Exam 2 Problems

1. (Areas and Volumes)

   (a) Use horizontal slices to find the area of a ladybug's shell (a semicircle) of radius 4 mm.

   (b) Some termites have been found living in a square pyramid of height 4 inches and base 12 inches with 10 inches of cylindrical tunnelling with radius 5 inches outside the home base of the pyramid. Find the total volume occupied by the termites.

   (c) Rotating the parabola given by $y = -x^2 + 2x$ and enclosed by the x-axis around the x-axis approximates your neighborhood beehive. Find the volume of the solid of revolution described above.

2. (Geometry) A special ant farm has the shape of the function $y = x^2$ bounded by the lines $y = 1$ and $x = 0$. Furthermore, the thickness of the ant farm perpendicular to the x-axis is given by equilateral triangles. Find the amount of sand we need to completely fill our ant farm.

3. A beehive can be thought of as a solid in 3-dimensional space. One horizontal cross-section of this solid is a circle of radius 1 sitting in the x, y-plane centered at the origin. Vertical cross sections perpendicular to the y-axis which sit above this circle are triangles whose heights are equal to the bases cubed, and vertical cross sections perpendicular to the y-axis below this circle are circles. What is the volume of the beehive?

4. (Polar Coordinates)

   The graph of the function $r = \sin(2\theta)$ looks kind of like a butterfly. Find the area of the region outside $r = \sin(2\theta)$ and inside the circle $r = 1$.

5. (Density) Your typical metropolitan ant colony is circular, and has population density higher at its center than at its edges. Suppose the population density is given by $15 - r$ ants per square inch at distance r from the center of the colony. What is the total population of the colony? (Don't forget to find the radius of the colony!)

6. (Work) A dung beetle is rolling its ball of dung along the ground, picking up more as it goes. Suppose the ball weighs 1lb to start (it is a very large beetle), and the ball picks up dung at a rate of .5lb/ft. If the beetle rolls the ball 3 feet, how much work is done on the dung ball?

7. The ants are building a great pyramid to celebrate their victory over the grasshoppers. They are building the pyramid out of stone that is 70 lb/m^3 and is 10 meters high. The base of the pyramid is a rectangle which is 5 by 40 meters. How much work will the ants do to build the pyramid?

8. (Sequences) Write the first four terms of the following sequences, then determine (with justification) their convergence

   (a) $a_n = 2n^2 - 3$.

   (b) $b_1 = \dfrac{1}{2}$ and $b_n = b_{n-1}^2$ for all $n \geq 2$.

   (c) $c_n = \sin(\pi n)$.

   (d) $d_n = 1 - \dfrac{1}{n}$.

   (e) $e_n = \dfrac{2(-1)^n}{n}$.

   (f) $f_n = \dfrac{3n^3 - 4n^2 + 7}{2n^3 - 2n^2 - 8n - 8}$.

9. (Geometric Series)

   (a) Find the sum of the following finite and infinite geometric series if they converge. If they diverge, say why.

i. The infinite geometric series with first term $a = \frac{5}{7}$ and common ratio $x = \frac{1}{2}$

ii. The infinite geometric series with first term $a = \frac{5}{7}$ and common ratio $x = 2$

iii. $\sum_{n=1}^{\infty} 4 \left(\frac{1}{7}\right)^{n-1}$

iv. $\sum_{n=1}^{8} (.15)^{n-1}$

v. $6 + 3 + \frac{3}{2} + \frac{3}{4} + ... + \frac{3}{16}$

(b) For which values of x and y will the geometric series

$$\sum_{n=1}^{\infty} 3y \left(\frac{-x}{2}\right)^{n-1}$$

converge?

(c) For which values of x and y will the geometric series

$$\sum_{n=1}^{8} y \left(\frac{2x}{5}\right)^{n-1}$$

converge?

(d) Suppose a bee flies in quickly to land on a flower, resulting in the flower swaying back and forth (similarly to an upside down pendulum). If the flower sways 1 *in* to the left on the bee's initial landing (recoiling all the way back to 1 *in* to the right), then the flower sways 25% less for each swing, how far will the bee travel while atop the flower?

10. Determine if the following series converge or diverge. State how you know.

(a) $\sum_{n=1}^{\infty} ((\frac{3}{4})^n + \frac{1}{n})$

(b) $\sum_{n=1}^{\infty} \frac{n}{n+1}$

(c) $\sum_{n=1}^{\infty} \frac{1}{n(1+\ln(n))}$

(d) $\sum_{n=1}^{\infty} \frac{1}{n^4 + e^n}$

(e) $\sum_{n=1}^{\infty} \frac{n+1}{n^2+2}$

(f) $\sum_{n=1}^{\infty} \frac{2^n}{3^n - 1}$

(g) $\sum_{n=2}^{\infty} \frac{1}{n \ln(n)}$

(h) $\sum_{n=1}^{\infty} \frac{1}{n^2 + n}$