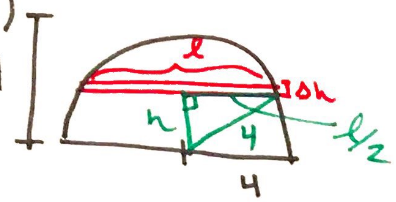
1) a)



Area of slice: $l\Delta h = \sqrt{4(16-h^2)}\,\Delta h$ ← rectangle

use pythagorean theorem:

$$4^2 = h^2 + \left(\frac{l}{2}\right)^2$$

$$16 - h^2 = \frac{l^2}{4}$$

$$4(16-h^2) = l^2$$

$$\sqrt{4(16-h^2)} = l.$$

Area of shell: $\int_0^4 \sqrt{4(16-h^2)}\,dh$

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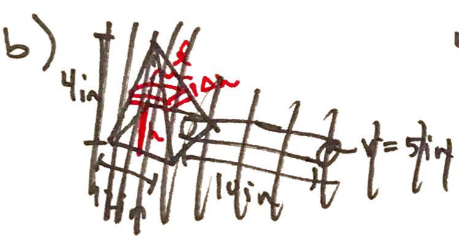
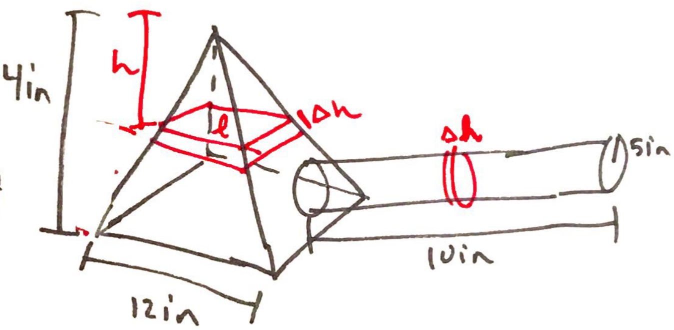
b)





Volume of pyramid:

rectangular prism → Volume of slice: $l^2\Delta h. = (3h)^2\Delta h$

Find l in terms of h: $\frac{l}{h} = \frac{12}{4} \Rightarrow l = 3h$

Volume of pyramid: $\int_0^4 (3h)^2\,dh.$

Volume of cylinder:

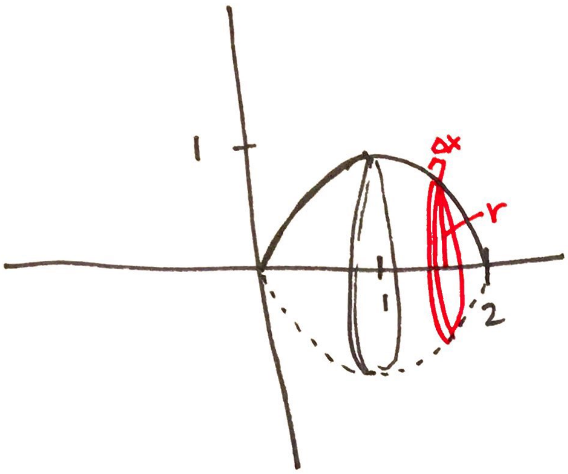
disk → Volume of slice: $\pi r^2 \Delta h = \pi(5^2)\Delta h = 25\pi\Delta h.$

↑ since radius is constant.

volume of cylinder: $\int_0^{10} 25\pi\,dh.$

Total Volume: $\int_0^4 9h^2\,dh + \int_0^{10} 25\pi\,dh.$

1) c)



disk.

Volume of slice: $\pi r^2 \Delta x$

Find r in terms of x:

the height of the function gives the radius!

$r = -x^2 + 2x$.

Volume of slice: $\pi(-x^2+2x)^2 \Delta x$.

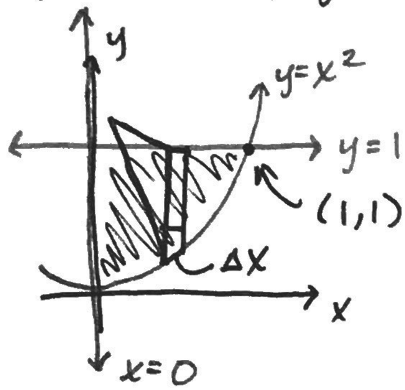
Volume of solid:

$$\int_0^2 \pi(-x^2+2x)^2 \, dx.$$

Note these are the possible x values.

Exam 2 Review

2. special ant farm



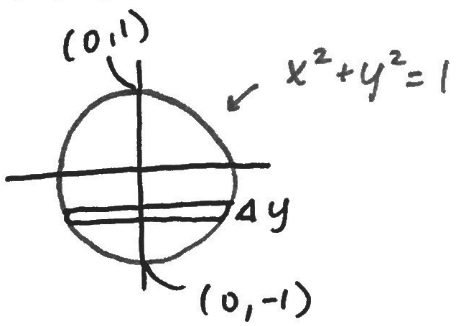
$$h^2 + \left(\frac{1-x^2}{2}\right)^2 = (1-x^2)^2$$

$$h = \sqrt{(1-x^2)^2 - \left(\frac{1-x^2}{2}\right)^2}$$

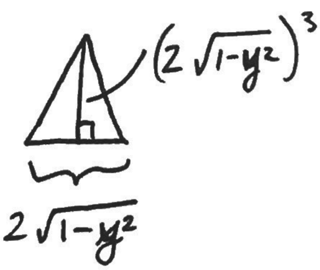
area of $\Delta = \frac{1}{2}(1-x^2)\sqrt{(1-x^2)^2 - \left(\frac{1-x^2}{2}\right)^2}$

Volume of ant farm $= \displaystyle\int_0^1 \frac{1}{2}(1-x^2)\sqrt{(1-x^2)^2 - \left(\frac{1-x^2}{2}\right)^2}\, dx$

3. beehive



above the circle:



area $\Delta =$

$$\frac{1}{2}\left(2\sqrt{1-y^2}\right)\left(2\sqrt{1-y^2}\right)^3$$

below the circle:

area ◠ $=$

$$\frac{1}{2}\pi\left(\sqrt{1-y^2}\right)^2$$

total volume:

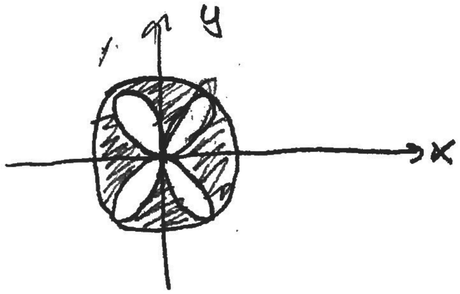
$$\int_{-1}^{1} \frac{1}{2}\left(2\sqrt{1-y^2}\right)\left(2\sqrt{1-y^2}\right)^3 + \frac{1}{2}\pi\left(\sqrt{1-y^2}\right)^2\, dy$$

~~$\frac{1+2^n \ln(2)}{2^n + n2^n \ln(2)}$~~

~~$\frac{n}{n2^n} + \frac{2^n}{n2^n} \to 0$~~

4) $r = \sin(2\theta) \quad r = 1$

Area inside circle - outside cardiod



$$A = \frac{1}{2}\int_0^{2\pi} (1)^2 - (\sin(2\theta))^2 \, d\theta$$

(5) radius of colony: when density = 0

$$15 - r = 0.$$
$$r = 15$$

slice at distance r from center:

a very thin ring

area of a slice: $2\pi r \Delta r$

population ~~density~~ of a slice: $(15-r) \cdot 2\pi r \Delta r$

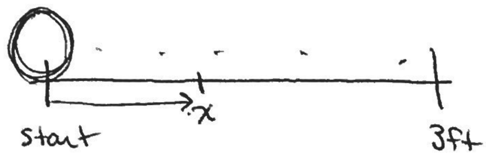
integral: $$\int_0^{15} 2\pi r(15-r)\,dr$$

$$= \int_0^{15} 2\pi(15r - r^2)\,dr$$

$$= 2\pi\left(\frac{15}{2}r^2 - \frac{r^3}{3}\right)\Big|_0^{15}$$

$$= \pi\left(15^3 - 2\frac{15^3}{3}\right)$$

(6)



slice along x's... Δx is how far the ball is moved when it has a certain weight

Weight of the ball x feet from the start:

$$1 \text{ lb} + \tfrac{1}{2} \text{ lbs/ft} \times x \text{ feet} = 1 + \tfrac{1}{2}x \text{ lbs}$$

integral:

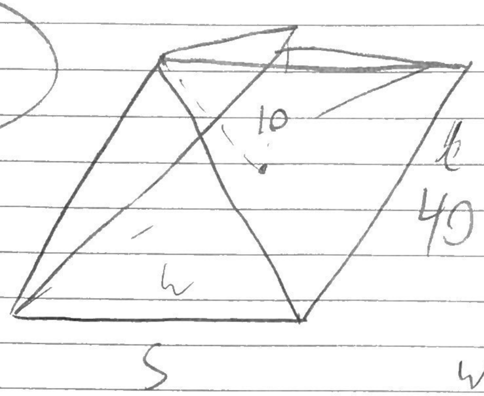
$$\int_0^3 \underbrace{\left(1 + \tfrac{1}{2}x\right)}_{\text{lbs}} \underbrace{dx}_{\text{ft}} \qquad = \text{lb} \cdot \text{ft}$$

$$= \left(x + \frac{x^2}{4}\right)\Big|_0^3$$

$$= 3 + \frac{9}{4}$$

8)

70 kg/m^3



$wl\,dh$

$w = 5 - \frac{1}{2}h$

$l = 40 - 4h$

$$\int_0^{10} 9.8\left(5-\frac{1}{2}h\right)(40-4h)\,h\,dh$$

$$\int_0^{10} (9.8)(70)\left(5-\frac{1}{2}h\right)(40-4h)\,h\,dh$$

Exam 2 Reiview

7

(a) $a_n = 2n^2 - 3$

$a_1 = -1$

$a_2 = 5$

$a_3 = 13$

$a_4 = 19$

diverges because

$\lim_{n\to\infty} 2n^2 - 3 = \infty$

(b) $b_1 = 1/2, \; b_n = b_{n-1}^2$ for all $n \geq 2$

$b_1 = 1/2$

$b_2 = 1/4$

$b_3 = 1/8$

$b_4 = 1/16$

converges because $0 < b_n < 1$ for all n and $b_n \geq b_n^2 = b_{n+1}$

(c) $c_n = \sin(\pi n)$

$c_1 = 0$

$c_2 = 0$

$c_3 = 0$

$c_4 = 0$

converges because

$\sin(\pi n) = 0$ for all n

Exam 2 Review

Problem 7

ⓓ $d_n = 1 - \frac{1}{n}$

$d_1 = 0$

$d_2 = 1/2$

$d_3 = 2/3$

$d_4 = 3/4$

converges because

$\lim_{n\to\infty} 1 - \frac{1}{n} = 1$

ⓔ $e_n = \frac{2(-1)^n}{n}$

$e_1 = -2$

$e_2 = 1$

$e_3 = -2/3$

$e_4 = 1/2$

converges by the squeeze theorem.

$$\frac{-2}{n} \leq \frac{2(-1)^n}{n} \leq \frac{2}{n}$$

and $\lim_{n\to\infty} \frac{-2}{n} = 0 = \lim_{n\to\infty} \frac{2}{n}$

so $\lim_{n\to\infty} \frac{2(-1)^n}{n} = 0$

ⓕ $f_n = \frac{3n^3 - 4n^2 + 7}{2n^3 - 2n^2 - 8n - 8}$

$f_1 = -3/8$

$f_2 = -15/16$

$f_3 = 13$

$f_4 = 135/56$

converges because

$$\lim_{n\to\infty} \frac{3n^3 - 4n^2 + 7}{2n^3 - 2n^2 - 8n - 8} = \frac{3}{2}$$

Exam 2 Review

8 a

i. infinite geometric series with $a = 5/7$, $x = 1/2$

since $|1/2| < 1$; $\dfrac{5/7}{1 - 1/2} = 10/7$

ii. infinite geometric series with $a = 5/7$, $x = 2$

diverges because $|2| > 1$

iii. $\sum_{n=1}^{\infty} 4\left(\frac{1}{7}\right)^{n-1}$

since $|1/7| < 1$; $\dfrac{4}{1 - 1/7} = \dfrac{4 \cdot 7}{6} = \dfrac{42}{6}$

iv. $\sum_{n=1}^{8} (.15)^{n-1}$

$$\frac{1 - (0.15)^n}{1 - 0.15} = \frac{1 - (0.15)^8}{1 - 0.15}$$

v. $6 + 3 + \frac{3}{2} + \frac{3}{4} + \dots + \frac{3}{16}$

$= 6 + \sum_{n=1}^{5} 3\left(\frac{1}{2}\right)^{n-1}$

$= 6 + 3\left(\dfrac{1 - (1/2)^5}{1 - 1/2}\right)$

Exam 2 Review

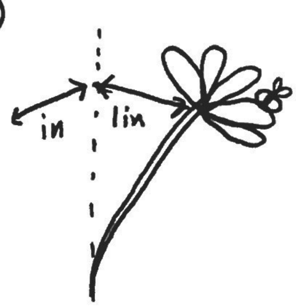
8

(b) $\left|\frac{-x}{2}\right| < 1$ so $-2 < x < 2$

y can be any value

(c) Since the sum is finite, it will converge for any values of x and y

(d)



distance travelled: $2 + 2(.75) + 2(.75)^2 + 2(.75)^3 \ldots$

$$= \sum_{n=1}^{\infty} 2(.75)^{n-1}$$

Since $|.75| < 1$, we have

$$\sum_{n=1}^{\infty} 2(.75)^{n-1} = \frac{2}{1-.75} = 8 \text{ in}$$

10) a) $\sum_{n=1}^{\infty}\left(\left(\frac{3}{4}\right)^n + \frac{1}{n}\right)$ diverges - compare to $\frac{1}{n}$

b) $\sum_{n=1}^{\infty} \frac{n}{n+1}$ diverges - use n^{th} term test

c) $\sum_{n=1}^{\infty} \frac{1}{n(1+\ln(n))}$ diverges - integral test

d) $\sum_{n=1}^{\infty} \frac{1}{n^4+e^n}$ converges - compare to $\frac{1}{n^4}$

e) $\sum_{n=1}^{\infty} \frac{n+1}{n^2+2}$ diverges - use limit comparison with $\frac{1}{n}$

f) $\sum_{n=1}^{\infty} \frac{2^n}{3^n-1}$ converges - use limit comparison with $\frac{2^n}{3^n} = \left(\frac{2}{3}\right)^n$, which is a geometric series

g) $\sum_{n=1}^{\infty} \frac{1}{n\ln(n)}$ diverges - integral test

h) $\sum_{n=1}^{\infty} \frac{1}{n^2+n}$ converges - limit comparison or comparison with $\frac{1}{n^2}$.