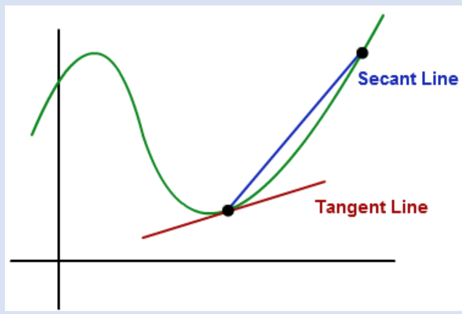
Tangent Lines & Derivatives

The **tangent line** to the graph of $f(x)$ at the point $(a, f(a))$ is the line through this point having the *slope*

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

provided this limit exists. If the limit does not exist, then there is no tangent line at the point.



$\frac{f(x+h) - f(x)}{h}$	AROC	Slope of secant line
$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$	IROC	Slop of tangent line

The Limit Definition of the Derivative

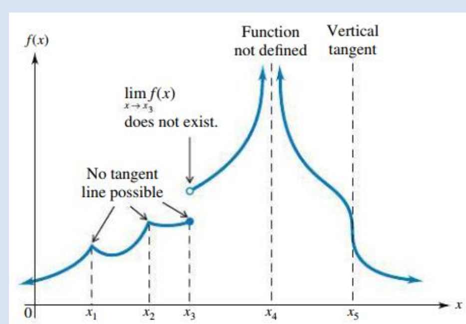
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Step 1: $f(x+h)$

Step 2: $f(x+h) - (f(x))$

Step 3: $\frac{f(x+h) - f(x)}{h}$ $(factor/cancel\ the\ h)$

Step 4: $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

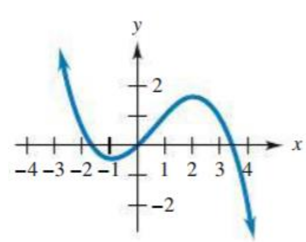


The derivative does not exist at “corners” or “sharp points” on a graph.

1) Let $f(x) = x^2 + 2$. Find the equation of the tangent line to the graph at the point where $x = 1$.

2) Let a be any real number. Find the equation of the tangent line to the graph of $f(x) = 7x + 3$ at the point where $x = a$.

3) The graph of a function g is shown below. Determine whether the given numbers are positive, negative, or zero.



a) $g'(0)$

b) $g'(-1)$

c) $g'(3)$

4) A student brings a cold soft drink to a 50-minute math class but is too busy during class to drink it. If $C(t)$ represents the temperature of the soft drink (in degrees Fahrenheit) t minutes after the start of class, interpret the meaning of the following statements, including units.
    a) $C(50) = 68$

    b) $C(50) - C(25) = 7.6$

    c) $\frac{C(50)-C(25)}{50-25} = 0.3$

    d) $C'(50) = 0.2$

5) Let $f(x) = -5x^2 + 4$. Find $f'(x)$ and $f'(3)$.

6) Let $f(x) = -\frac{5}{x}$. Find $f'(x)$ and $f'(-1)$.

7) Let $g(x) = \sqrt{x}$. Find $g'(x)$.

8) Find all points where the function whose graph is shown does not have a derivative.