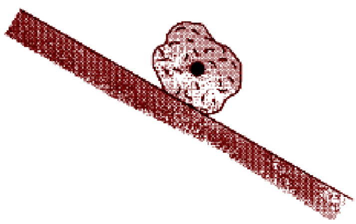
Subject Code	PHY 1	**Physics 1**
Module Code	7.0	**Applications of Newton's Laws of Motion**
Lesson Code	7.5	**Free-body Diagrams of Inclined Planes**
Time Frame		30 minutes

Components	**Tasks**	**TA[1] (min)**	**ATA[2] (min)**
Target 	By the end of this learning guide, the student should be able to: ● construct free-body diagrams, representing forces using appropriately labeled vectors, for situations involving an inclined plane ● write expressions for the summation of forces along axes parallel and perpendicular to the incline	1	
Hook 	In a space diagram, forces involved in the interactions present in a physical situation are all accounted for. These are drawn at different locations in the diagram to indicate their points of application. But in analyzing the motion of an object or a component in the given physical system, a ***free-body diagram*** (***FBD***) is used to isolate the forces acting on the object. These forces are drawn to be emanating from a common point which represents the object. The common point is usually set to be the origin of orthogonal axes denoted as x and y. (Buffa & Wilson, 2010). Can you draw the free body diagram for the object on an inclined plane, illustrated in Figure 1 below, given that friction prevents it from sliding?  ***Figure 1***. *Object in an inclined plane.* *(Court, Free-body diagrams revisited)*	2	

[1] Time allocation suggested by the teacher.
[2] Actual time allocation spent by the student (for information purposes only).

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In drawing free-body diagrams (FBDs), the object is represented as a point and the forces acting on it are represented as vector arrows. Force vectors do not have to be drawn exactly to scale but it helps to show in terms of relative sizes whether the forces balance each other or not. This way, one can also visually represent if the net force is in a particular direction.

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Recall from Newton's second law of motion that an object accelerates in the direction of the net force acting on it. As such, if the direction of the net force is known, it is convenient to set one of the axes along this direction. The components of the individual forces along the two axes will then be determined in order to come up with expressions for the summation of forces along each axis.

Here are the general steps in constructing and using FBDs on an object on an inclined plane (adopted from Buffa & Wilson, 2010).

1. Draw an illustration, or space diagram, of the given situation and identify the forces acting on the components of the system (Figure 2).

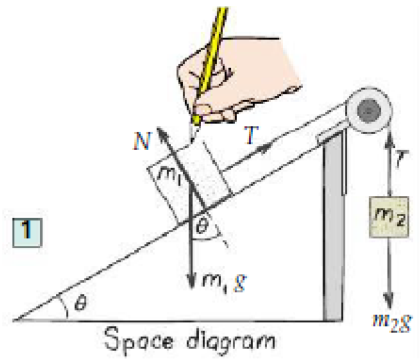

Figure 2*. Space (force) diagram on a body on an inclined plane. (Buffa & Wilson, 2010)*

2. Isolate the object for which the free-body diagram is to be constructed. Draw a set of orthogonal axes, labeled *x*- and *y*-, with the origin at the object and with one of the axes along the direction of the object's acceleration. In this case, one axis will be set parallel to the incline.

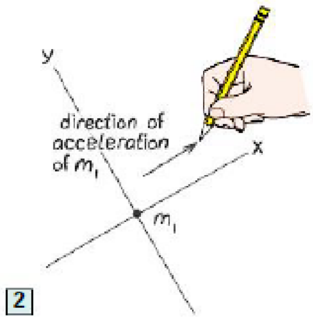

Figure 3. *Isolation of the Body for FBD.*
(Buffa & Wilson, 2010)

3. Draw a vector to represent the acceleration. If the acceleration is not known, you may just assume a direction. (*"Note: If you assume that the acceleration is in one direction, and in the solution it comes out with the opposite sign, then the acceleration is actually in the opposite direction from that assumed. For example, if you assume that* ***a*** *(acceleration) is in the +x-direction, but you get a negative answer, then* ***a*** *is in the -x-direction."*)

4. Draw force vectors, taking note of the proper direction of each, such that they emanate from the origin of the axes. Be sure to include only those forces that act on the object of interest. If the force is not along either *x*- or *y*-axis, specify the angle in your illustration.

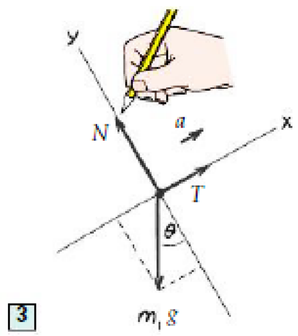

Figure 4. *FBD of the object on an inclined plane.*
(Buffa & Wilson, 2010)

5. Resolve any forces that are not directed along the *x*- or *y*-axis into *x*- or *y*-components (use plus and minus signs to indicate direction). Applying Newton's second law of motion, write expressions for the summation of force along each axis. (Figure 5).

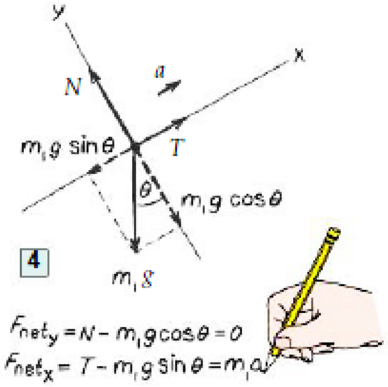

Figure 5. *Resolution of vectors into components. (Buffa & Wilson, 2010)*

What does the FBD of the object shown in Figure 1 look like?

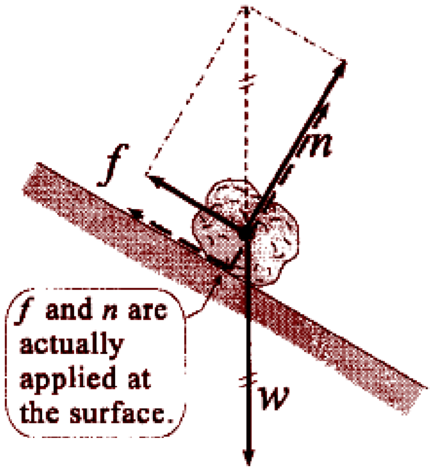

Figure 6*. FBD of the object on an inclined plane. (Court, Free-body diagrams revisited)*

As shown in Figure 6, there are three forces acting on the object: (1) weight (**w**) which is directed downwards, (2) normal force (**n**) which is directed towards the object and perpendicular to the inclined plane, and (3) static friction (**f**). Since the friction prevents the object from sliding, it must be directed up the incline.

As the object is in equilibrium, we can conclude that the summation of all these three forces is zero. This is illustrated in Figure 6, where the vector sum of the normal force and friction is shown to be of the same length as the weight vector. That is, the combination of normal force and friction cancels out the weight. Equivalently, the vector sum of the weight and the normal force can be shown to be down the incline, with a magnitude equal to that of friction. Without friction, the object will accelerate down the incline.

Alternatively we can write Newton’s second law of motion along axes parallel and perpendicular to the inclined plane. Since the weight is not directly along either of the axes parallel and perpendicular to the incline, we first resolve it into components denoted as $\mathbf{w}_{\parallel}$ **and** $\mathbf{w}_{\perp}$.

The summation of forces can then be written as:

$$\Sigma F_{\parallel} = w_{\parallel} - f = 0$$
$$\Sigma F_{\perp} = n - w_{\perp} = 0$$

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Do A and B for each of the three situations 1) to 3) where a rock is on an inclined plane.

A) Draw accurate free-body diagrams showing all the forces acting on the rock. *An example is given using the parallelogram method to show that the object is in equilibrium or with no net force (Figure 7).* For convenience, you may draw all forces acting at the center of mass, even though friction and normal reaction force act at the point of contact with the surface. Please use a ruler, and do it in pencil so you can correct mistakes. Label forces using the following symbols: **w** = weight of rock, **T** = tension, **n** = normal reaction force.

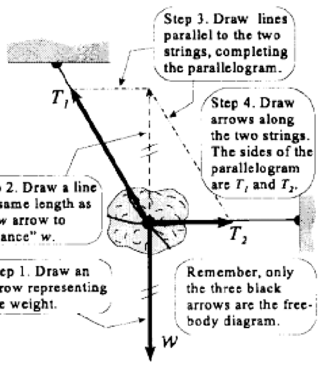
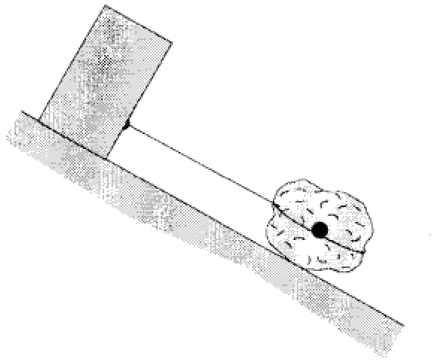


***Figure** 7. Example of the FBD of the rock in equilibrium. (Court, Free-body diagrams revisited)*

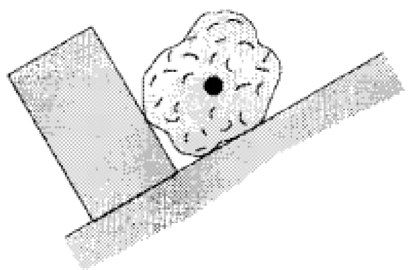
B) Give the expression for the net force along *x*- and *y*-axes. Hint: Have the *x*- and *y*-axes fitted for the incline.

1) The rock is in equilibrium.



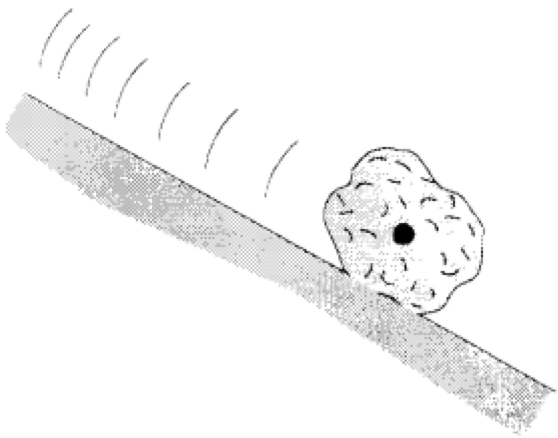
(Court, Free-body diagrams revisited)

2) The rock is at rest.



(Court, Free-body diagrams revisited)

3) Rock is sliding on a frictionless incline.



(Court, Free-body diagrams revisited)

Knot 	In summary, ● A free-body diagram represents the object as a particle (point) and shows all the forces acting on the object. ● The FBD of an object on an inclined plane utilizes the tilted *x*- and *y*-axes wherein one of the axes is parallel to the inclined plane and usually gives the direction of the acceleration (or net force) if any. ● Vectors must be resolved into *x*- and *y*-components before applying Newton's second law of motion on each axis. Net force is equated to zero for zero acceleration (equilibrium, at rest or moving at constant velocity) and will be equated to mass $\times$ acceleration (**ma**) for the component with **net acceleration or net force.**	2	

References:

1. Wilson, J. D., Buffa, A. J., & Lou, B. (2010). *College physics* (7th ed.). Pearson Prentice Hall.
2. Court, J. E. (1999). Free-body Diagrams Revisited - I. *The Physics Teacher, 37*, 427-433

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