Homework 0

Due Jan 18, 2013,
Instructions for submission into your
class SVN repository will be included with lab01.

The purpose of this assignment is to give you a chance to refresh the math skills we expect you to have learned in prior classes. These particular skills will be essential to mastery of CS225, and we are unlikely to take much class time reminding you how to solve similar problems. Though you are not required to work independently on this assignment, we encourage you to do so because we think it may help you diagnose and remedy some things you might otherwise find difficult later on in the course.

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Section (circle one): Wed 7-9,

Thurs 9-11, Thurs 11-1, Thurs 1-3, Thurs 3-5, Thurs 5-7, Thurs 7-9,

Fri 9-11, Fri 11-1, Fri 1-3, Fri 3-5, Fri 5-7

Grade		Out of 60
Grader		

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1. (3 points) Using 140 characters or less, post a synopsis of your favorite movie to the course piazza space under the "HW0 tell me something!" notice, so that your post is visible to everyone in the class, and tagged by #HW0num1. Also, describe someplace you traveled this winter break to a private post to course staff, also with the tag #HW0num1. Finally, please write the 2 post numbers corresponding to your posts here:

Favorite Movie Post (Public) number:	@17
Summer Travel Post (Private) number:	@165

2. (12 points) Simplify the following expressions as much as possible, **without using an calculator (either hardware or software)**. Do not approximate. Express all rational numbers as improper fractions. Show your work in the space provided, and write your answer in the box provided.

(a) $\prod_{k=2}^{n}(1-\frac{1}{k^2})$

Answer for (a):	$\frac{n+1}{2n}$

(b) $3^{1000} \bmod 7$

$3^0 \bmod 7 = 1$
$3^1 \bmod 7 = 3$
$3^2 \bmod 7 = 2$
$3^3 \bmod 7 = 6$
$3^4 \bmod 7 = 4$
$3^5 \bmod 7 = 5$
$3^6 \bmod 7 = 1$

Repeating 6-cycle

$6\overline{)1001}$ = 166 remainder 5
-6
40
-36
41
-36
5

$3^{1000} \bmod 7$ is 5th element in cycle $= 4$

Answer for (b):	4

(c) $\sum_{r=1}^{\infty}(\frac{1}{2})^r$

$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}\dots+\frac{1}{2^r}=1$

OR

since the series converges, we can use the formula

$$\frac{\frac{1}{2}}{1-\frac{1}{2}} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1$$

Answer for (c):	1

(d) $\dfrac{\log_7 81}{\log_7 9}$

$\log_9 81$

$9^x = 81$

Answer for (d):	2

(e) $\log_2 4^{2n}$

$2^x = 4^{2n}$

$2^x = 2^{4n}$

Answer for (e):	4n

(f) $\log_{17} 221 - \log_{17} 13$

$= \log_{17}\left(\frac{221}{13}\right)$

$= \log_{17}(17)$

$= 1$

Answer for (f):	1

3. (8 points) Find the formula for $1+\sum_{j=1}^{n} j!j$, and show work proving the formula is correct using induction.

Formula:	$(n+1)!$

Claim: $1+\sum_{j=1}^{n} j!j = (n+1)!$

Base Case: $n=1$; $1+\sum_{j=1}^{1} 1!\cdot 1 = (1+1)!$

$1 + 1!\cdot 1 = 2!$

$2 = 2$

Induction: Suppose $1+\sum_{j=1}^{n} j!j = (n+1)!$ for $n = 1,2,3,\dots,k-1$

We need to show that $1+\sum_{j=1}^{n} j!j = (n+1)!$ for $n=k$

So From IH: $1+\sum_{j=1}^{k-1} j!j = k!$

$$1+\sum_{j=1}^{k} j!j = \left(1+\sum_{j=1}^{k-1} j!j\right) + k!k$$
$$= k! + k!\cdot k$$
$$= k!(k+1)$$

$1+\sum_{j=1}^{k} j!j = (k+1)!$

which is what we wanted to show.

$\square$

4. (8 points) Indicate for each of the following pairs of expressions $(f(n), g(n))$, whether $f(n)$ is O, Ω, or Θ of $g(n)$. Prove your answers to the first two items, but just GIVE an answer to the last two.

(a) $f(n) = 4^{\log_4 n}$ and $g(n) = 2n+1$.

$= n$ $= 2n+1$

$0 \le n \le (2n+1)\cdot C$

$n \le (2n+1)\cdot C$ for $n \ge k$

$n \le 2n+1$ $C=1, k=1$

So n is $O(2n+1)$

Also $2n+1 \le C\cdot n$ for $n \ge k$

$2n+1 \le 3n$ $C=3, k=1$

So $2n+1$ is $O(n)$

If $f(n)$ is $O(g(n))$ and $g(n)$ is $O(f(n))$ then $f(n)$ is $\Theta(g(n))$

Answer for (a):	$f(n)$ is $\Theta(g(n))$

(b) $f(n) = n^2$ and $g(n) = (\sqrt{2})^{\log_2 n}$.

$f(n) = n^2$ $g(n) = \sqrt{n}$

Let's show that $g(n) \le f(n)$

$\sqrt{n} \le C\cdot n^2$ for $n \ge k$

$C=1, k=1$

$\sqrt{n} \le n^2$

$\sqrt{1} \le 1^2$

So $\sqrt{n}$ is $O(n^2)$

and n^2 is $\Omega(\sqrt{n})$

Answer for (b):	$f(n)$ is $\Omega(g(n))$

(c) $f(n) = \log_2 n!$ and $g(n) = n \log_2 n$.

Answer for (c):	$f(n)$ is $O(g(n))$

(d) $f(n) = n^k$ and $g(n) = c^n$ where k, c are constants and c is >1.

Answer for (d):	$f(n)$ is $O(g(n))$

5. (9 points) Solve the following recurrence relations for integer n. If no solution exists, please explain the result.

(a) $T(n) = T(\frac{n}{2}) + 5$, $T(1) = 1$, assume n is a power of 2.

Answer for (a):	$5 \log_2 n + 1$

(b) $T(n) = T(n-1) + \frac{1}{n}$, $T(0) = 0$.

Answer for (b):	No solution, it is a harmonic series

that does not converge

(c) Prove that your answer to part (a) is correct using induction.

Claim: $T(n) = 5\log_2 n + 1$ Recursive Definition: $T(n) = T(\frac{n}{2}) + 5$

$T(1) = 1$

Base Case: $T(1) = 5\log_2 1 + 1$

$= 5 \cdot 0 + 1$

$T(1) = 1$

Induction: Suppose $T(n) = 5\log_2 n + 1$ for $n = 1, 2, \ldots, k-1$

We must show that $T(k) = 5\log_2 k + 1$

So $T(k) = T(\frac{k}{2}) + 5$ (from recursive definition)

$T(k) = 5\log_2(\frac{k}{2}) + 1 + 5$ · from I.H.

$\log_2(\frac{k}{2}) = \log_2 k - \log_2 2$

$T(k) = 5(\log_2(\frac{k}{2}) + 1) + 1$

$= 5(\log_2 k - 1 + 1) + 1$

$T(k) = 5\log_2 k + 1$

$\square$

6. (10 points) Suppose function call parameter passing costs constant time, independent of the size of the structure being passed.

(a) Give a recurrence for worst case running time of the recursive Binary Search function in terms of n, the size of the search array. Assume n is a power of 2. Solve the recurrence.

Recurrence:	$T(n) = T(\frac{n}{2}) + 1$
Base case:	$T(1) = 1$
Recurrence Solution:	$\log_2 n + 1$

(b) Give a recurrence for worst case running time of the recursive Merge Sort function in terms of n, the size of the array being sorted. Solve the recurrence.

Recurrence:	$T(n) = 2T(\frac{n}{2}) + n$
Base case:	$T(1) = 1$
Running Time:	$O(n \log_2 n)$

7. (10 points) Consider the pseudocode function below.

```
derp( x, n )
    if( n == 0 )
        return 1;
    if( n % 2 == 0 )
        return derp( x^2, n/2 );
    return x * derp( x^2, (n - 1) / 2);
```

(a) What is the output when passed the following parameters: $x = 2$, $n = 12$. Show your work (activation diagram or similar).

$d(2,12) \rightarrow d(4,6) \rightarrow d(16,3) \rightarrow 16 \cdot d(16^2, 1) \rightarrow 16^2 \cdot d(16^4, 0) \rightarrow 1$

Answer for (a):	4096

(b) Briefly describe what this function is doing.

Computing x^n

(c) Write a recurrence that models the running time of this function. Assume checks, returns, and arithmetic are constant time, but be sure to evaluate all function calls. [Hint: what is the *most* n could be at each level of the recurrence?]

$T(0) = 1$

$T(n) = T\left(\lfloor \frac{n}{2} \rfloor\right) + 1$

(d) Solve the above recurrence for the running time of this function.

$T(n) = \log_2 n$