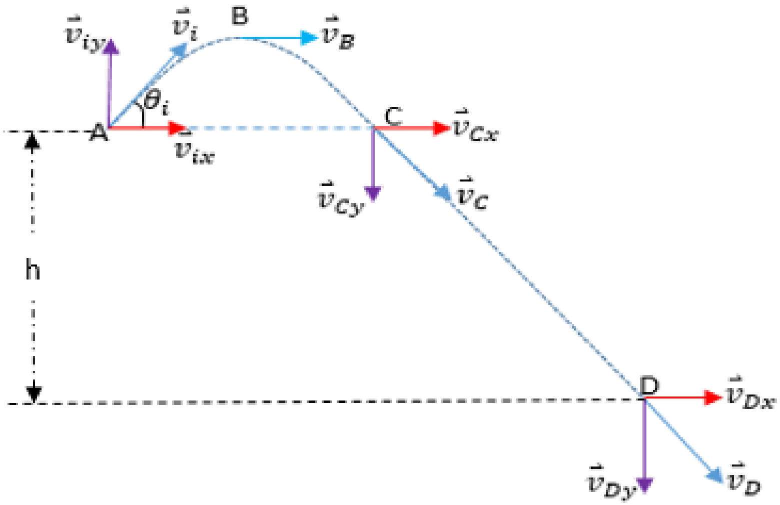
PROPERTY OF THE PHILIPPINE SCIENCE HIGH SCHOOL SYSTEM

Subject Code	PHY 1	**Physics 1**
Module Code	6.0	**Projectile Motion**
Lesson Code	6.3	**Projectile Motion: Problem Solving (Part 1)**
Time Frame		30 minutes

Components	**Tasks**	**TA[1] (min)**	**ATA[2] (min)**
Target	By the end of this learning guide, the student should be able to: • apply kinematic equations for uniform acceleration to projectile motion	1	
Hook	Consider the graphical representation of your answer in the Navigate part of Lesson 6.1 as shown below.  ***Figure 1**: The graphical representation of the velocities at different points from the Navigate part of Lesson 6.1.* We can solve for the numerical values of the velocities at different points in the projectile motion using kinematic equations for uniform acceleration and applying other principles you've learned before like the Pythagorean theorem and trigonometric functions.	1	
Ignite	The kinematic equations you've learned from Lesson 3.1 can be used whether the motion is along the horizontal or vertical, as long as the acceleration is uniform or constant.	20	

[1] Time allocation suggested by the teacher.
[2] Actual time allocation spent by the student (for information purposes only).

***Table 1**: General kinematic equations for uniform acceleration*

Horizontal	Vertical
$\vec{x}_f - \vec{x}_i = \vec{v}_{ix}t + \frac{1}{2}\,\vec{a}_x t^2$	$\vec{y}_f - \vec{y}_i = \vec{v}_{iy}t + \frac{1}{2}\,\vec{a}_y t^2$
$\vec{x}_f - \vec{x}_i = \frac{\vec{v}_{ix} + \vec{v}_{fx}}{2}\,t$	$\vec{y}_f - \vec{y}_i = \frac{\vec{v}_{iy} + \vec{v}_{fy}}{2}\,t$
$\vec{v}_{fx} = \vec{v}_{ix} + \vec{a}_x t$	$\vec{v}_{fy} = \vec{v}_{iy} + \vec{a}_y t$
${\vec{v}_{fx}}^2 = {\vec{v}_{ix}}^2 + 2\vec{a}_x(\vec{x}_f - \vec{x}_i)$	${\vec{v}_{fy}}^2 = {\vec{v}_{iy}}^2 + 2\vec{a}_y(\vec{y}_f - \vec{y}_i)$

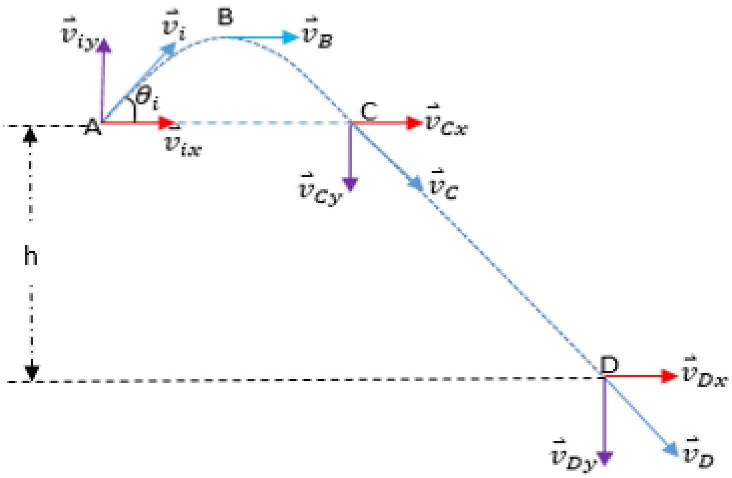
Noting that there is no acceleration along x, $\vec{a}_x = 0$, and the acceleration along y is just the acceleration due to gravity, $\vec{a}_y = -g$ where $g = 9.81\ m/s^2$, the kinematic equations for projectile motion can be written as in Table 2.

***Table 2**: Kinematic equations for projectile motion*

Horizontal	Vertical
$\vec{x}_f - \vec{x}_i = \vec{v}_{ix}t$	$\vec{y}_f - \vec{y}_i = \vec{v}_{iy}t - \frac{1}{2}\,gt^2$
$\vec{x}_f - \vec{x}_i = \frac{\vec{v}_{ix} + \vec{v}_{fx}}{2}\,t$	$\vec{y}_f - \vec{y}_i = \frac{\vec{v}_{iy} + \vec{v}_{fy}}{2}\,t$
$\vec{v}_{fx} = \vec{v}_{ix}$	$\vec{v}_{fy} = \vec{v}_{iy} - gt$
	${\vec{v}_{fy}}^2 = {\vec{v}_{iy}}^2 - 2g(\vec{y}_f - \vec{y}_i)$

Sample Problem

Suppose the projectile in Figure 1 is launched from the top of the building (at point A), of height 10.0 m, with an initial velocity of 7.00 m/s directed at an angle of 30.0º above the horizontal. Find the (a) initial horizontal velocity (b) initial vertical velocity (c) time of flight going to point B, horizontal velocity at B, vertical velocity at B, resultant velocity at B (d) time of flight going to point C, horizontal velocity at C, vertical velocity at C, resultant velocity at C, (e) time of flight going to point D, horizontal velocity at D, vertical velocity at D, resultant velocity at D.



Given:

h = 10.0 m, $\vec{v}_i = 7.00\frac{m}{s}$, 30.0^o above the horizontal

Required:

$\vec{v}_{ix}$, $\vec{v}_{iy}$

t_B , $\vec{v}_{Bx}$, $\vec{v}_{By}$, $\vec{v}_B$

t_C , $\vec{v}_{Cx}$, $\vec{v}_{Cy}$, $\vec{v}_C$

t_D , $\vec{v}_{Dx}$, $\vec{v}_{Dy}$, $\vec{v}_D$

Equations and Solutions:

a. $|\vec{v}_{ix}| = |\vec{v}_i|\, cos\ \theta$

$|\vec{v}_{ix}| = (7.00\frac{m}{s}) cos\ 30.0^o$

$|\vec{v}_{ix}| = 6.06\frac{m}{s}$

$\vec{v}_{ix} = 6.06\frac{m}{s},\ horizontal$

b. $|\vec{v}_{iy}| = |\vec{v}_i|\, sin\ \theta$

$|\vec{v}_{iy}| = (7.00\frac{m}{s}) sin\ 30.0^o$

$|\vec{v}_{iy}| = 3.50\frac{m}{s}$

$\vec{v}_{iy} = 3.50\frac{m}{s},\ upward$

c. $\vec{v}_{By} = 0$

$\vec{v}_{Bx} = \vec{v}_{ix}$

$\vec{v}_{Bx} = 6.06\frac{m}{s},\ horizontal$

$$|\vec{v}_B| = \sqrt{|\vec{v}_{Bx}|^2 + |\vec{v}_{By}|^2}$$

$$|\vec{v}_B| = \sqrt{|6.06\tfrac{m}{s}|^2 + |0\tfrac{m}{s}|^2} = 6.06\frac{m}{s}$$

$\vec{v}_{By} = \vec{v}_{iy} - g t_B$

$t_B = \frac{\vec{v}_{iy} - \vec{v}_{By}}{g}$

$t_B = \frac{3.50\frac{m}{s} - 0\frac{m}{s}}{9.81\frac{m}{s^2}} = 0.357\ s$

d. $t_C = 2t_B = 2\,(0.357\ s) = 0.714\ s$

$\vec{v}_{Cx} = \vec{v}_{ix} = 6.06\ \frac{m}{s},\ horizontal$

$\left|\vec{v}_{Cy}\right| = \left|\vec{v}_{iy}\right| = 3.50\ \frac{m}{s}$

$\vec{v}_{Cy} = 3.50\ \frac{m}{s},\ downward$

$\left|\vec{v}_C\right| = \left|\vec{v}_i\right| = 7.00\ \frac{m}{s}$

$\theta_C = 30^o\ below\ the\ horizontal$

$\vec{v}_C = 7.00\frac{m}{s},\ 30^o\ below\ the\ horizontal$

e. $t_D = t_{upward\ motion} + t_{downward\ motion}$

$t_{upward\ motion} = t_B = 0.357\ s$

To get the displacement from the building to the peak point:

$\vec{y}_B - \vec{y}_A = \frac{\vec{v}_{iy} + \vec{v}_{By}}{2}\ t_B$

$\vec{y}_B = \frac{\vec{v}_{iy} + \vec{v}_{By}}{2}\ t_B + \vec{y}_A$

$\vec{y}_B = \left(\frac{3.50\frac{m}{s} + 0\frac{m}{s}}{2}\right)\ (0.357\ s) + \ (\ 10.0\ m) \ = \ 10.625\ m$

Note that we set the ground to be the reference point, so the top of the building $\vec{y}_A = 10.0\ m$.

To get the time for the downward motion:

$\vec{y}_D - \vec{y}_B = \ \vec{v}_{By}t - \frac{1}{2}g(t_{downward\ motion})^2$

$\vec{y}_D - \vec{y}_B = \ -\frac{1}{2}g(t_{downward\ motion})^2$

$t_{downward\ motion} = \sqrt{2\left[\frac{\vec{y}_D - \vec{y}_B}{-g}\right]} = \sqrt{2\left[\frac{-10.625\ m}{-9.81\frac{m}{s^2}}\right]} = 1.472\ s$

$t_D = t_{upward\ motion} + t_{downward\ motion}$

$t_D = 0.357\ s + \ 1.472\ s \ = 1.83\ s$

$\vec{v}_{Dx} = \vec{v}_{ix}$

$\vec{v}_{Dx} = 6.06\frac{m}{s},\ horizontal$

$\vec{v}_{Dy} = \ \vec{v}_{iy} - gt_D$

$\vec{v}_{Dy} = \ (3.50\frac{m}{s}) + (-\ 9.81\frac{m}{s^2})(1.83\ s)$

$\vec{v}_{Dy} = \ \ -14.5\frac{m}{s}\ or\ 14.5\ \frac{m}{s},\ downward$

$\left|\vec{v}_D\right| = \sqrt{\left|\vec{v}_{Dx}\right|^2 + \left|\vec{v}_{Dy}\right|^2}$

<table>
<tr><td></td><td>

$$|\vec{v}_D| = \sqrt{\left|6.06\tfrac{m}{s}\right|^2 + \left|-14.5\tfrac{m}{s}\right|^2} = 15.7\tfrac{m}{s}$$

$$\theta_D = tan^{-1}\frac{|\vec{v}_{Dy}|}{|\vec{v}_{Dx}|} = tan^{-1}\frac{14.5\frac{m}{s}}{6.06\frac{m}{s}} = 67.3^o\ below\ the\ horizontal$$

$$\vec{v}_D = 15.7\tfrac{m}{s},\ 67.3^o\ below\ the\ horizontal$$

</td><td></td><td></td></tr>
<tr><td>Navigate
</td><td>Now it’s your time to shine ☺

Using the situation in Figure 1 and your understanding in solving word problems on projectile motion, find the range of the projectile or the horizontal displacement from point A to D.</td><td>6</td><td></td></tr>
<tr><td>Knot
</td><td>Word problems on projectile motion can be solved using kinematic equations for uniformly accelerated motion.
<table>
<tr><th>Horizontal</th><th>Vertical</th></tr>
<tr><td>$\vec{x}_f - \vec{x}_i = \vec{v}_{ix}t$</td><td>$\vec{y}_f - \vec{y}_i = \vec{v}_{iy}t - \frac{1}{2}\,gt^2$</td></tr>
<tr><td>$\vec{x}_f - \vec{x}_i = \frac{\vec{v}_{ix} + \vec{v}_{fx}}{2}\ t$</td><td>$\vec{y}_f - \vec{y}_i = \frac{\vec{v}_{iy} + \vec{v}_{fy}}{2}\ t$</td></tr>
<tr><td>$\vec{v}_{fx} = \vec{v}_{ix}$</td><td>$\vec{v}_{fy} = \vec{v}_{iy} - gt$</td></tr>
<tr><td></td><td>${\vec{v}_{fy}}^2 = {\vec{v}_{iy}}^2 - 2g(\vec{y}_f - \vec{y}_i)$</td></tr>
</table>
</td><td>2</td><td></td></tr>
</table>

Prepared by: Marjun M. Tugade	Reviewed by: Joseph P. Hortezuela
Position: SST II	Position: SST IV
Campus: SOCCSKSARGEN Region Campus	Campus: Central Visayas Campus