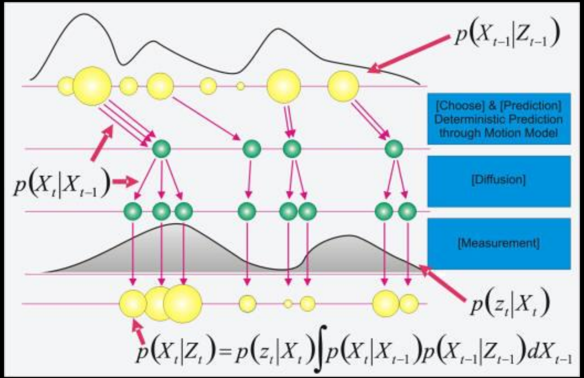
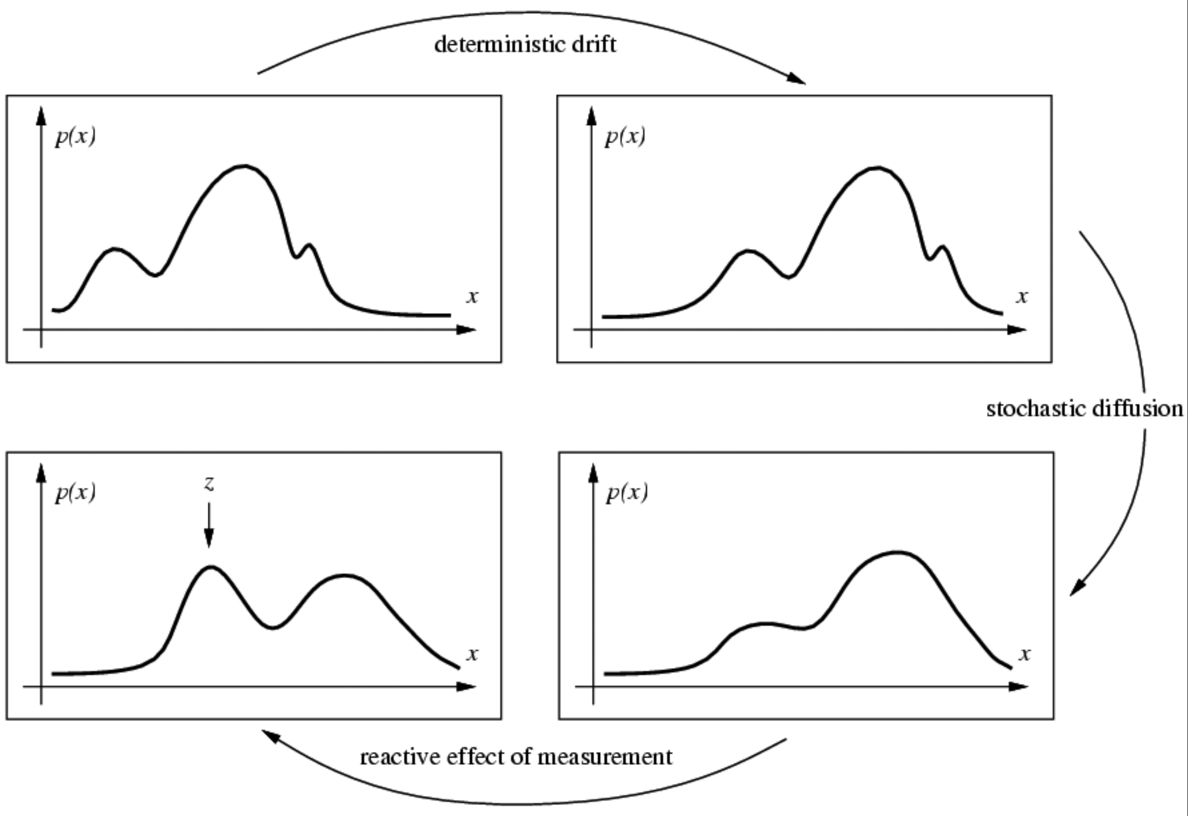
CS4495/6495
Introduction to Computer Vision

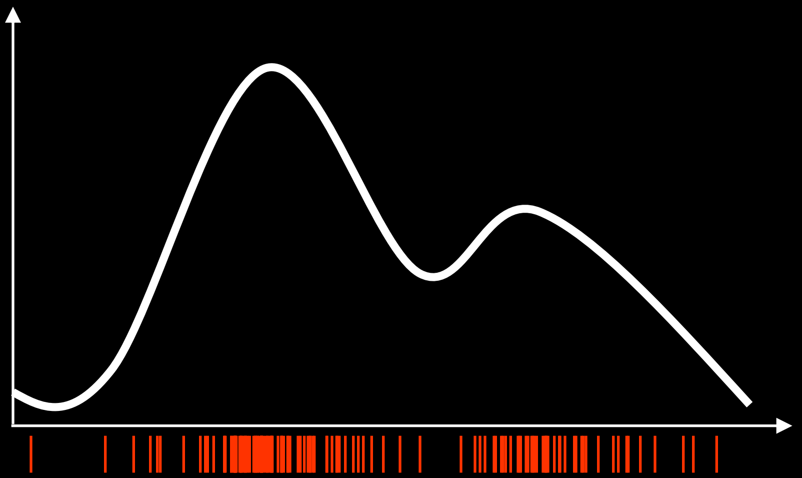
7C-L2 *Particle filters*



Recall: Propagation of general densities



Particle Filters: Basic Idea



Density is represented by both **where** the particles are and their **weight**.

$p(x = x_0)$is now probability of drawing an *x* with value (really close to) x_0.

set of *n* (weighted) particles X_t

Goal: $p(x_t \in X_t) \approx p\left(x_t \middle| z_{\{1\ldots t\}}\right)$ with equality when $n \to \infty$

Last time: Bayes Filters

z = observation
u = action
x = state

Likelihood

$$Bel(x_t) = \eta \; P(z_t \mid x_t) \underbrace{\int P(x_t \mid u_{t-1}, x_{t-1}) \; Bel(x_{t-1}) \; dx_{t-1}}_{}$$

prediction *before* taking measurement

Prior probability $P^{-}(x_t)$

Imagine a robot with only a simple map of a hallway :

The robot also has a sensor that looks to the side and detects whether it sees a “hole” or “wall”

Quiz: Door detector

Hallway with doors:

Simplified representation with discrete positions:

W	**W**	**D**	**D**	**W**	**W**	**W**	**D**	**W**	**W**
0	1	2	3	4	5	6	7	8	9

Current position: x (unknown), sensor reading: z

Direction of movement: Left to Right

Quiz: Door detector

		W	W	D	D	W	W	W	D	W	W
t	**z**	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
0	W	☑	☑	☐	☐	☑	☑	☑	☐	☑	☑
1	D	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
2	W	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
3	W	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐

Check those boxes where you think you might be at time t

Quiz: Door detector [answer]

		W	W	D	D	W	W	W	D	W	W
t	**z**	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
0	W	☑	☑	☐	☐	☑	☑	☑	☐	☑	☑
1	D	☐	☐	☑	☐	☐	☐	☐	☑	☐	☐
2	W	☐	☐	☐	☐	☐	☐	☐	☐	☑	☐
3	W	☐	☐	☐	☐	☐	☐	☐	☐	☐	☑

Check those boxes where you think you might be at time t

A more realistic example

Properties of the real world:

- Position of robot is not discrete (a real number)
- Sensor is noisy, so some readings may be false

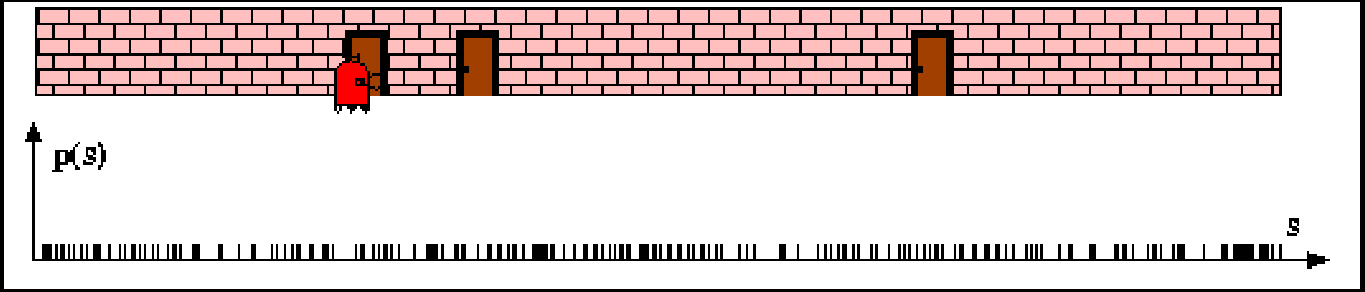
As a result, the predicted position given sensor readings is a *probability distribution* over space

Why Particle Filters?

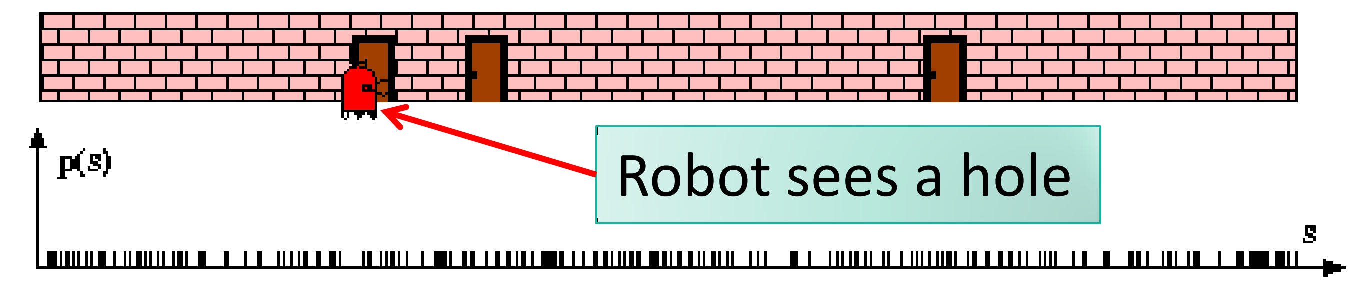
- At any point in time we'll have a density but it's unlikely to be anything like a Gaussian or any other parameterized density
- It typically won't be unimodal, for example
- Perfect for a *particle filter*

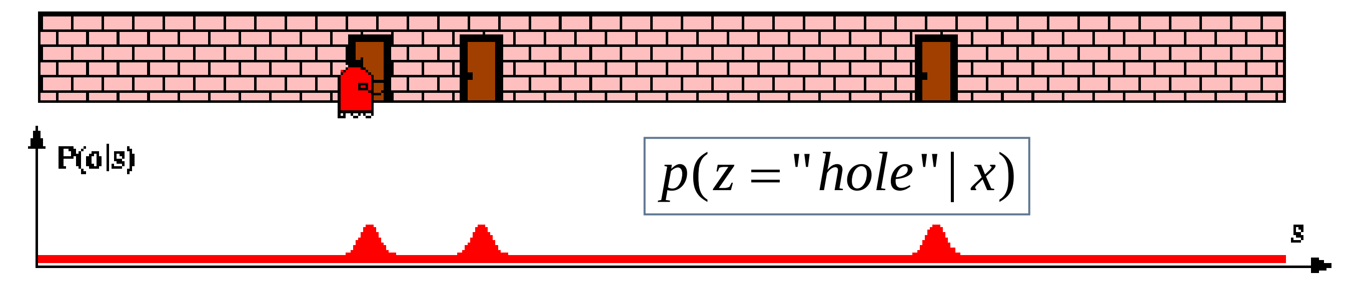
Particle Filters

The prior density:



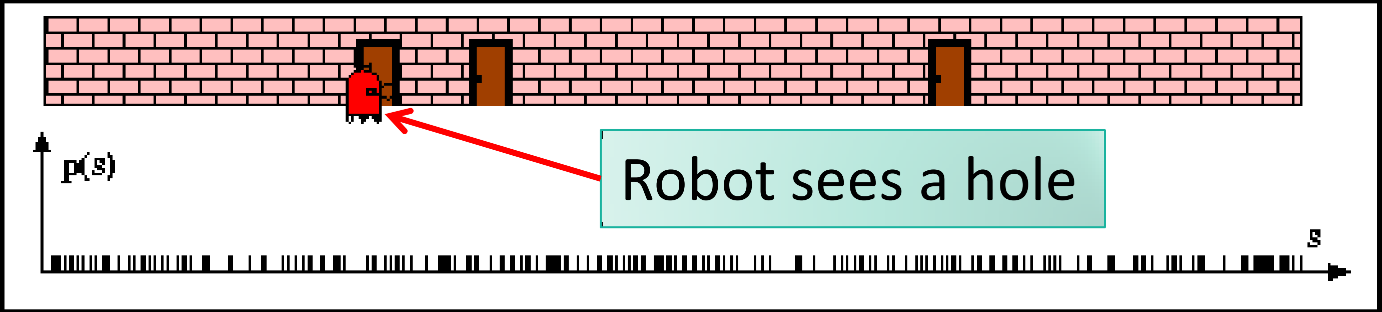
Sensor Information

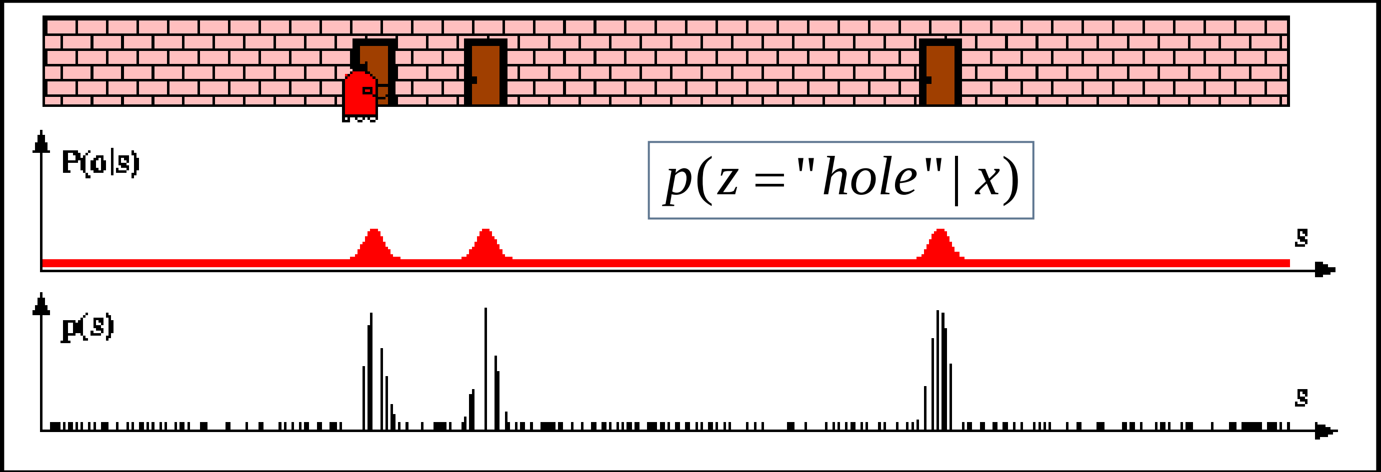




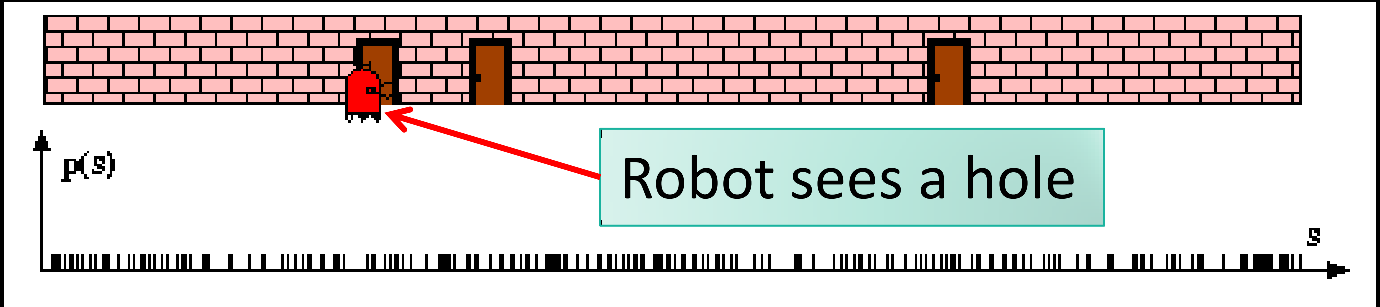
$$p(z = \text{"}hole\text{"} \mid x)$$

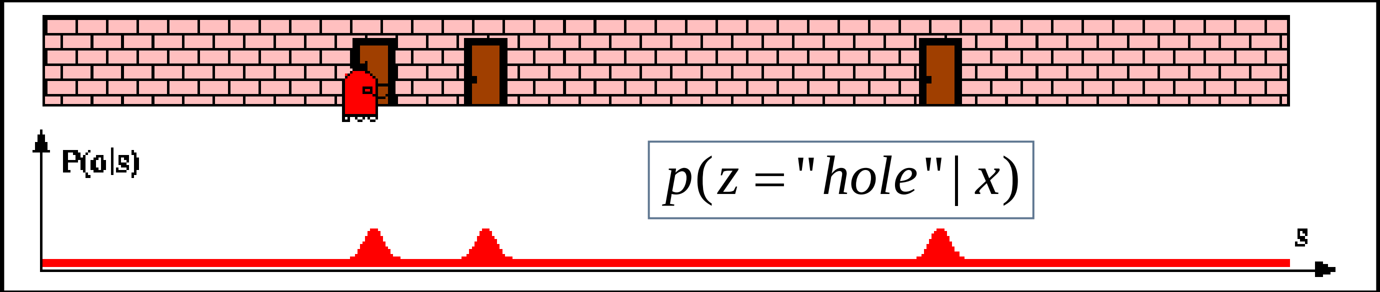
Sensor Information





Sensor Information

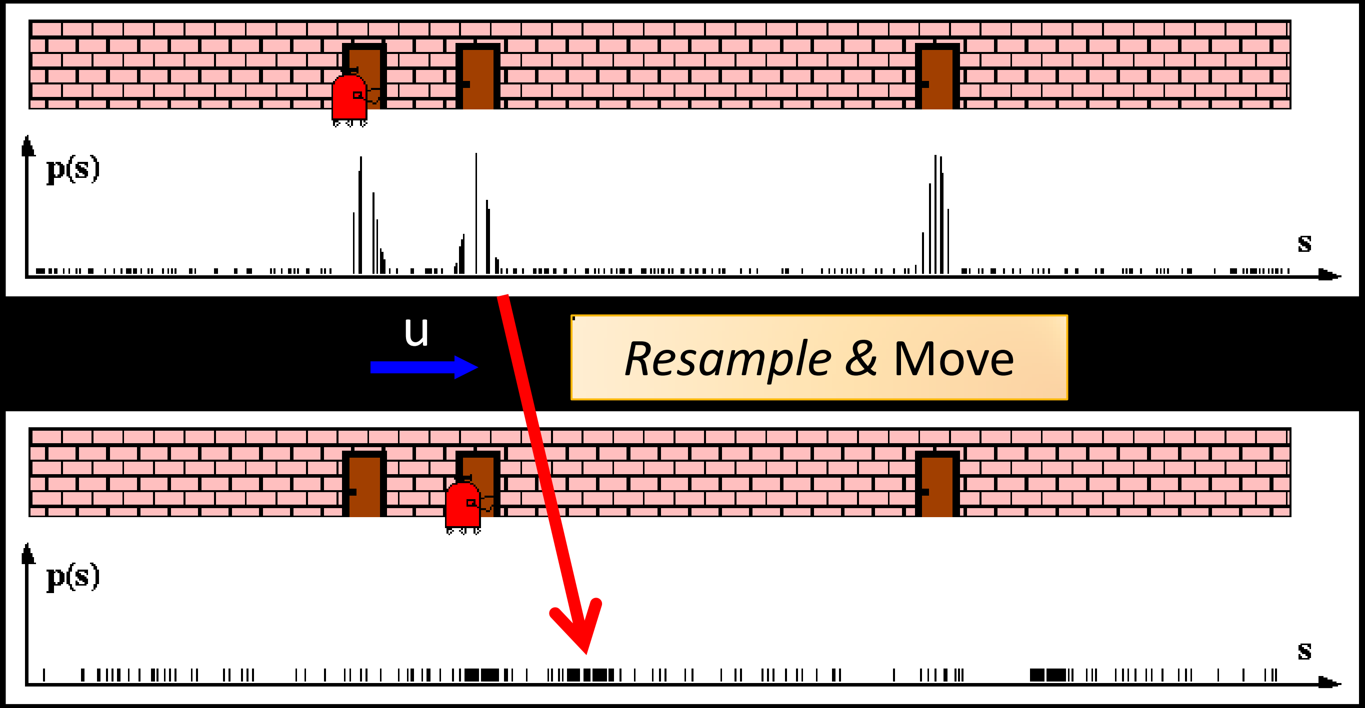




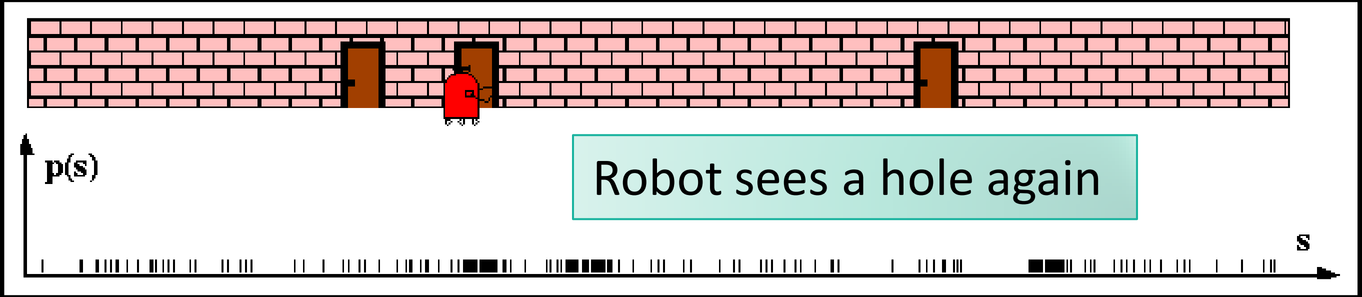
$$Bel(x_t) = \eta \; P(z_t \mid x_t) \;\; \mathrm{Pred}(x_t)$$

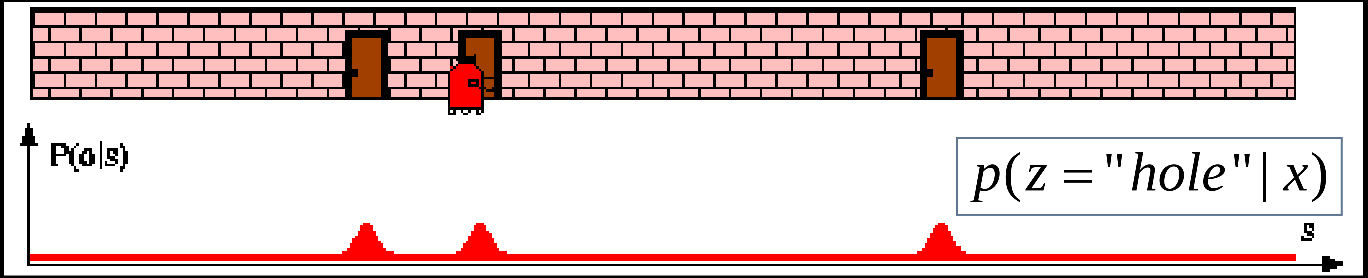
Robot Motion

$$Bel^-(x) \leftarrow \int p(x \mid u, x')\, Bel(x')\, \mathrm{d}x'$$

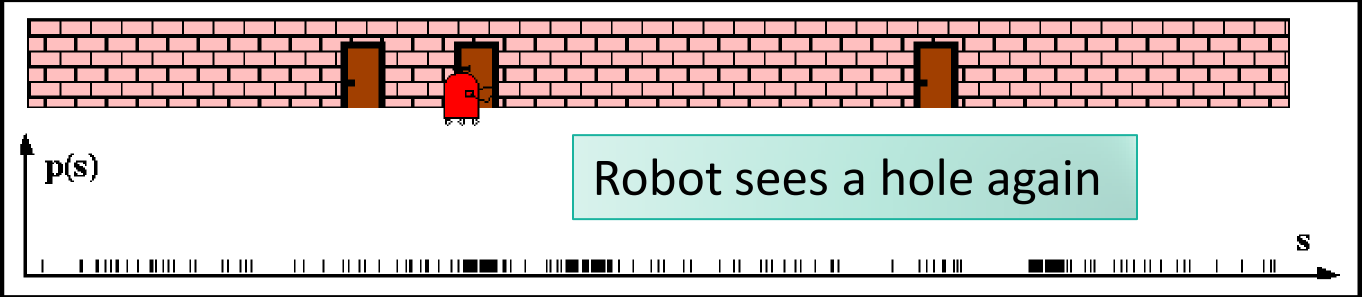


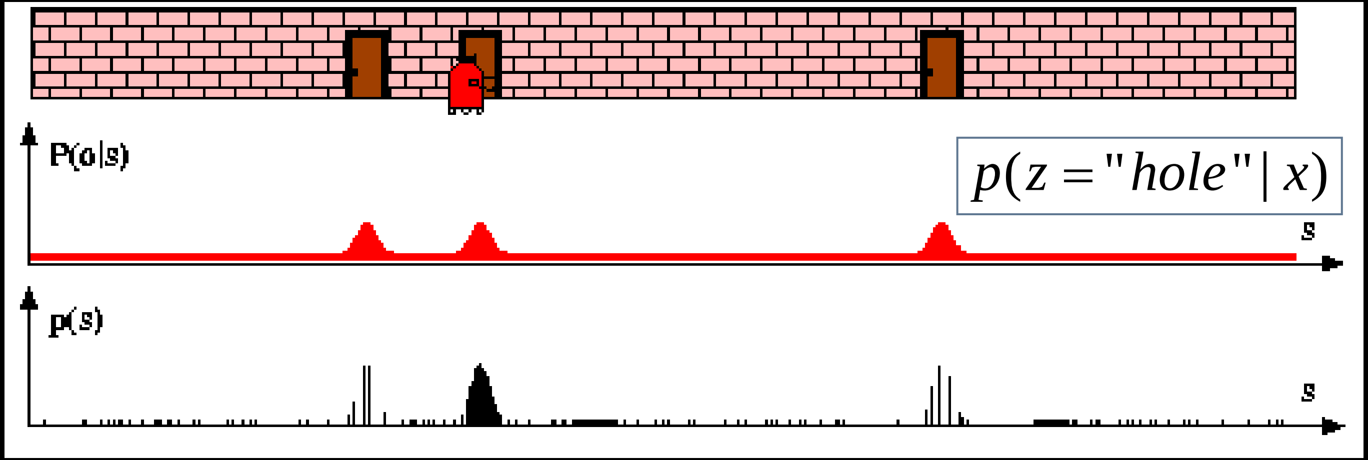
Next Sensor Reading



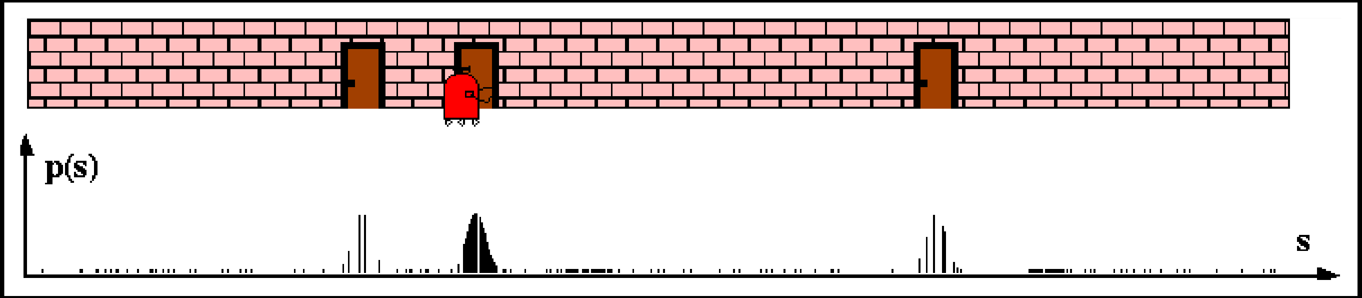


Next Sensor Reading

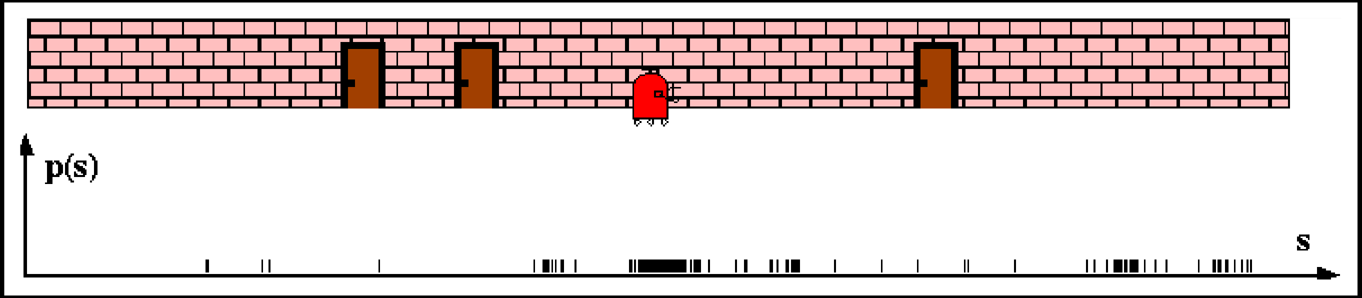




Robot Moves Again



Move & *Resample again*



Algorithm particle_filter $\{S_{t-1} = < x^j_{t-1}, w^j_{t-1} >, u_t, z_t\}$

1. $S_t = \emptyset, \quad \eta = 0$
2. **For** $i = 1 \ldots n$ ***Resample (generate i new samples)***
3. Sample index *j(i)* from the discrete distribution given by w_{t-1}
4. Sample x^i_t from $p(x_t \mid x_{t-1}, u_t)$ using $x^{j(i)}_{t-1}$ and u_t ***Control***
5. $w^i_t = p(z_t \mid x^i_t)$ ***Compute importance weight (reweight)***
6. $\eta = \eta + w^i_t$ ***Update normalization factor***
7. $S_t = S_t \cup \{< x^i_t, w^i_t >\}$ ***Insert***
8. **For** $i = 1 \ldots n$
9. $w^i_t = w^i_t / \eta$ ***Normalize weights***

Graphical steps