





Subject Code	PHY 1	<b>Physics 1</b>
Module Code	4.0	<b>Vectors</b>
Lesson Code	4.4	<b>Vector Components: Applying SOHCAHTOA</b>
Time Frame		30 minutes

Components	Tasks	TA <sup>1</sup> (min)	ATA <sup>2</sup> (min)
<b>Target</b> 	By the end of this learning guide, the student should be able to: <ul style="list-style-type: none"> <li>resolve a vector into its components using SOHCAHTOA</li> </ul>	1	
<b>Hook</b> 	<p>We recall from previous lessons that a <b>vector</b> is a quantity that has both <i>magnitude</i> and <i>direction</i>. Quantities like displacement, velocity, acceleration, and force are under this category.</p> <p>In situations in which vectors are directed at angles to the conventional coordinate plane, a useful mathematical trick may be employed to transform the vector into two parts with each part being directed along the coordinate axes.</p> <p>Take for example the case of Fluffy, a well-loved dog. Whenever his owner goes for a stroll, Fluffy always comes with him. If Fluffy's dog chain is stretched this way by its owner, how do we resolve the components of the tension in the chain?</p>  <p><i>Figure 1. An example of an angled vector. Reprinted from Physics classroom n.d. Retrieved July 12, 2020 from: <a href="https://www.physicsclassroom.com/class/vectors/Lesson-1/Vector-Components">https://www.physicsclassroom.com/class/vectors/Lesson-1/Vector-Components</a> Copyright 2020.</i></p>	2	
<b>Ignite</b> 	<p>If the magnitude and direction of a vector are known, it is possible to find the components of the vector. The process of finding the components is called “<i>resolving vectors into its components</i>”. This process can be carried out with the aid of trigonometry.</p>	18	

<sup>1</sup> Time allocation suggested by the teacher.

<sup>2</sup> Actual time allocation spent by the student (for information purposes only).

*Trigonometry* is the branch of mathematics dealing with the relations of the sides and angles of triangles and with the relevant functions of any angles.

We will be using trigonometry to resolve vectors into its components.

First, let us recall the anatomy of a right-angled triangle:

**Adjacent** is the side of the right triangle that is adjacent to the angle ( $\theta$ ),

**Opposite** is the side opposite to the angle ( $\theta$ ),

and the longest side is the **Hypotenuse**.

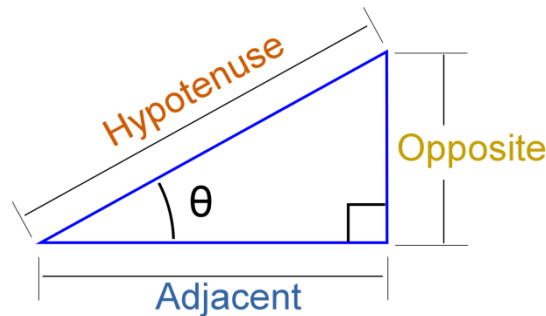


Figure 2. Anatomy of a right-angled triangle. Reprinted from *MathIsFun.com*. n.d.

Retrieved July 12, 2020 from:

<https://www.mathsisfun.com/algebra/trig-finding-angle-right-triangle.html>. Copyright 2017

Now that we have already recalled the names of the three sides of a right triangle, let us now use the concept of the SOH-CAH-TOA in resolving the components of a vector.

The SOH-CAH-TOA is a mnemonic device to help you remember the three basic trigonometric ratios used to solve for missing sides and angles in a right triangle.

It is defined as:

$$\text{SOH: } \sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\text{CAH: } \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\text{TOA: } \tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

Let us now go back to our example of tension in the chain of the dog named Fluffy. Given that the dog chain is stretched upward and rightward and pulled tight by his owner, then the tension force in the chain has two components - an upward component and a rightward component.

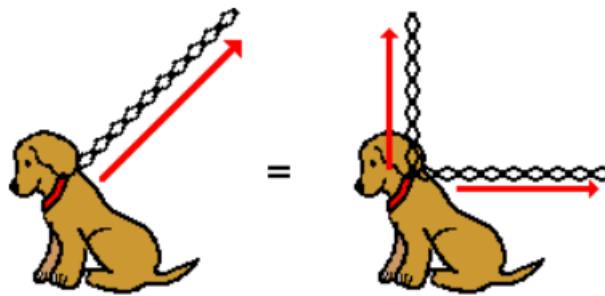


Figure 3. The upward and rightward force of the chain is equivalent to the upward and rightward force by two chains. Reprinted from *Physics classroom* n.d. Retrieved July 12, 2020 from: <https://www.physicsclassroom.com/class/vectors/Lesson-1/Vector-Components>. Copyright 2020.

The rightward and upward components of the force correspond to the x- and y- components respectively. We may illustrate it this way,

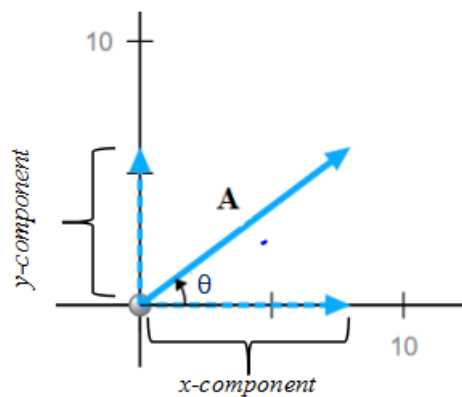


Figure 4. The x and y components of a vector

Let us call the tension as vector **A**. Vector **A** has an x-component oriented along the positive x-axis and a y-component oriented along the positive y-axis. We will move the y-component so it will be connected to the head of the x-component to form a right triangle.

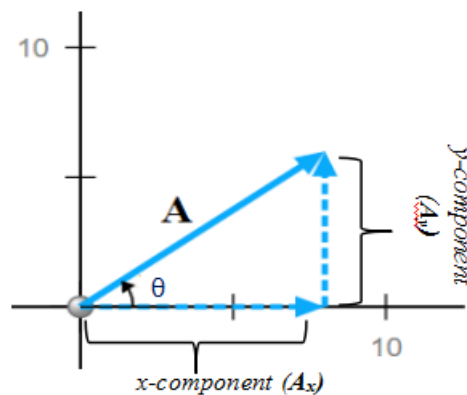


Figure 5. The x and y components of a vector forming a right triangle

Considering the location of  $\theta$ , the y-component now becomes the opposite side, while x-components is the adjacent side. Vector A is the hypotenuse of the triangle.

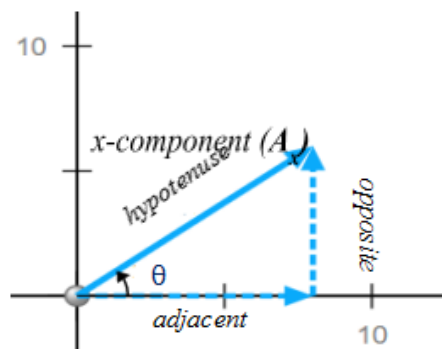


Figure 6. The different sides of a right triangle

Let us provide some numerical values. Vector A has a magnitude of 10.0N and is oriented at  $37.0^\circ$  with respect to the x-axis. Resolve this vector into its x and y components.

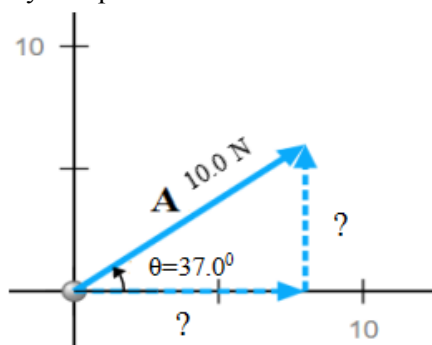


Figure 7. Illustration of the problem

Given are the following:


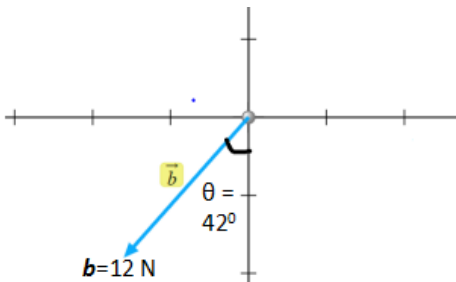
$A$  (hypotenuse) = 10.0 N

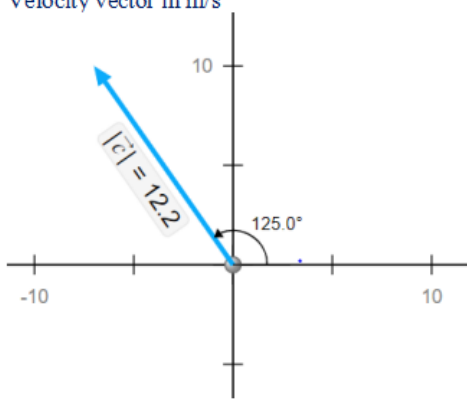

$\theta = 37.0^\circ$

To determine the magnitude of the x-component (adjacent side), we use the trigonometric ratio, CAH since it is the adjacent side (x-component) that is unknown.

$$\text{CAH: } \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

Substituting the values,

	$\cos 37.0^\circ = \frac{A_x}{10.0N}$ $A_x = 10.0N(\cos 37.0^\circ)$ $A_x = 7.99N$ <p>To determine the magnitude of the y-component (opposite side), we use the trigonometric ratio, SOH since it is the opposite side (y-component) that is unknown.</p> <p>SOH: <math>\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}</math></p> <p>Substituting the values,</p> $\sin 37.0^\circ = \frac{A_y}{10.0N}$ $A_y = 10.0N(\sin 37.0^\circ)$ $A_y = 6.02N$		
<p><b>Navigate</b></p> 	<p>It is your turn to resolve vectors into its x and y components. Write your answers (with complete solutions) on a clean sheet of paper. Follow your teacher's instructions regarding submission. All items will be graded.</p> <p>Find the x and y components of the following vectors.</p> <p>1. A displacement vector <b>r</b> has a magnitude of <math>r=175</math> m at an angle <math>50.0^\circ</math> relative to the x-axis.</p> <p>2.</p> 	7	

	<p>3. Velocity vector in m/s</p> 		
<p><b>Knot</b></p> 	<p>In summary, If the magnitude and direction of a vector are known, it is possible to find the components of the vector. The process of finding the components is called “<i>resolving vectors into its components</i>”. This process can be carried out with the aid of trigonometry, specifically through trigonometric ratios SOH-CAH-TOA.</p> <p>The SOH-CAH-TOA is a mnemonic device to help you remember the three basic trigonometric ratios used to solve for missing sides and angles in a right triangle. It is defined as:</p> <p><b>SOH:</b> <math>\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}</math></p> <p><b>CAH:</b> <math>\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}</math></p> <p><b>TOA:</b> <math>\tan \theta = \frac{\text{opposite}}{\text{adjacent}}</math></p>	2	

#### References:

1. Giancoli, Douglas C. (2014). *Physics Principles with Applications 7th ed.* United States of America: Pearson Education, Inc.
2. Cutnell, John D. and Johnson, Kenneth W. (2012). *Physics 9th ed.* United States of America: John Wiley & Sons, Inc.
3. Vector components. n.d. Retrieved July 12, 2020 from <https://www.physicsclassroom.com/class/vectors/Lesson-1/Vector-Components>

Prepared by: Carolyn Mae V. Solamo	Reviewed by: Jericho V. Narvasa
Position: SST IV	Position: SST I
Campus: Southern Mindanao Campus	Campus: Ilocos Region Campus