Overview SCR-1 PIC

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1 Introduction

This is the main outline for the SCR-1 Particle-In-Cell code, it specifies the main theoretical and computational concepts to carry out this type of simulation. The two computational infrastructures proposed are AMReX and the one presented in "Plasma Simulations by Example". This document will not give specific details on how to apply these technically, it will rather give out the physical and numerical abstraction on the physical model.

2 Physical model

This physical model is based on the Lorentz force equation which describes particle motion in the presence of electric and magnetic fields. The dynamical aspect of this equation lies in the change of the electric and magnetic fields, which is given by the Maxwell equations of electromagnetics. The motion of the particles is obtained with the solution of these equations, ç

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \tag{1}$$

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \tag{2}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{3}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{4}$$

$$\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{j} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right) \tag{5}$$

With the following definitions,

$$\mathbf{B} = -\nabla \phi_B + \nabla \times \mathbf{A}_B \tag{6}$$

$$\mathbf{E} = -\nabla \phi_E \tag{7}$$

2.1 Computational Domain

These equations will be defined in a torus with the approximate dimensions of the SCR-1 device. The boundaries selected will not be a representation of the device, but a boundary of confinement confirmation. Figure 1 shows the boundary to be used. The first implementation of the domain will be on the box surrounding the torus, in next steps, the mesh will need to be adjusted to the toroidal geometry to ignore areas outside the boundary.

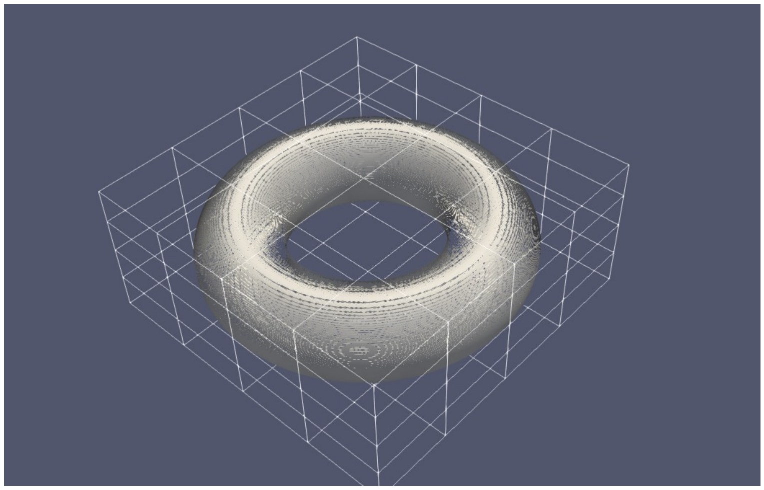

Figure 1: Computational toroidal boundary.

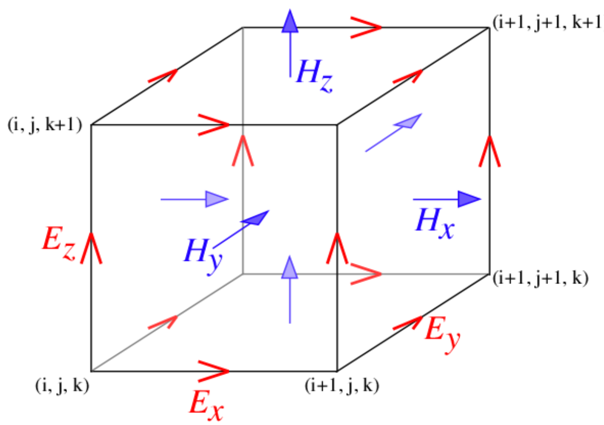

Figure 2: Staggered grid approach for PIC methods.

2.1.1 Non-uniform magnetic field equation of motion

For any non-uniform, inhomogenous magnetic field the equation of motion has some additional force terms associated with the curvature and the gradient of the magnetic field. Because of this, the equation of motion for the particles has to be modified to take into account those effects. The equation of motion then takes the form,

$$\mathbf{F} = \mathbf{F}_{lorentz} + \mathbf{F}_{\nabla} + \mathbf{F}_{cf} \;\; (+\mathbf{F}_E)$$

$$\mathbf{F} = m\frac{d\mathbf{v}}{dt} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) - \frac{mv_{\perp}^2}{2}\frac{\nabla\mathbf{B}}{B} + mv_{||}^2\frac{}{R_c^2} \quad \left(+\mathbf{F}xt \propto \frac{\mathbf{E} \times \mathbf{B}}{B^2}\right); \;\; \mathbf{v} = \mathbf{v}_{\perp} + \mathbf{v}_{||} \tag{8}$$

To solve for these equations numerically, the most convenient solution that has been found is to implement the staggered grid discretization approach [1] shown in figure 2. This method of calculation has the advantage of working closely with a stable numerical scheme for the curl operator which is defined, for example, on the edges of the cube.

This equation of motion is solved altogether with Maxwell's equations expressed in the following form for time and space discretization,

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} \tag{9}$$

$$\frac{\partial \mathbf{E}}{\partial t} = c^2(\nabla \times \mathbf{B} - \mu_0 \mathbf{j}) \tag{10}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} \tag{11}$$

References

[1] L. Brieda, *Plasma simulations by example*. 6000 Broken Sound Parkway NW, Suite 300 Boca Raton, FL 33487-2742: CRC Press, 2020.