Subject Code	PHY 1	**Physics 1**
Module Code	4.0	**Vectors**
Lesson Code	4.8	**Relative Motion**
Time Frame		30 minutes

Components	**Tasks**	**TA[1] (min)**	**ATA[2] (min)**
Target 	By the end of this learning guide, the student should be able to: ● determine the velocity of a moving object with respect to a reference frame ● analyze word problems by applying velocity vector equations for relative motion	1	
Hook 	Have you ever wondered why a car or a train approaching you from the opposite direction seems faster than when it moves along with you in the same direction? To find the answer, kindly watch the video from the link below. Enjoy!  https://www.youtube.com/watch?v=lQ_tQYLbVwE You have just realized that, so far, the velocity that we have taken into account to describe the motion of an object is actually the velocity of an object with respect to the earth. But the description of motion is not only restricted to the perspective of the earth. It's the same as saying that your velocity right now is zero while you are just sitting there and reading this learning guide but for an observer standing still outside the earth, you are actually rotating at a speed of 107,000 km/h. This gives rise to what we call in Physics as *relative motion*.	3	
Ignite 	Relative motion is just like two persons facing each other and presenting where the left and the right directions are, as shown in Figure 1.	15	

[1] Time allocation suggested by the teacher.
[2] Actual time allocation spent by the student (for information purposes only).

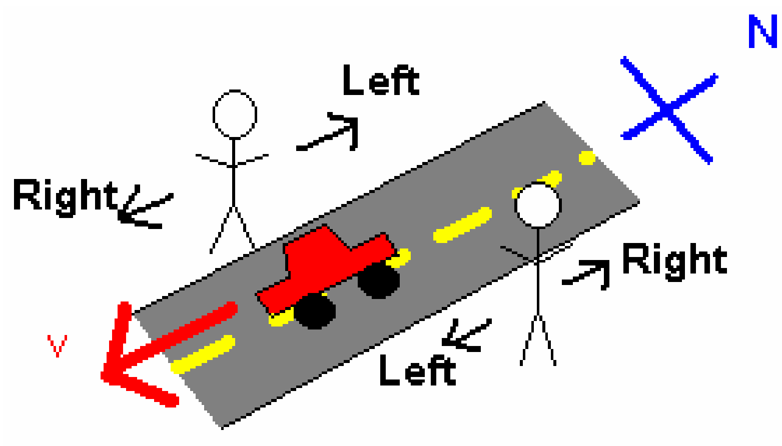

Figure 1: Observer A is saying that the car is heading right with respect to its reference frame but observer B, on the other hand, is saying that the car is heading left with respect to its reference frame.

In Figure 1, person A and person B might have different perspectives about the car's velocity but this difference doesn't mean that one of them is right and the other is wrong. Actually, both of them are correct with respect to their reference frame. A reference frame is like a window through which an observer is looking.

This is a good analogy for the idea about relative motion. Relative means "*it depends*". But whose perspective is the motion dependent upon? The answer is the motion depends upon the observer. So if the velocity of A is being described by person B, we say velocity of A with respect to B. This is visually shown in Figure 2.

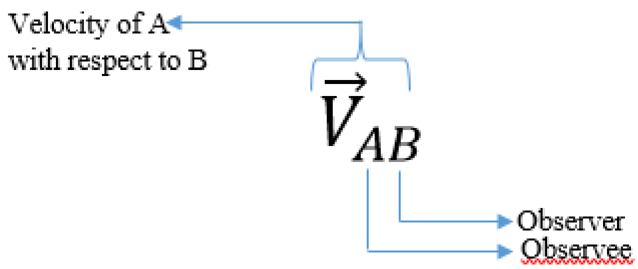

Figure 2: This notation is read as the velocity of A with respect to B. The subscripts indicate the observee and the observer in sequence. The first letter represents an object being observed and the second letter represents the observer to which the reference frame is based upon.

It is very important to write the arrow above the letter V to distinguish that velocity is a vector quantity. Also, notice that the first letter on the subscript represents the observee and the second letter is the observer on which we base our reference frame. On the other hand, if the letters on the subscript are being interchanged, it will be read as the velocity of B with respect to

A and will just be equal to the negative of $\vec{V}_{AB}$, or

$$\vec{V}_{BA} = -\vec{V}_{AB} \qquad \text{[eqn. 1]}$$

Equation 1 simply implies that $\vec{V}_{BA}$ has the same magnitude but pointing in the opposite direction as the $\vec{V}_{AB}$. Thus, if $\vec{V}_{AB} = 30\frac{km}{h},\ east$, then $\vec{V}_{BA} = 30\frac{km}{h},\ west$; consequently, if $\vec{V}_{AB} = 30\frac{km}{h},\ west$, then $\vec{V}_{BA} = 30\frac{km}{h},\ east$. By convention, we consider east and north as the positive directions while west and south are the negative directions.

One-Dimensional Relative Motion

Problem 1: Trains approaching each other

Suppose we let "A" represent train 1, "B" for train 2, and "E" for the earth or ground. Both velocities of train 1 and 2 are given with respect to the Earth, such that the velocity of train 1 is moving at 30 km/h heading east with respect to the earth ($\vec{V}_{AE} = 30\frac{km}{h},\ east)$ and velocity of train 2 is 60 km/h heading west with respect to the earth ($\vec{V}_{BE} = 60\frac{km}{h},\ west$). What will be the velocity of train 2 with respect to train 1 ($\vec{V}_{BA}$)?

Solution:

To get the velocity of B with respect to A, we need to use the general equation for relative motion, which is

$$\vec{V}_{BA} = \vec{V}_{BE} + \vec{V}_{EA} \qquad \text{[eqn. 2]}$$

Notice the sequence of the subscripts on the right-hand side of the equation: B-E-E-A and the sequence for the left-hand side: B-A. To acquire the subscript BA, the subscripts on the right-hand side of the equation must be in such a way that B will be the first in the sequence, A will be the last, and the intermediate reference frame E should be arranged consecutively for it to be "cancelled out", leaving the outer letters A and B in sequence.

We cannot yet directly plug in the values in equation 2 since we are not given $\vec{V}_{EA}$. We need to rearrange first the subscripts on the right-hand side. Noting that $\vec{V}_{EA} = -\vec{V}_{AE}$ as in equation 1, equation 2 will become

$$\vec{V}_{BA} = \vec{V}_{BE} - \vec{V}_{AE} \qquad \text{[eqn. 3]}$$

Then we are now ready to substitute the values in equation 3:

$$\vec{V}_{BA} = (60\frac{km}{h},\ west) - (30\frac{km}{h},\ east)$$

$$= (60\frac{km}{h},\ west) + (30\frac{km}{h},\ west) = 90\frac{km}{h},\ west$$

The above vector relations can be graphically visualized as follows

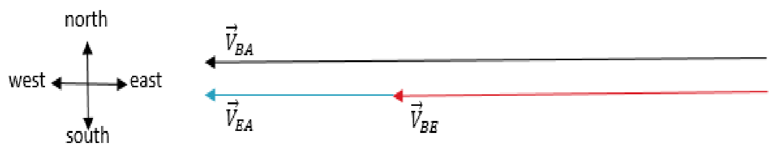

Figure 3: $\vec{V}_{BA} = \vec{V}_{BE} + \vec{V}_{EA}$, *where* $\vec{V}_{EA}$ *is opposite the given direction of motion of train 1 with respect to the earth. Since both* $\vec{V}_{BE}$ *and* $\vec{V}_{EA}$ *are directed west, hence the resultant is also directed to the west and greater in magnitude to either vectors*

Consequently, the velocity of A with respect to B will be

$$\vec{V}_{AB} = -\vec{V}_{BA} = 90\frac{km}{h},\ east$$

which is graphically shown as follows

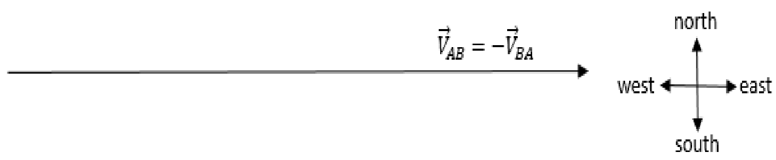

Figure 4: Vector representation of $\vec{V}_{AB}$, *which is directed to the east given that* $\vec{V}_{BA}$ *is directed to the west.*

Problem 2: Case 2 Trains moving in the same direction

Suppose both trains 1 and 2 are heading East, such that $\vec{V}_{AE} = 30\frac{km}{h},\ east$ and $\vec{V}_{BE} = 60\frac{km}{h},\ east$. What is the velocity of B with respect to A?

Solution:

Substituting the values to equation 3, we get

$$\vec{V}_{BA} = (60\frac{km}{h}, east) - (30\frac{km}{h},\ east) = 30\frac{km}{h},\ east$$

which can be graphically shown as follows

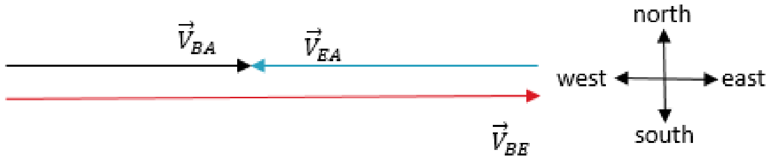

Figure 5: $\vec{V}_{BA} = \vec{V}_{BE} + \vec{V}_{EA}$, *where* $\vec{V}_{EA}$ *is opposite the given direction of motion of train 1 with respect to the earth. Since* $\vec{V}_{BE}$ *and* $\vec{V}_{EA}$ *are opposite, the magnitude of the resultant is between those of the given velocities.*

Consequently, the velocity of A with respect to B will be

$$\vec{V}_{AB} = -\vec{V}_{BA} = 30\frac{km}{h},\ west$$

which can be graphically shown as follows

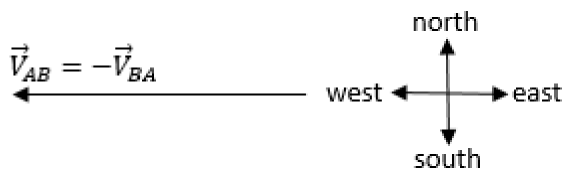

Figure 6: Vector representation of $\vec{V}_{AB}$*, which is directed to the west given that* $\vec{V}_{BA}$ *is directed to the east.*

Now the question why a car seems faster if it is approaching you rather than going along with you has been answered by comparing the answers to problems 1 and 2.

Relative Motion in Two-Dimension

We can see that Case 1 and case 2 are very straightforward ways of vector addition since there is only one coordinate involved. It's as simple as you just sum up all the values going to the East; sum up all the values going to the west, then take the difference of the East and West, and KABOOM! You have the resultant velocity.

On the other hand, two dimensional motion is not as straightforward as the one-dimensional motion. You need to use the principle of Pythagorean theorem to get the magnitude of the velocity and trigonometric functions (SOH CAH TOA) to get the direction of the velocity.

Problem 3: Case 3 Trains moving perpendicular to each other

Suppose that we retain all the conditions in Problem 1 except that we change the direction of train 2 to the North. What would be the velocity of train 2 with respect to train 1?

Solution:

Equation 2 gives us

$$\vec{V}_{BA} = \vec{V}_{BE} + \vec{V}_{EA}$$

where $\vec{V}_{EA} = -\vec{V}_{AE} = -\left(30\frac{km}{h}, east\right) = 30\frac{km}{h}, west$

Substituting this values to equation 2, we will have

$$\vec{V}_{BA} = \left(60\frac{km}{h},\ north\right) + \left(30\frac{km}{h}, west\right)$$

<table>
<tr>
<td></td>
<td>

which can be graphically shown as follows

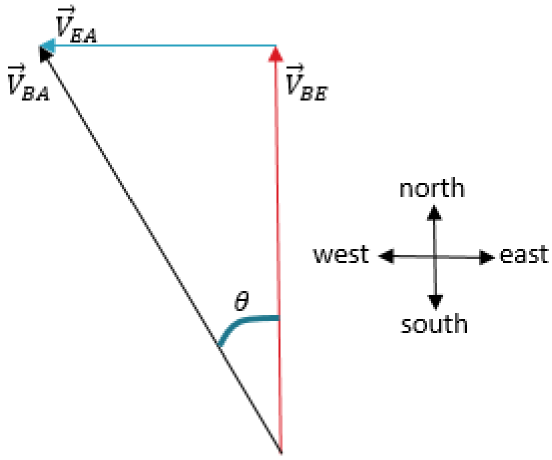

Figure 7: $\vec{V}_{BA} = \vec{V}_{BE} + \vec{V}_{EA}$ *. In this case, the given vectors are actually components of the resultant* $\vec{V}_{BA}$ *.*

The magnitude of $\vec{V}_{BA}$, written as $|\vec{V}_{BA}|$, will be solved using Pythagorean theorem,

$$|\vec{V}_{BA}| = \sqrt{|\vec{V}_{EA}|^2 + |\vec{V}_{BE}|^2} \qquad \text{[eqn. 4]}$$

$$|\vec{V}_{BA}| = \sqrt{\left(30\frac{km}{h}\right)^2 + \left(60\frac{km}{h}\right)^2}$$
$$|\vec{V}_{BA}| = 67\ km/h$$

$$\theta = tan^{-1}\left(\frac{|\vec{V}_{EA}|}{|\vec{V}_{BE}|}\right)$$

$$\theta = tan^{-1}\left(\frac{30\frac{km}{h}}{60\frac{km}{h}}\right) = 27^o,\ \ W of\ N$$

Therefore, the velocity of train 2 with respect to train 1 is

$$\vec{V}_{BA} = 67\frac{km}{h},\ 27^o,\ \ W\ of\ N$$

</td>
<td></td>
<td></td>
</tr>
<tr>
<td>

Navigate

</td>
<td>

Write your answers with complete solutions on a clean sheet of paper. Follow your teacher's instructions regarding submission. All items will be graded.

1. There are three cars moving differently with respect to the earth. Suppose that Car A is moving $20\frac{km}{h}$ to the east, Car B is moving $8\frac{km}{h}$ to the east, while car C is moving $5\frac{km}{h}$ to the west.
    a. What is the velocity of car A with respect to car B?
    b. What is the velocity of car C with respect to car B?
    c. Which among the cars A and C moves faster with respect to car B?

2. Suppose all the conditions in Problem 3 are retained.

</td>
<td>10</td>
<td></td>
</tr>
</table>

	a. What is the magnitude of the velocity of train 1 with respect to train 2? b. What is the direction of the velocity of train 1 with respect to train 2?		
Knot 	The description of motion of an object is dependent upon the chosen reference frame. The velocity of object B with respect to A is the same in magnitude but opposite in direction to the velocity of object A with respect to B. $\vec{V}_{BA} = -\vec{V}_{AB}$ In case there is an intermediate reference frame in between these two objects, relationship among the relative velocities is given by the following equations: $\vec{V}_{BA} = \vec{V}_{BE} + \vec{V}_{EA}$ or $\vec{V}_{AB} = \vec{V}_{AE} + \vec{V}_{EB}$ Take note that the relative velocities should be in such a way that the arrangement of the subscripts on the right-hand side starts with the first subscript on the left-hand side and ends with the last subscript on the left-hand side while keeping the same letter in between.	2	

References:

1. SHArPEdgeGlobal. July 10, 2013https://www.youtube.com/watch?v=lQ_tQYLbVwE
2. John Horwat. (2014). Frames of reference. https://sites.google.com/site/apphysics1online/unit-02b-2-dimensional-kinematics/3-relative-motion
3. R Nave. Relative Velocity. http://hyperphysics.phy-astr.gsu.edu/hbase/relmot.html

Prepared by: Marjun M. Tugade	Reviewed by: Jericho V. Narvasa
Position: SST II	Position: SST I
Campus: SOCCSKSARGEN Region Campus	Campus: Ilocos Region Campus