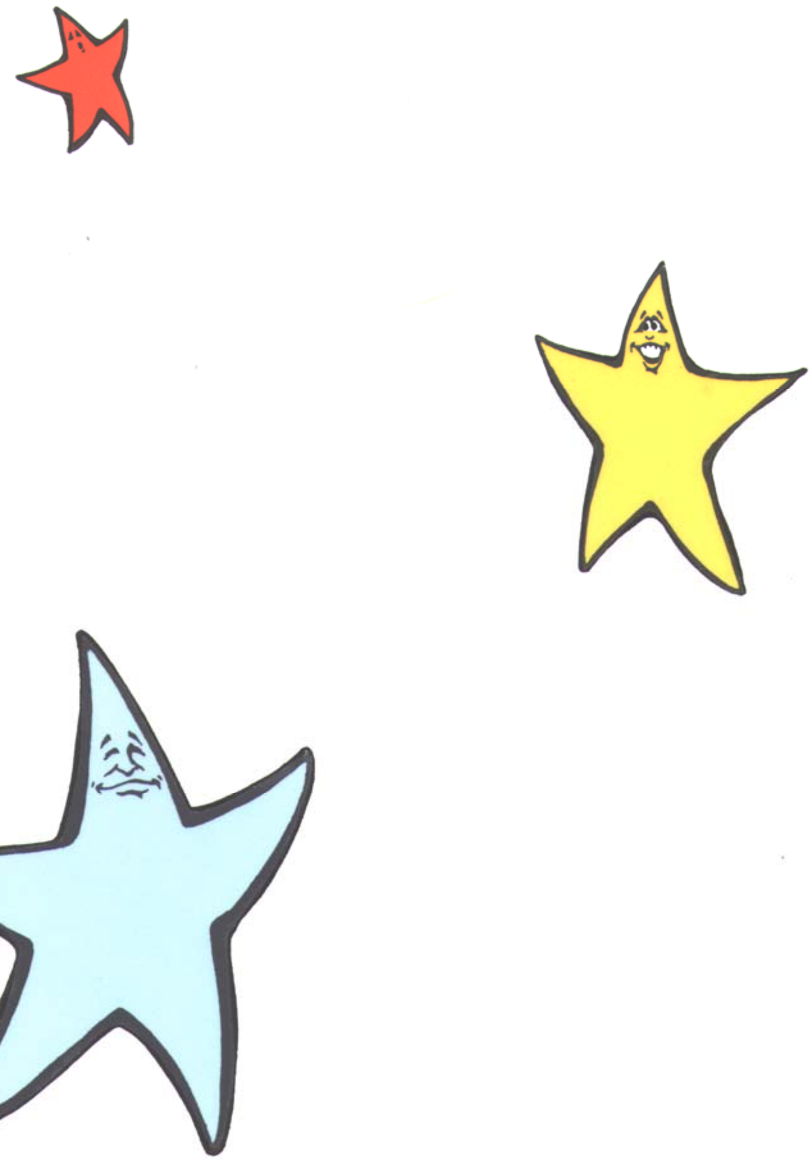
Binary Stars



Key Characteristics

About half of all stars are binary or multiple star systems

Key Fundamentals

Binary Stars

Used to determine stellar masses

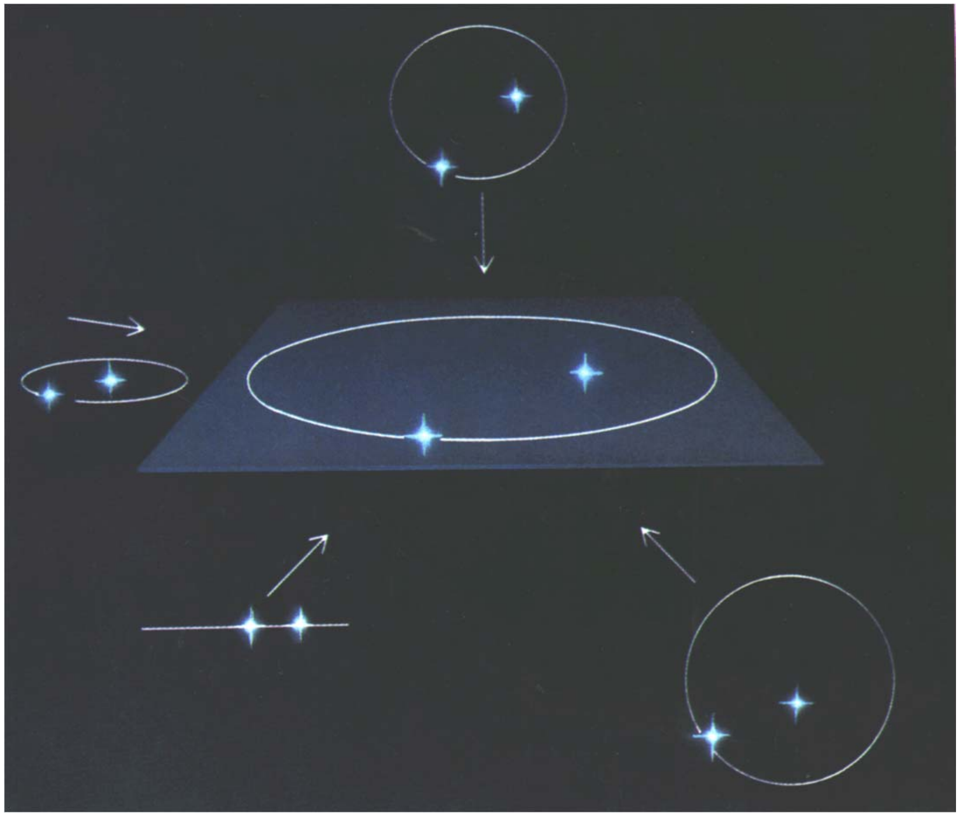
Used to determine stellar diameters

Major Types of Binary Stars

Visual Binaries

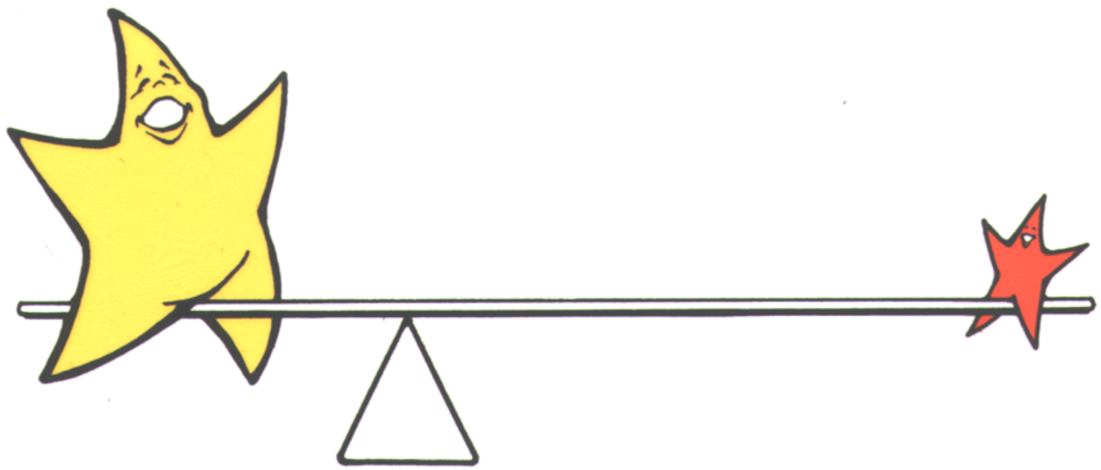
Spectroscopic Binaries

Eclipsing Binaries



Visual Binaries

Visual Binaries



Visual Binaries are Resolved

Skinny Triangle Approximation

$$D = d \tan \theta$$

Resolution

$$\alpha = 2.5 \times 10^5 \lambda / D$$

Example: GT 16-inch telescope

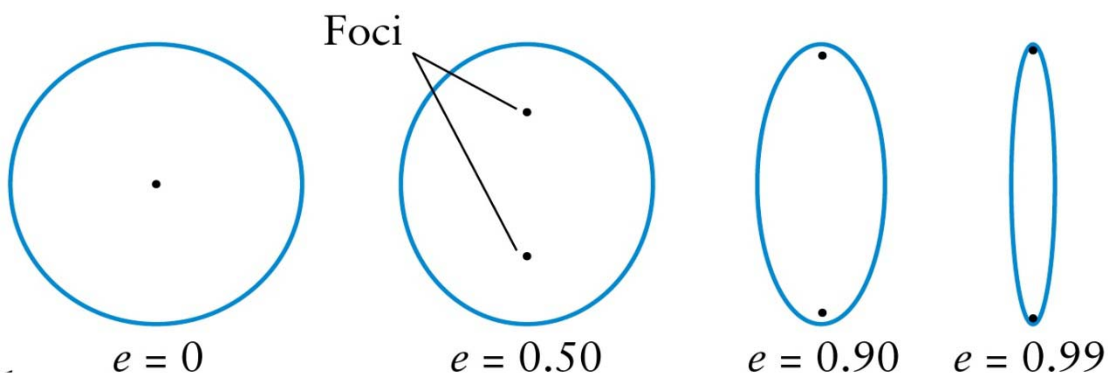
$$\alpha = 2.5 \times 10^5 (500 \times 10^{-9} \text{ m}) / (0.4 \text{ m}) = 0.3 \text{ arcsec}$$

The atmosphere limits all telescopes to a resolution of ~1.0 arcseconds.

Shapes of Orbits

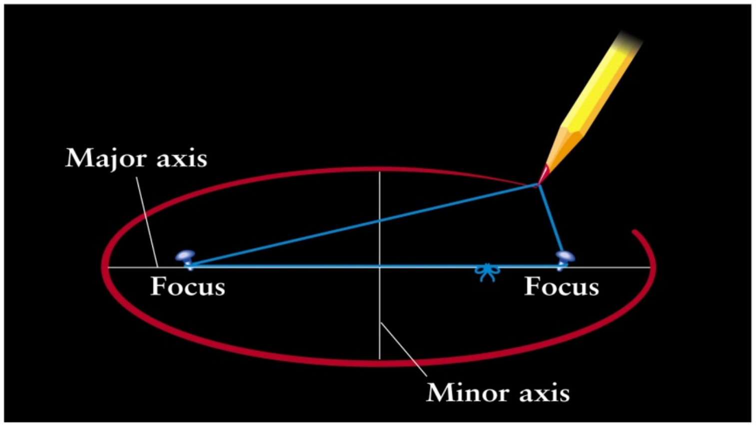
First Kepler tried circles, equants, ovals, etc. Finally, after years, he tried an ellipse. Found that the orbit of Mars is an ellipse with the Sun at a focus.

The sum of the distances to the two foci is always constant for all points on the ellipse. Ellipses are described by their **semi-major axis** and by their **eccentricity**.   e = (Δ foci / major axis)



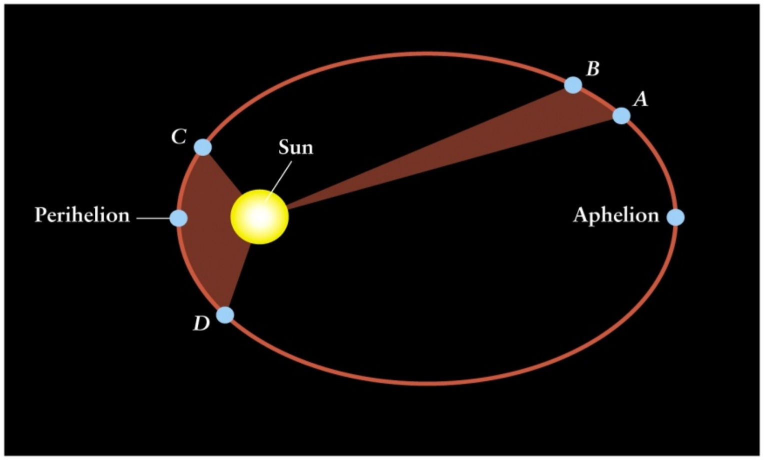
Kepler's Three Laws

1. All planets have elliptical orbits with the Sun at a focus (conic sections).



2. Law of Equal Areas:

Equal areas are swept out in equal time intervals.



[Kepler's Second Law Interactive]

Kepler’s Three Laws

3. Harmonic Law (published in *The Harmony of the Worlds)*:

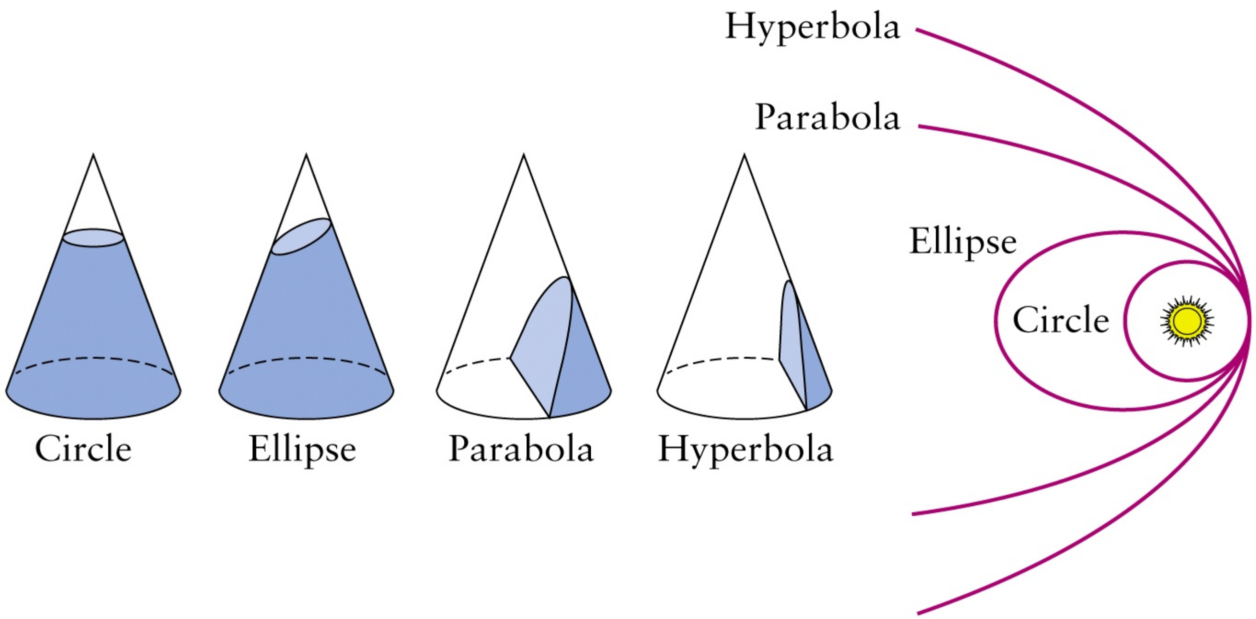
$$P^2 = k\,a^3,$$

where k = 1 if P is in earth years and *a* is in **AU**s.

table 4-3 A Demonstration of Kepler’s Third Law

Planet	Sidereal period P (years)	Semimajor axis a (AU)	P^2	a^3
Mercury	0.24	0.39	0.06	0.06
Venus	0.61	0.72	0.37	0.37
Earth	1.00	1.00	1.00	1.00
Mars	1.88	1.52	3.53	3.51
Jupiter	11.86	5.20	140.7	140.6
Saturn	29.46	9.54	867.9	868.3
Uranus	84.01	19.19	7,058	7,067
Neptune	64.79	30.06	27,160	27,160
Pluto	248.54	39.53	61,770	61,770

Modification of Kepler's Laws



All orbiting bodies have a conic-section orbit,
with the massive body (i.e., the Sun) at a focus.

Modification of Kepler's Laws

a Ball moves at a high speed in a small circle

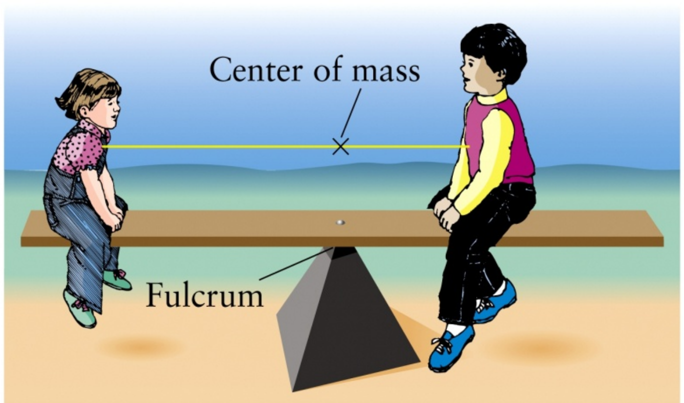
b Ball moves at a low speed in a large circle

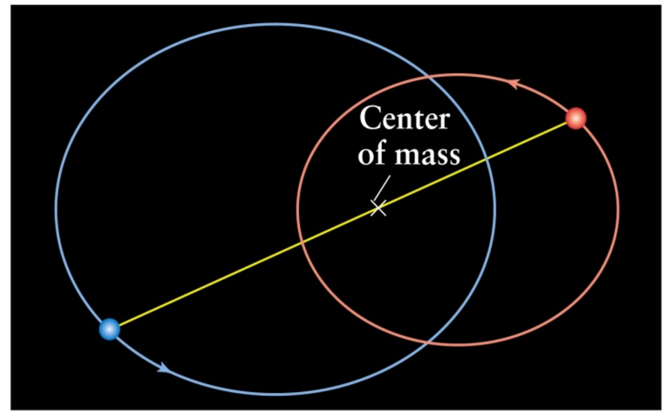
Law of Equal Areas: Equal areas are swept out in equal time intervals.

This is explained by Conservation of Angular Momentum.

$$r_1 v_1 = r_2 v_2$$

Modification of Kepler's Laws





The Third Law needs to have the sum of the masses included.

$$(\mathcal{M} + m)\, P^2 = k\, a^3,$$

where k = 1 if P is in earth years, *a* is in AUs, and $(\mathcal{M} + m)$ is in solar masses.

For objects orbiting the Sun, $(\mathcal{M} + m) = 1$.

Visual Binaries

Kepler's 3rd Law

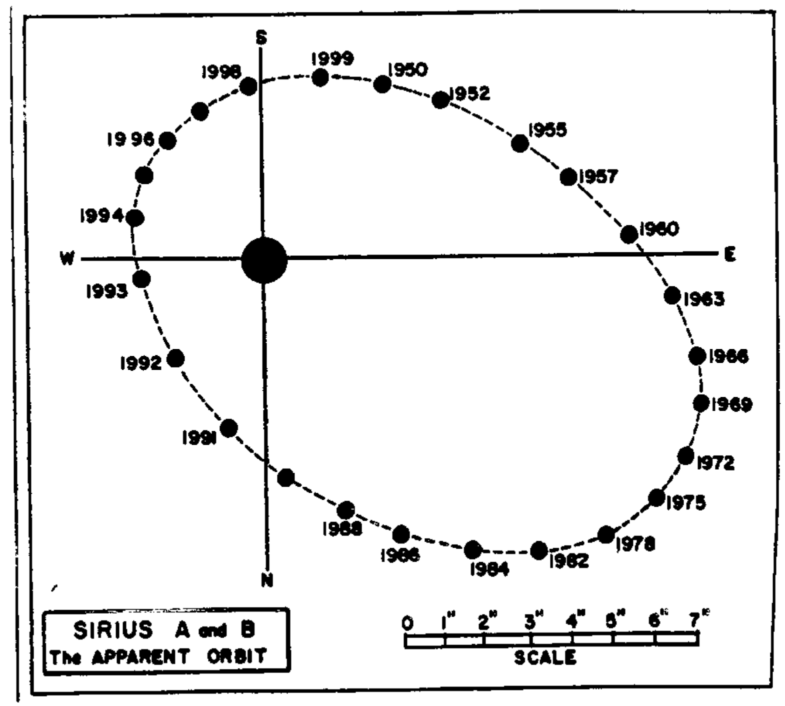
$$(\mathcal{M}_1 + \mathcal{M}_2)\, P^2 = a^3$$

$\mathcal{M}$ is in solar masses

P is in years, and

a is in Astronomical Units

(1 AU = mean Earth-Sun distance)



[Kepler's Third Law Interactive]

PRS Question

1. If two stars with the mass of the Sun had a circular orbit (radius = 5 AU), the orbital period would be about

   a. 4.0 years
   b. 5.5 years
   c. 8.0 years
   d. 12.0 years
   e. 16.0 years

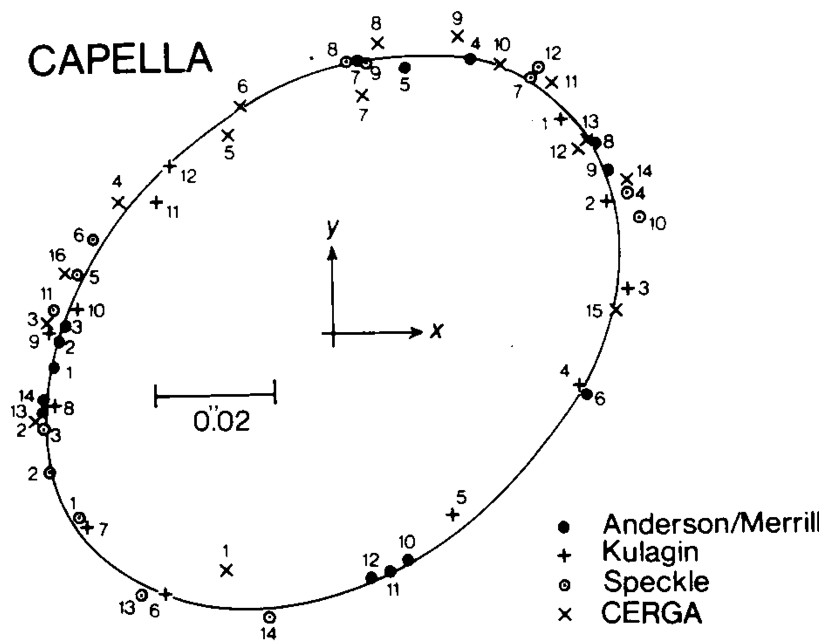
Resolution Improvements

Speckle Techniques

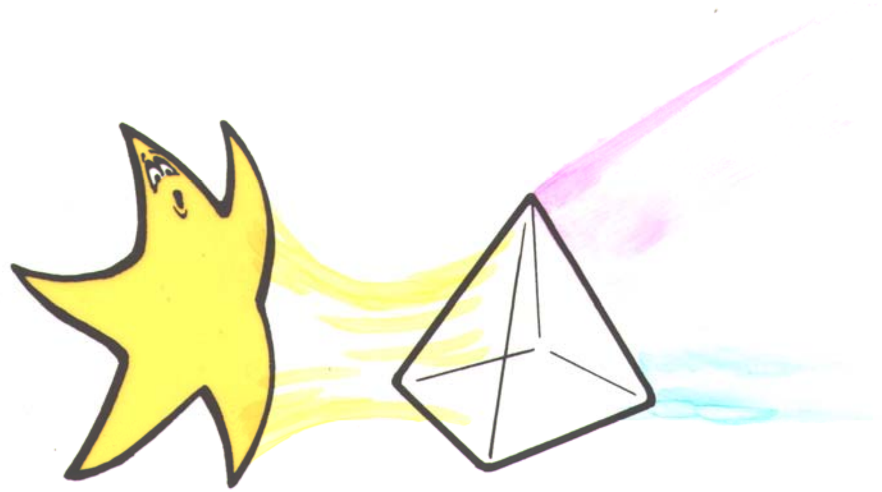
Resolutions to 0.02 arcsec

Interferometric Techniques

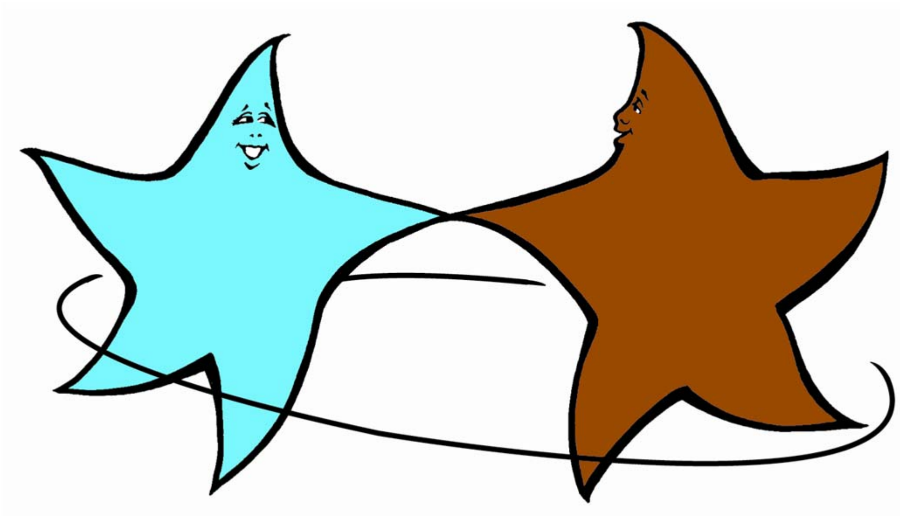
Resolutions to 0.001 arcsec



Spectrum Binaries



Spectroscopic Binaries



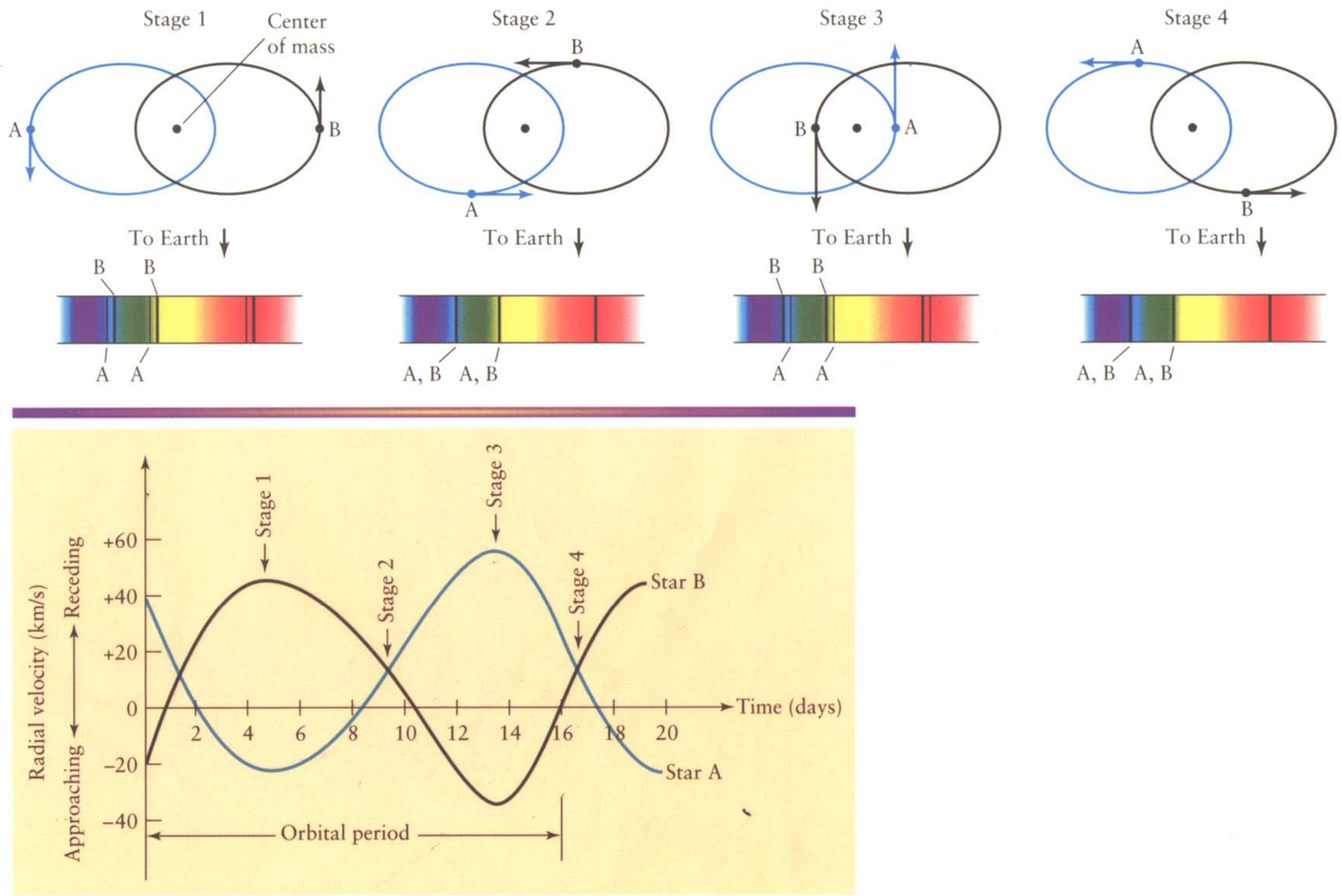
Spectroscopic Binaries

Conservation of Angular Momentum

$$\mathcal{M}_1 / \mathcal{M}_2 = v_2 / v_1 = r_2 / r_1$$

Doppler Shift

$$\Delta\lambda / \lambda = v / c$$



[Radial Velocity Interactive]

PRS Question

2. If the orbital velocity of Star A is 5 times that of Star B, then Star A is how many times as massive as Star B?

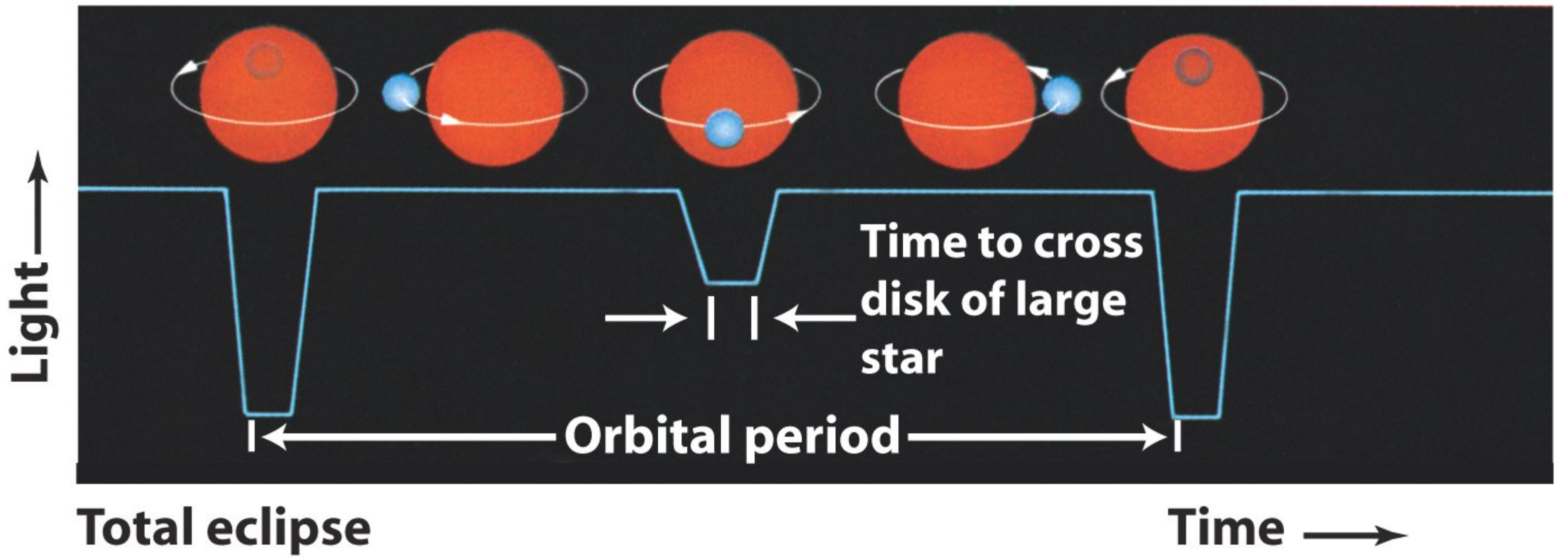
   a. The same
   b. 5 times
   c. 25 times
   d. 1/5 times
   e. 1/25 times

Eclipsing Binaries

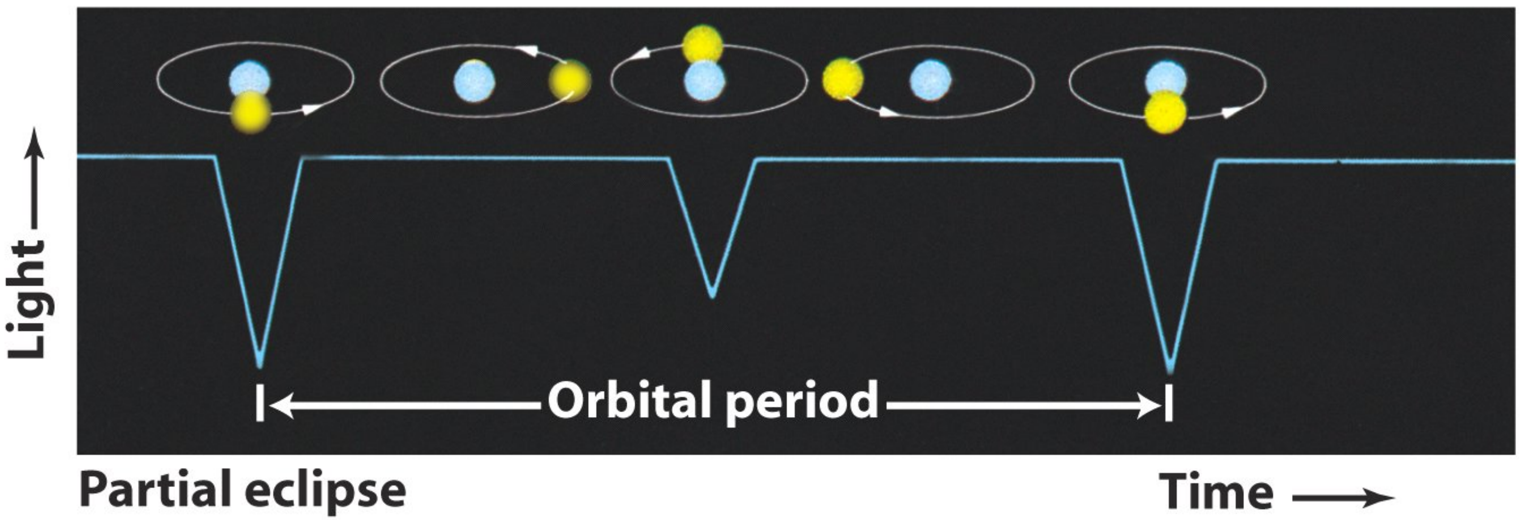
Eclipsing Binaries

Unresolved – appear as a single star

Orbital plane lies close to our line-of-sight



Partial Eclipse



Binary Star Interactive

Phase

Horizontal Axis

$$\frac{\text{Observed Time} - \text{Reference Time}}{\text{Orbital Period}} = XX.\underbrace{\boxed{XXXX}}_{\text{Phase}}$$

Textbook Binary

Observational

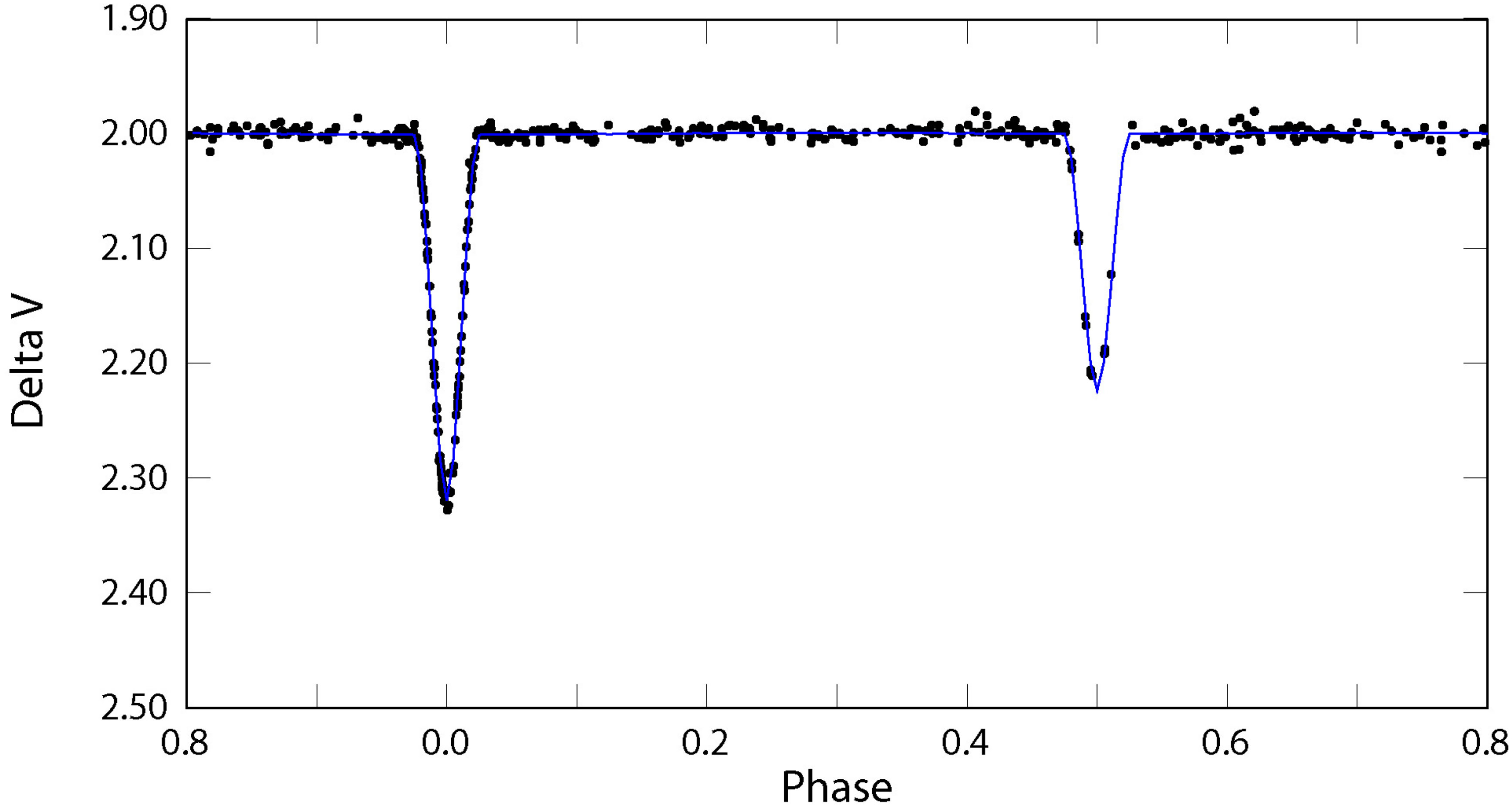
HD 71636 Partial

P = 5.0 days

a = 17.4 $R_\odot$

$\mathcal{M}_1 = 1.5\ \mathcal{M}_\odot$ $\mathcal{M}_2 = 1.3\ \mathcal{M}_\odot$

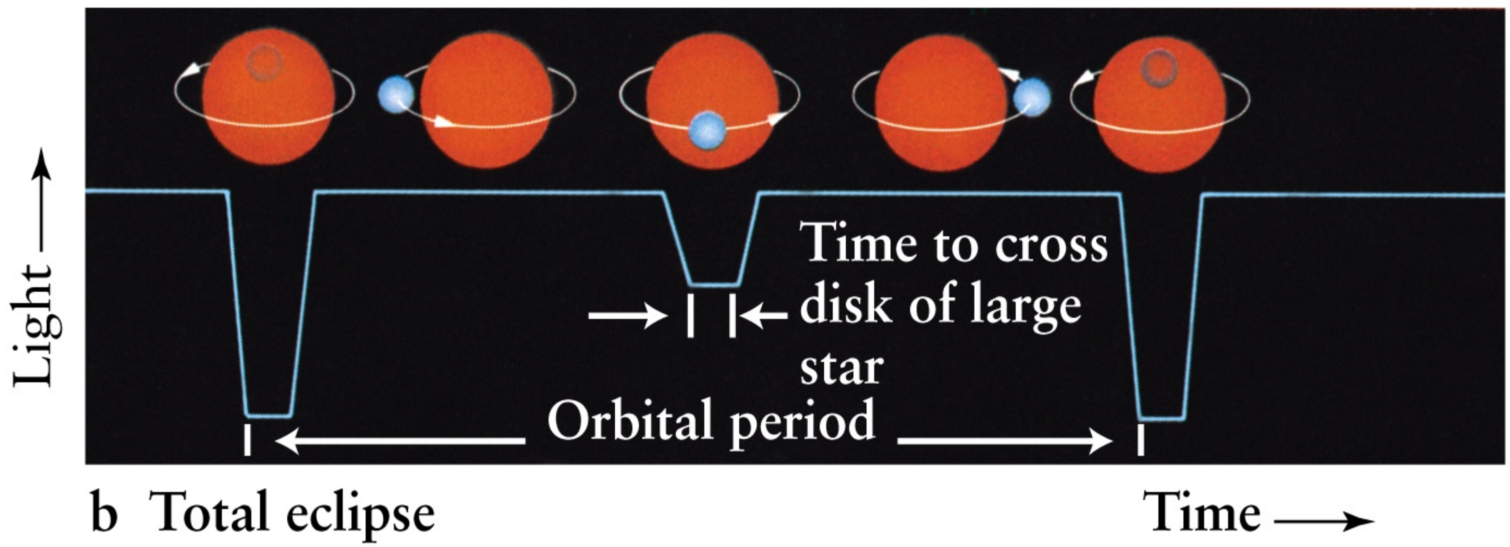
$R_1 = 1.4\ R_\odot$ $R_2 = 1.6\ R_\odot$


1.90
2.00
2.10
2.20
2.30
2.40
2.50
Delta V
0.8
0.0
0.2
0.4
0.6
0.8
Phase

Diameters of Stars

$$L = 4 \pi R^2 \sigma T^4$$

$$L \propto R^2 T^4$$



b Total eclipse

PRS Question

3. Two stars in a binary system have the same temperature but the diameter of Star A is twice that of Star B. How many times brighter is Star A than Star B?

a. 1 X
b. 2 X
c. 4 X
d. 16 X
e. 32 X

Mass-Luminosity Relationship

$$L \propto \mathcal{M}^{4.0}$$

$$0.08 \text{ solar} < \mathcal{M} < 50 \text{ solar}$$