Bayesian Change Point Analysis of Real GDP Growth

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Abstract

Monitoring the Real GDP Growth is an important process for assessing the state of a country's economy. Estimating when a change has occurred or is going to occur can help a country in determining what policies should be undertaken for a healthy economy. In this paper two different methods are presented in the Bayesian framework for detecting change points. The first method is a Hidden Markov Mixture Model by Koop and Potter (2004) based on the work of Chib (1998). For this method there will be a discussion of the statistical framework and the simulated data. The second method discussed is the Bayesian Change Point (bcp) package in R Erdman and Emerson (2007) which is based on the work of Barry and Hartigan (1993). Lastly, the results of the bcp's detection of change points will be compared with historical knowledge and possible factors for the change in GDP Growth for the countries of the United States, Great Britain, South Korea, Australia, Canada, Norway, Japan, and Ireland.

1 Introduction

The study of change point detection has long been a popular topic in many fields. For this paper we are going to consider studying change point detection in the field of economics. In particular, the study of Real GDP Growth, which has many practical applications and can have lasting impacts on economic policy either Regionally or Globally. There are many factors that can influence Real GDP growth and understanding when a change occurs can help notify governments and businesses of what steps they can take to best function in this new regime/segment in time. There are many different methods of Change point detection, two such differences are the approaches of offline and online change point detection models. The idea for each is like what their names bring to our mind. For offline change point models, you are working with data in which you consider having the complete data. On the other hand, for online change point methods, the data is not complete, and data is constantly being steamed to the database. The advantage of online methods is that they allow for real-time analysis and detection of a change in the data. The disadvantage is that online methods may be less accurate than that of offline methods because you are only seeing the data as it appears and do not get to see the full picture like with offline methods. For this paper the two methods discussed are both offline methods. A good review of some different offline change methods can be found in Truong et al. (2020). In the context of time series change point analysis there has been a good amount of research in the area of Time Varying Parameter (TVP) models. A good paper that introduces this idea can be found in Huber et al. (2019), which describes the process and how to extend it to a Vector Autoregression Model (VAR) Framework. The TVP model is an important part of the first method discussed in this paper.

2 Data

The data used in this paper was downloaded from the Organization for Economic Co-operation and Development OECD (2023). The OECD is an international organization made up of twenty countries that promote national accounting standards and economic advice and strategies around the globe. The economic data of OECD countries that we are using complies with the 2008 System of National Accounts (SNA). The SNA is a statistical framework that allows for consistent accounting of economic data IMF (2009). The definition for Quarterly GDP is given as

> Gross domestic product (GDP) is the standard measure of the value added created through the production of goods and services in a country during a certain period. As such, it also measures the income earned from that production, or the total amount spent on final goods and services (less imports). While GDP is the single most important indicator to capture economic activity, it falls short of providing a suitable measure of people's material well-being for which alternative indicators may be more appropriate. This indicator is based on real GDP (also called GDP at constant prices or GDP in volume), i.e. the developments over time are adjusted for price changes. The numbers are also adjusted for seasonal influences. The indicator is available in different measures: percentage change from the previous quarter, percentage change from the same quarter of the previous year and volume index (2015=100). All OECD countries compile their data according to the 2008 System of National Accounts (SNA). OECD (2023)

The data analyzed in this paper is the Quarter-to-Quarter percent change in Real GDP (GDP Growth) for the countries of the United States, Great Britain, South Korea, Australia, Canada, Norway, Japan, and Ireland. The goal of studying this data is to see when the change points occur in the 8 countries listed above and to see whether the countries experience similar changes in economic growth that is detected by the bcp model.

2.1 How to Obtain the Data

To download the same data used for this paper you can follow the link in the Reference Section OECD (2023). Once you have gone to the link there are a few steps you need to take to get the correct data. First go down the page to the chart for quarterly GDP, you need to adjust a few chart options. Now, under perspectives make sure Total and Percentage change previous period are selected, under Countries for select background set to None and uncheck show baseline:OECD - Total, then make sure the following countries are highlighted Australia, Canada, United Kingdom, Ireland, Japan, Korea, Norway, and the United States. Next, under Time click quarterly, uncheck latest data available, and select the data range of Q2 1947 – Q2 2023. Finally, above the chart/table there is a button for download click and select selected data only .csv. Once the data is obtained a little preprocessing is needed for Ireland, Japan, and Norway because there were estimated values that extend the time series farther back in time, but that data is too smooth, so I drop the beginning of each of these time series with the flag E. As a result, there are 8 time series with Quarterly GDP percent change, period over period.

3 Hidden Markov Mixture Model Framework

The method developed by Koop and Potter (2004) was an internal staff report that was released by the Federal Reserve Bank in New York. The main building blocks of this paper is built on the work of Chib (1996) who developed a MCMC algorithm to calculate posterior distributions and model estimates by incorporating Markov Mixture Models, and in another paper by Chib (1998) which extends the model to change point detection by way of a Hidden Markov model. The idea of the Koop and Potter (2004) method is to nest a Time Varying Parameter (TVP) model and a change point model with few change points in one model by setting M the number of regimes as $\leq T$. By our definition of the transition probabilities, we do not restrict the number of regimes in the sample to be equal to M. This allows for an unknown number of regimes to be in sample which allows for several regimes to lie outside of our sample. This is a desirable property for forecasting out of sample.

Data on a scalar time series variable, y_t for $t = 1, \ldots, T$ and let $Y_i = (y_1, \ldots, y_i)^\top$ denote the history through time i and denote the future by $Y^{i+1} = (y_{i+1}, \ldots, y_T)^\top$. Regime changes depend on a discrete random variable, s_t, which takes on values $1, 2, \ldots, M$. We let $S_i = (s_1, \ldots, s_i)^\top$ and $S^{i+1} = (s_{i+1}, \ldots, s_T)^\top$.

The Likelihood function is defined by assuming $p(y_t|Y_{t-1}, s_t = m) = p(y_t|Y_{t-1}, \theta_m)$ for a parameter vector θ_m for $m = 1, \ldots, M \leq T$. Thus, change-points occur at times τ_m defined as $\tau_m = \{t : s_{t+1} = m+1, s_t = m\}$ for $m = 1, \ldots, M-1$.

Also, that the P Matrix or transition Probability matrix is defined by: P is MxM matrix $Pr[s_T = M|s_{T-1} = M] = p_M$, $Pr[s_T = M|s_{T-1} = M-1] = 1 - p_{M-1}$

3.1 HMM Poisson Hierarchical Prior for duration's in regimes

We adopt a Poisson Hierarchical prior for the length of the segments in our model.

$$\begin{aligned}
d_m &\text{ denotes regime duration/segment} \\
d_m|\lambda_m &\sim Poisson(\lambda_m) \\
\lambda_m|\beta_\lambda &\sim Gamma(\underline{\alpha_\lambda}, \beta_\lambda) \text{ where } \underline{\alpha_\lambda} \text{ is a hyperparameter} \\
\beta_\lambda^{-1} &\sim Gamma(\underline{\xi_1}, 1/\underline{\xi_2}) \text{ where } \underline{\xi_1} \text{ and } \underline{\xi_2} \text{ are hyperparameters}
\end{aligned}$$

3.2 HMM Priors for the Parameters in Each Regime

We adopt a structure based on an autoregressive model with stochastic volatility, and a state space framework where our time series satisfies the measurement equation

$$y_t = X_t\phi_{st} + exp(\sigma_{st}/2)\epsilon_t, \tag{1}$$

where $\epsilon_t \sim N(0,1)$ and the (K+1) state vector $\theta = \{\phi_{st}, \sigma_{st}\}$ satisfies the state transition equations:

$$\begin{aligned}
\phi_m &= \phi_{m-1} + U_m, && (2) \\
\sigma_m &= \sigma_{m-1} + u_m, && (3) \\
& && (4)
\end{aligned}$$

where $U_m \sim N(0, V)$, $u_m \sim N(0, \eta)$ and X_t is a K-dimensional row vector containing lagged dependent or other explanatory variables.

ϕ_0 and σ_0 are initialized in the state space algorithm the forward filter step is initialized with a diffuse prior.

The state equations allow for a use of a standard Kalman filter and smoother techniques to draw the parameters in each regime. Define Conditionally conjugate prior for the innovation variances:

$$\begin{aligned}
V^{-1} &\sim W(\underline{\nu}_V, \underline{V}_V^{-1}) \text{ where } \underline{\nu}_V, \underline{V}_V \text{ are hyperparameters} && (5) \\
\eta^{-1} &\sim Gamma(\underline{\alpha}_\eta, \underline{\beta}_\eta) \text{ where } \underline{\alpha}_\eta, \underline{\beta}_\eta \text{ are hyperparameters} && (6) \\
& && (7)
\end{aligned}$$

There is also a prior found in Koop and Potter (2004) in which V and η change with each regime that may be property that could be of interest in some instances.

3.3 HMM Posterior distributions

Calculating the transition probability matrix P involves $O(T^3)$ calculations. To simulate the drawing of states S_T you need to use a recursive strategy starting at the initial state and then draw each state in reverse order. The details can be found in Koop and Potter (2004). To calculate the parameters of the autoregressive model with stochastic volatility the use of a standard state space model with stochastic volatility is acceptable. $p(\Theta|Y_T, S_T, \lambda, V, \eta, \beta_\lambda)$ is drawn from some standard state space algorithms with stochastic volatility, one-step ahead accept rejection, and Kalman filter.

Posterior full conditional distributions:

$$\lambda_m|Y_T, S_T, V, \eta, \beta_\lambda \sim G(\underline{\alpha}_\lambda + d_m, [\beta_\lambda^{-1} + 1]^{-1})$$

$$V^{-1}|Y_T, S_T, \Theta, \lambda, \eta \sim W(\underline{\nu}_V + M, [\underline{V}_V + \sum_{m=1}^{M}(\phi_m - \phi_{m-1})(\phi_m - \phi_{m-1})^\top]^{-1})$$

$$\eta^{-1}|Y_T, S_T, \Theta, \lambda \sim G(\underline{\alpha}_\eta + \frac{M}{2}, 1/(\underline{\beta}_\eta + \frac{1}{2}\sum_{m=1}^{M}(\sigma_m - \sigma_{m-1})^2))$$

$$\beta_\lambda^{-1}|Y_T, S_T, \Theta, \lambda, V, \eta \sim G(M\underline{\alpha}_\lambda + \underline{\xi}_1, [\sum_{m=1}^{M}\lambda_m + \frac{1}{\underline{\xi}_2}]^{-1})$$

The outline of the MCMC algorithm can be found in the appendix of Koop and Potter (2004). When studying this method we found while it is very interesting and has some very desirable properties, the actual implementation is complex especially in reference to the state space parameters and autoregressive parameters.

4 Product Partition Model Framework

The framework for Product Partition Model was developed by Barry and Hartigan (1993) and then implemented into an R package called bcp by Erdman and Emerson (2007). The main idea of this model is to partition the data into mutually independent blocks. For the Normal error model they assume that $X_i \overset{\text{ind}}{\sim} N(\mu_i, \sigma^2)$ and the probability of a change point at point i is p, in which $p_i \perp p_j$ where $i \neq j$. They further mention that any other parametric assumption can similarly replace the normal assumption. In the Barry and Hartigan (1993) they also say that this independent assumption can be relaxed and that given the parameters and partition, observations in different blocks are mutually independent. We also partition the data as $\rho = (U_1, \ldots, U_n)$ where $U_i = 1$ indicates a change point at $i + 1$.

Prior: (i<j)

$$\pi(\mu_0)\pi(\sigma)\pi(p)\pi(w) \propto 1 \cdot \frac{1}{\sigma^2} \cdot \frac{1}{p_0} \cdot \frac{1}{w_0}, \text{ where } w = \frac{\sigma^2}{\sigma_0^2 + \sigma^2}$$

$$\mu_{ij} \sim N(\mu_0, \sigma_0^2/(j - i)), \text{ is the block prior from points } [i + 1, j]$$

$$X_i \overset{\text{ind}}{\sim} N(\mu_i, \sigma^2)$$

The Posterior of $\hat{\mu_{ib}}$ need to be computed iteratively by a system of 3 equations found in Barry and Hartigan (1993). To calculate the odds of a change point Barry and Hartigan (1993) used $\frac{p_i}{1-p_i} = \frac{P(U_i=1|\boldsymbol{X}, U_j, i\neq j)}{P(U_i=0|\boldsymbol{X}, U_j, i\neq j)}$, but Erdman and Emerson (2007) found this ratio to be numerically unstable for long sequences and suggest a simplification using incomplete beta integrals. The new odds of a change point can be calculated by

$$\frac{p_i}{1-p_i} = \frac{P(U_i = 1|\boldsymbol{X}, U_j, i \neq j)}{P(U_i = 0|\boldsymbol{X}, U_j, i \neq j)} = \left(\frac{W_0}{W_1}\right)^{\frac{n-b-2}{2}} \left(\frac{B_0}{B_1}\right)^{\frac{b+1}{2}} \sqrt{\frac{W_1}{B_1}}\cdot$$
$$\cdot \frac{\int_0^{\frac{B_1\lambda/W_1}{1+B_1\lambda/W_1}} p^{(b+2)/2}(1-p)^{(n-b-3)/2}dp}{\int_0^{\frac{B_0\lambda/W_0}{1+B_0\lambda/W_0}} p^{(b+1)/2}(1-p)^{(n-b-2)/2}dp} \cdot \frac{\int_0^{\gamma} p^b(1-p)^{n-b-1}dp}{\int_0^{\gamma} p^{b-1}(1-p)^{n-b}dp}$$

A hurdle that is based on the formulation of this method is that we cannot estimate the probability of a change at the last observation because there is no next observation to compare it with. One advantage found through simulations and Real Data analysis is the choice of the prior on the probability of a change at point i had little impact on the posterior probabilities of a change point at point i. This is a very desirable feature in that it lets the data speak for itself.

5 Simulation

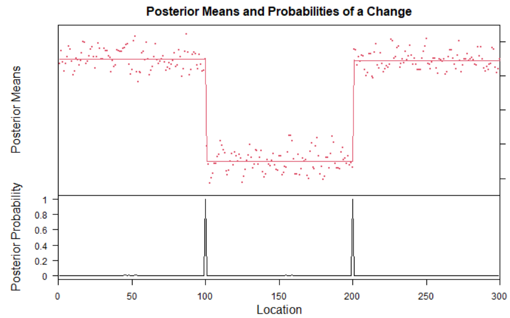

Figure 1: Simulated Change point data of AR(2) with independent segments. Each segment is 100 observations and each segment has a mean and scale of $\mu = (3, -3, 3)$ and $\sigma^2 = c(.5, .5, .5)$

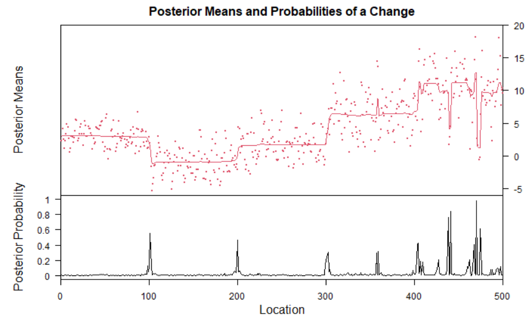

Figure 2: Simulated Change point data of AR(2) with dependent segments according to the model framework of Koop and Potter (2007). Each segment has a length of 100 observations and an initial mean and scale of $\mu = (3, -4, 3, 4, 3)$ and $\sigma^2 = c(.5, .5, .5, .5, .5)$

For each section of the simulated data, an Autoregressive (AR) model with two lags was considered. For the AR(2) coefficients we considered $\text{ar}_1 = -0.3$ and $\text{ar}_2 = 0.5$, which satisfies the 3 Yule Walker equations for AR(2) model: $\text{ar}_1 + \text{ar}_2 < 1$, $\text{ar}_2 - \text{ar}_1 < 1$, $|\text{ar}| < 1$.

For the simulated data in Figure 1 each segment has independent mean, variance, and ar coefficients which is in accordance of the model assumptions of the Partition model described in Barry and Hartigan (1993). In the top figure of Figure 1 we can see that the estimated mean remains constant across each segment. We can also see from the bottom figure of figure1 that the posterior probability of a change point is correctly identified at time equal to 100 and 200. The bcp model prior settings are kept at default for the prior on the probability of a change at each observation $p_0 = 0.2$.

For the simulated data according the Koop and Potter (2004) model framework shown in the top figure of figure2 that incorporates the information of the previous state/segment by adding the previous mean and variance to the new state/segment's mean and variance times $\exp(\sigma/2)$. Setting the segment $\mu = (3, -4, 3, 4, 3)$ and $\sigma = c(.5, .5, .5, .5, .5)$ and then by applying the Koop and Potter (2004) structure we then get a $\mu = (3, -1, 2, 6, 9)$ and $\sigma = c(.5, 1, 1.5, 2, 2.5)$. In Figure 2 we can observe that with the additive property of the state space equations the partition model implemented through the bcp package has a harder time with the last two segments. This makes sense because by applying the bcp model are looking for a change in the mean of the data, so it has a harder time picking up changes due to high variance. The bcp model correctly picks up possible change points at time 100, 200 and 300, but it has a hard time in the last segment because of the high variance.

Overall, the result of the simulations corresponds to what we expect from the bcp model. It also works very well for when data follows the model assumption of block mutual independence, but even in the case for the Koop and Potter (2004) data structure where the independence assumption of segments is not fully met the bcp model still preforms fairly well.

6 Real GDP Growth Data

For this data we will perform change point analysis using the bcp package and we will only look at the means and not consider an autoregressive framework.

6.1 United States of America (USA) (1947 Q2 - 2023 Q2) 305 Observations

It can be seen in Figure 3 and Table 2 that Covid had a big impact of about 8% negative and positive change in the GDP on the economy from 2020-Q1 to 2020-Q3. It seems from the figure that after the Covid period the level of percent change goes back to about the same level as before Covid. We can also see that a change is detected in 2008 in Figure 3 the same time as the housing crash. It can also be seen that there is quite a bit of estimated change points around 1978 - 1982 there are couple possible factors for this: the first is that inflation went as high as 11.1%, rising Fed Rates from 10.5% in 1979 to 17.5% in 1980, lastly was Iranian Revolution that caused oil prices to double. Our model also detected a change from 1973-1975, the time of the oil embargo that quadrupled oil prices and a

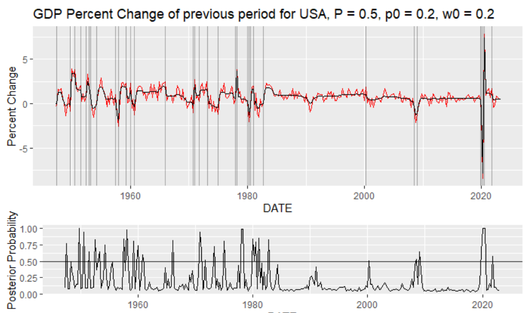

Figure 3: USA GDP Growth and estimated Change Points

	DATE	Post.Prob	Pct.Chg
1	1949-Q4	1.000	-0.84
11	2020-Q1	1.000	-1.18
12	2020-Q2	1.000	-8.48
13	2020-Q3	1.000	7.85
7	1978-Q1	0.990	0.32
8	1978-Q2	0.986	3.86
5	1958-Q1	0.978	-2.60
2	1950-Q3	0.942	3.86
6	1970-Q4	0.942	-1.07
10	1981-Q1	0.854	1.96
4	1957-Q3	0.840	0.98
9	1980-Q1	0.834	0.31
3	1952-Q3	0.832	0.72

Table 2: Detected Change Points of GDP Growth for USA with Posterior Probability greater than 0.83

time of increasing inflation. The period of 1957-Q3 to 1958-Q1 is marked as the investment Bust because global investment surged in the wake of the end of the Korean War, and the 1957 Asian Flu pandemic that killed 70 - 100 thousand Americans in 1957 that hurt US industrial production. The early 1950's changes detected can be attributed to the Korean War. The change point detected in 1949-Q4 can be attributed to the high inflation rate of 19% in 1947 and the policies implemented to try to curb the high inflation that led to a mild recession. Overall, the bcp model does a good job of picking up the known instances of recessions in the USA's economy. The background knowledge of the events for the estimated change points is based on the article The Investopedia (2023).

6.2 Great Britain (GBR) (1955 Q2 - 2023 Q2) 273 Observations

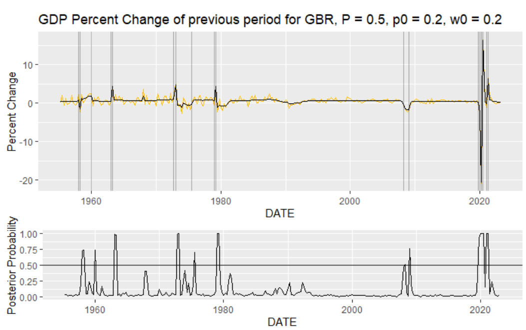

Figure 4: GBR GDP Growth and estimated Change Points

	DATE	Post.Prob	Pct.Chg
4	1973-Q1	1.000	4.91
6	1979-Q2	1.000	4.43
8	2020-Q1	1.000	-2.64
9	2020-Q2	1.000	-20.99
10	2020-Q3	1.000	16.61
11	2021-Q1	1.000	-1.05
12	2021-Q2	1.000	6.53
5	1979-Q1	0.998	-0.52
1	1963-Q1	0.992	0.46
3	1972-Q4	0.992	1.58
2	1963-Q2	0.980	4.42
7	2019-Q4	0.932	-0.03

Table 4: Detected Change Points of GDP Growth for GBR with Posterior Probability greater than 0.8

Great Britain experienced the greatest change in economic Growth during the Covid Pandemic out of the 8 countries studied in this paper. With a probability of a change occurring close to 100% the decline in GDP Growth of 23.63% from 2020-Q1 to 2020-Q2 is a very significant drop in the total economy of GBR. In the following quarter 2020-Q3 there is an increase of 16.61% in GDP Growth, but it is not until the end of 2021-Q2 that the Covid Recession is fully over, lasting about 2 more Quarters of a Covid recession than the USA. In addition, to the Covid recession GBR seems to experience similar changes in economic activity as the USA as seen in Figure 4. With change points detected around the USA housing crisis of 2008, 1978-1979 Oil shortage due to the Iranian Revolution, 1972 - 1975 the period of the Oil embargo and the impact of High interest rates of USA. From the Figure 4 it seems that GBR's economy is fairly stable with little variance, but this is caused by the huge change during the Covid recession that massively affects the scale of the graph. Overall, though, our bcp model detects fewer change points than for the USA, this indicates that GBR's economy experiences a relatively stable economic activity.

6.3 South Korea (KOR) (1960 Q2 - 2023 Q2) 253 Observations

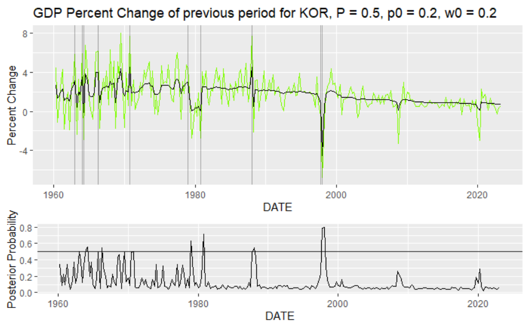

Figure 5: KOR GDP Growth and estimated Change Points

	DATE	Post.Prob	Pct.Chg
10	1998-Q1	0.798	-6.82
9	1997-Q4	0.792	-0.46
7	1980-Q4	0.718	-2.77
6	1979-Q1	0.628	4.49
3	1964-Q2	0.556	-2.87
4	1966-Q2	0.548	6.19
8	1988-Q1	0.546	7.70
2	1964-Q1	0.524	5.97
1	1963-Q1	0.504	5.86
5	1970-Q4	0.502	7.70

Table 6: Detected Change Points of GDP Growth for KOR with Posterior Probability greater than 0.5

For the South Korean GDP Growth shown in the Figure 5 we can see that while there is a spike in both the 2020 Covid and 2008 USA housing Crisis both instances did not have a detectable change point by the bcp model. It is surprising in the instance of the Covid impact because in comparison with the other economies discussed in this paper all experienced change points in their GDP Growth during this time except South Korea. One instance of a change point that does not appear in the other economies is the period of 1997-Q4 to 1998-Q1 where South Korea suffered a 6.82% decrease in GDP Growth. This contraction can be attributed to many factors, but the main factors included the bankruptcy of multiple large chaebol's (Korean Industrial Conglomerates) and foreign investment leaving en masse. The effect of this was that foreign banks did not extend credit and South Korea had to ask for a rescue package from the International Monetary Fund (IMF). More details on this incident can be found in Seliger (2005). For the rest of the change points, they mostly correspond with change points discovered for the USA GDP Growth such as the 1979-1982 USA high inflation and the high oil prices. We also know that there are a few change points discovered in the early 1960's that can be attributed to a weak balance of payments and foreign exchange reserves at low levels in reference to previous years. It was also the start of many industrial development programs to increase economic growth.

6.4 Australia (AUS) (1960 Q2 - 2023 Q2) 253 Observations

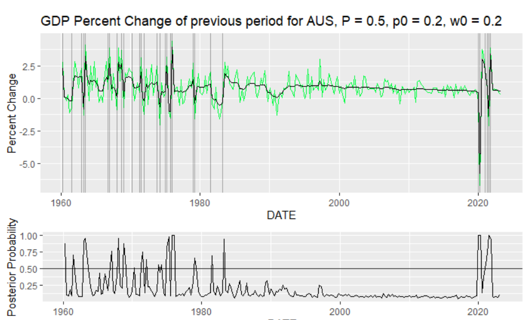

Figure 6: AUS GDP Growth and estimated Change Points

	DATE	Post.Prob	Pct.Chg
7	1975-Q4	1.000	-1.51
10	2020-Q1	1.000	-0.21
11	2020-Q2	1.000	-6.71
8	1976-Q1	0.996	4.42
12	2021-Q3	0.996	-2.08
6	1975-Q2	0.974	3.16
3	1963-Q2	0.958	-1.30
4	1968-Q1	0.956	-0.89
13	2021-Q4	0.946	3.88
9	1983-Q2	0.938	-0.20
2	1963-Q1	0.926	2.31
1	1960-Q2	0.872	2.77
5	1968-Q4	0.872	3.73

Table 8: Detected Change Points of GDP Growth for AUS with Posterior Probability greater than 0.85

For Australia's GDP Growth shown in Figure 6 it for the most part resembles the USA GDP Growth. With similar incidents such as the Covid Recession but lasting a longer length of time from 2020-Q1 to 2021-Q4. AUS also experienced a change in the economic growth during the 1982 - 1983 which may be in response to the high USA inflation and the Iranian Revolution increasing Global Oil prices. The time period from 1973 - 1975 can be attributed to political turmoil and a series of bad economic policies to try and fight high inflation with election promises Price (1976).

6.5 Canada (CAN) (1961 Q2 - 2023 Q2) 249 Observations

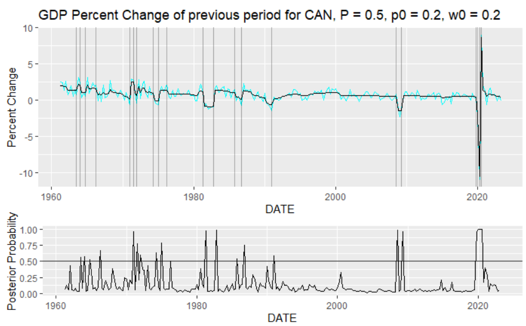

Figure 7: CAN GDP Growth and estimated Change Points

	DATE	Post.Prob	Pct.Chg
10	2020-Q1	1.000	-2.13
11	2020-Q2	1.000	-10.93
12	2020-Q3	1.000	9.02
5	1982-Q4	0.992	-0.92
7	2008-Q3	0.984	0.82
4	1981-Q2	0.978	1.12
1	1971-Q1	0.966	-0.57
8	2009-Q2	0.966	-1.09
9	2019-Q4	0.966	0.33
3	1975-Q1	0.788	-0.63
2	1971-Q3	0.780	2.78
6	1986-Q4	0.752	-0.72

Table 10: Detected Change Points of GDP Growth for CAN with Posterior Probability greater than 0.7

For Canada the Figure 7 showing the GDP Growth resembles a less turbulent GDP Growth pattern than in the USA. This makes intuitive sense, because Canada and the USA are very close geographically and have many trade agreements. Therefore, we notice change points for Canada from 2020-Q1 to 2020-Q3 Covid recession, 2008 - 2009 housing market crash in the USA, 1981 - 1982 high inflation and high Oil prices, and the early 1970's a time of high inflation and the Oil Embargo. The Canadian Economic Growth seems to be closely related to the USA's economic Growth.

6.6 Norway (NOR) (1978 Q1 - 2023 Q2) 182 Observations

The Norway GDP Growth Chart and estimated Change points shown in Figure 15 is very interesting in that the time series has a constant mean except a slight change detected in 2020-Q2 with a posterior probability of 0.506 and a percent change of -5.34%. Therefore, the Norwegian economy seems to be very stable with only one change point detected.

6.7 Japan (JPN) (1994 Q1 - 2023 Q2) 118 Observations

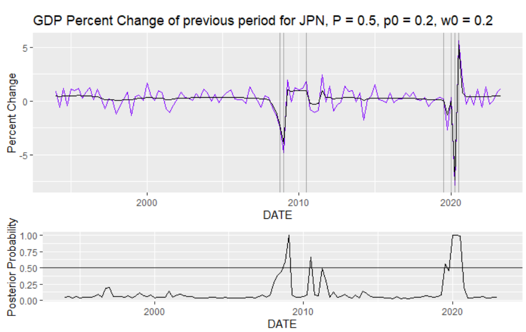

Figure 8: JPN GDP Growth and estimated Change Points

	DATE	Post.Prob	Pct.Chg
2	2009-Q1	1.000	-4.82
5	2020-Q1	1.000	0.37
6	2020-Q2	1.000	-7.86
7	2020-Q3	0.992	5.64
3	2010-Q3	0.666	1.80
1	2008-Q4	0.594	-2.44
4	2019-Q3	0.560	0.19

Table 12: Detected Change Points of GDP Growth for JPN with Posterior Probability greater than 0.5

For Japan's GDP Growth shown in Figure 8 there are only two time periods where there was a significant change in GDP Growth using a 0.5 cutoff. One of them was the Covid Recession from 2019-Q1 to 2020-Q3 with the largest change found in 2020-Q2 with a decrease of 7.86% in GDP Growth and a posterior probability of a change occurring equal to or close to 100%. The other period of a change that was detected was between 2008-Q4, 2009-Q1, and 2010-Q3 which can be attributed to the 2008 USA housing crash that severely affected the all auto industries world wide. In the 2008-Q4 and 2009-Q1 the USA gave a bailout to US automakers that hurt Japanese auto companies who had to compete with domestic USA auto companies. Japan's export of cars to the United States fell by 75% in 2008-Q4. Sommer (2009)

6.8 Ireland (IRL) (1995 Q1 - 2023 Q2) 113 Observations

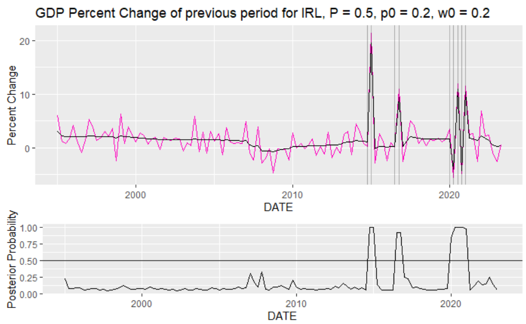

Figure 9: IRL GDP Growth and estimated Change Points

	DATE	Post.Prob	Pct.Chg
1	2014-Q4	1.000	0.19
2	2015-Q1	1.000	21.38
6	2020-Q2	1.000	-5.61
7	2020-Q3	1.000	11.91
8	2020-Q4	1.000	-4.82
9	2021-Q1	0.978	11.49
3	2016-Q3	0.922	0.13
4	2016-Q4	0.918	10.90
5	2020-Q1	0.842	3.41

Table 14: Detected Change Points of GDP Growth for IRL with Posterior Probability greater than 0.5

Lastly, Ireland's GDP Growth shown in Figure 9 there are three incidents of a change in the mean of the GDP Growth of Ireland. The Covid recession time frame is like the other countries, lasting from 2020-Q2 to 2021-Q1. The next two changes are interesting because they are specific to Ireland in 2016-Q4 Ireland saw a 10.90% increase in GDP Growth and in 2015-Q1 a 21.38% increase in GDP Growth. The reason for this is the economic policies by Ireland that lowered the corporate tax rate, which prompted many large multinational companies to relocate their economic activities and intellectual property to Ireland. OECD (2016) As result, Ireland experienced a huge increase in their GDP Growth.

7 Future Work and Conclusion

In the future, I would like to implement the Koop and Potter (2004) model once I have a better understanding of State Space modeling approaches and how to incorporate Kalman filters into a model. I also saw later that there is an R package called shrinkTVP Knaus et al. (2021) with different shrinkage choices such as spike and slab priors and horseshoe priors. The Koop and Potter (2004) model nests a Time Varying Parameter (TVP) model into their model, so this package may be a good starting point. From the simulation and real data analysis done throughout this paper, it seems that a partition product model (bcp) performs very well. If you consider that the observations of GDP growth are not independent given the parameters and block, we can see from historical knowledge of events at the change points estimated, that the model still preforms well with the mutual independence assumption being violated. We have also seen how many incidents of change points in one economy, are present in other countries' economies as well.

8 Figures

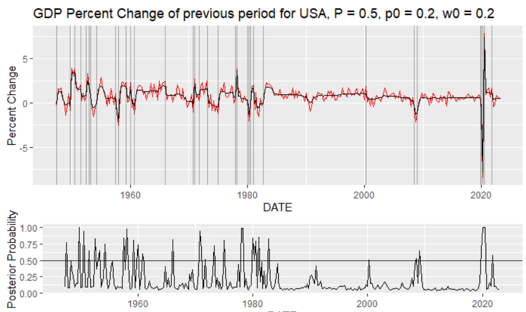

Figure 10: United States (USA) (1947 Q2 - 2023 Q2) 305 Obs

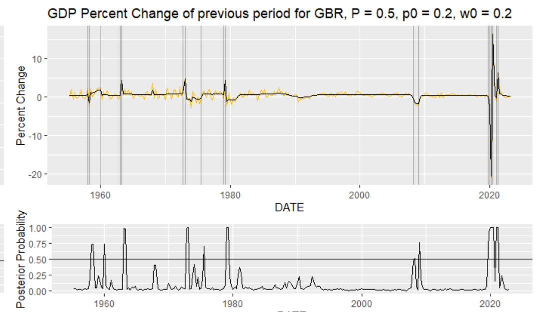

Figure 11: Great Britain (GBR) (1955 Q2 - 2023 Q2) 273 Observations

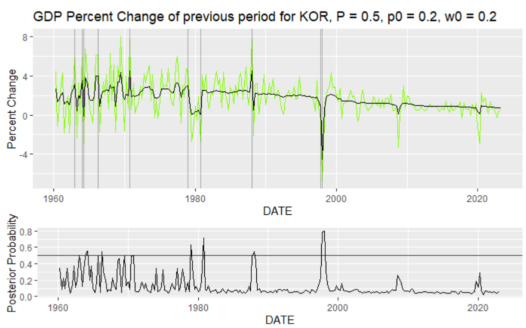

Figure 12: South Korea (KOR) (1960 Q2 - 2023 Q2) 253 Observations

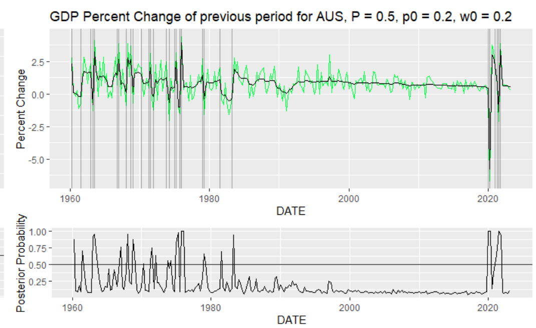

Figure 13: Australia (AUS) (1960 Q2 - 2023 Q2) 253 Observations

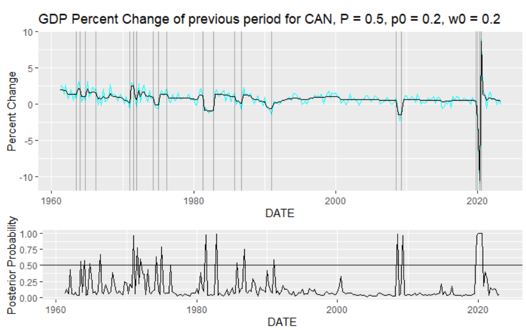

Figure 14: Canada (CAN) (1961 Q2 - 2023 Q2) 249 Observations

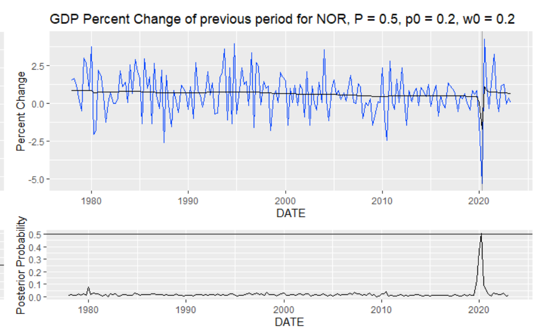

Figure 15: Norway (NOR) (1978 Q1 - 2023 Q2) 182 Observations

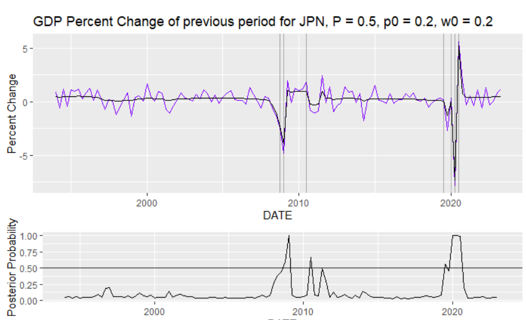

Figure 16: Japan (JPN) (1994 Q1 - 2023 Q2) 118 Observations

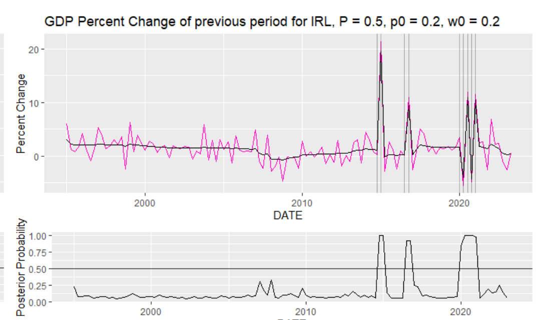

Figure 17: Ireland (IRL) (1995 Q1 - 2023 Q2) 113 Observations

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