

given

$$\frac{dC_A}{dt} = -k C_A^\alpha$$

a common way to estimate $k + \alpha$ is through

$$\ln\left(-\frac{dC_A}{dt}\right) = \ln k + \alpha \ln C_A$$

This is not mathematically legitimate, because the natural log of units are not defined. The right way to do this is to make everything dimensionless. we define. We define these dimensionless quantities;

$$C = \frac{C_A}{C_{A0}} \quad \tau = \frac{t}{t_{1/2}}$$

$$C_A = C_{A0} \cdot C \quad t = \tau \cdot t_{1/2}$$

then,

$$\frac{C_{A0} dC_A}{t_{1/2} d\tau} = -k (C_{A0} \cdot C)^\alpha \Rightarrow \frac{dC}{d\tau} = -\frac{k t_{1/2}}{C_{A0}} C_{A0}^\alpha C^\alpha$$

$$\text{So, } \ln\left(-\frac{dC}{d\tau}\right) = \ln(k t_{1/2} C_{A0}^{\alpha-1}) + \alpha \ln C$$

Now, all the terms in the natural log terms are dimensionless, and this is mathematically well posed.

The only thing we have to do is figure out what the characteristic dimensions for making the dimensionless quantities. For the concentration, the inlet concentration is a natural characteristic value. For time, we can use the time to react $\frac{1}{2}$ the inlet concentration away.

Now we use the units package to show everything is actually dimensionless.