Conceptual problems week 3

1) a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ POSSIBLE

b) Not possible, the set will remain linearly dependent. There will still be a non-trivial solution if another vector is added to the set

c)

possible $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ Exactly 3 vectors can be in this set

possible $\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{bmatrix}$ 4 or more vectors can be in this set

possible $\begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}$ 2 or less vector can be in this set

possible $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 5 \end{bmatrix}$ There can be an infinite amount of vectors in this set

2)

a) This is true, there will continue to be a pivot in every row if a second vector is added

$n \leq 3$

b) This is true, since the vectors are linearly independent b/c they span R^n, then replacing linear combinations will also be linearly independent

c) This is false,

take: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and add $\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

Now there is not a pivot in every column and the matrix is in echelon form thus making the system linearly indepent

d) This is true, no vectors can be made a linear combination of the other thus making this statement true.

3) a) $n=3$, matrix has 3 rows

b) $m=4$ matrix has 4 columns

c) No, there is not a pivot in every row

d) No, there is a zero row so the set can only span R^2

e) $\{u_3, u_4\}$, these vectors are not scalar multiples of each other thus making them linearly independent

f) No, since the matrix is in echelon form, we can deduce that any linear combination of the vectors can only span R^2

g) Yes, we know that set is linearly dependent, meaning that all vectors can be represented as linear combinations of one another. Thus all vectors are in the same span

h) Yes, see explanation for g.

4) a) $T: R_2 \to R_3$

thus

$$A = \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix} = \begin{matrix} a+d=1 \Rightarrow a=2 \\ b+e=0 \Rightarrow b=-1 \\ c+f=1 \Rightarrow c=1 \end{matrix}$$

$$\begin{bmatrix} 0 \\ 2 \end{bmatrix} \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ 0 \end{bmatrix} \quad \begin{matrix} d=-1 \\ e=1 \\ f=0 \end{matrix}$$

$$\begin{bmatrix} 2 & -1 \\ 1 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\Downarrow$$

$$\begin{bmatrix} 2 & -1 \\ -1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} -1 \\ -1+3 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$$

Not possible, only linear transformation that satisfies ① & ② doesn't satisfy ③

Not possible

b) my answer changes to $T(\vec{x}) = \begin{bmatrix} 2 & -1 \\ -1 & 1 \\ 1 & 0 \end{bmatrix}$ as it satisfies all conditions

5) $T(x) = Ax$

a) not possible, vectors of matrix cannot span all of R^3

b) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 5 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$ pivot in every column

d) Not possible, $n < m$

e) Not possible, unifying theorem states the transformation will either be both or neither

f) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ No pivot in every row & col.