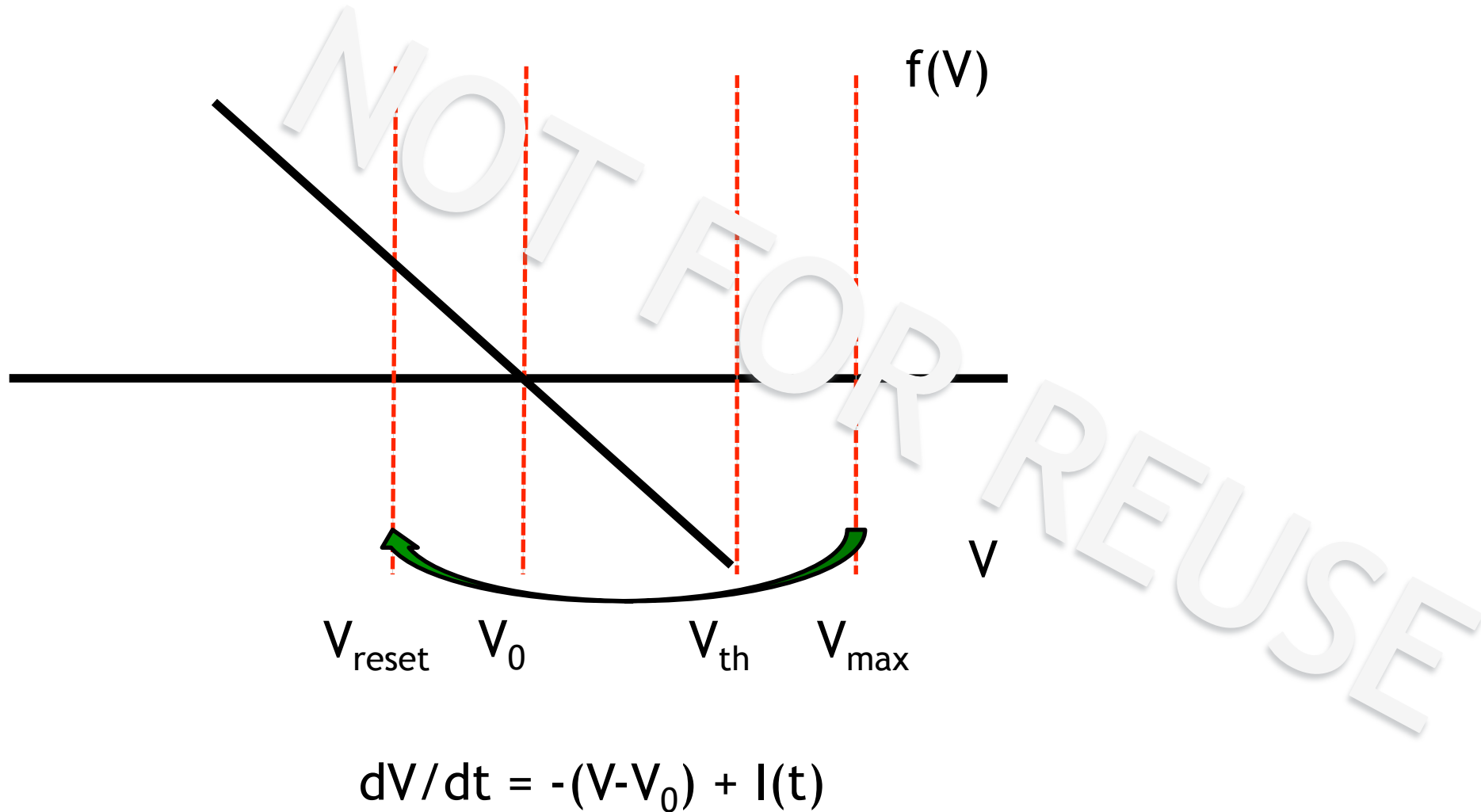
# Neuron, what are you trying to tell us?



# Capturing the basic dynamics of neurons

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# Capturing the basic dynamics of neurons



# The integrate-and-fire neuron

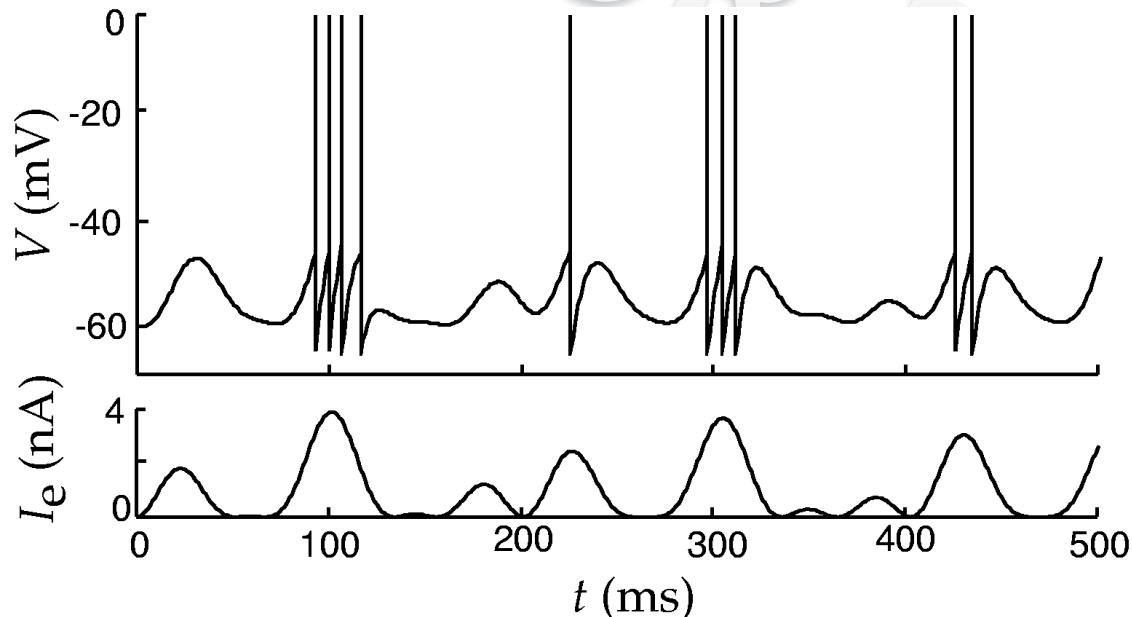
Like a passive membrane:

$$C_m \frac{dV}{dt} = -g_L(V - E_i) - I_e$$

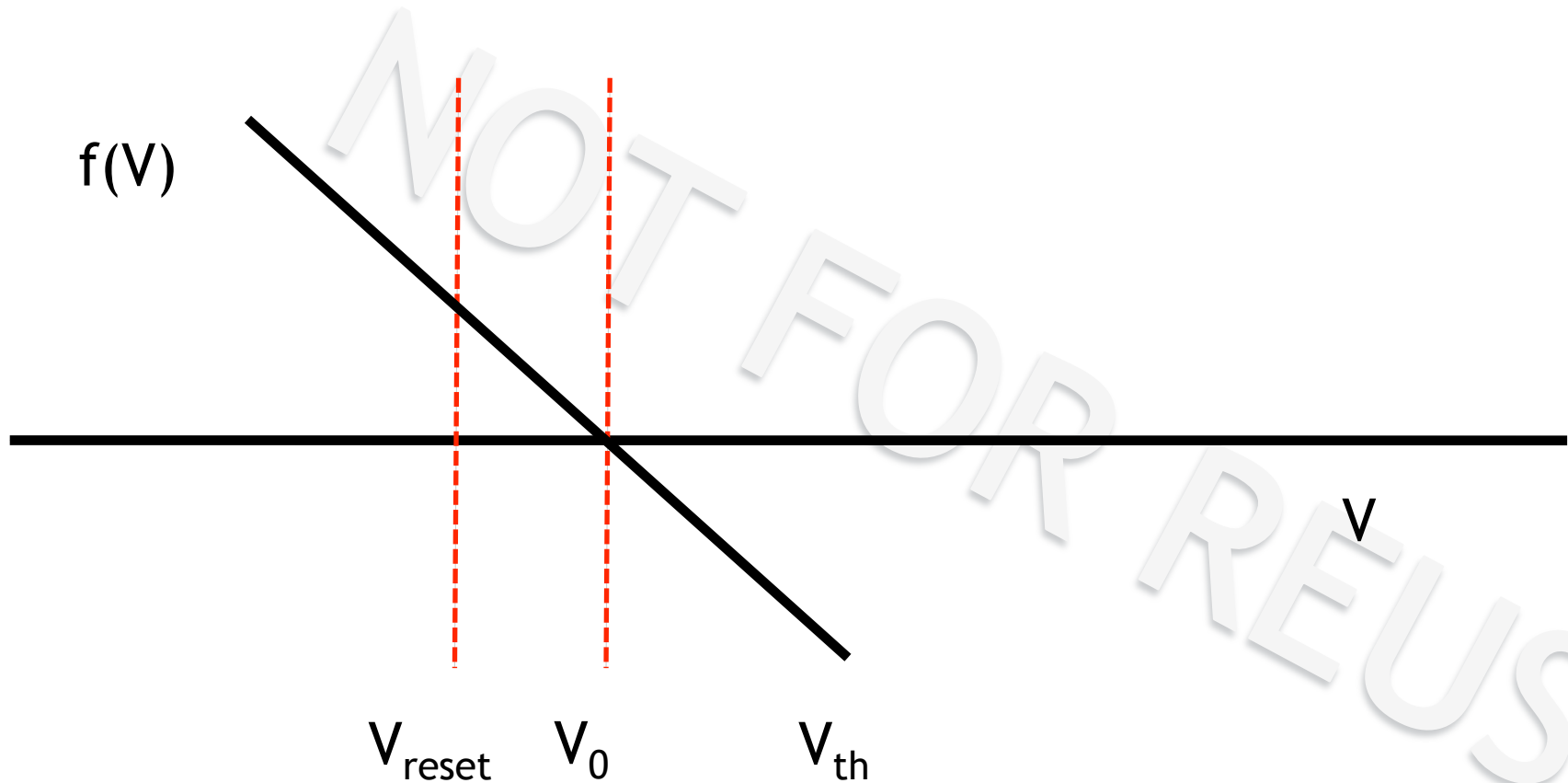
but with the additional rule that

when  $V \rightarrow V_T$ , a spike is fired  
and  $V \rightarrow V_{\text{reset}}$ .

$E_L$  is the resting potential of the “cell”.

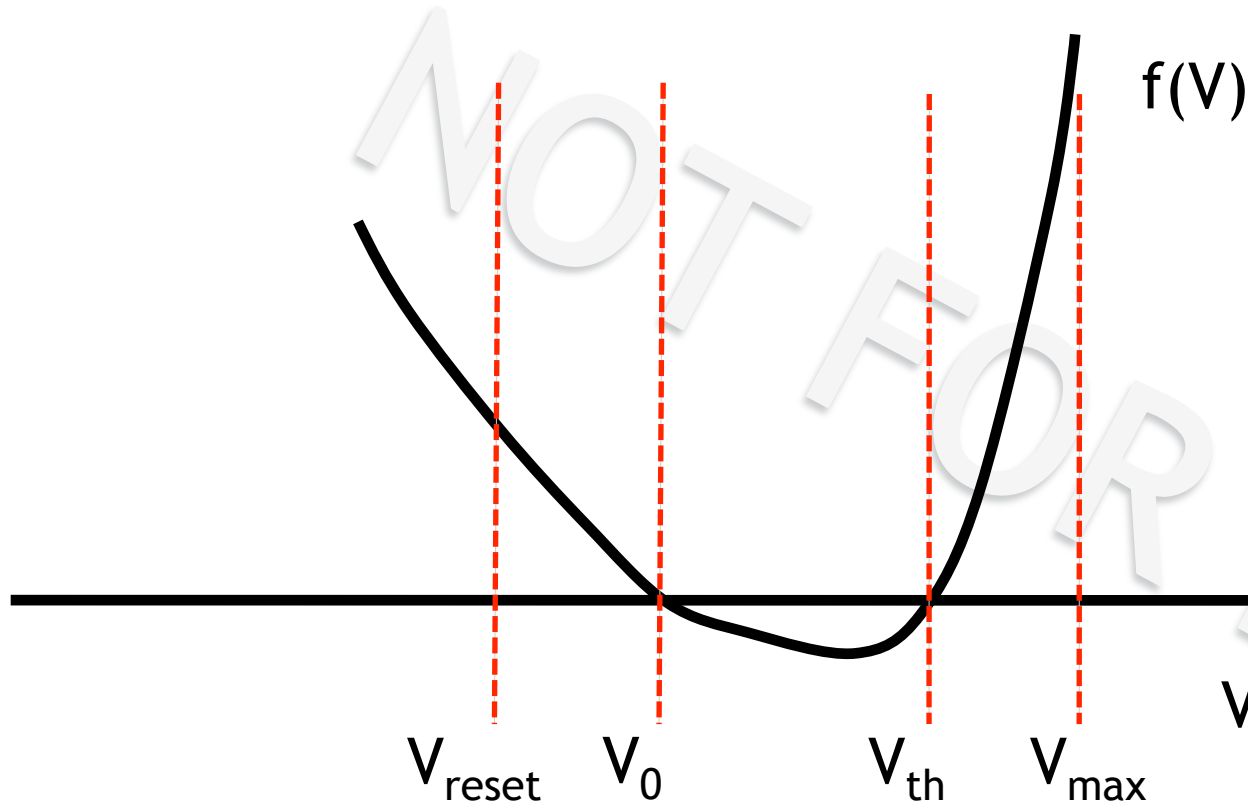


# How do we make it excitable?



$$dV/dt = -a(V - V_0) + I(t)$$

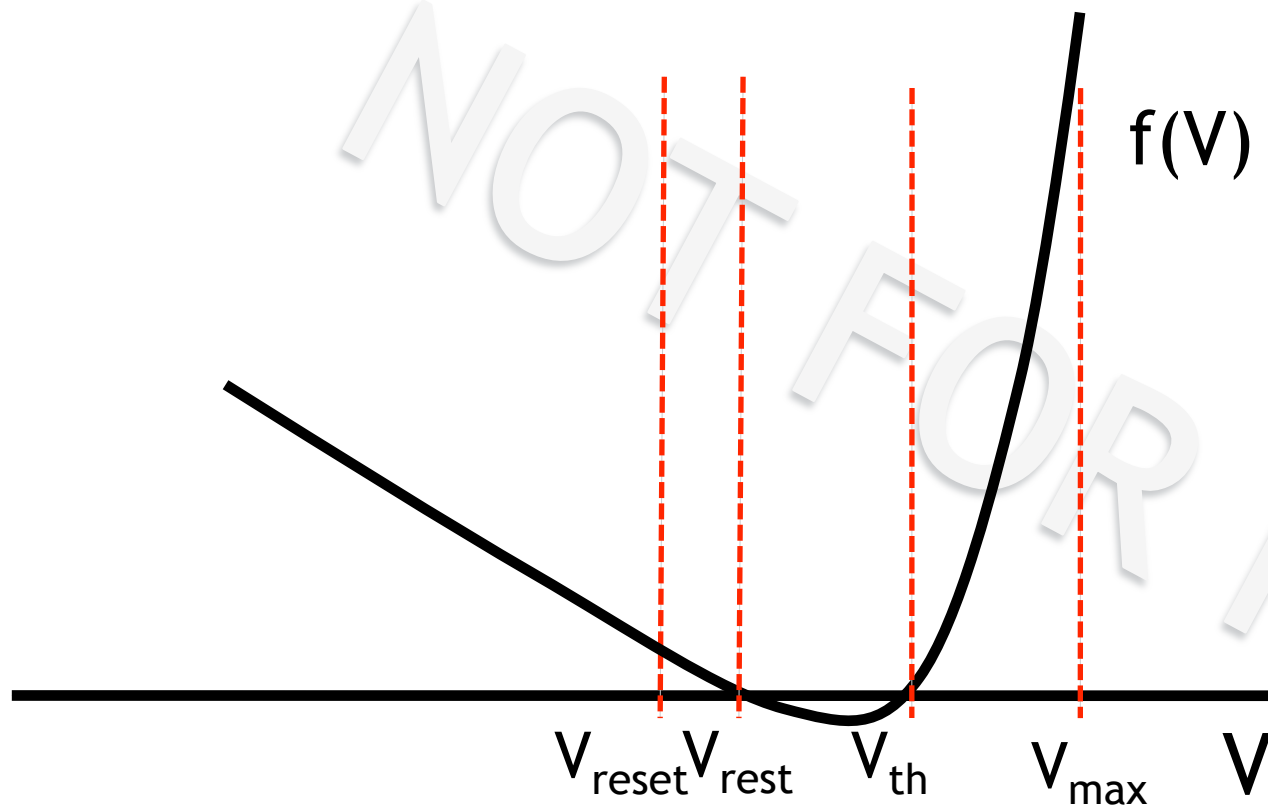
# Capturing the basic dynamics of neurons



$$dV/dt = f(V) + I(t)$$



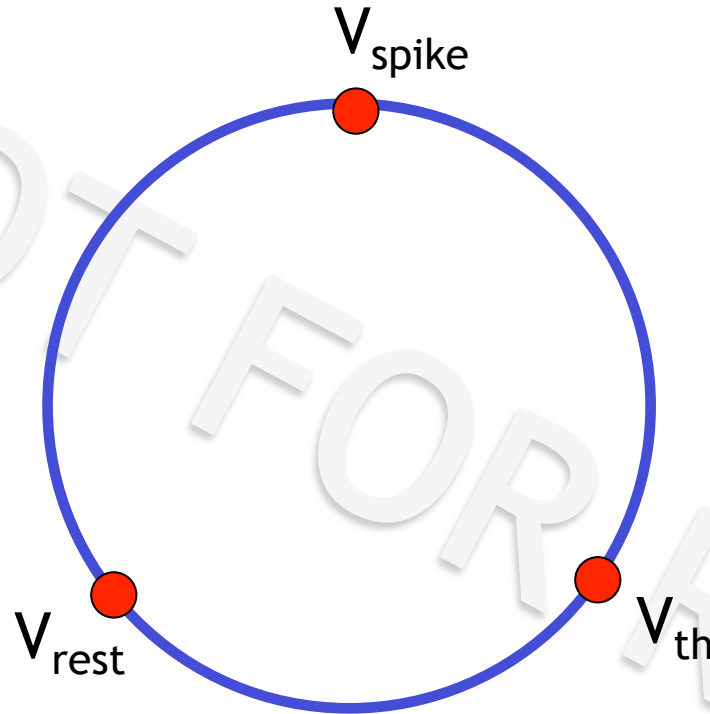
# Exponential integrate-and-fire neuron



$$f(V) = -a(V - V_0) + \exp([V - V_{th}] / \Delta)$$

# The theta neuron

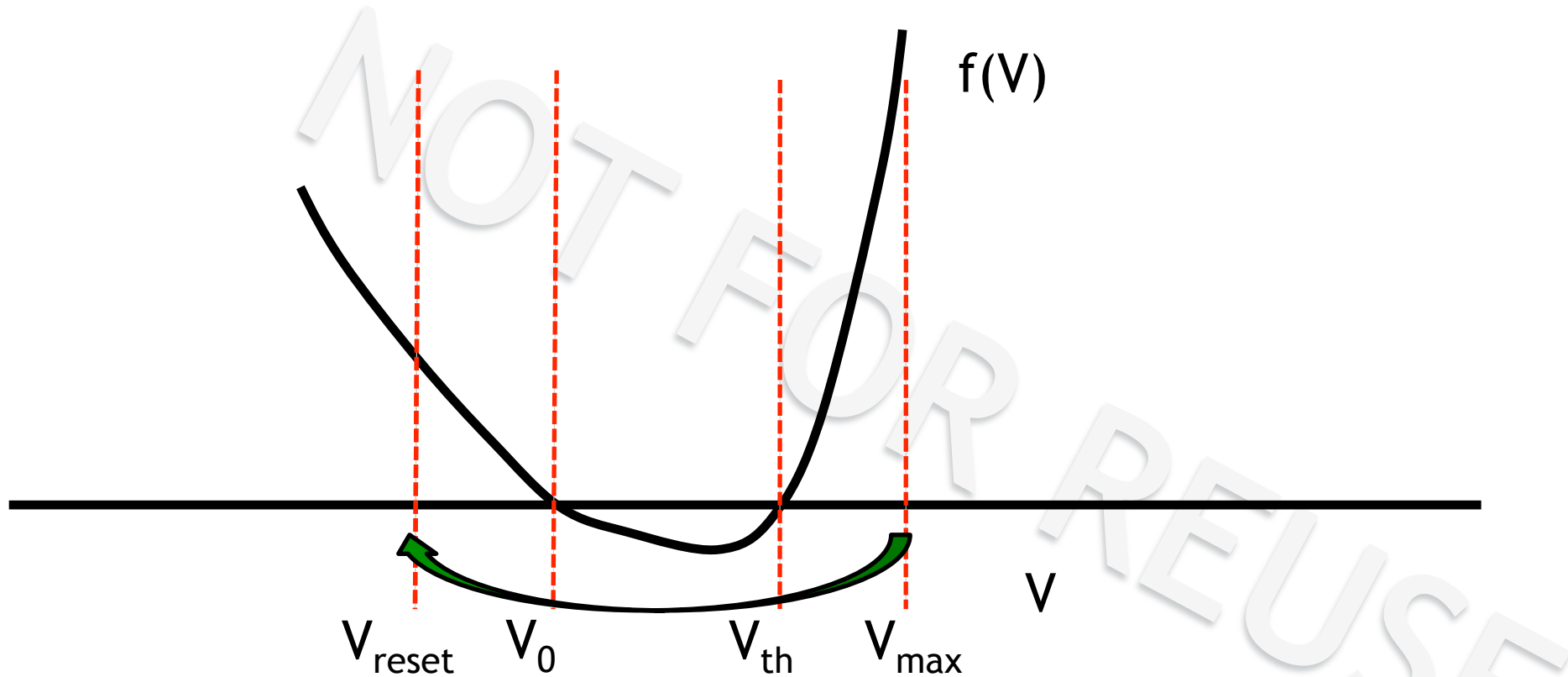
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$$d\theta/dt = 1 - \cos \theta + (1 + \cos \theta) I(t)$$

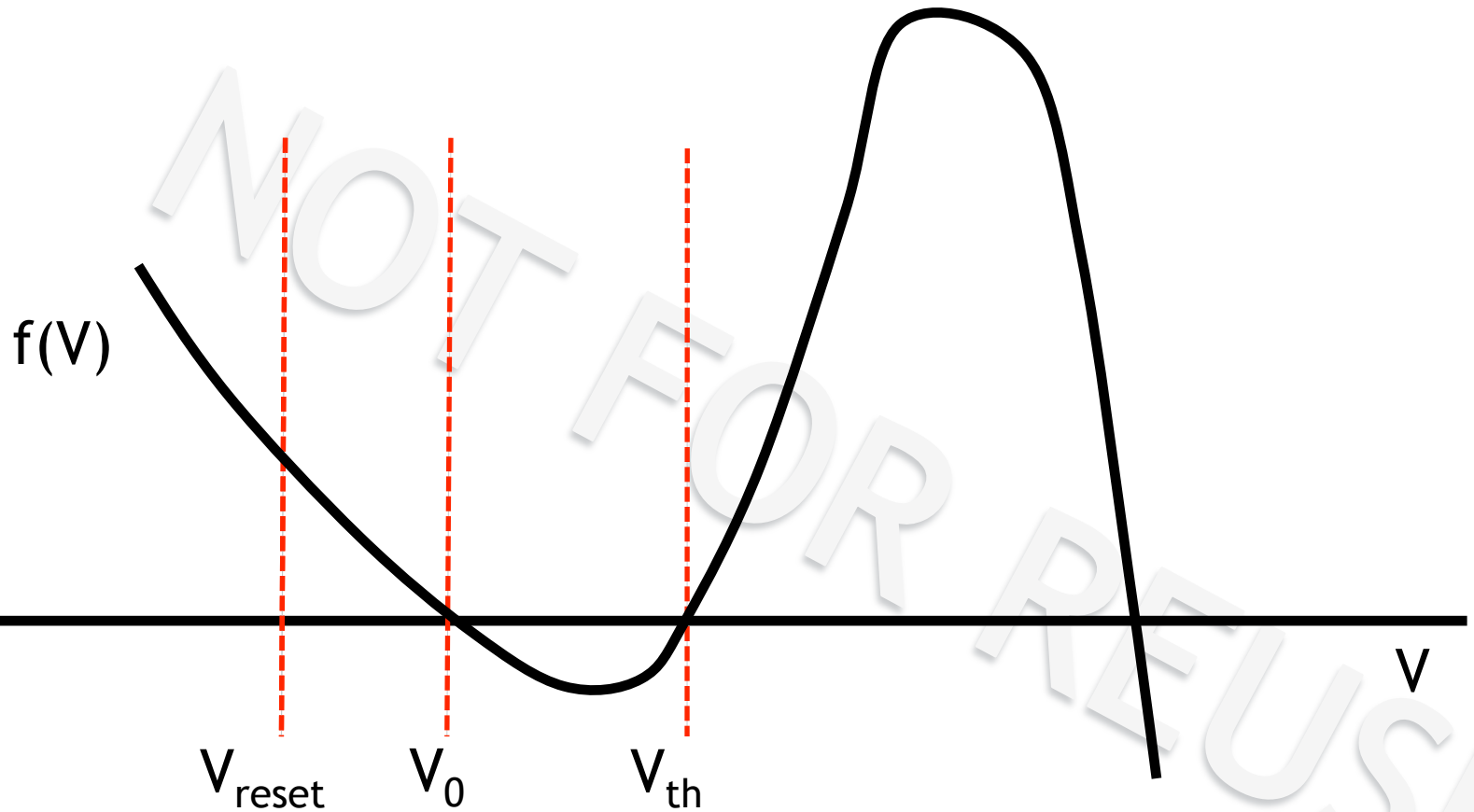
Ermentrout and Kopell

# Capturing the basic dynamics of neurons

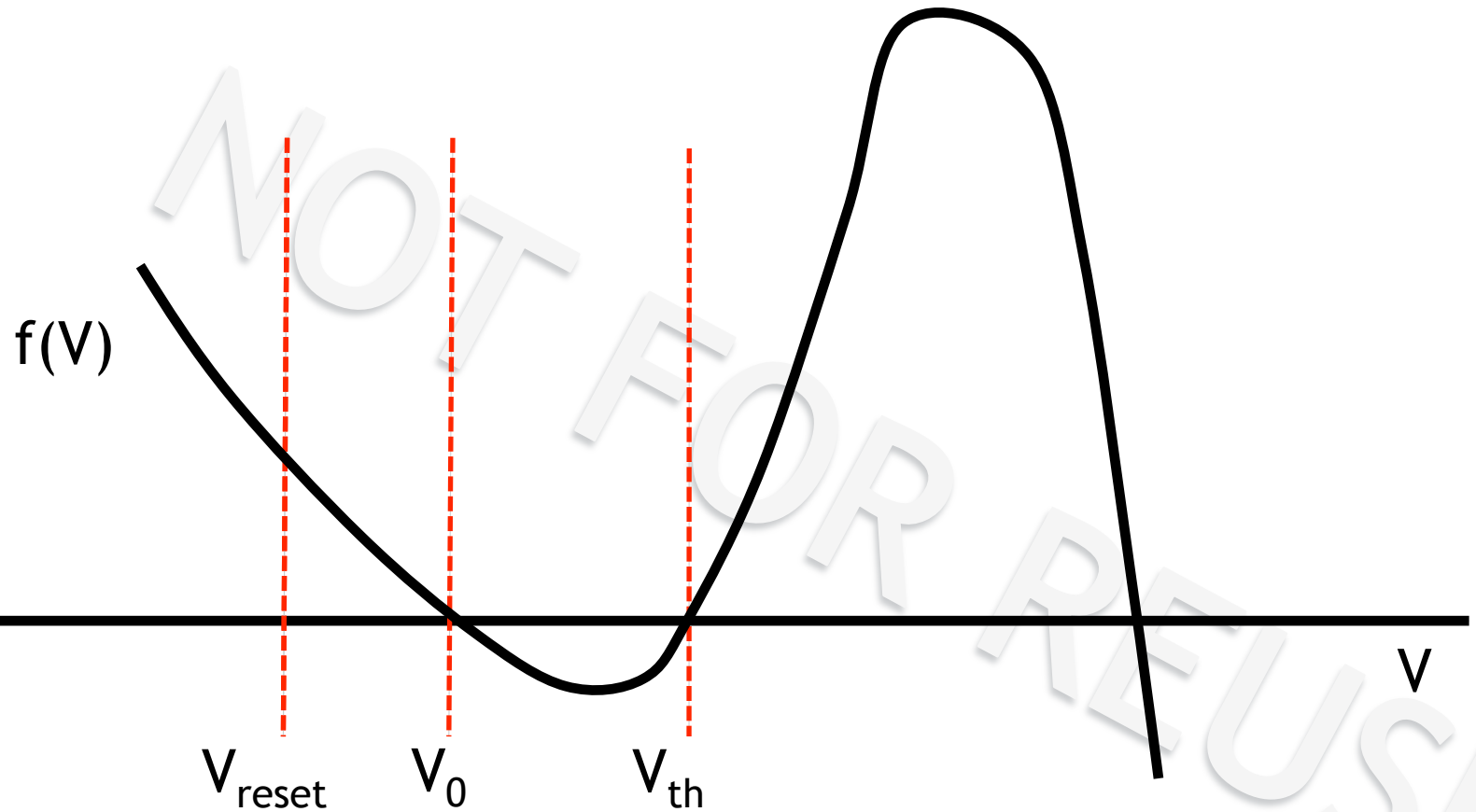


$$dV/dt = f(V) + I(t)$$

# Capturing the basic dynamics of neurons



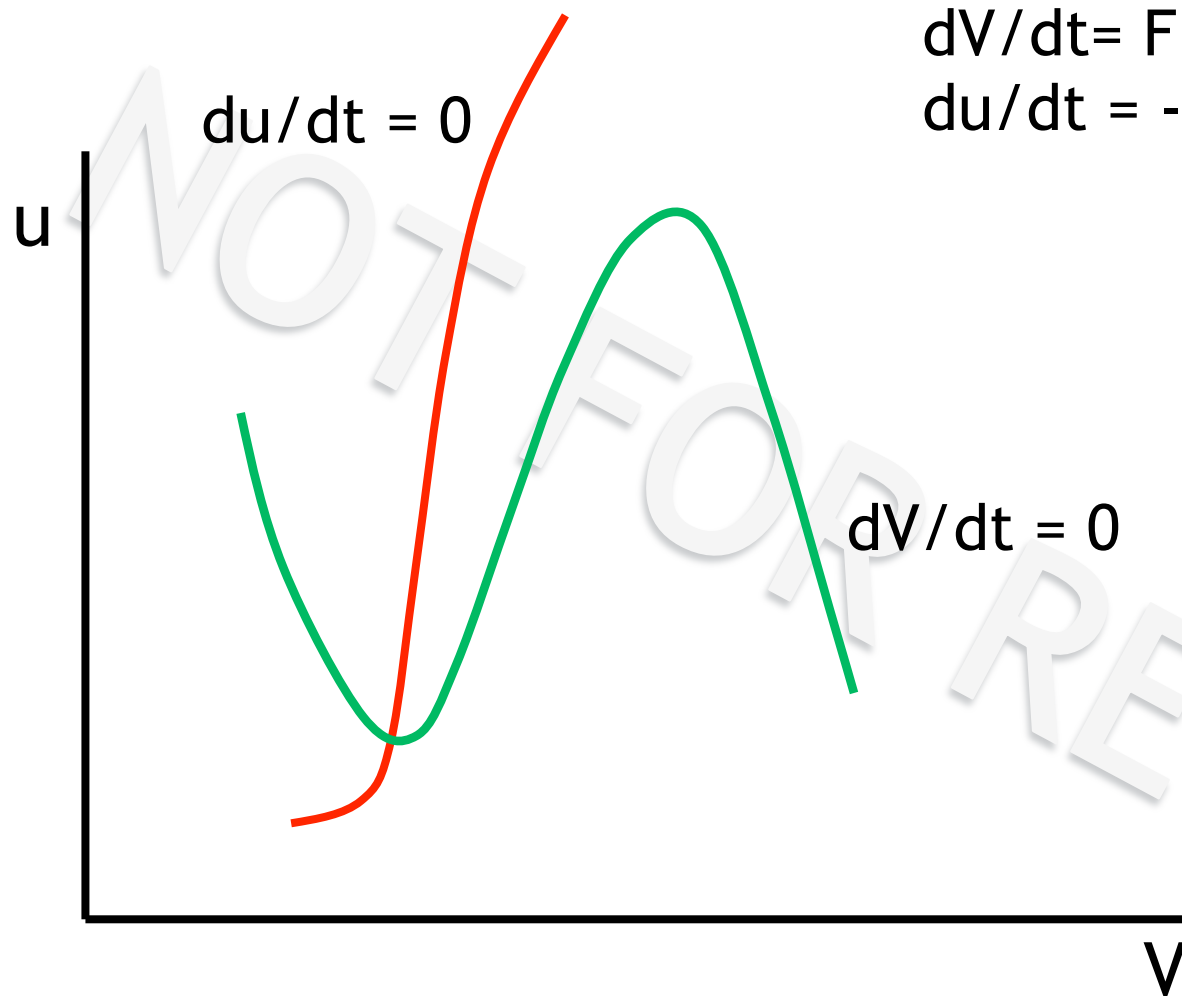
# Capturing the basic dynamics of neurons



$$dV/dt = f(V) + I(t) \rightarrow$$

$$\begin{aligned} dV/dt &= F(V) + G(u) + I(t) \\ du/dt &= -u + H(V) \end{aligned}$$

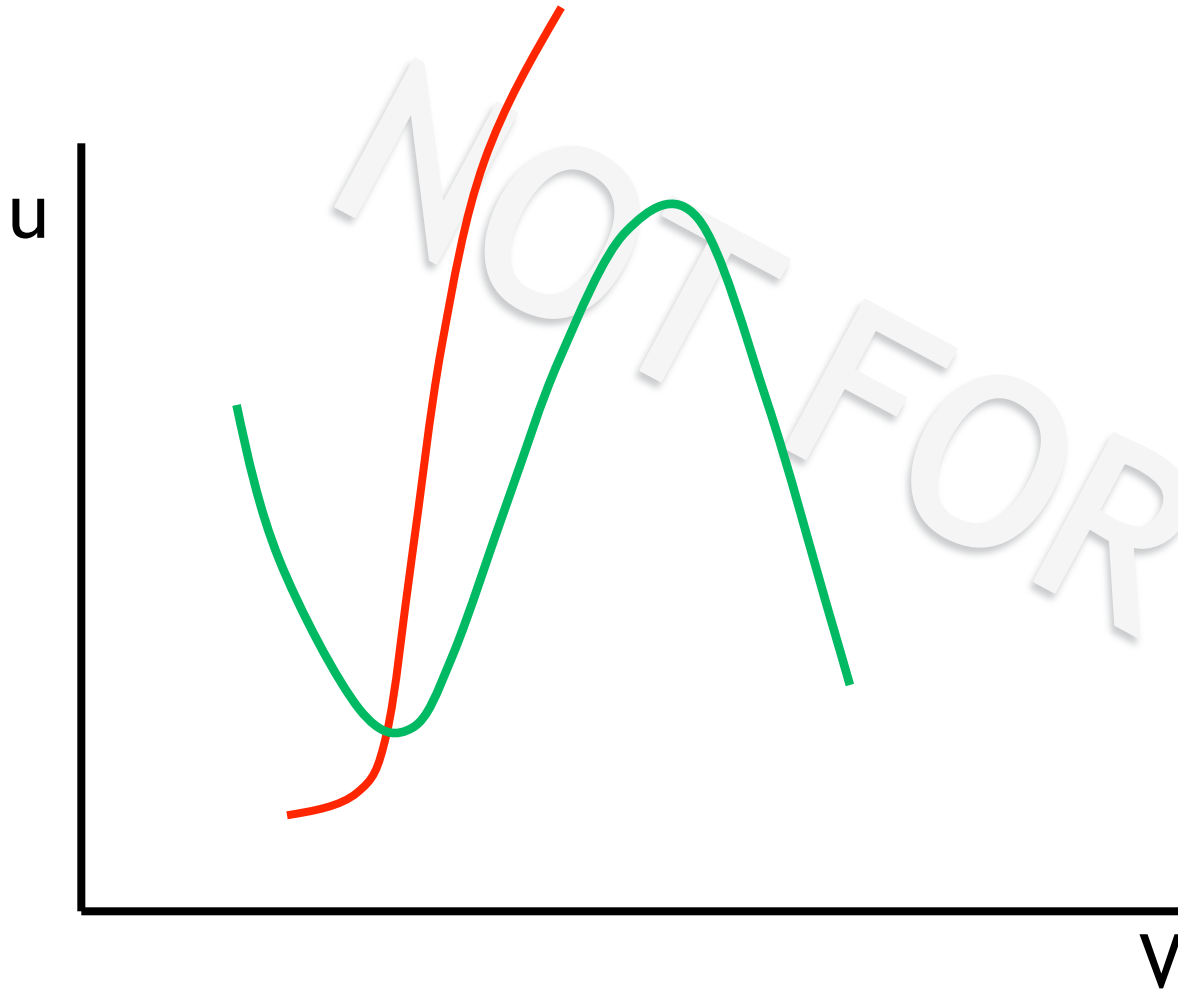
# Two-dimensional models



$$\begin{aligned} dV/dt &= F(V) + G(u) + I \\ du/dt &= -u + H(V) \end{aligned}$$

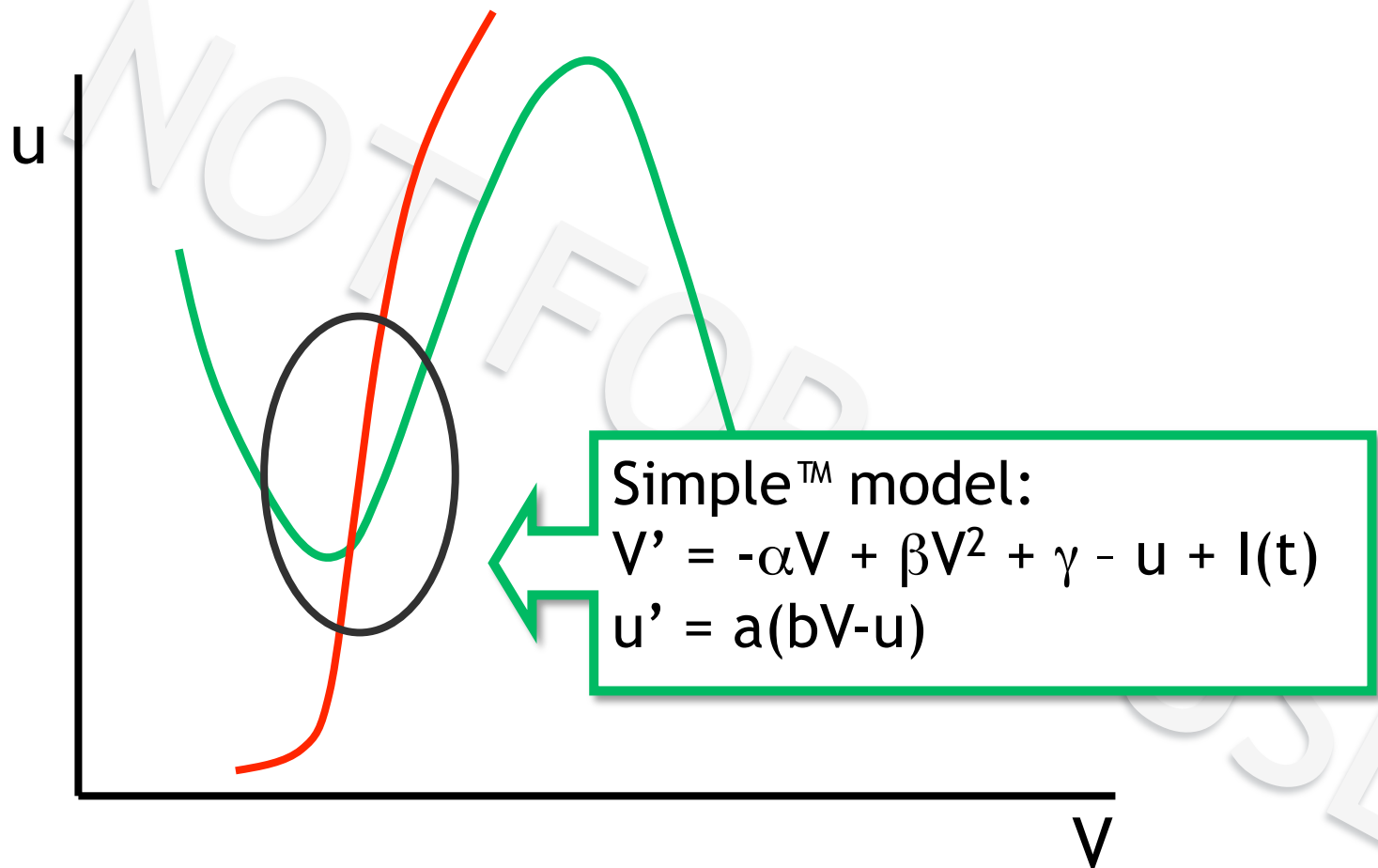
# On-line resources about phase plane analysis

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- Wulfram Gerstner
- Bard Ermentrout
- Scholarpedia

# Two-dimensional models

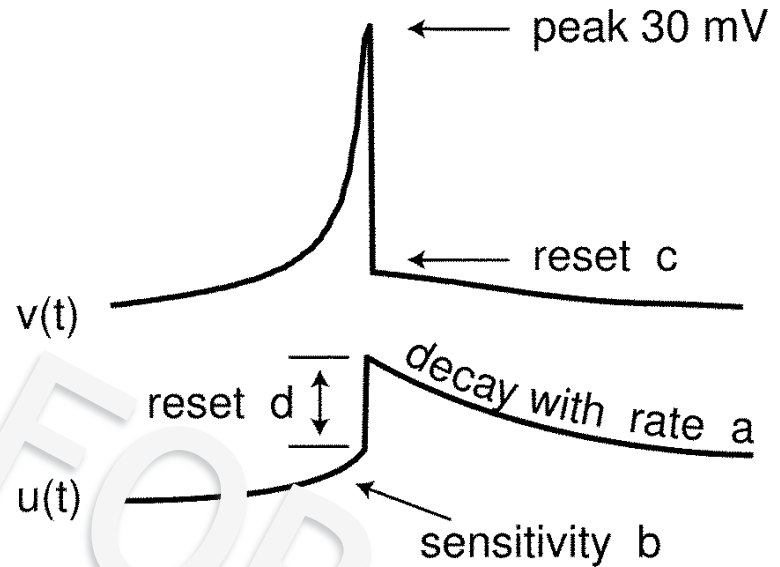




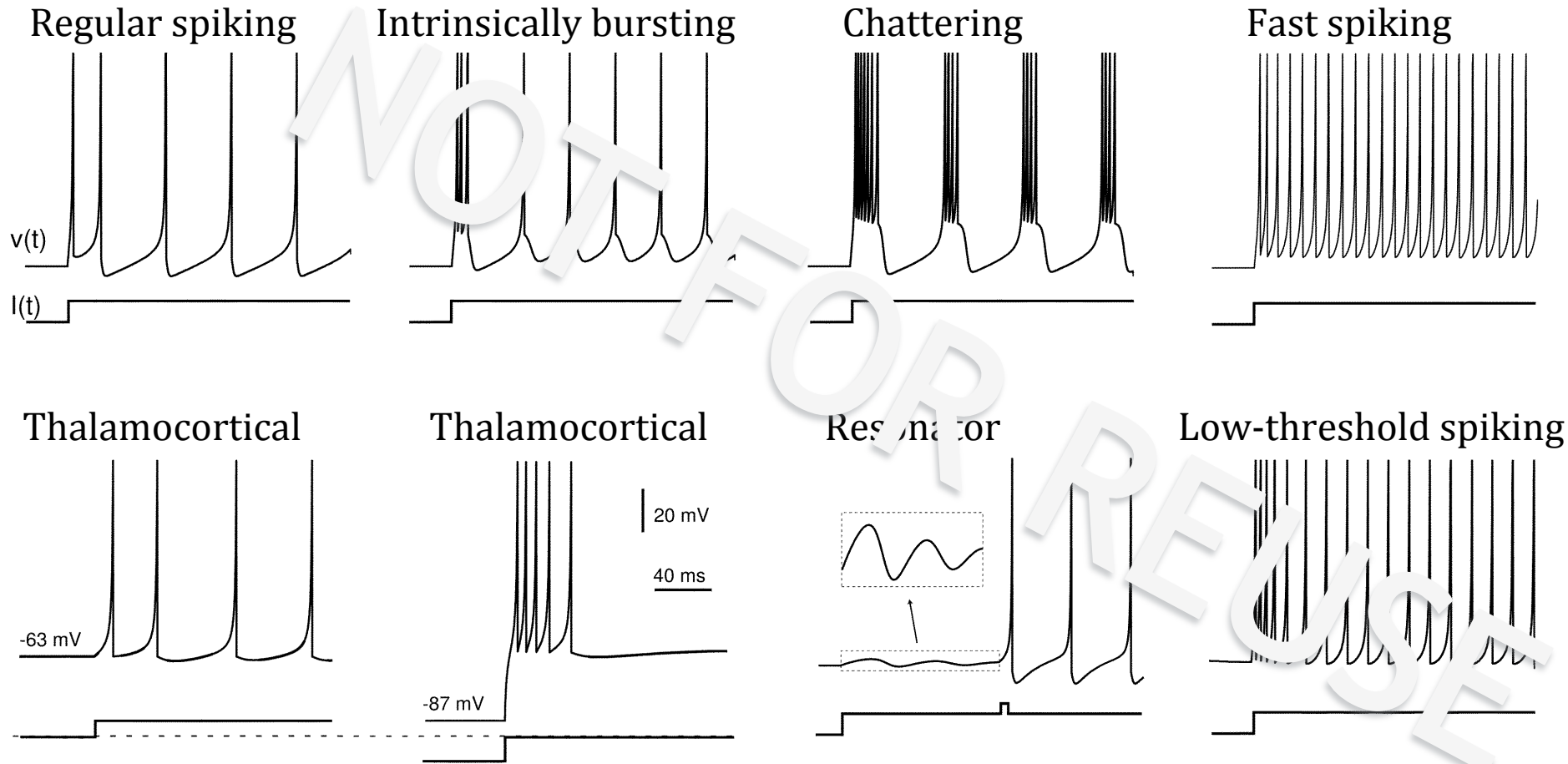
# The simple model

$$v' = 0.04v^2 + 5v + 140 - u + I$$
$$u' = a(bv - u)$$

**if**  $v = 30$  mV,  
**then**  $v \leftarrow c$ ,  $u \leftarrow u + d$



# The simple model



# Where to from here?

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The diagram features a central red-outlined oval at the top labeled 'Hodgkin-Huxley'. Two large green arrows point downwards from this central oval to two separate red-outlined ovals below it. The left oval is titled 'Biophysical realism' and lists 'Ion channel physics', 'Additional channels', and 'Geometry'. The right oval is titled 'Simplified models' and lists 'Fundamental dynamics' and 'Analytical tractability'. A faint, large watermark 'CR' is visible in the background.

**Hodgkin-Huxley**

**Biophysical realism**  
Ion channel physics  
Additional channels  
Geometry

**Simplified models**  
Fundamental dynamics  
Analytical tractability