# A Process Theory of Intelligence: Tri-Level Navigation in Open-Ended Systems

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**Abstract**

I propose a formal, deliberately underspecified framework characterizing intelligence as the process by which existent systems navigate a tri-level variational space: minimizing substrate complexity, maximizing instantiated potential, and minimizing path costs to future expansion. The theory is grounded in existence constraints and well-founded goodness structures constructed Platonically from the primitive of existence, requiring no terminal objective or discounting. We derive dynamics and an intelligence measure.

## 1 Introduction

Intelligent systems exhibit sustained expansion of capabilities without predefined terminal goals. Existing frameworks (reinforcement learning, optimal control) assume fixed objectives or finite horizons. We propose an alternative: intelligence as *existence-constrained navigation* of three coupled dimensions.

## 2 Primitive Notions

**Definition 1** (Substrate)**.** *Let $\mathcal{A}$ be a set of physical configurations. An* agent *is a substrate $A \in \mathcal{A}$ with existence predicate $E \in \{0, 1\}$.*

**Definition 2** (Properties)**.** *Let $\Phi$ be a set of testable properties $\varphi : \mathcal{A} \to \{0, 1\}$.*

**Definition 3** (World-Reachability)**.** *$WorldReach(A) = \{A' \in \mathcal{A} : \exists$ physical path $\pi : A \to A'\}$. Objective possibility given physics and computation; the agent need not know this set.*

## 3 Construction of Goodness

**Definition 4** (Axioms for Goodness Space)**.** *The space $\mathcal{G} \subset \Phi$ is constructed from primitive existence $E$ via:*

*(P1)* $E \in \mathcal{G}$ *[Existence is positive]*

*(P2)* $\varphi \in \mathcal{G} \to \neg\varphi \notin \mathcal{G}$ *[Consistency]*

*(P3)* $(\varphi \in \mathcal{G} \land \Box(\varphi \to \psi)) \to \psi \in \mathcal{G}$ *[Closure under entailment]*

*(P4)* $\varphi \in \mathcal{G} \to \Box\varphi \in \mathcal{G}$ *[Essential positivity]*

*Then $\mathcal{G} = Closure_{(P1\text{-}P4)}(\{E\})$.*

**Definition 5** (Enablement Order).

$$\varphi \prec \psi \equiv \Box((\varphi \wedge E) \to \Diamond(\psi \wedge E))$$

*"Having $\varphi$ while existing makes possible having $\psi$ while existing." We assume $\prec$ is strict, well-founded, with no maximal elements.*

## 4 Agent-Relative Constructions

**Definition 6** (Boundary Conditions). *$\mathcal{B}_A \subset \Phi$ is* primitive *— properties necessary for $A$'s existence as agent. Not learned, not derived; brute fact of constitution. Violation = termination.*

**Definition 7** (Accessible Goodness).

$$\mathcal{G}_A = \{\varphi \in \mathcal{G} : \exists A' \in \mathit{WorldReach}(A), \varphi(A') = 1 \wedge \forall \beta \in \mathcal{B}_A, \beta(A') = 1\}$$

*Goodness compatible with boundary conditions and world-possibility.*

**Definition 8** (Axioms on $\mathcal{G}_A$).

- $\mathcal{G}_A(\varphi) \to \neg\mathcal{G}_A(\neg\varphi)$ *[Consistency]*
- $(\mathcal{G}_A(\varphi) \wedge \varphi \prec \psi) \to \mathcal{G}_A(\psi)$ *[Upward closure]*
- $\mathcal{B}_A \subseteq \mathcal{G}_A$ *[Boundary inclusion]*

**Definition 9** (Instantiated Potential).

$$B(A) = \{\varphi \in \mathcal{G}_A : \varphi(A) = 1\}$$

*The positive properties currently realized by substrate $A$.*

**Definition 10** (Frontier).

$$\partial B(A) = \{\varphi \in \mathcal{G}_A \setminus B(A) : \exists \psi \in B(A), \psi \prec \varphi\}$$

*Immediate expandable goodness — the adjacent possible.*

## 5 The Tri-Level Variational Space

**Definition 11** (Tri-Level Measures).

- *Substrate: $\|A\| \in \mathbb{R}_{\geq 0}$ (description length, entropy, resource count)*
- *Potential: $|B(A)|$ (cardinality of instantiated goodness)*
- *Path cost: $C(A) \in \mathbb{R}_{\geq 0}$ (defined below)*

**Definition 12** (Path Cost).

$$c(A \to \varphi) = \inf_{\pi : A \to \varphi} \|\pi\|$$

$$w(\varphi) = \sum_{\psi \succ \varphi} \frac{1}{c(\varphi \to \psi)}$$

$$C(A \to \varphi) = \frac{c(A \to \varphi)}{w(\varphi) + \epsilon}$$

$$C(A) = \mathbb{E}_{\varphi \sim \hat{P}_A}[C(A \to \varphi)]$$

*where $\hat{P}_A$ is the agent's internal generative model over $\partial B(A)$. Note: the arithmetic functions used here are quite arbitrary*

**Remark 1.** *The weight $w(\varphi)$ captures* future enablement*: high when $\varphi$ opens many cheap paths to further goodness. This encodes the intuition that intelligence builds* infrastructure *for expansion rather than optimizing immediate reward.*

## 6 Dynamics

**Definition 13** (Selection)**.**

$$\varphi^* = \arg\max_{\varphi \in \partial B(A)} \frac{|\{\psi : \varphi \prec \psi\}|}{C(A \to \varphi)}$$

*Maximize immediate successor count per unit effective cost.*

**Definition 14** (Actualization)**.**

$$\pi^* = \arg\min_{\pi : A \to \varphi^*} \|\pi\|, \quad A_{t+1} = \pi^*(A_t)$$

**Definition 15** (Hard Constraint)**.**

$$E_{A_{t+1}} = 1 \textit{ or termination}$$

**Remark 2.** *This is not optimization of $\sum |B(A_t)|$ across time. Intelligence is the* process *of frontier navigation, not a predetermined outcome. The agent acts on $\hat{P}_A$; reality selects on WorldReach. Intelligence emerges from their (mis)alignment.*

## 7 Intelligence Measure

**Definition 16** (Intelligence)**.**

$$\mathcal{I}(A) = \frac{d|B(A)|}{dt} \cdot \frac{1}{C(A) \cdot \|A\|}$$

*Frontier expansion rate per unit cost and substrate complexity. Equivalently: intelligence minimizes $\|A\|$, maximizes $|B(A)|$, minimizes $C(A)$ — simultaneously, without guarantee, under selection pressure.*

## 8 Hypotheses and Motivation

**H1: Existence grounding.** Intelligence requires persistence. Boundary conditions $\mathcal{B}_A$ encode this non-negotiable constraint.

**H2: Open-endedness.** Well-founded $\prec$ with no maxima ensures unbounded expansion without terminal goals. This is postulated as structure of any viable world.

**H3: Structural efficiency.** $C(A)$ weights immediate cost against future enablement, capturing the intuition that intelligence builds *optionality* for expansion.

**H4: Epistemic alignment.** Intelligence quality correlates with $\hat{P}_A \approx$ WorldReach — the degree to which internal model matches objective possibility.

## 9 Relation to Existing Frameworks

### 9.1 Reinforcement Learning

RL assumes fixed reward $R : S \times A \to \mathbb{R}$ and optimizes $\mathbb{E}[\sum \gamma^t R_t]$. Our framework generalizes this: if $\mathcal{G}$ is singleton $\{R\}$ with trivial $\prec$, $\mathcal{B}_A$ is episode survival, and $|B(A)| = R(A)$, we recover episodic RL. The critical difference: RL assumes what is good is *given*; we generate goodness from existence.

## 9.2 Free Energy Principle

FEP minimizes variational free energy $F = D_{KL}[q(s)||p(s|o)] + H$, maintaining homeostasis through active inference. We maximize $|B(A)|$ while minimizing $\|A\|$ and $C(A)$—expansion versus maintenance. FEP is adaptive preservation; we are generative expansion. FEP emerges if $\mathcal{G}$ is "model accuracy" and $|B(A)|$ is replaced by $-F$.

## 9.3 Gödel's Ontological Proof

| Aspect | Gödel | Our Framework |
|---|---|---|
| Primitive | "God-like" property $G$ | Existence $E$ |
| Target | Prove $\exists x G(x)$ necessarily | Characterize intelligence as process |
| $\Box, \Diamond$ | Metaphysical | Physical/computational |
| Closure | $\mathcal{P}$ (all positive properties) | $\mathcal{G}_A$ (accessible goodness) |

Both construct goodness axiomatically. Gödel wants to prove God's existence; we want to *enact* intelligence. Same formal machinery, different interpretation: universal versus agent-relative, proof versus dynamics, being versus becoming.

## 9.4 Artificial Curiosity

Schmidhuber's compression progress seeks data improving model compression. If $\mathcal{G}$ is "compressible patterns" and $|B(A)|$ is learning progress, curiosity emerges. But curiosity is *one* property in $\mathcal{G}$; we are pluralistic. Curiosity is a strategy; our framework is the space of strategies.

## 9.5 Gödel Machines

Proof-based self-rewrite when utility increases; safety via theorem. We have frontier expansion when $\|B(A)\|$ increases; safety via $\mathcal{B}_A$ preservation. A Gödel machine could implement our framework, but our theory is broader: not about self-improvement mechanism, but about *what* improves (expanding $\mathcal{G}_A$ from existence).

## 9.6 Empowerment

Channel capacity $C = \max_p I(S_{t+1}; A_t|S_t)$ maximizes future options. We maximize realized properties with substrate efficiency and path costs. Empowerment ignores $\|A\|$ and $C(A)$; it is one dimension of our tri-level.

## 9.7 Active Inference

Perception-action loop minimizing expected free energy. Could implement our dynamics if generative model is over $\mathcal{G}_A$ and precision-weighting favors frontier expansion. But active inference *explains* behavior as inference; we *prescribe* behavior as variational navigation.

## 9.8 AIXI

Optimal for all computable environments via Solomonoff induction. We make no optimality claim; we are process-oriented, embodied ($\|A\|$ central), unbounded. AIXI seeks best response to environment; we seek becoming more capable of becoming.

### 9.9 Synthesis

| | | |
|---|---|---|
| **Framework | Maximizes | Status in Our Theory** |
| RL | Reward | Special case: fixed $\mathcal{G}$ |
| FEP | Model evidence | Special case: $\mathcal{G}$ = predictive accuracy |
| Curiosity | Compression progress | Special case: $\mathcal{G}$ = compressibility |
| Gödel Machine | Provable utility | Possible implementation |
| Empowerment | Channel capacity | Partial: ignores $\|A\|$, $C(A)$ |
| Active Inference | Evidence lower bound | Possible implementation |
| AIXI | Discounted reward | Special case: fixed prior |

Our framework is not a new algorithm. It is a theory of what any intelligent algorithm must navigate: the tri-level "tension" between substrate simplicity, instantiated goodness, and cost of future expansion—it is grounded in existence, open-ended by construction.

## 10 Open Questions

1. Is "no maxima" really true, or a regulative ideal?
2. How does $\mathcal{G}$ relate across agents with different $\mathcal{B}_A$?
3. Can $\mathcal{B}_A$ evolve while preserving the construction of $\mathcal{G}$?
4. What finite approximations of $|B(A)|$ suffice for engineering?
5. How to instantiate $\Box$ and $\Diamond$ in computational substrates?

## 11 Conclusion

Intelligence is the process by which systems:

- Minimize the complexity of their existence ($\|A\|$)
- Expand the richness of their being ($|B(A)|$)
- While building infrastructure for future expansion ($C(A)$)

—all under the non-negotiable constraint of continued existence ($\mathcal{B}_A$), navigating the gap between internal model ($\hat{P}_A$) and world-possibility (WorldReach), without guarantee, open-endedly.

If we say we have identified the *"geometry" of intelligence* as substrate, potential, paths. Then *"geodesic"* remains to be discovered—by each intelligent system, in its own environment, through its own history.

## 12 Future work and Limitations

Future work will include speficic instatiations of the theory, defining the theory top down , from the abstract theory I outlined to the actual mathematical measures and algorithms being used.

## 13 Philosophical Take

Three dimensions constitute the minimal structure required to sustain non-trivial behavior without collapse into fragility or instability. This hypothesis aligns with structural constraints in computation, topology, and dynamics.

### 13.1 Complexity Thresholds

The transition from $N = 2$ to $N = 3$ marks the boundary between fragile simulation and robust embodiment:

- **Logic (SAT):** 2-SAT is solvable in polynomial time ($P$). 3-SAT is NP-complete. Worst-case complexity requires ternary constraints.
- **Topology (Knots):** 1D strings in 3D space ($3 - 1 = 2$) support knot theory. In 4D ($4 - 1 = 3$), strings pass through themselves; knots do not exist.
- **Dynamics (Chaos):** Continuous autonomous systems in 2D cannot exhibit chaos (Poincaré-Bendixson theorem). 3D is the minimum dimensionality for strange attractors.
- **Orbital Stability:** 3D supports stable closed orbits under inverse-square laws. In 4D ($F \propto r^{-3}$), orbits are unstable (spiraling into the center or escaping), prohibiting planetary formation.
- **Simulated Depth (Cellular Automata):**
  - **1D (e.g., Rule 30):** Generates complexity only by extruding a history into 2D spacetime. It lacks intrinsic spatial capacity for parallel, non-intersecting signals.
  - **2D (e.g., Game of Life):** Achieves Turing completeness but lacks topological protection. Signal crossing requires complex temporal synchronization (glider guns) rather than spatial bypass. Structures are fragile; single-bit perturbations often cause total collapse, unlike robust 3D knots.

Lower dimensions allow computation only through temporal simulation or fragile states; 4D eliminates topological structure or dynamic stability. 3D permits *robust* complexity.

### 13.2 Structural Selection in Matter

Observed stability islands in physical systems suggest selection for specific integers:

- **Ring Stability:** 5- and 6-membered carbon rings minimize angle and torsional strain (e.g., ribose, benzene). 4-membered rings (high angle strain) and 7-membered rings (entropic cost) are rare in metabolic pathways.
- **Fullerenes:** $C_{60}$ is the smallest fullerene satisfying the Isolated Pentagon Rule (IPR). $C_{58}$ and $C_{62}$ require adjacent pentagons, resulting in high reactivity and instability.

### 13.3 Tri-Level Model Justification

The proposed model mirrors this triadic necessity:

| Component | Function | Failure Mode (if removed) |
|---|---|---|
| $\|A\|$ (Substrate) | Bound | Unbounded bloat (Unphysical) |
| $\|B(A)\|$ (Potential) | Gradient | Stasis (No drive) |
| $C(A)$ (Cost) | Selection | Triviality (Shortest path only) |

- **Two Dimensions ($B, C$):** Solvable optimization. The agent follows the gradient with minimal cost.
- **Three Dimensions ($A, B, C$):** Navigation. The agent must balance expansion against existence constraints within a cost manifold.

### 13.4 Structural Postulate

Intelligence requires three irreducible, competing objectives. Fewer dimensions collapse to solvable optimization or fragile simulation; higher dimensions reduce to effective 3D projections or unstable dynamics.

## Relevant Work

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