Technical Report: MAE 373

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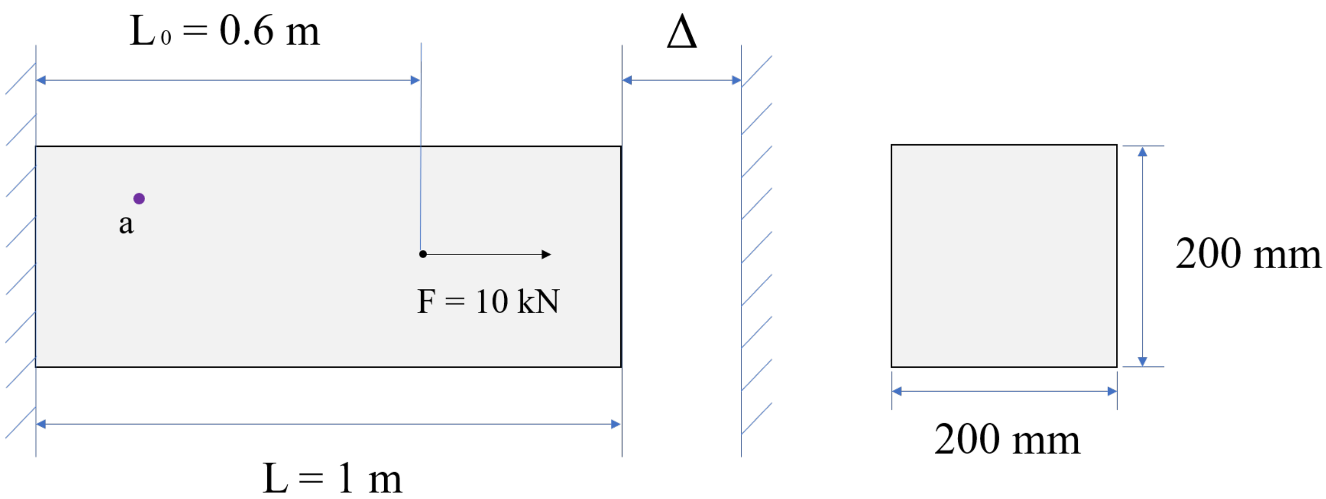
Here is the diagram we are tasked with analyzing:



From this diagram, we will calculate stress at point A and the total strain of the bar when:
1)$\Delta = 0.3mm$
2)$\Delta = 0.2mm$
3)$\Delta = 0.1mm$

The bar experiences a temperature increase of 20°C. We are also given:
$E = 100GPa$
$\nu = 0.25$
$\alpha = \frac{10^{-5}}{C^{\text{o}}}$

Some necessary assumptions to make before we analyze the free body diagram is that there is no thermal stress associated with thermal strain. Thermal stress is a result of the loading condition. In this particular situation, a statically indeterminate body will result in thermal stress. The necessary equations and relations we employed will be listed below and referred to throughout the documentation.

Stress due to load:

$$\sigma = \frac{P}{A} \tag{1}$$

Where P is the applied load and A is the cross sectional of a homogeneous body.

Strain due to deformation:

$$\epsilon = \frac{\delta}{L\text{o}} \tag{2}$$

Where δ is the total deformation and Lo is the original length of the body.

Deformation due to loading:

$$\delta_{load} = \frac{PL}{EA} \tag{3}$$

Where E is Young's Modulus which is a material property. Since the body is homogeneous, E is constant. In order to find L, we must consider the following conditions:

1. The internal loading must be constant
2. The cross-sectional area must be constant
3. E, Young's Modulus is constant

$\therefore$ L is the corresponding length during which the 3 other parameters are constant.

Deformation due to thermal expansion:

$$\delta_{thermal} = \alpha \cdot \Delta T \cdot L \tag{4}$$

α is the thermal expansion coefficient, ΔT is the change in temperature, and L is the length of the body.

Poissons's Ratio:

$$\nu = \frac{-\epsilon_y}{\epsilon_x} = \frac{-\epsilon_z}{\epsilon_x} \tag{5}$$

Poisson's ratio is a material property that is given but will be important because during thermal expansion, deformation occurs in three directions: x, y, and z.

Stress-Strain relationship:
We can come to this relationship by looking at equation 2 and substituting δ with 3.

$$\epsilon = \tfrac{\delta}{L\text{o}} = \tfrac{\frac{PL}{EA}}{L\text{o}}$$

L cancels, which will give:

$$\epsilon = \frac{P}{A}E = \sigma E$$

Therefore:

$$\epsilon = \sigma E \tag{6}$$

Thermal Stress:
If the system is statically indeterminate and it experiences a change in temperature, the resulting expansion will push against it's constraints and create compression. This compression is a reaction of the constraints. This reaction creates stress, or rather thermal stress. It is important to distinguish that thermal stress is not a component of thermal strain. We can quantify this by equating 4 and 3 because when it's statically indeterminate, $\delta = 0$:

$$\delta = \alpha \cdot \Delta T \cdot L = \frac{PL}{EA}$$

The L cancels and as a result:

$$\alpha \cdot \Delta T = \frac{P}{A}E$$

Recall in equation 1 the relation for stress. By substituting $\frac{P}{A}$ with σ and solving for it, we get:

$$\sigma_{thermal} = E \cdot \alpha \cdot \Delta T \tag{7}$$

Another important set of equations to know are the static equilibrium equations:

$$F_x = 0$$
$$F_y = 0$$
$$F_z = 0$$
$$\Sigma M = 0$$

Now that we have the necessary equations and relations, we may begin to analyze the structure.

The first part of the problem is when $\Delta = 0.3mm$

We will measure the total deformation of the beam by calculating the δ due to thermal expansion and applied loading.

$$\delta_{total} = \delta_{thermal} + \delta_{load} \tag{8}$$

Calculation of thermal deformation using equation 4:

$$\begin{aligned}\delta_{thermal} &= \alpha \cdot \Delta T \cdot L \\ &= \frac{10^{-5}}{C^{\text{o}}} \cdot 20^{\text{o}} \cdot 1000mm \\ &= 0.2mm\end{aligned}$$

Calculation of deformation due to concentrated load using equation

$$\begin{aligned}\delta_{load} &= \frac{PL}{EA} \\ &= \frac{10kN \cdot 600mm}{100GPa \cdot (200 \cdot 200)mm^2} \\ &= 0.0015mm\end{aligned}$$

The total deformation is calculated using equation 8

$$\begin{aligned}\delta_{total} &= \delta_{thermal} + \delta_{load} \\ &= 0.2mm + 0.0015mm \\ &= 0.2015mm\end{aligned}$$

Because δ_{total} is less than Δ, we can calculate strain in the x direction using equation 2:

$$\begin{aligned}\epsilon_{total} &= \frac{\delta_{total}}{L_{\text{o}}} \\ &= 201.5 \cdot 10^{-6}\end{aligned}$$

Due to the Poisson's ratio, we can find strain in y and z using equation 5:

$$\begin{aligned}(201.5 \cdot 10^{-6})(0.25) &= -\epsilon_z = -\epsilon_y \\ &-50.375 \cdot 10^{-6}\end{aligned}$$

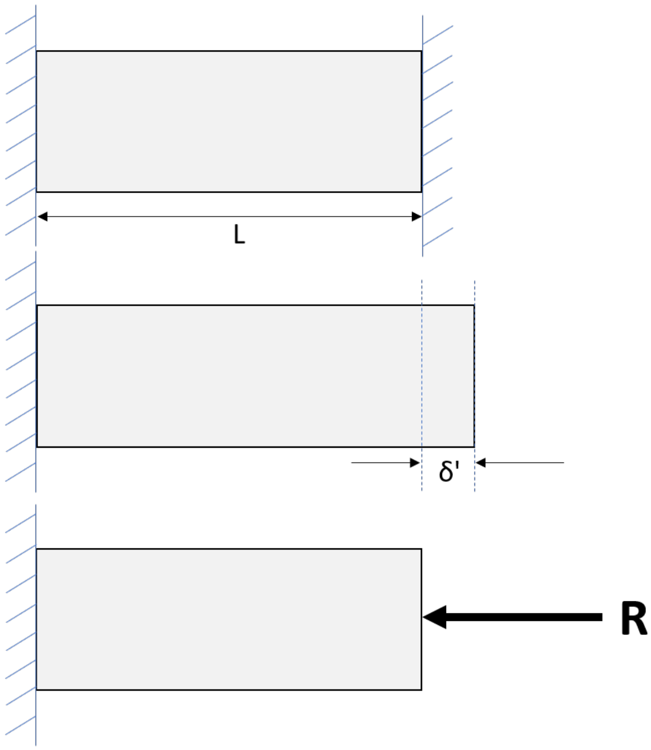
Total stress at A can be found using equation 1

$$\sigma = \frac{10kN}{200 \cdot 200mm^2} = 2.5 \cdot 10^{-4}GPa$$

Next, we will analyze when $\Delta = 0.2mm$

δ_{total} was calculated to be 0.2015mm which is greater than 0.2mm. This loading condition will render it statically indeterminate. We will take the difference of 0.0015mm and calculate the strain using equation 2 like so:

$$\epsilon^{'} = \frac{0.0015mm}{1000mm} = 1.5 \cdot 10^{-6} \tag{9}$$



However, because this body cannot deform any more due to it's horizontal constraints, we must use the super-position method referenced in the above figure and remove the constraint and replace it with an equivalent reaction force.

To find strain in the x-direction, we will use equation 2. Because total deformation is constrained, δ will be equal to Δ.

$$\epsilon_x = \frac{0.2mm}{1000mm} = 200 \cdot 10^{-6}$$

In order to find strain in the z and y direction, we must use Poisson's ratio(equation 5). Because it is not constrained in these directions, we can use the actual strain in x.

$$0.25 = \frac{-\epsilon_y}{201.5 \cdot 10^{-6}}$$
$$= -50.375 \cdot 10^{-6}$$

Because the beam is homogenous and isotropic: $\epsilon_y = \epsilon_z$

In order to find stress at A, we must calculate the reactionary forces due to the constraints as shown in the diagram. We will consider the displacement due to the compressive thermal stress as δ'. However, we cannot solve for the reactionary forces through statics alone; we must use complementary equations:

$$\Delta = \delta_{total} + \delta'$$
$$0.2mm = 0.2015mm + \delta'$$
$$\delta' = -0.0015mm$$

From here we will use equation 2 to solve for the force with δ' and R being the reactionary compressive force:

$$\delta' = \frac{RL}{EA}$$
$$-0.0015mm = \frac{R \cdot 1000mm}{100GPa \cdot 200^2mm^2}$$

Solve for R

$$\therefore R = -6kN$$

To solve for stress at A we create a cross section at A, resolve the internal force and then use equation 1:

$$\sigma = 150 \cdot 10^{-6}GPa$$

Our last loading condition is when $\Delta = 0.1mm$

We will use the same approach as the last problem due to the deformation of the material being statically indeterminate. To find horizontal strain we will use equation 2 with $\Delta = \delta$ like so:

$$\epsilon_x = \frac{0.1mm}{1000mm} =$$

To find strain in z and y, we will use Poisson's ratio:

$$0.25 = \frac{-\epsilon_y}{201.5 \cdot 10^{-6}}$$
$$= -50.375 \cdot 10^{-6}$$

Because the beam is homogenous and isotropic: $\epsilon_y = \epsilon_z$

To find stress at A, we will employ the same method of super-position as used when $\Delta = 0.2mm$.

$$\delta' = \frac{RL}{EA}$$
$$-1.015 = \frac{R \cdot 1000mm}{100GPa \cdot 200^2mm^2}$$

Solve for R

$$\therefore R =$$

By creating a cross section at A and resolving the internal forces we get:

$$\sigma =$$