1 Summary of Results

MEMO TO DEPARTMENT OF TRANSPORTATION:

Carbon emissions from personal vehicles are one of the greatest drivers of climate change. As such, a primary focus from policy-makers has been on encouraging costly alternative transportation methods like electric cars and public transit systems. Alternatively, electric bicycles (e-bikes) have been a fast-growing option for environmentally conscious consumers and should be a central pillar of your department's strategy.

We project that the American user base for e-bikes will increase near-exponentially in the next few years. Specifically, we can predict that approximately 1,323,000 e-bikes will be sold in 2 years and 2,202,000 e-bikes will be sold in 5 years. Though these figures might seem far-fetched, they correspond with the explosive growth observed in European markets. Furthermore, our growth projection model can be applied to other geographic regions and novel consumer technologies, giving your department a strong analytical tool.

To explain the popularity of e-bikes, we modeled population trends using differential equations of five different modes of transportation: walking, acoustic bikes, e-bikes, cars, and public transit. We considered several factors that would impact people's decisions to switch their primary mode of transportation; these included the terrain of a city, environmental awareness, relative costs, and the so-called "coolness factor": the mechanism by which increased visibility of the new technology accelerates demand. We found that these factors accounted for the growth in e-bikes mainly by providing an alternative to cars. We also determined that the most important factor in increasing e-bike usage was the environmental awareness of a population, which can increase e-bike market share by up to 30%.

The effects of a decrease in car users—as a result of commuters' transitions to e-bikes from other modes of transportation—can be measured in the sphere of traffic through congestion levels and minor crash probability. As represented by car density, a road is considered congested once its density surpasses a certain value, and an increase in e-bike users was found to have an obvious positive impact on road congestion. Minor accidents—namely those caused by accordion collisions and tailgating—could be modeled based on the deceleration of cars earlier on in a given lane. With a decrease in car density and thereby the number of cars in an observed lane segment, the number of crashes (modeled by the number of intersections between solution curves) will concomitantly be minimized.

Overall, e-bikes provide an alternative transportation method to cars, and their adoption would result in widespread benefits: less traffic congestion, fewer car accidents, and lower carbon emissions—the ultimate goal in the fight against climate change. We hope our results are helpful to the Department of Transportation.

Sincerely,
Team #16738

Contents

2 Introduction

As the climate crisis unfolds, the need for alternative energy vehicles continues to grow. Although electric cars are at the forefront of the electric vehicle revolution, other cheaper renewable vehicle options—such as electric bikes (e-bikes)—provide a compelling product in the eyes of climate-friendly consumers. Any reduction in potential emissions through the continued transition to e-bikes presents a possibility of reducing carbon dioxide emissions. In the United States alone, the transportation industry is responsible for approximately 27% of all carbon emissions [1].

2.1 Background

2.2 Problem Restatement

1. **The Road Ahead:** Using a model for future e-bike sales, predict the number of e-bikes sold in the United States two and five years from now. Use historical time-series data as training data for the chosen model.
2. **Shifting Gears:** There are several factors that could have led to e-bike sales increasing. Use a mathematical model to determine the most important factor(s) in e-bike usage growth.
3. **Off the Chain:** Quantify the effects of e-bike usage on emissions, traffic congestion, health, and other factors deemed important.

3 Problem-wide Definitions, Assumptions, and Variables

3.1 Assumptions and Justifications

These are assumptions that we use several times in our paper, so we include them here for clarity.

Assumption 1: The population of a city is constant.
Justification 1: Over the short time scales that we work with, a growing population should have negligible effects on the adoption of e-bikes.

Assumption 2: People do not bike outside of bike lanes.
Justification 2: Not only does this assumption allow for increased safety of all involved, but it also clearly distinguishes lanes meant for motorists and those meant for bikes (both acoustic and electronic).

Assumption 3: People generally use one mode of transport.
Justification 3: This simplifies the models, allowing us to focus on population-level trends instead of modeling individual decisions on a day-to-day basis.

3.2 List of Variables

Table 1: List of Variables

Variable	Description	Units
$P_i(t)$	Time-dependent number of people using mode i of transportation	People
L_i	Lifespan of mode i of transportation	Years
$\mathbf{C}$	Cost ratio matrix	-
E	Ecological footprint	Global hectares
H	Scaled Mean Slope	-
R_0	Basic reproduction number for e-bikes & People	-
ρ	Car density	$\frac{1}{m}$
l	Average car length	m
d	Equilibrium distance between cars	m
v	Equilibrium velocity	$\frac{m}{s}$
t	Time	s
i	Car number	-
x_i	Position of ith car in a right-moving line	m
v_i	Velocity of ith car in a right-moving line	$\frac{m}{s}$
a_i	Acceleration of ith car in a right-moving line	$\frac{m}{s^2}$
ε_i	Perturbation of ith car	m
τ	Average human reaction time	s
Δt	Moment of acceleration after braking	s

4 The Road Ahead

4.1 Overall Approach

We fit a logistic curve to the American data provided in the problem statement and fill in holes with another study [2][3]. We validated against the European data before extracting specific results, which confirmed that the rates of e-bike ownership will increase in the coming years. See 4.1 for a schematic of our approach.

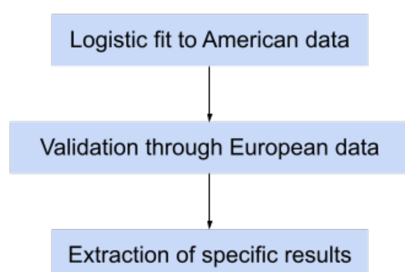

Figure 1: Schematic for The Road Ahead

4.2 Logistic Fit

The introduction of new technologies tends to follow a logistic trend. At first, technology is rapidly adopted at near-exponential rates. This is not sustainable, as it implies a degree of infinite growth. The logistic curve allows us to correct for this, as it slows down the exponential curve to linear growth, then sub-linear growth, and finally an asymptotic ceiling. We fit a logistic curve to the sales of e-bikes in the United States.

Our logistic fit is

$$f(t) = \frac{c}{1 + a\exp(-bt)},$$

where t is the years after e-bikes were introduced to the market and a, b, c are constants determined in the logistic regression.

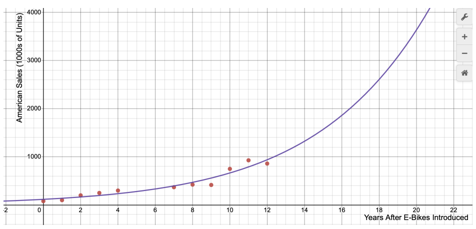

Figure 2: Logistic Fit to American E-bike Sales

Though the logistic fit was quite powerful, with an R^2 value of 0.92, this did not indicate much predictive power with such sparse data. To test whether e-bikes followed a logistic trend, we also fit a logistic curve to the European data.

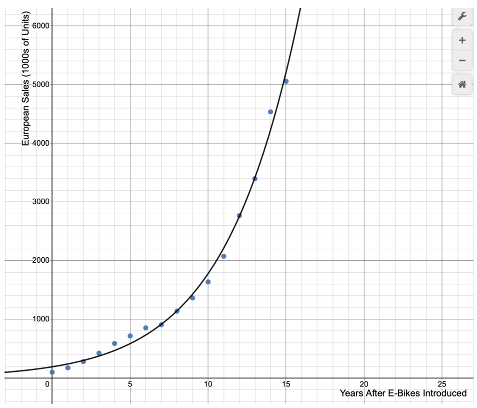

Figure 3: Logistic Fit to European E-bike Sales

We can see that e-bike sales in Europe continue to follow the logistic curve—even past the stage of exponential growth, showing that e-bikes have followed the expected slowing trend that new technologies tend to. Assuming that e-bike sales will follow the same trend in the United States as they have in Europe, this lends credence to the following results extrapolated beyond the range of the original data.

4.3 Results

For the American logistic fit, we had parameters of $a = 548, b = 0.175, c = 64200$. These are the results obtained by extrapolating our model two years and five years out from the most recent data point.

Table 2: Predicted E-Bike Sales

Years in Future	Predicted E-Bike Sales (1000s of Units)
2	1323
5	2202

4.4 Discussion

Our results show that the rate of e-bikes sales will continue increasing in the United States. Furthermore, the data points' relatively early positions on the logistic curve demonstrate that we are still in a period of near-exponential growth. For at least a decade, this near-exponential growth will continue. However, the logistic curve does not predict continuous exponential growth—eventually, linear growth will occur as the market is saturated with e-bikes.

4.5 Strengths and Weaknesses

4.5.1 Strengths

- **Accuracy**: With the confirmation from the European data, we can be reasonably sure that the American e-bike sales will follow the logistic trends we predict.
- **Generalizability**: Our approach of fitting a logistic curve is generalizable to other geographic regions and novel consumer technologies as long as data is available.

4.5.2 Weaknesses

- **Long-term Prediction**: Logistic fits predict upper-bounds far in the future once the period of sub-linear growth is reached. Though a useful conceptualization for the slowing rate of acquisition of a new technology, the exact quantitative predictive power is diminished.

5 Shifting Gears

5.1 Overall Approach

We model the population as existing in several "buckets," categorized by mode of transport. Every year, some proportion of people using each mode of transport (walking, acoustic bikes, e-bikes, cars, and public transit) become susceptible to changing their preferred mode and make the switch with some probability. That probability is dependent on relative costs, environmental awareness, terrain, and the coolness factor. The exact degree to which these factors affect the probability of switching is specific to each mode of transport, which we will describe in the next sections. Finally, we collect all this information in a system of coupled differential equations. This allows us to both model the user base of each mode of transport and determine the most important factors through a sensitivity analysis. See 5.1 for a schematic.

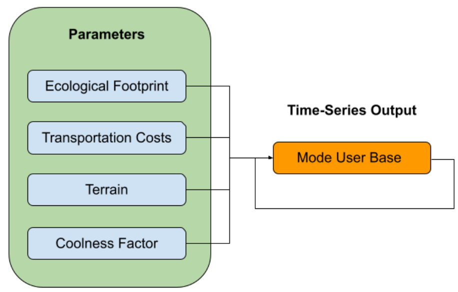

Figure 4: Schematic for Shifting Gears

5.2 Internal Organization

Our model simulates on a discrete per-year basis. Each mode of transport i has an associated population $P_i(t)$: the number of people using that mode of transport in that year. After each year, some proportion of the people become dissatisfied with their mode of transport. We denote this proportion as $\frac{1}{L_i}$. Yet, not every dissatisfied person will actually make the decision to switch to another mode of transport. Each transition probability takes input from the cost matrix:

$$\mathbf{C} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} \\ c_{21} & c_{22} & c_{23} & c_{24} & c_{25} \\ c_{31} & c_{32} & c_{33} & c_{34} & c_{35} \\ c_{41} & c_{42} & c_{43} & c_{44} & c_{45} \\ c_{51} & c_{52} & c_{53} & c_{54} & c_{55} \end{bmatrix},$$

where each c_{ij} is the ratio of the cost-per-unit-time of mode of transport i to mode of transport j. This gives us a general differential equation for $P_i(t)$. For now, we keep this as a proportionality statement with the expectation that constants must be tuned later.

$$f(i,j) = \frac{1}{L_j} \cdot P_j(t) \cdot c_{ji},$$

$$\frac{\mathrm{d}P_i}{\mathrm{d}t} \propto \sum_{j \neq i} f(i,j) - f(j,i)$$

This formulation encodes the user base of one mode of transport as constantly in flux and dependent on other user bases for growth, which is represented by function f which is how many from bucket j moves to bucket i. The subtracted term serves to keep the population constant from the reverse transfer of population, as we assume that our time scale is small enough that population trends have a negligible effect on the e-bike user base.

However, this equation treats all modes of transport as symmetric and functionally identical to each other. In reality, there are specific factors that impact one's decision to use an e-bike more so than a car—environmental constraints, the terrain of the city, etc. We discuss these mode-specific modifications next.

5.3 Environmental Awareness

Furthermore, there is an environmental modifier E according to the ecological footprint of the region in question. Ecological footprint is conceptualized as the "number of Earths" required to support a population with a certain region's habits of living (E is internally coded in the standard Global Hectares) [11]. We use this as a proxy for environmental awareness; a city with higher sustainability and a lower ecological footprint is likely to have a population that is more willing to make pro-environment transport choices. We categorize walking, acoustic bikes, e-bikes, and public transit as environmentally-friendly, so their equations will include a $\frac{1}{E}$ term.

5.4 Terrain

An oft-cited reason for switching to electric bikes is the hilliness of the region [3]. Naturally, this affects those people using walking and acoustic bikes as their primary modes of transport. To quantify the hilliness of a city, we use a scaled mean slope percentage H (for example, this takes a value of 100 in Honolulu because it is the American city with the most varied terrain)[12]. Hence, the transitions from acoustic bikes and walking to e-bikes are governed by an H term. In addition, hills might prompt a cyclist to use a car or public transportation instead, so we include the H term in the acoustic bike to car or public transportation transition and scale them based on ease of use for sloped transportation.

5.5 Coolness Factor

Finally, the arrival of a new technology like e-bikes can induce more popularity, boosting susceptibility of switching to e-bikes. We model an analogue of the susceptibility as a scalar R_0 for e-bikes.

5.6 Costs

To generate $\mathbf{C}$, we gave walking an implied annual fee of $1 a day or $365 per year for the loss of time, and for the other modes of transportation, we calculated the cost per year by totaling any initial cost required for vehicles divided by lifetime expectancy in years and the annual cost of maintaining that mode of transportation, such as fares for public transportation or maintenance fees for bikes and cars. We aggregated this relative cost data in sequence from several sources [4][5][6][7][8][9][10]. With this final addition, we have a fully fleshed out system of coupled differential equations.

5.7 Revised Model

Incorporating all of our mode-specific modifications, we arrived at the following system of equations where the i's are in order of walking, acoustic bikes, e-bikes, cars, and public transportation:

$$\frac{\mathrm{d}P_1}{\mathrm{d}t} \propto \frac{1}{E} \sum_{j \neq 1} \frac{1}{L_j} \cdot P_j(t) \cdot c_{j1} - f(1, j)$$

$$\frac{\mathrm{d}P_2}{\mathrm{d}t} \propto \frac{1}{E} \sum_{j \neq 2} \frac{1}{L_j} \cdot P_j(t) \cdot c_{j2} - f(2, j)$$

$$\frac{\mathrm{d}P_3}{\mathrm{d}t} \propto R_0 \frac{1}{E}(0.2H) \sum_{j \neq 3} \frac{1}{L_j} \cdot P_j(t) \cdot c_{j3} - f(3, j)$$

$$\frac{\mathrm{d}P_4}{\mathrm{d}t} \propto H \sum_{j \neq 4} \frac{1}{L_j} \cdot P_j(t) \cdot c_{j4} - f(4, j)$$

$$\frac{\mathrm{d}P_5}{\mathrm{d}t} \propto \frac{1}{E}(0.7H) \sum_{j \neq 5} \frac{1}{L_j} \cdot P_j(t) \cdot c_{j5} - f(5, j)$$

5.8 Euler's Method via Python

With an analytical approximation provided by Euler's method, the circumvention of an analytical solution to the differential system was left to computational processing. We originally designed our model on a discrete time-frame, so we can be sure that we do not lose information in the transformation between continuous differential and discrete difference equations. In order to expedite testing and model tuning, we used Python. The code for solving the system can be found in the Appendix.

5.9 Results

Table 3: Percentage of Population for Transit Modes Over Time

Year	Walking	Bicycle	E-bike	Car	Public Transit
2020	0.035204	0.025820	0.066799	0.845249	0.026929
2028	0.037975	0.079424	0.201322	0.658187	0.023093
2036	0.035575	0.109801	0.250493	0.582528	0.021603

5.10 Strengths and Weaknesses

5.10.1 Strengths

- **Ease of Testing**: Our model allows for simple testing through varying the input parameters and observing the relative change in outputs. This allowed us to determine the most important factors in e-bike usage.
- **Ease of Data Transformation**: Our model allows for simple portability of data to other models. Output data contrived through the differential equations can be easily used as input data for further simulations (see Off the Chain).

- **Adaptability**: Adding another mode of transport to the model only involves defining its costs and transition parameters. If new consumer technology emerges, it can easily be added to our model and its impact on e-bike usage can be devised.

5.10.2 Weaknesses

- **Single Transport Each Year**: For simplicity, our model assumed that people only used one mode of transport within a year. This does not allow for individual-level flexibility. Further extensions of this model might consider individual decisions as more central, rather than as probabilistic outcomes affecting the population.
- **Year-period Constrained**: Due to the lack of continuous time-period data, the model is restricted to an incremented, yearly basis. If further data were available, the differential equations model could be extrapolated and fit to said new data.

5.11 Sensitivity Analysis

For sensitivity analysis of each independent variable E, H, the model constant, R_0, and the average cost of e-bikes per year, we scaled them by $0.4, 0.6, 0.8, 0.9, 1, 1.1, 1.2, 1.4$, and 1.6 in order to analyze the effect of the variables on the resulting percentage of the population with e-bikes.

Table 4: Percentage of Population of E-bikes for Scaled Independent Variables in 10 Years

Scaling factor	E	H	Model Constant	R_0	Cost of E-bike per Year
0.4	0.378170	0.104948	0.110571	0.088538	0.191487
0.6	0.296073	0.143229	0.147884	0.128364	0.199511
0.8	0.240282	0.175010	0.177681	0.165908	0.201229
1.0	0.201322	0.201322	0.201322	0.201322	0.201322
1.2	0.172927	0.223040	0.219917	0.234743	0.200955
1.4	0.151434	0.240917	0.234404	0.266300	0.200473
1.6	0.134646	0.255589	0.245566	0.296111	0.199988

Table 4 shows that E, the inverse of environmental awareness, had the most impact on the resulting percentage of the population that used e-bikes after 10 years with roughly an increase of 18% of the total population with a scaling factor of 0.4. Furthermore, the values seem to match an inverse exponential function; therefore, its impact will only increase faster as environmental awareness increases and E decreases. Furthermore, R_0 also has an outstanding impact, second to inverse environmental awareness with an approximate increase of 9.4% of the total population with a scaling factor of 1.6. This means that social factors can be a major factor in transportation choices.

6 Off the Chain

Congestion, particularly in the United States, is a quintessential element of transportation dynamics. A significant dependence on single-vehicle transportation solutions has unequivocally resulted in severe congestion in nearly every major urban hub across the country.

The rise in popularity of e-bikes, alongside increased investments in public transportation, promises to reduce congestion and traffic in major metropolitan hubs.

In order to simulate the effects of the changes in e-bike usage relative to other forms of transportation, three specific case studies representing various city dynamics are tested using a programmatic model in conjunction with our time-series predictions from the differential equations model in Shifting Gearing.

6.1 Model Overview

Our programmatic model simulates the presence of cars and e-bicycles in a given city environment, specifically on two unique transportation pathways: highways and surface (residential streets). For each hour in a day (24 hours), we used a probability distribution to calculate the total percentage of commuters traveling at any given hour of the day. Commuters are referred to as "traveling" as our model takes into account both cars and cyclists. Using data collected from each respective case study scenario, the total number of commuters is used to calculate the amount of motorists using highways and surface streets. Finally, we determine the traffic density and congestion of the network, ultimately as a result of the urban planning of the city.

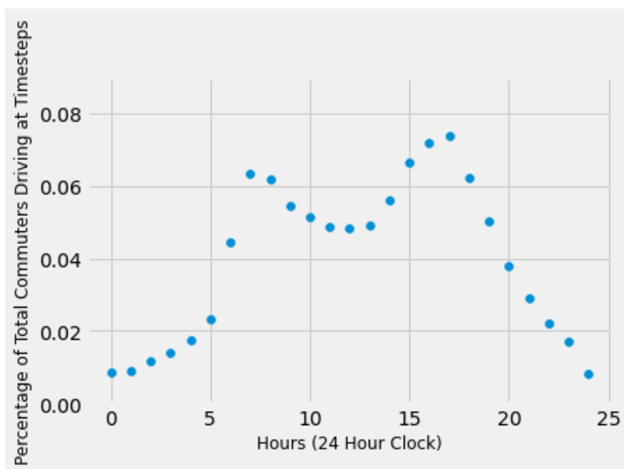

Figure 5: Probability Distribution for Commuter Traveling

6.1.1 Highway-Surface Street Distribution

Depending on the given location, and thus scenario, of the simulation, the ratio of drivers on highways to those on surface streets will change. In a given traffic system, each commuter will always try to minimize the total travel time, without regard for others. Using this simplifying assumption, we can determine the number of drivers on highways and surface streets:

$$N_r S_r L_r,$$

where N_r is the ratio of highway miles to street miles, S_r is the ratio of the highway speed limit to the street speed limit, and L_r is the ratio of the number of highway lanes to street lanes. A useful byproduct is the proportion of all vehicles using the highway:

$$\frac{N_r S_r L_r}{N_r S_r L_r + 1}.$$

For all scenarios, the ratio of speed limit between surface streets and highways is a fixed constant of $\frac{25mi/h}{55mi/h}$. This ratio is a simplifying assumption made about all four locations, although it represents the basic speed limits for residential streets and highways in the United States.

6.2 Congestion Assumptions and Justifications

The following are significant assumptions present in the congestion analysis.

Model-specific Assumption 1: Commute time is less than or equal to one hour.
Model-specific Justification 1: The average U.S. commute time is under one hour, so our decision to limit for simplicity does not create significant error.

Model-specific Assumption 2: Humans are disposed to minimize their travel time without regard for others.
Model-specific Justification 2: It is cost-efficient and self-beneficial to expedite one's commute.

6.3 Scenario #1: Dallas Model

Dallas, one of the largest metropolitan areas in the United States, represents the first car-dependent scenario. Dallas was chosen due to its sprawling suburban landscape and its large commuter population. Dallas also illustrates an important occurrence in American urban planning: though only 3% of Dallas County's roads are freeways, nearly half its metro area is served by freeways and only 2% of its commuting population uses public transit [15][16].

Our results for Dallas traffic are shown in 6.3 [20] [21]. We see that highway congestion is much greater than surface street congestion, and overall congestion is quite high. Yet, the adoption of e-bikes in subsequent years does decrease the amount of congestion on the highways.

6.4 Scenario #2: San Francisco Model

San Francisco is the antithesis of Dallas with respect to its transportation infrastructure planning. While Dallas is extremely highway-dependent, San Francisco accomplishes its motorist transport objectives through a dense grid of surface streets. Furthermore, San Francisco has a much greater proportion of its population that uses public transit. This is shown in the result of the simulation, as the amount of congestion at any given time is greatly reduced [22] [23].

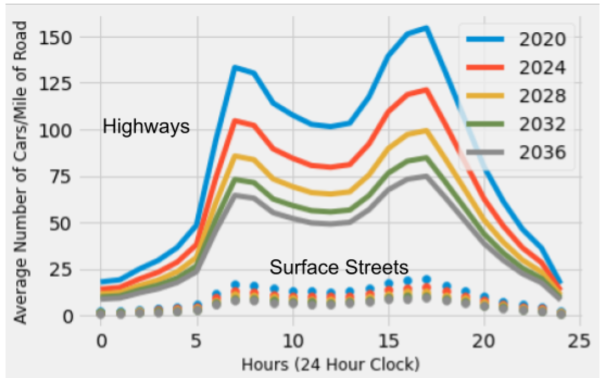

Figure 6: Dallas Traffic Results

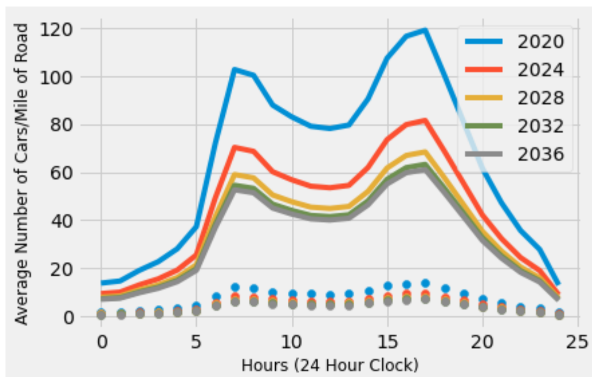

Figure 7: San Francisco Traffic Results

6.5 Scenario #3: Barcelona Method

Barcelona has taken an innovative approach to urban planning compared to the United States: they have neighborhoods arranged in "superblock" structures that do not permit motorized traffic inside, resulting in a strikingly low amount of highways[17]. Meanwhile, long-range transport occurs through a more even split of public and private transport networks[18]. We model a hypothetical American city that has adopted the Barcelona model using San Francisco population parameters [24]. Note that Barcelona has a strong biking history that we incorporate into this scenario[24].

6.6 Congestion Results

Year	Dallas Highways	Dallas Streets	SF Highways	SF Streets	Barcelona Highways
2020	0.26	0.05	0.19	0.04	0.64
2024	0.2	0.04	0.13	0.03	0.44
2028	0.17	0.03	0.11	0.02	0.37
2032	0.14	0.03	0.1	0.2	0.34
2036	0.13	0.03	0.1	0.2	0.33

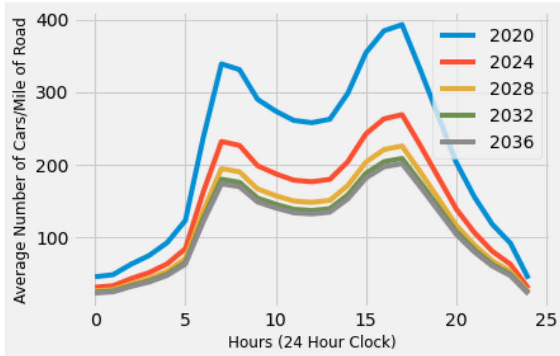

Figure 8: Barcelona Traffic Results

When a city is car-dependent city strives to reduce congestion, its primary solution to congestion is to increase the number of highways, as the Dallas example shows. However, the existence of alternative options, such as e-bikes, allows cities to invest in alternate congestion reduction methods. As the Barcelona example shows, significant dependence on private, single-vehicle transportation significantly increases the total amount of traffic congestion. However, in the long term, congestion significantly decreases as the total number of electric vehicles increases.

6.7 Strengths and Weaknesses

6.7.1 Strengths

- **Rigor**: The model accounts for multi-year trends in product changes.
- **Dynamism**: The model simulates rush hour traffic throughout a continuous, 24-hour period.
- **Range**: The model examines both highways and surface streets, as well as the dynamic between the two.

6.7.2 Weaknesses

- **Density Base**: The simulation only models density-based congestion, disregarding route-based congestion.
- **E-Bike Congestion**: The simulation cannot model e-bike congestion.
- **Time Limits**: The model generalizes all traffic to hour-long buckets.

6.8 Testing

Because we have provided specific scenarios with our model applied, our model could be tested through a prospective analysis of San Francisco, Dallas, and Barcelona as their e-bike user bases grow in the coming years.

6.9 Sensitivity Analysis

The congestion of particular cities is dependent on the city-wide parameters (those defining highway and surface-street ratios). A sensitivity analysis would be performed by varying these parameters across a range and finding the impact on congestion accordingly.

6.10 Traffic and Congestion

6.10.1 Overview

In 6.7.2 and 6.7.3, we do not account for variance among car lengths and instead denote the average car length l. In 6.7.3, we do not account for variance among response time and instead denote the average human reaction time τ.

6.10.2 Traffic Congestion

Falling under the categories of traffic management and safety, our model addresses changes in road congestion and the changes in crash analysis that accompany a decrease in cars. To define congestion, we take a region L and restrict it to a one-dimensional line segment equivalent to a single road lane. With this simplification, we measure congestion as the quotient of the total number of vehicles within a lane segment L and the length of L. We then define road congestion as

$$\text{congestion} = \rho(t) = \frac{\text{Volume}}{\text{Capacity}} = \frac{nl}{L}$$

where n is the number of cars within L and l is the average car length (congestion is clearly bounded by $0 \leqslant \rho \leqslant 1$). Evidently, a decrease in the number of active cars leads to a decrease in car density and therefore congestion.

6.10.3 Accordion Accidents

To analyze minor traffic accidents, we develop a delayed differential model and analyze the effects of decreased car numbers on accordion accidents. Considering a line of cars moving with the same direction and velocity in a lane, the event of collision can be determined by investigating the growing oscillatory results of small perturbations in the front. This state of equal velocity and distance is called the equilibrium.

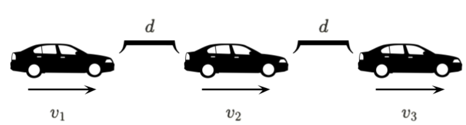

Figure 9: Equilibrium State of Accordion Situation

In Figure 5, $v_1 = v_2 = v_3 = v$. Denoting the front of each car as $x_i(t)$, the relative velocity of car i to the one behind it is is $|x_i'(t) - x_{i+1}'(t)|$. We consider a slight perturbation in velocity from equilibrium as represented by the distance $\varepsilon_i(t)$. Notice that, during an accordion event, the distance $\varepsilon_i(t)$ is strictly less than $\varepsilon_{i+1}(t)$ because human response time is greater than zero seconds (this "delay" is the eponymous progenitor of the "delay differential equations" we will use for this model). In order to achieve differential form, we begin by noting the relationship between braking force (the force necessary to decelerate before collision), relative velocity, and car gap (the distance between cars i and $i+1$). Primarily, as the relative velocity increases, the braking force must also increase. As the gap increases, the braking force must decrease because there is more distance over which the car can decelerate. We have:

$$\begin{aligned} ma_i(t) &\propto \left|\frac{x_i'(t) - x_{i+1}'(t)}{x_i(t) - x_{i+1}(t)}\right| \\ a_i(t) &\propto \left|\frac{x_i'(t) - x_{i+1}'(t)}{x_i(t) - x_{i+1}(t)}\right| \\ x_i''(t) &\propto \left|\frac{x_i'(t) - x_{i+1}'(t)}{x_i(t) - x_{i+1}(t)}\right| \\ x_i''(t+\tau) &\propto \left|\frac{x_i'(t) - x_{i+1}'(t)}{x_i(t) - x_{i+1}(t)}\right| \\ x_i''(t+\tau) &= \lambda\left|\frac{x_i'(t) - x_{i+1}'(t)}{x_i(t) - x_{i+1}(t)}\right| \end{aligned}$$

where τ represents the response delay observed in the ith car after a change in distance behind the $i+1$th car (resulting from a perturbation in the velocity of the first observed car). By integration we obtain a delayed function of velocity:

$$x_i'(t+\tau) = v_i(t+\tau) \propto \ln|x_i(t) - x_{i+1}(t)| + C.$$

Next, we model the effects of perturbations $\varepsilon_i(t)$ as delayed functions of velocity. Since these are displacements from equilibrium, we can call

$$\varepsilon_i(t) = x_i(t) - (vt - (i+1)|x_i(t) - x_{i+1}(t)|).$$

This expression, however, can be simplified by noticing that the latter terms are constant. At equilibrium, we have $|x_i(t) - x_{i+1}(t)| = \text{car length} + d = l + d$. Thus,

$$\begin{aligned} \varepsilon_i(t) &= x_i(t) - (vt - (i+1)(l+d)) \\ \varepsilon_i'(t) &= x_i'(t) - v. \end{aligned}$$

At this stage, we complete the differential by adding its delay, which accounts for average human response time (the time it takes for the driver of car $i+1$ to react to the perturbation from car i):

$$\varepsilon_i'(t+\tau) = x_i'(t+\tau) - v.$$

At this point, we need only to simplify our equation for $x_i'(t+\tau)$ and substitute it into the function above. As a result, we end up with a final delayed differential whose constants can be found with minimal analysis on the equilibrium state:

$$\varepsilon_i'(t+\tau) = v\left(\ln\left|\frac{1}{l}(l+d+\varepsilon_i(t)-\varepsilon_{i+1}(t))\right| - 1\right).$$

This delayed differential model does require initial conditions, however, which are granted by observing simple properties of the accordion situation as well as modeling a function of the first car. To do this, we can use any number of different models that encapsulate the properties of the first car. Here we construct a piecewise function, first noting that the car will have equilibrium velocity v up until and at time $t = 0$ (therefore we will generate a velocity function):

$$x_1'(t) = \begin{cases} v & \text{if } t = 0 \\ & \text{if } t > 0 \end{cases}.$$

For the rest of the function's curve, we expect to observe a dip and recovery to model the initial perturbation as seen in a deceleration. We decide to model this with an altered normal distribution curve (functional PDF), and condition this function with the addition of the time marker Δt. This time will denote the moment when the first car begins to accelerate after braking. This provides:

$$x_1'(t) = \begin{cases} v & \text{if } t = 0 \\ \frac{1}{-\sigma\sqrt{2\pi}} e^{-0.5\left(\frac{t-\Delta t}{-\sigma}\right)^2} + v & \text{if } t > 0 \end{cases}$$

for $\sigma > 0$. To clean up our expressions, we both translate $x_1'(t)$ to $\varepsilon_1(t)$ and endow the system with $\varepsilon_i(0) = 0$ because at equilibrium there is no perturbation. This delay differential can be solved numerically and whose solutions $\varepsilon_i(t)$ follow damped harmonic oscillations, as shown below for solutions to a very similar system.

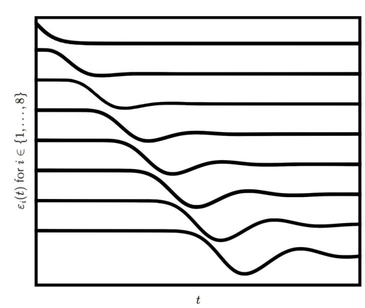

Figure 10: Functionally Similar $\varepsilon_i(t)$ [14]

In our model of commuters transitioning from car usage to e-bike transportation, we observe a decrease in the car density and therefore the number of crashes (which are represented by curve intersections). This number is quantifiable and can be modeled accurately using the system of delay differential equations and data on the new e-bike population.

6.11 Carbon Emission in the United States

To look at the change in carbon emitted by modes of transportation in the United States, we calculate the expected carbon emitted per passenger-kilometer by each of the modes of transportation (Cars, E-Bikes, Acoustic Bikes, Public Transit, and Walking) in 2023 and 2033 using data generated from our model in the Shifting Gears section.

- Cars: 77.014% → 61.748%
- E-Bikes: 13.007% → 23.633%
- Acoustic Bikes: 5.383% → 10.272%
- Public Transit: 0.894% → 0.767%
- Walkers: 3.702% → 3.580%

Noting that the population of the United States is 334,233,854 [25] and our assumption that it remains constant, we use the following table of carbon footprint for each mode of transportation to calculate the total expected carbon per passenger-kilometer emitted in 2023 and 2033 [26][27].

Mode of Transportation	Carbon Footprint per Passenger-Kilometer (g)
Gasoline-Powered Car	192
Bus (Public Transit)	105
Train (Public Transit)	41
Walking	0
Acoustic Bike	0
E-Bike	3

<u>Final calculations for 2023:</u>

- Gas-Powered Car: $192 \times 334{,}233{,}85 \times 0.77014 = 4{,}942{,}211{,}658.9888$g
- Public Transit: $(105 + 41) \times 334{,}233{,}85 \times 0.00894 = 43{,}625{,}539.0374$g
- Walking: $0 \times 334{,}233{,}85 \times 0.03702 = 0$g
- Acoustic Bike: $0 \times 334{,}233{,}85 \times 0.05383 = 0$g
- E-Bike: $0.13007 \times 334{,}233{,}85 \times 3 = 13{,}042{,}139.06085$g

<u>Final calculations for 2033:</u>

- Gas-Powered Car: $192 \times 334{,}233{,}85 \times 0.61748 = 3{,}962{,}548{,}179.8016$g
- Public Transit: $(105 + 41) \times 334{,}233{,}85 \times 0.00767 = 37{,}428{,}174.9907$g
- Walking: $0 \times 334{,}233{,}85 \times 0.03580 = 0$g
- Acoustic Bike: $0 \times 334{,}233{,}85 \times 0.10272 = 0$g
- E-Bike: $0.23633 \times 334{,}233{,}85 \times 3 = 23{,}696{,}845.73115$g

Adding up the total carbon footprint per passenger-kilometer in 2023, we get $4,942,211,658.9888 + 43,625,539.0374 + 0 + 0 + 13,042,139.06085 = 4,998,879,337.08705$g. Adding up the total carbon footprint per passenger-kilometer in 2033, we get $3,962,548,179.8016 + 37,428,174.9907 + 0 + 0 + 23,696,845.73115 = 4,023,673,200.52345$g. With a difference of 975,206,136.5636g, we see there is a drastic difference in total carbon footprint per passenger-kilometer when mostly switching modes of transportation from cars to e-bikes over a span of 10 years (Cars: 77.014% → 61.748%, E-Bikes: 13.007% → 23.633%).

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8 Code

8.1 Solution Code to Part 2: Shifting Gears

This code solves the system of differential equations defining the transitions between modes of transportation. It generates time-series data of the modes of transport of a population.

```
import matplotlib.pyplot as plt
import numpy as np

# Order is Walking, Acoustic Bike, Ebike, Car, Public transit
R = [1, 1, 2, 1, 1] # Coolness factor

# Pairwise relative cost
# walking: 20 per day? maybe 1 per day to make it realistic
#           compared to other forms
# acoustic bikes: 100/5 year lifespan + 350/year maintenance
#           = 1.02 per day
# ebike: 1300/5 year lifespan + 418/year maintenance
#           = 1.85 per day
# cars: 20000/12 year lifespan + 1776 / year maintenance
#           = 9.43 per day
# public transit: 120/month = 4 per day
ebike_cost = 4
C = [
    [1, 1/1.02, 1/1.85, 1/9.43, 1/ebike_cost],
    [1, 1, 1.02/1.85, 1.02/9.43, 1.02/ebike_cost],
    [1, 1, 1, 1.85/9.43, 1.85/ebike_cost],
    [1, 1, 1, 1, 9.43/ebike_cost],
    [1, 1, 1, 1, 1]
]
# now conjugate on bottom triangle
for i in range(5):
    for j in range(i):
        C[i][j] = 1 / C[j][i]

constant = 1/10
E = 8.34 # gha per capita
```

```
HS = 14.2042 # Hilliness scale
# Car is better than ebikes and public transit is mostly better
H = np.array([0, 0, 0.2, 1, 0.7]) * HS + np.array([1, 1, 0, 0, 0])
L = [1, 5, 5, 12, 1] # Lifespan
P = np.array([8605272, 1654860, 928000, 280995228, 17210544],
            dtype=float) # Initial populations

T = 20 # Simulation time
P_over_t = np.array([P])

# Euler's method
for t in range(T):
    diff_mat = np.zeros((5,5)) # transfer from i to j

    for i in range(5): # Iterate through receiver
        for j in range(5): # Iterate through giver
            if j == i: continue
            transfer = constant * R[i] * 1/(E if not i == 3 else 1)
            transfer *= H[i] * 1/L[j] * P[j] * C[j][i]
            diff_mat[j][i] += transfer
            diff_mat[i][j] -= transfer

    # Sum all transfers for dP/dt
    diff_P = np.sum(diff_mat, axis=0)
    P += diff_P
    P_over_t = np.append(P_over_t, np.array([P]), axis=0)

tot = np.sum(P)

# Stacked area plot of modes of transportation
P_over_t = np.swapaxes(P_over_t, 0, 1)

labels = ["Walking", "Acoustic bike", "Ebike",
            "Car", "Public transit"]
plt.stackplot(np.arange(np.shape(P_over_t)[1]), P_over_t,
                labels=labels)
plt.legend(loc='upper right')
```

```
plt.show()

for i in range(5):
    if i == 2:
        plt.plot(np.arange(np.shape(P_over_t)[1]), P_over_t[i],
                label=labels[i])
plt.legend(loc='upper right')
plt.show()
```

8.2 Solution Code to Part 3: Off the Chain

This is our highway vs. surface-street model that calculates the amount of congestion in a city given population parameters and the time-series data from Part 2.

```
import sys
import pandas as pd
import gdown
import os
import gzip
import json
import numpy as np
import matplotlib.pyplot as plt
import statsmodels.api as sm
import seaborn as sn
from google.colab import drive
import random
import itertools
plt.style.use('fast')


highway_speed = 55
street_speed = 25
speed_ratio = highway_speed/street_speed
speed_ratio

df_traffic_24_hr_distribution = pd.read_excel("file_path") #data
 ↪  downloaded from sources referenced
df_car_bike_ratios_problem_2_dallas = pd.read_csv("file_path")
```

```
df_car_bike_ratios_problem_2_sf = pd.read_csv("file_path")

plt.scatter(df_traffic_24_hr_distribution['HOUR'],\
        df_traffic_24_hr_distribution['RATIO'])
ax = plt.gca()
ax.set_xlabel("Hours (24 Hour Clock)",fontsize=12)
ax.set_ylabel("Percentage of Total Commuters Driving at
 ↪  Timesteps",fontsize=12)
ax.set_ylim([0, 0.09])
plt.savefig('time_distribution.png', dpi=300)
plt.show()


#DALLAS DALLAS DALLLAS !!!!


dallas_commuters = 1362000


def
 ↪  mapping_dallas(grid_size_x,grid_size_y,highway_ratio,highway_lanes,\
        traffic_table,total_commuters):

  nodes = []

  for x in range(grid_size_x+1):
    for y in range(grid_size_y+1):
      nodes.append((x,y))

  combinations = [(a, b) for idx, a in enumerate(nodes) for b in
   ↪  nodes[idx + 1:]]

  unique_combinations = []

  for count, ((xc1,yc1),(xc2,yc2)) in enumerate(combinations):
    diagonal_counter = abs(xc1-xc2)+abs(yc1-yc2)
```

```
    if abs(xc1-xc2) <= 1 and abs(yc1-yc2) <= 1 and diagonal_counter <= 1:
      unique_combinations.append(combinations[count])

  highways = []
  surface_streets = []

  for count, value in enumerate(unique_combinations):
    if random.random() > highway_ratio:
      surface_streets.append(unique_combinations[count])
    else:
      highways.append(unique_combinations[count])
  fig, ax = plt.subplots()
  for index, car, bike in zip(df_car_bike_ratios_problem_2_dallas['Index'], df_car_bike_ratios_problem_2_dallas['Car'],df_car_bike_ratios_problem_2_dallas[' EBike']):
    index = 2019+index
    #____________________________________________#
    #highway ratio and car distributions
    temp = speed_ratio*highway_lanes*(highway_ratio/(1-highway_ratio))
    highway_advantage = temp/(temp+1)

    #print("highway advantage = ", highway_advantage)

    #print("total edges =", len(unique_combinations))
    highway_density = 0
    street_density = 0

    if (index % 4) == 0:
      hr = []
      hd = [] #list of cars on highway at any given point element point
      sd = []
```

```
        for hour, ratio in
         ↪  zip(df_traffic_24_hr_distribution['HOUR'],
         ↪  df_traffic_24_hr_distribution['RATIO']):
          people_per_hour = ratio*total_commuters*car
          #highway
          highway_density =
           ↪  (highway_advantage*people_per_hour)/len(highways)
          #street
          street_density =
           ↪  ((1-highway_advantage)*people_per_hour)/len(surface_streets)

          hr.append(hour)
          hd.append(highway_density)
          sd.append(street_density)
        sum_hd = 0
        sum_sd = 0
        for k in hd:
          sum_hd += (14.5*k)/5280
        for p in sd:
          sum_sd += (14.5*k)/5280

        print(index, round(sum_hd/25,2))
        print(index, round(sum_sd/25,2))

        ax.plot(hr,hd,label=index,)
        ax.scatter(hr,sd)
    #plt.figure(figsize=(30,200))
    leg = plt.legend(loc='upper right')
    #plt.savefig('temp.png', dpi=300)
    plt.xlabel('Hours (24 Hour Clock)',fontsize=12)
    plt.ylabel("Average Number of Cars/Mile of Road",fontsize=12)
    plt.savefig('dallas.png', dpi=1200)
    plt.show()

mapping_dallas(50,36,0.03,4,df_traffic_24_hr_distribution,dallas_commuters)

#SF
```

```python

sf_commuters = 1500000
sf_highway = 1300
sf_streets = 3300
sf_highway_ratio = 0.04



def
→ mapping_sf(grid_size_x,grid_size_y,highway_ratio,highway_lanes,\
        traffic_table,total_commuters):

  nodes = []

  for x in range(grid_size_x+1):
    for y in range(grid_size_y+1):
      nodes.append((x,y))

  combinations = [(a, b) for idx, a in enumerate(nodes) for b in
  →  nodes[idx + 1:]]

  unique_combinations = []

  for count, ((xc1,yc1),(xc2,yc2)) in enumerate(combinations):
    diagonal_counter = abs(xc1-xc2)+abs(yc1-yc2)
    if abs(xc1-xc2) <= 1 and abs(yc1-yc2) <= 1 and
    →  diagonal_counter <= 1:
      unique_combinations.append(combinations[count])

  highways = []
  surface_streets = []

  for count, value in enumerate(unique_combinations):
    if random.random() > highway_ratio:
      surface_streets.append(unique_combinations[count])
    else:
      highways.append(unique_combinations[count])
```

```
fig, ax = plt.subplots()
for index, car, bike in zip(df_car_bike_ratios_problem_2_sf['Index'], df_car_bike_ratios_problem_2_sf[' Car'],df_car_bike_ratios_problem_2_sf[' EBike']):
  index = 2019+index
  #_____________________________________________#
  #highway ratio and car distributions
  temp = speed_ratio*highway_lanes*(highway_ratio/(1-highway_ratio))
  highway_advantage = temp/(temp+1)

  #print("highway advantage = ", highway_advantage)

  #print("total edges =", len(unique_combinations))
  highway_density = 0
  street_density = 0

  if (index % 4) == 0:
    hr = []
    hd = [] #list of cars on highway at any given point element point
    sd = []
    for hour, ratio in zip(df_traffic_24_hr_distribution['HOUR'], df_traffic_24_hr_distribution['RATIO']):
      people_per_hour = ratio*total_commuters*car
      #highway
      highway_density = (highway_advantage*people_per_hour)/len(highways)
      #street
      street_density = ((1-highway_advantage)*people_per_hour)/len(surface_streets)

      hr.append(hour)
      hd.append(highway_density)
      sd.append(street_density)
```

```
      sum_hd = 0
      sum_sd = 0
      for k in hd:
        sum_hd += (14.5*k)/5280
      for p in sd:
        sum_sd += (14.5*k)/5280

      print(index, round(sum_hd/25,2))
      print(index, round(sum_sd/25,2))

      ax.plot(hr,hd,label=index,)
      ax.scatter(hr,sd)
  #plt.figure(figsize=(30,200))
  leg = plt.legend(loc='upper right')
  plt.xlabel('Hours (24 Hour Clock)',fontsize=12)
  plt.ylabel("Average Number of Cars/Mile of Road",fontsize=12)
  plt.savefig('sf.png', dpi=1200)
  plt.show()

mapping_sf(40,59,sf_highway_ratio,4,df_traffic_24_hr_distribution,\
        sf_commuters)

#barcelona


barcelona_commuters = 250000
barcelona_highway = 3179
barcelona_streets = 431
barcelona_highway_ratio = barcelona_streets/barcelona_highway
print(barcelona_highway_ratio)

# for index, car, bike in
 ↪ zip(df_car_bike_ratios_problem_2_sf['Index'],
 ↪ df_car_bike_ratios_problem_2_sf['
 ↪ Car'],df_car_bike_ratios_problem_2_sf[' EBike']):
#
 ↪ mapping(40,59,sf_highway_ratio,3,df_traffic_24_hr_distribution,sf_commu
```

```
def
↪ mapping_barcelona(grid_size_x,grid_size_y,highway_ratio,highway_lanes,t

  nodes = []

  for x in range(grid_size_x+1):
    for y in range(grid_size_y+1):
      nodes.append((x,y))

  combinations = [(a, b) for idx, a in enumerate(nodes) for b in
   ↪ nodes[idx + 1:]]

  unique_combinations = []

  for count, ((xc1,yc1),(xc2,yc2)) in enumerate(combinations):
    diagonal_counter = abs(xc1-xc2)+abs(yc1-yc2)
    if abs(xc1-xc2) <= 1 and abs(yc1-yc2) <= 1 and
     ↪ diagonal_counter <= 1:
      unique_combinations.append(combinations[count])

  highways = []
  surface_streets = []

  for count, value in enumerate(unique_combinations):
    if random.random() > highway_ratio:
      surface_streets.append(unique_combinations[count])
    else:
      highways.append(unique_combinations[count])
  fig, ax = plt.subplots()
  for index, car, bike in
   ↪ zip(df_car_bike_ratios_problem_2_sf['Index'],
   ↪ df_car_bike_ratios_problem_2_sf['
   ↪ Car'],df_car_bike_ratios_problem_2_sf[' EBike']):
    index = 2019+index
    #____________________________________________#
```

```
  #highway ratio and car distributions
  temp = 55*highway_lanes
  #print("highway advantage = ", highway_advantage)

  #print("total edges =", len(unique_combinations))
  highway_density = 0
  street_density = 0

  if (index % 4) == 0:
    hr = []
    hd = [] #list of cars on highway at any given point element point
    sd = []
    for hour, ratio in zip(df_traffic_24_hr_distribution['HOUR'], df_traffic_24_hr_distribution['RATIO']):
      people_per_hour = ratio*total_commuters*car
      #highway
      highway_density = (people_per_hour)/len(highways)

      hr.append(hour)
      hd.append(highway_density)
    sum_hd = 0
    sum_sd = 0
    for k in hd:
      sum_hd += (14.5*k)/5280

    print(index, round(sum_hd/25,2))

    ax.plot(hr,hd,label=index,)
#plt.figure(figsize=(30,200))
leg = plt.legend(loc='upper right')
plt.xlabel('Hours (24 Hour Clock)',fontsize=12)
plt.ylabel("Average Number of Cars/Mile of Road",fontsize=12)
plt.savefig('barcelona.png', dpi=1200)
#plt.savefig('temp.png', dpi=300)
plt.show()
```

```

mapping_barcelona(50,57,sf_highway_ratio,4,df_traffic_24_hr_distribution,\
          sf_commuters)
```