Independent Study: Control Systems

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1 System

The system I chose to control for this independent study is the magnetic levitation system from [2] and [1]. This system is similar to the one used in magnetic bearings and Maglev trains. A diagram of the system is shown in Figure 1, and the equation of motion of the ball is (1).

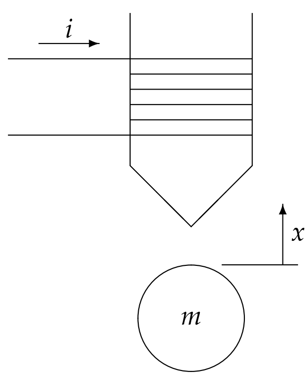

Figure 1: Diagram of the magnetic levitation system.

$$m\ddot{x} = f_m(x,i) - mg \tag{1}$$

The force on the ball from the magnet, $f_m(x,i)$ is shown in (2) [1], making the equation of motion (3).

$$f_m(x,i) = K_f\left(\frac{i}{x}\right)^2 \tag{2}$$

$$m\ddot{x} = K_f\left(\frac{i}{x}\right)^2 - mg \tag{3}$$

where K_f is the magnetic force constant.

In order to analyze the system and design a controller, it is necessary to linearize the system. Linearizing the system around the equilibrium point when the force from the electromagnet cancels out the force from gravity simplifies the system to (4), as derived in [1].

Constant	Value
K_f	$32\,654\,\mathrm{mN\,mm^2/A^2}$
g	$9800\,\mathrm{mm/s^2}$
m	$0.068\,\mathrm{kg}$

Table 1: Physical properties of the system

$$\partial\ddot{x} = \frac{2K_f i_{ss}^2}{x_{ss}^3 m}\partial x + \frac{2K_f i_{ss}}{x_{ss}^2 m}\partial i \tag{4}$$

where x_{ss} is the desired equilibrium position, i_{ss} is the current required to hold the ball at x_{ss} and

$$\partial x(t) \triangleq x(t) - x_{ss}$$
$$\partial i(t) \triangleq i(t) - i_{ss}$$

In order to simulate the system, it is rewritten in state space form, shown in (5)

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ b & 0 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ a \end{bmatrix}u \tag{5}$$

where

$$\mathbf{x} \triangleq \begin{bmatrix} \partial x \\ \partial \dot{x} \end{bmatrix}$$
$$u \triangleq \partial i$$
$$a \triangleq \frac{2K_f i_{ss}}{x_{ss}^2 m}$$
$$b \triangleq \frac{2K_f i_{ss}^2}{x_{ss}^3 m}$$

Using the physical properties listed in Table 1 [1] makes the state equation (6).

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ 2800 & 0 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 19600 \end{bmatrix}u \tag{6}$$

The poles of the system are shown in Figure 2. Since one of the poles is in the right-hand plane (52.92), the system is unstable. In physical terms this means that any deviation from equilibrium current or position will cause the ball to either fall to the floor or attach to the magnet.

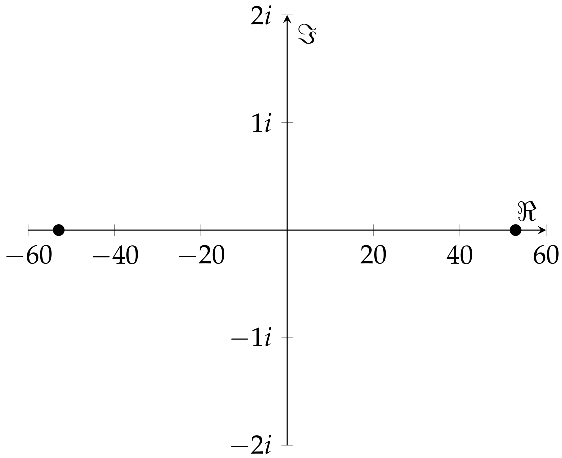

Figure 2: Open loop poles for the magnetic levitation system.

In order to model the disturbance response of the system, a second input must be added to the system. Since a disturbance to this system is a force applied directly to the ball, it can be modeled by adding $\frac{F_d}{m}$ to the acceleration term of the ball. This manifests as an additional column in the **B** matrix, shown below.

$$\mathbf{B} = \begin{bmatrix} 0 & 0 \\ a & \frac{1}{m} \end{bmatrix}$$

2 Control

2.1 Objectives

This system is commonly used in applications such as magnetic bearings and suspending Maglev trains. As such, a controller that accepts a reference input doesn't make much sense for this system. Instead, the controller should be able to effectively reject step and impulse disturbances such as a Maglev train being loaded with cargo or a bearing taking an impact. The location of the ball should not deviate more than 10 % (700 µm) from the equilibrium point (7 mm), since this could disrupt the operation of the bearing or Maglev system. In order to save energy, and reduce the risk of damaging any electrical systems, the control effort of the system should not deviate more than 25 % (250 mA) from the equilibrium point (1 A)

Additionally, this controller should deal with real world constraints such as not being able to measure complete state and handling noisy signals.

2.2 Dominant Second Order

One method of pole-placement control is dominant second order. This involves selecting the desired poles such that the response of the system is dominated by a second order response, with the rest of the poles far enough to the left that they have little effect on the response. There are only two poles to place, since this is a second order system, so the response can be exactly that of the selected second order poles.

2.2.1 Pole Selection

The poles used for this controller are that of the second order system with the desired response characteristics. The desired response characteristics for this system can be extrapolated from the potential uses of this system: magnetic bearings and Maglev trains. In both cases it is important for the control system to quickly reject impulse and step disturbances without straying to far from the desired ball position. This can be quantified as a settling time of less than or equal to 250 ms with a percent overshoot of less than 5 %. This coincides with $\zeta = 0.7$ and $\omega_n = 28.4$. With the desired ζ and ω_n we can find the poles for the second order system with the desired response, shown in (7).

$$p = -\zeta\omega_n \pm i\omega_n\sqrt{1-\zeta^2} = -19.85 \pm 20.25i \quad (7)$$

Using the `place` command in Matlab gives the matrix of control gains, **K**.

$$\mathbf{K} = \begin{bmatrix} 0.1839 & 0.0020 \end{bmatrix} \quad (8)$$

With full state feedback, the state space representation of the control system is (9).

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} - \mathbf{B}\mathbf{K}\mathbf{x} \quad (9)$$

2.2.2 State Estimation

The controller designed in the previous section assumes that the complete state of the system can be measured at any time. In practice, this is highly unlikely. Realistically, the only state that would be measured for this system is position. This can be added to the controller by adding a state estimator to the system. A state observer uses a feedback and an estimator gain, L, to estimate the unknown states given the input, output, and known state of the system. The gain is chosen to have much faster poles than closed loop system, this is acceptable because the state estimator doesn't represent a physical system and can response arbitrarily fast. The closed loop poles of the system are shown in (7). The poles of the estimator are chosen to be 5 times faster than the poles of the system, shown in (10).

$$q = -99.92 \pm 10.12i \tag{10}$$

Because of the duality between control and estimator problems, described Section 7.7.1 of [2], the same MATLAB function used to find the control gain can be used to find the estimator gain.

```
L = place(A', C', q).'
```

The estimator gain found using the above function is shown in (11).

$$\mathbf{L} = \begin{bmatrix} 198.47 \\ 22897.92 \end{bmatrix} \tag{11}$$

Using the MATLAB `reg` creates the regulator shown in Figure 3 and the `feedback` command creates the complete control system shown in Figure 4.

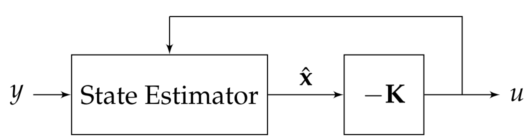

Figure 3: Block diagram for the dominant second order regulator.

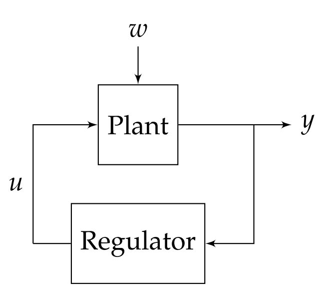

Figure 4: Block diagram for complete dominant second order control system.

Figure 5 compares the impulse response of the full-state feedback control system to the system with the state estimator. The system with estimated state settles at roughly the same time as the full-state feedback system, but peaks higher. Since the higher peak of the estimated state system is still within design constraints, and the settling time is the same, the estimated state feedback control system is a valid design.

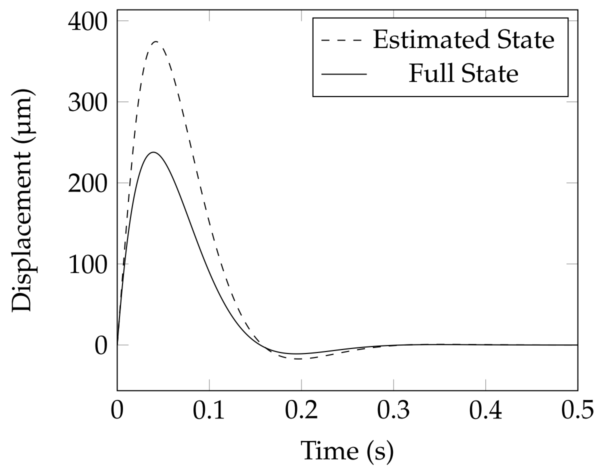

Figure 5: Comparison of full-state feedback with state estimation for the dominant second order control system.

2.2.3 Disturbance Rejection

The response and control effort of the system to an impulse disturbance is shown in Figure 6. The peak response of the system is 374 µm at 42 ms. This represents a very small deviation from equilibrium (5.3 %), and is within design constraints. The settling time is 252 ms, which is very close to the the designed settling time. The peak control effort is −75 mA, which is within the bounds for an acceptable control effort. Note that a negative value for control effort is of no concern, as the control effort u is defined as the deviation from the input required to keep the system at equilibrium.

The response and control effort of the control system to a step disturbance is shown in Figure 7. The peak response of the system is 30.1 µm at 158 ms, this is within the design constraints. The percent overshoot is 4.6 %, which matches with the selected poles but isn't relevant to the design of this control system. The settling time is 213 ms, quicker than the

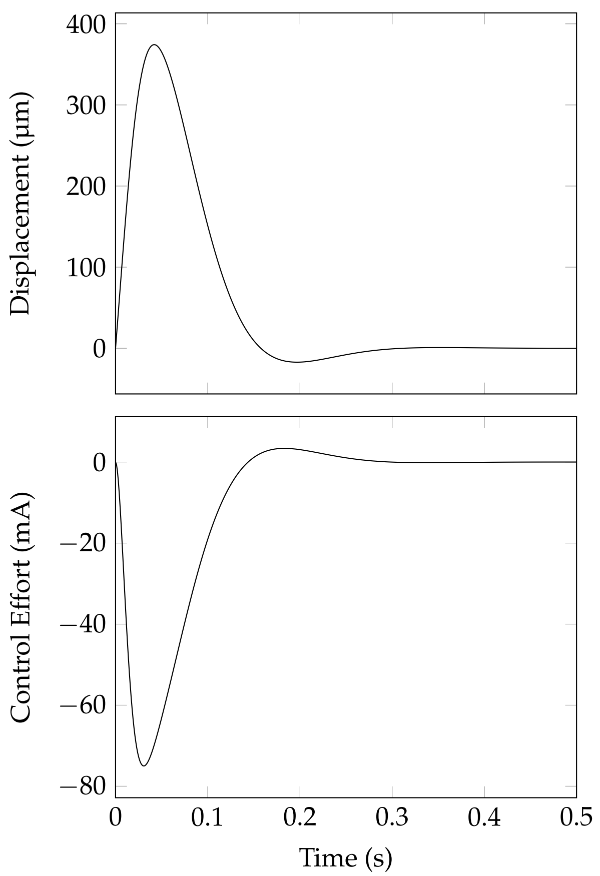

Figure 6: Response and control effort of the dominant second order control system to an impulse disturbance.

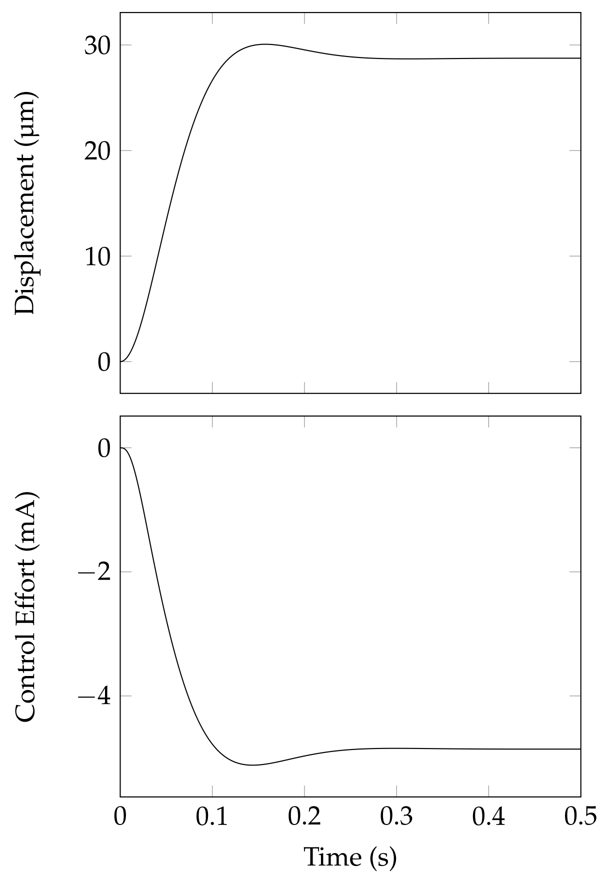

Figure 7: Response and control effort of the dominant second order control system to a step disturbance.

designed settling time but still acceptable. Note the steady state error of 28.7 µm, which is within the allowable displacement from equilibrium. The peak control effort is -5.12 mA, which is well within the bounds for an acceptable control effort.

2.3 Linear Quadratic Regulation

Linear Quadratic Regulation, or LQR, is a popular method for optimal control design. LQR design finds the control such that the performance index

$$\mathcal{J} = \int_0^\infty (\mathbf{x}^T\mathbf{Q} + \mathbf{u}^T\mathbf{R}\mathbf{u})\,\mathrm{d}t \tag{12}$$

is minimized for the system

$$\begin{aligned} \mathbf{x} &= \mathbf{A}\mathbf{x} + \mathbf{B}u \\ z &= \mathbf{C}_1\mathbf{x} \end{aligned}$$

where $\mathbf{Q}$ and $\mathbf{R}$ are weighting matrices and z is the tracking error [2].

2.3.1 Weighting Factor Selection

Using Bryson's Rule [2], initial guesses for the LQR weighting matrices, $\mathbf{Q}$ and $\mathbf{R}$, can be found by using the maximum acceptable state values and the maximum acceptable control efforts. Bryson's Rule is shown in (13).

$$\begin{aligned} Q_{ii} &= 1/x_{i,max}^2 \\ R_{ii} &= 1/u_{i,max}^2 \end{aligned} \tag{13}$$

Using the values specified in the control objectives, the weighting matrices become (14).

$$\begin{aligned} \mathbf{Q} &= \begin{bmatrix} 2.0408 & 0 \\ 0 & 0 \end{bmatrix} \\ R &= 16 \end{aligned} \tag{14}$$

Using the `lqr` command in MATLAB gives the matrix of control gains, **K**.

$$\mathbf{K} = \begin{bmatrix} 0.5275 & 0.0073 \end{bmatrix} \tag{15}$$

With full state feedback, the state space representation of the control system is (9), same as the dominant second order system.

2.3.2 State Estimation

While the method used to design the state estimator for the dominant second order controller can be used to design a estimator for the LQR controller, a common solution is to use a Kalman state estimator, making the system a linear quadratic Gaussian regulator. The Matlab command `kalman` creates a Kalman state estimator given the system, the disturbance noise variance Q_n, and the measurement noise variance R_n [3]. Note that $Q_n = R_n = 1$ in this case, meaning the variance (and therefore power) of both the disturbance noise and measurement noise is 1. Using the MATLAB command `lqgreg` with the Kalman estimator and the LQR gain gives the regulator shown in Figure 8. Using the `feedback` function on the regulator and the plant creates the complete control system, shown in Figure 9.

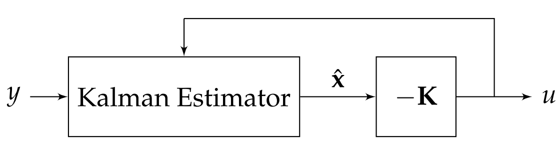

Figure 8: Block diagram for the linear quadratic gaussian regulator.

Figure 10 compares the impulse response of the full-state feedback control system to the system with the state estimator. While the LQG system has a much higher peak and settles slower than the full-state LQR system, it still exceeds the design requirements.

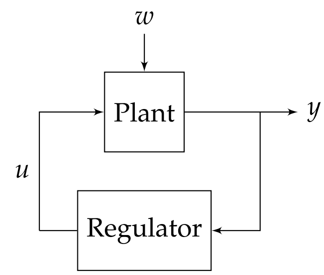

Figure 9: Block diagram for complete linear quadratic Gaussian control system.

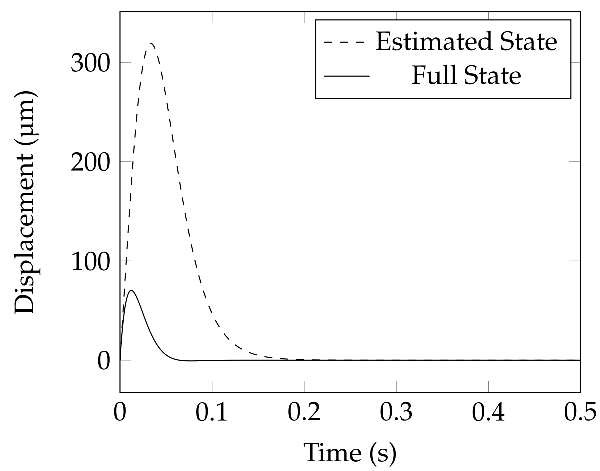

Figure 10: Comparison of full-state feedback system with state estimation for the LQR/LQG control system.

2.3.3 Disturbance Rejection

The response of the system to an impulse disturbance is shown in Figure 11. The peak response of the system is 319 μm at 33 ms. This reprents a small deviation from equillibrium (4.6 %), and is well within design constraints. The settling time is 145 ms, which fits the design constraints. The peak control effort is −77.8 mA, which is within the bounds for an acceptable control effort.

The response and control effort of the system to a step disturbance is shown in Figure 12. The settling time is 122 ms, quicker than the designed settling time but still acceptable. Note that this control system has a steady state error of 19.8 μm, which is generally undesirable, but is acceptable in this case as the steady state value is within the design constraints. The peak control effort is −3.6 mA, which is well within the bounds for an acceptable control

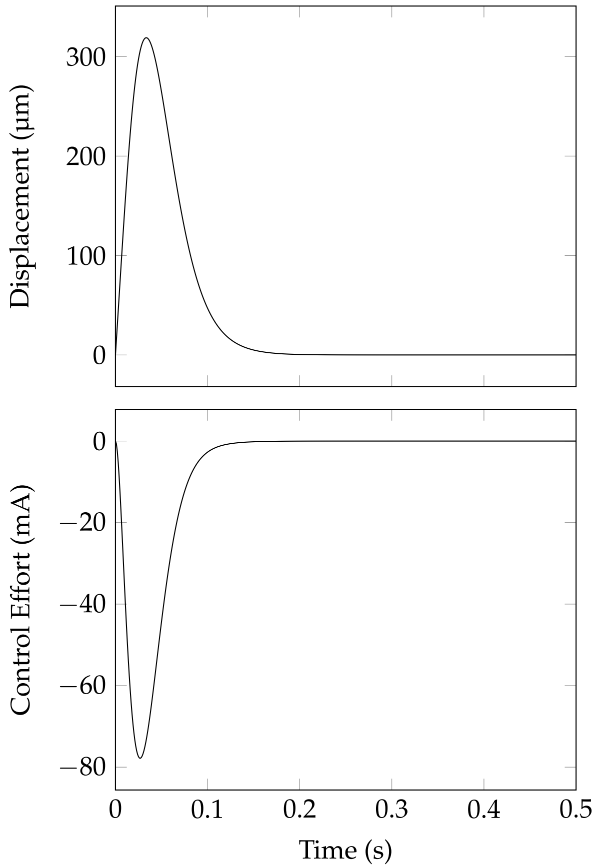

Figure 11: Response and control effort of the LQG control system to an impulse disturbance.

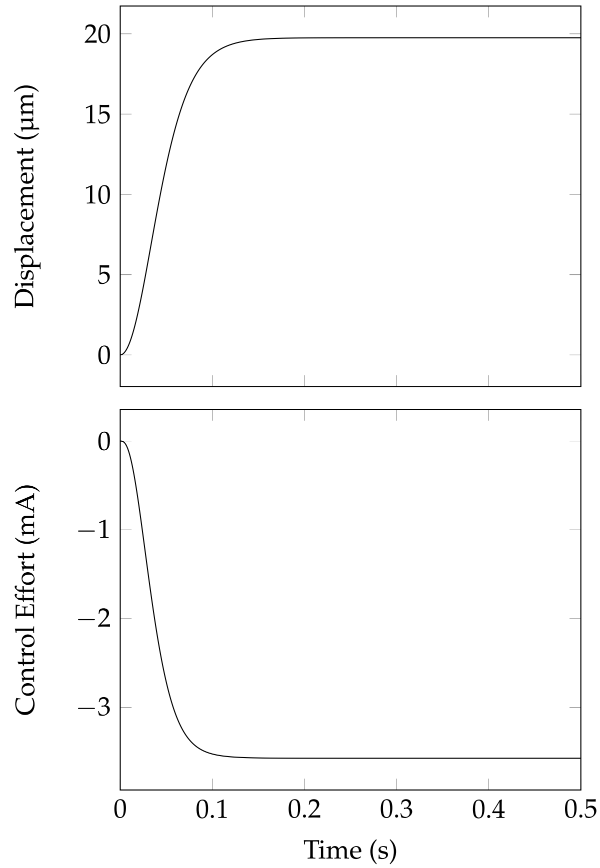

Figure 12: Response and control effort of the LQG control system to a step disturbance.

effort.

2.3.4 Adding and Integrator to the LQG System

While it's not stated as an objective for the control system, in some cases it might be necessary to eliminate steady state error. MATLAB provides a simple command to create a LQG regulator with integral action, `lqgtrack`, which should remove any steady state error from a step disturbance. The `lqgtrack` command takes a Kalman estimator and a state feedback gain and creates the regulator shown in Figure 13. Since the control objectives don't require a reference input, r can be ignored in this case (set to 0). This makes the `lqgtrack` regulator have the same input and output as the LQG regulator, meaning it can be used as a drop-in replacement. Before creating the regulator with `lqgtrack`, a new feedback gain has to be found, since the K from the previous LQG regulator doesn't take into account the added integral state. This is done by first modifying the LQR weighting matrix, Q, to account for the addi-

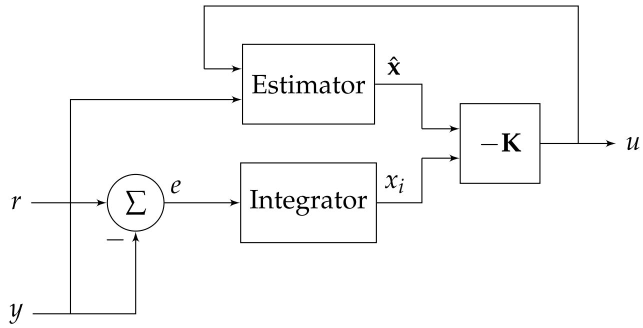

Figure 13: Block diagram for the linear quadratic gaussian regulator with integral action.

tional state, shown in (16). The new Q was chosen such that the settling time for a step disturbance was 250 ms. Then the new gain can then be found by calling the `lqi` method with the system and weighting matrices as arguments. The new gain is shown in (17).

$$Q = \begin{bmatrix} 2.0408 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1500 \end{bmatrix} \tag{16}$$

$$K = \begin{bmatrix} 0.7012 & 0.0085 & -9.6825 \end{bmatrix} \tag{17}$$

After the new K is found, the `lqgtrack` function can be called to give the new regulator, at which point the new control system can be made with the same method as the previous LQG system. Figure 14 compares the step disturbance response of the LQG regulator vs the LQG plus integral system.

The LQG plus integral system has a peak response of 9.58 µm and a peak control effort of -2.53 mA, both much lower than the LQG system whil having no steady state error and a settling time of 250 ms.

The response of the LQG plus integral system vs the plain LQG system for an impulse response is shown in Figure 15. While the integral action had a benefit for the step disturbance response, the impulse disturbance response doesn't show much benefit. The LQG plus integrator has a slightly lower peak response of 257 µm and a slightly higher peak control effort of -82 mA, with a settling time of 249 ms. The LQG plus integral system also has more of an oscillation than the LQG system, which may or may not be detrimental. In this case, since the settling time is within spec, and the oscillations don't bring either the control effort or displacement out of bounds, the oscillation is neither beneficial or detrimental.

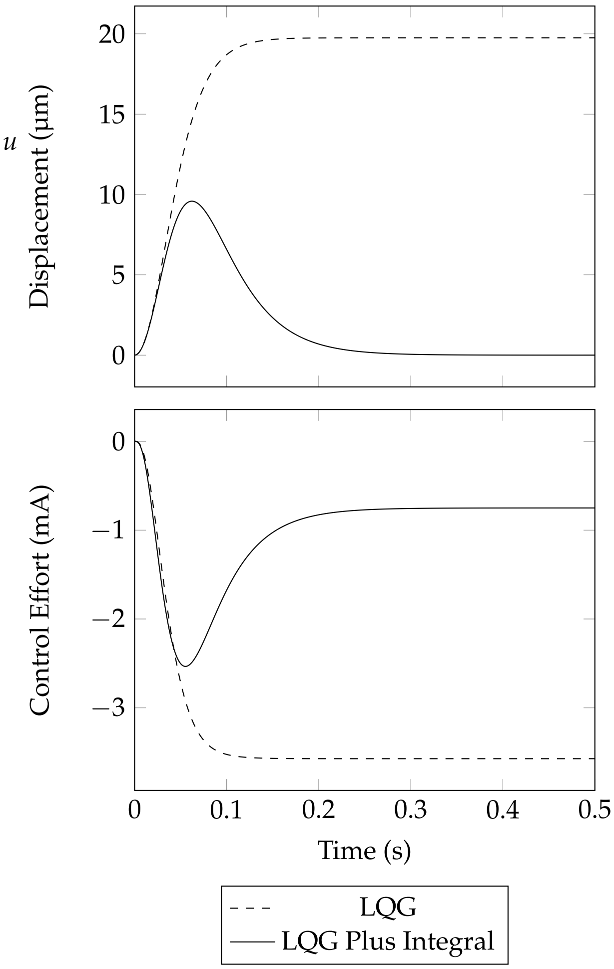

Figure 14: Response and control effort of the LQG control system vs the LQG plus integral control system to a step disturbance.

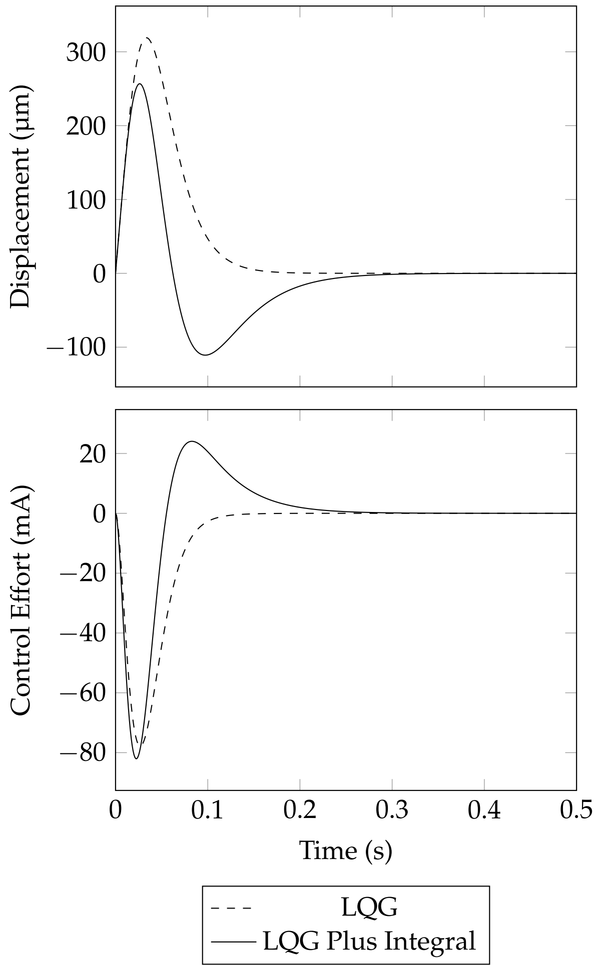

Figure 15: Response and control effort of the LQG control system vs the LQG plus integral control system to an impulse disturbance.

2.4 PID Tuning

Using MATLAB's built in PID tuning utility, `pidtool`, it is possible to easily design a PID controller for this system. The PID tuner gives two sliders, one for response time, and one for Transient Behavior. In order to design a PID controller for the magnetic levitation system, I changed the values of the sliders until the settling time of the controller to a disturbance input was less than or equal to 250 ms. The resulting controller gains are shown in (18).

$$k_P = 6.236 \qquad k_I = 102 \qquad k_D = 0.0954 \tag{18}$$

With the controller gains found, the next step is to create the control system using MATLAB's `feedback` and `connect` commands. The block diagram for the resulting system is shown in Figure 16.

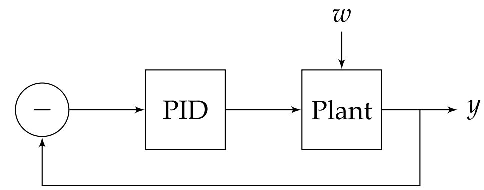

Figure 16: PID control system block diagram.

2.4.1 Disturbance Rejection

The response of the system to an impulse disturbance is shown in Figure 17. The peak response of the system is 7.19 μm at 1.89 ms. This is a 0.1 % deviation from equilibrium , and is therefore well within design constraints. The settling time is 167 ms, which fits the design constraints. The peak control effort is −1.4 A, which is far out of the range stated in the design requirements. The control effort drops to within constraints after 945 μs, however, so it most likely won't have an effect on a real world system.

The response of the control system to a step disturbance is shown in Figure 18. The response reaches a peak of 91 nm at 31 ms and settles down after 201 ms. Both the peak response and the settling time for this input are well within the ranges set in the control objectives. Unlike the previous controllers, the PID control system does not have a steady state error to a step disturbance, this is because of the integral term in the controller. The peak control effort is 777 μA, which is well within the design constraints, not to mention much less than the previous two controllers.

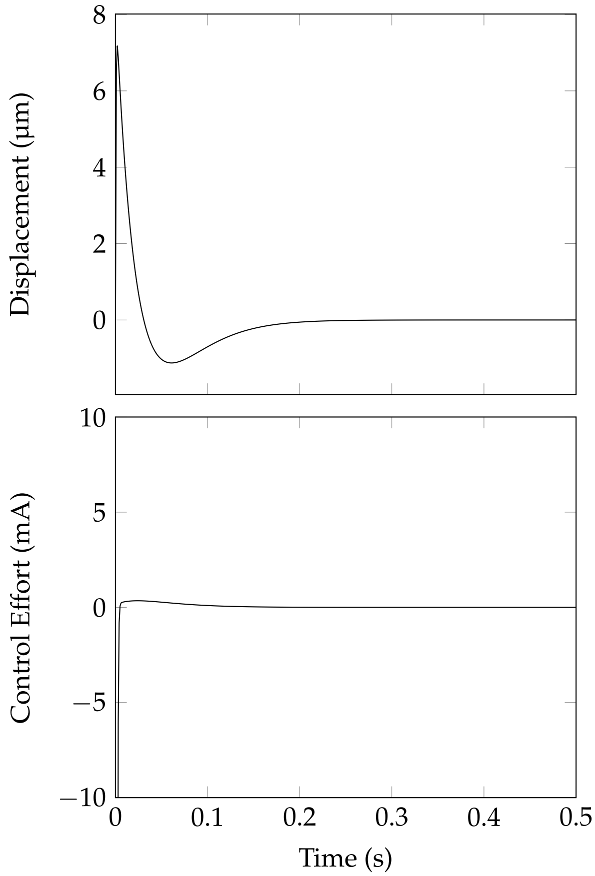

Figure 17: Response and control effort of the PID control system to an impulse disturbance.

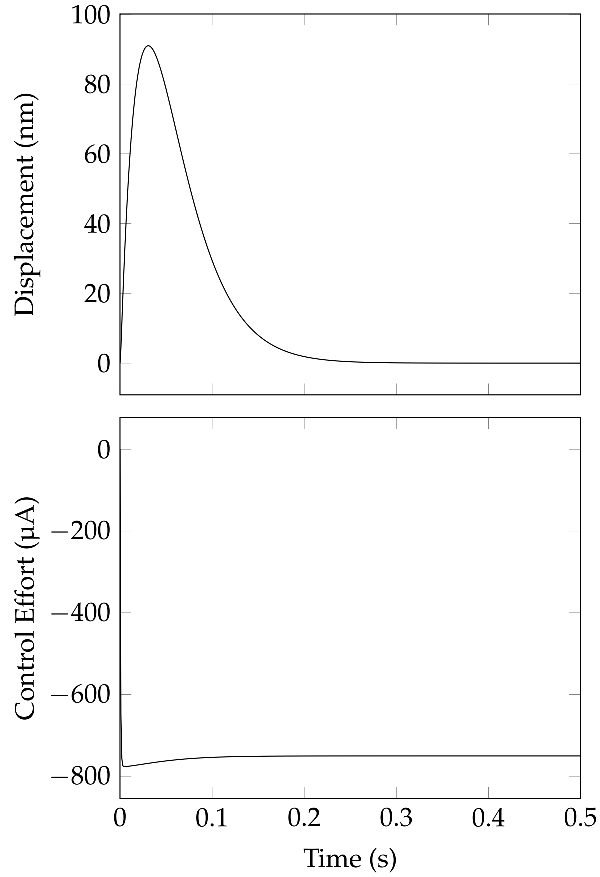

Figure 18: Response and control effort of the PID control system to a step disturbance.

2.5 Controller Comparison

Figure 19 compares the step responses and control efforts for the four controllers, and Figure 20 shows the impulse responses and control efforts. Looking at the step responses, while all systems meet the design requirements, the PID controller and the LQG plus integral controller have the least displacement from equilibrium, the lowest control effort, and zero steady state error. For the impulse responses the PID controller again performs the best with the lowest displacement and controller effort while the other three controllers have much greater displacements (between 250 μm to 400 μm) and control efforts.

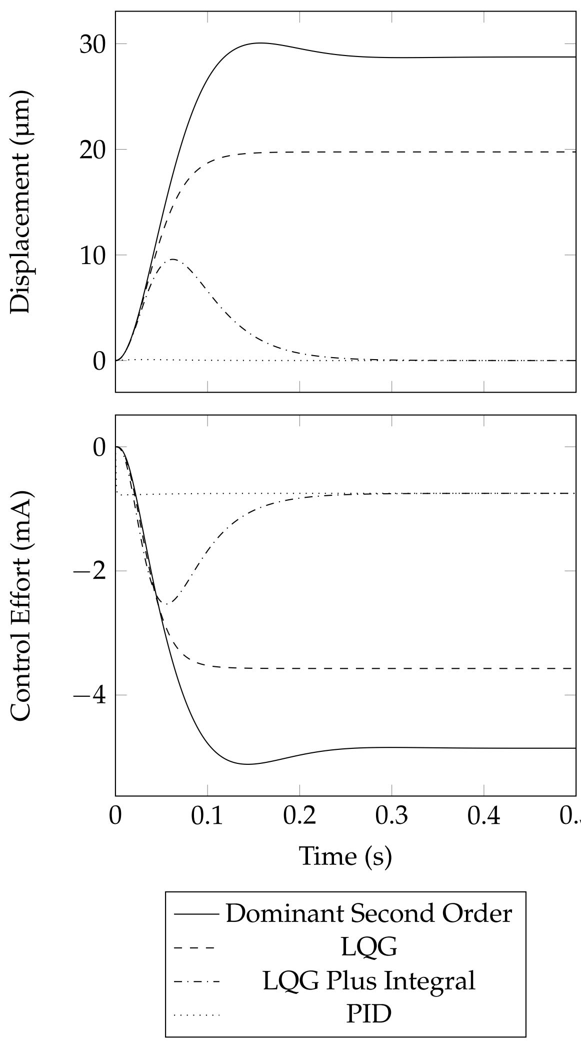

Figure 19: Comparison of the step response for the control systems.

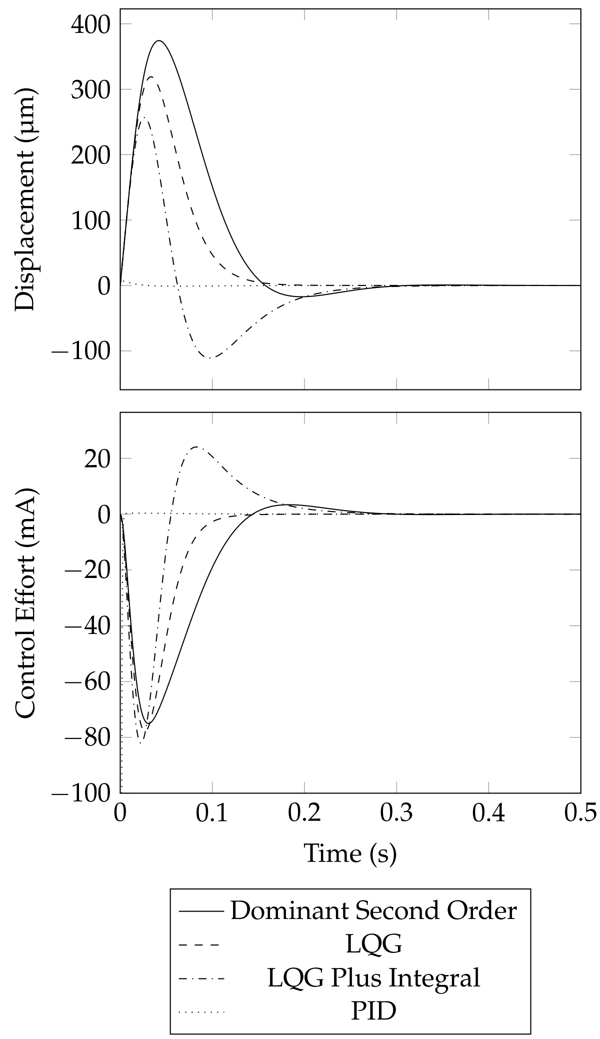

Figure 20: Comparison of the impulse response for the control systems.

2.6 Noise Response

In the real world the control system would have to be able to deal with noisy signals. To test each controller, random noise was added to the signal right before entering the controller and right after exiting the controller. The noise added before the controller representing sensor noise, and the noise after the controller representing process noise. The sensor noise was picked such that the value of the sensor was generally accurate to within ±100 μm, and the process noise was chosen such that the input to the system generally stayed within ±10 mA of the nominal value. An example of the noise is shown in Figure 21. Figure's 22, 23, 24, and 25 show the response of the dominant second order, LQG, LQG plus integral, and PID controllers (respectively) to the noise input.

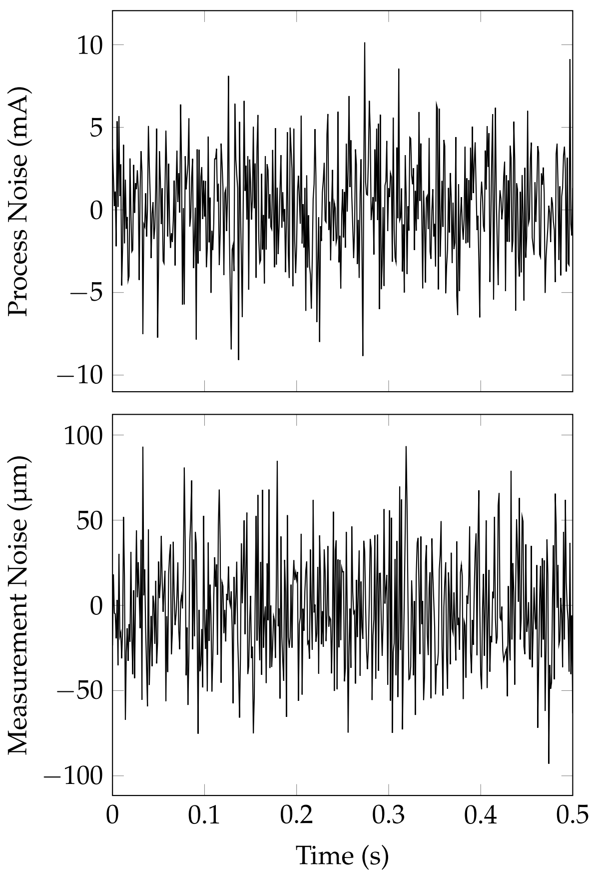

Figure 21: Noise inputs used for responses.

The dominant second order controller (Figure 22) is able to maintain the ball within the tolerances given in the control objectives (±700 μm) and doesn't exceed the specified range of control effort (±250 mA), but the control effort changes very rapidly which a real controller may not be able to replicate. This controller would not be an ideal choice to deal with a system with noise of this magnitude.

The LQG controller (Figure 23) also maintains the ball within the specified range for displacement and control effort. But the controller also suffers from the same rapidly changing control effort values as the dominant second order controller, making the LQG controller a poor choice as well. Adding integral action to the LQG controller only slightly changes the response in regards to displacement but manages to do so with a much smoother and lower magnitude control effort (Figure 24).

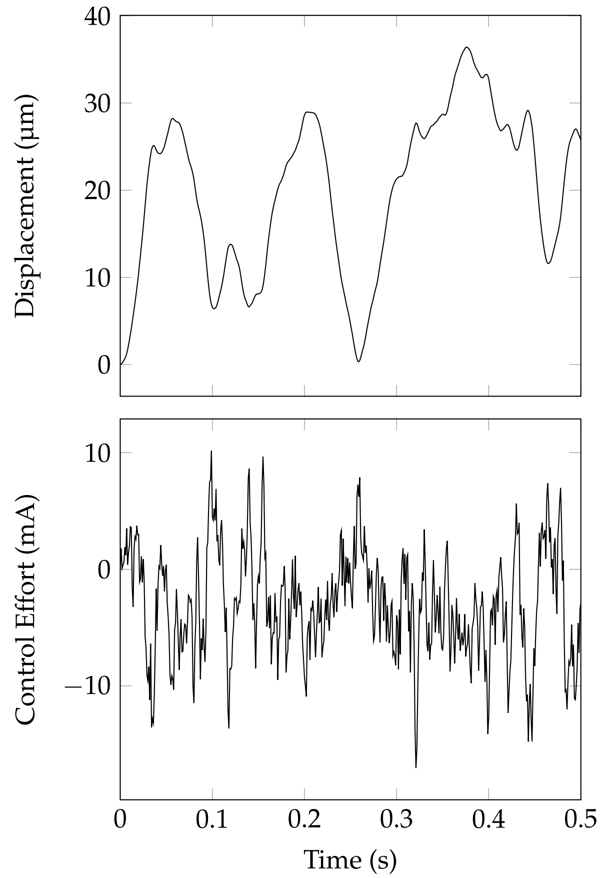

Figure 22: Noise response for dominant second order controller.

The PID controller (Figure 25) keeps the displacement of the ball in a much tighter tolerance than any of the other systems, but fails to filter out the noise, making the position of the ball oscillate wildly. This could have a negative effect on the system as having the ball rapidly oscillate could cause resonance or any number of ill effects. The control effort of the PID controller is also unacceptable as it peaks near 200 A, which is far beyond the specified range of ±250 mA.

The best choice for a controller if the system will be subjected to noisy signals is the LQG controller with integral action, shown in Figure 24. While all the controllers manage to keep the displacement in bounds, the LQG plus integral controller does it while also having the lowest control effort. In addition to having a low control effort, the LQG plus integral control effort doesn't contain the noise that

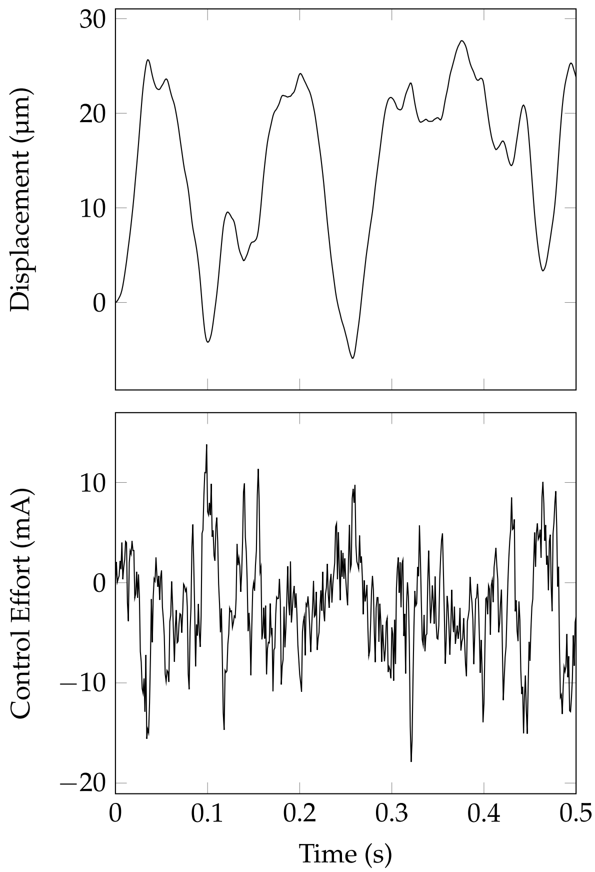

Figure 23: Noise response for LQG controller.

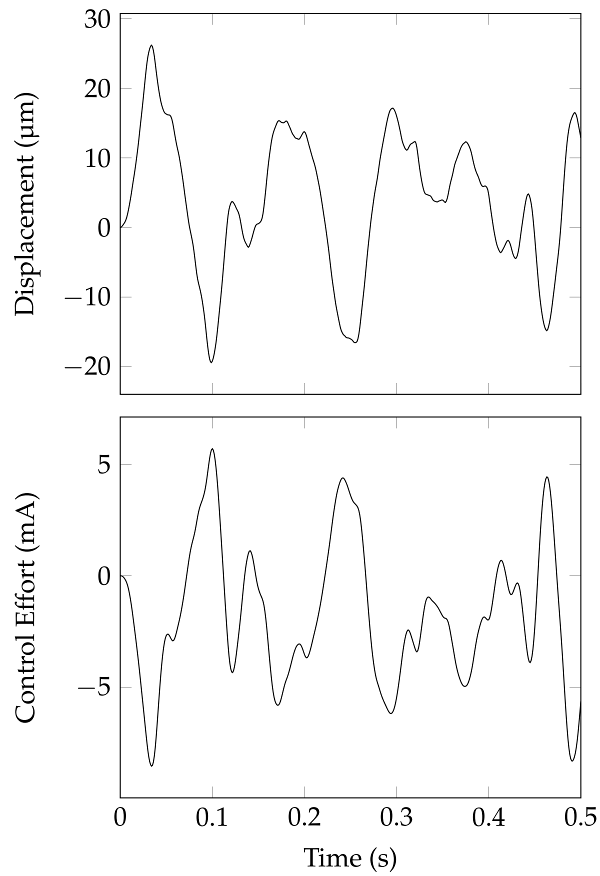

Figure 24: Noise response for LQG plus integral controller.

the other control efforts do. This is a benefit as a real controller wouldn't be able to change values quick enough to match the theoretical responses of the dominant second order, LQG, and PID controllers, while the control effort for the LQG plus integral controller changes much more slowly and smoothly than the others.

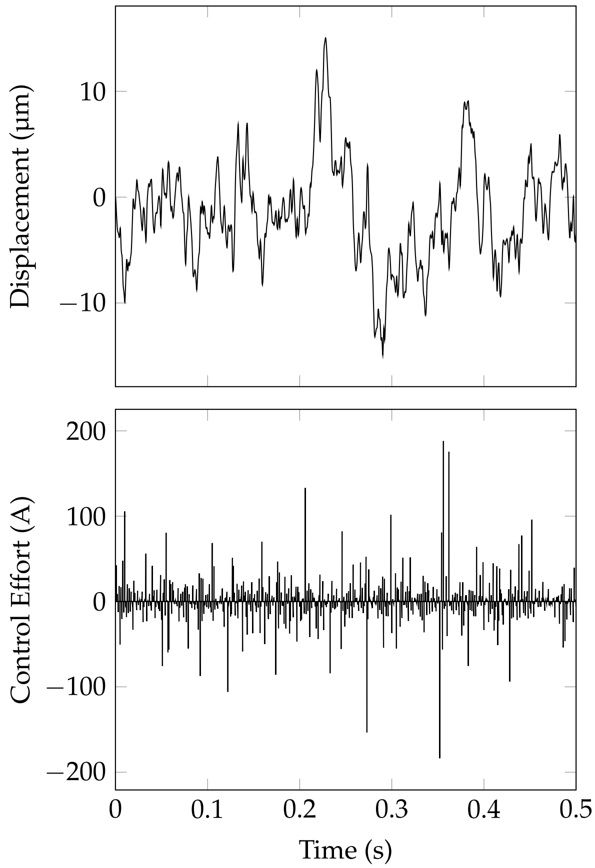

Figure 25: Noise response for PID controller.

3 Conclusion

Comparing controllers made with dominant second order pole placement, linear quadratic Gaussian regulation, linear quadratic Gaussian regulation with integral action, and PID tuning shows that, at least for the initial iterations, PID tuning creates the most effective controller. The metrics used to gauge the effectiveness of each controller were: the maximum displacement, the maximum control effort for both a step and impulse disturbance. The PID controller performs orders of magnitude better than the other methods for this system, as shown in Figures 19 and 20. While the tuned PID controller is more effective for a first iteration, the LQG plus integral design is flexible enough that it should be able to match or even improve on the response of the PID controller, given sufficient iteration.

When process and sensor noise is added, the PID controller is unable to eliminate the noise from the response of the system and requires tremendous control efforts to attempt to do so. With noise added, the linear quadratic Gaussian controller with integral action becomes the best choice, as it eliminates the noise from the response and control effort and keeps the control effort low.

If I were to choose one of these controllers to implement in a production system, I would choose the LQG plus integral system. Even with the slightly worse performance characteristics compared to the PID system for step and impulse disturbances, the LQG plus integral offers many more benefits such as being able to pick the LQR weighting values to put more emphasis on the state or control effort and the ability to tailor the state estimator to counteract the specific noise that the system is subject to.

References

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