Artificial & Computational Intelligence

DSE **ZG557

A* Algorithm

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Informed Search
A*

A* Search

Expands the node which lies in the closest path (estimated cheapest path) to the goal

Evaluation function **f(n) = g(n) + h(n)**

- g(n) – the cost to reach the node / path cost
- h(n) – the expected cost to go from node to goal
- f(n) – estimated cost of cheapest path through node n

Optimality of A*

Test for Admissibility

Expands the node which lies in the closest path (estimated cheapest path) to the goal

Evaluation function **f(n) = g(n) + h(n)**

- g(n) – the cost to reach the node
- h(n) – the expected cost to go from node to goal
- f(n) – estimated cost of cheapest path through node n

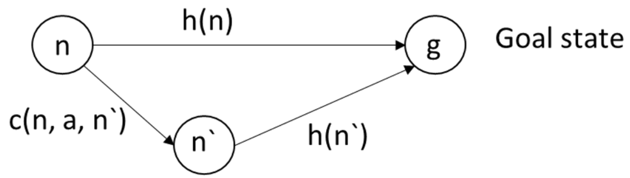
A heuristic is admissible or optimistic if , $\mathbf{0 \leq h(n) \leq h^*(n)}$, where h*(n) is the actual cost to reach the goal

A* Search

Optimal on condition

h(n) must satisfies two conditions:

- **Admissible Heuristic** – one that never overestimates the cost to reach the goal
- **Consistency** – A heuristic is consistent if for every node n and every successor node n` of n generated by action a, h(n) <= c(n, a, n`) + h(n`)



Consistency

In A*, the triangle inequality is related to the **consistency** property of the heuristic function. For a heuristic h(n), the triangle inequality ensures that:

$$h(n) \leq c(n,n') + h(n')$$

Where:

- h(n) is the estimated cost from node n to the goal,
- c(n,n′) is the actual cost between node n and its neighbor n′,
- h(n′) is the estimated cost from n′ to the goal.

Triangle inequality

It states that for any three points A, B, and C, the direct distance between two points cannot be greater than the sum of the distances through a third point.

$$d(A,C) \leq d(A,B) + d(B,C)$$

Example

Imagine three cities on a map: A, B, and C. The direct distance from A to C is shorter or equal to the sum of the distances from A to B and B to C. This is the triangle inequality in action.

- $d(A,C)=10$
- $d(A,B)=6$
- $d(B,C)=5$

Here:

$$d(A,C) \leq d(A,B)+d(B,C) \qquad (10 \leq 6+5)$$

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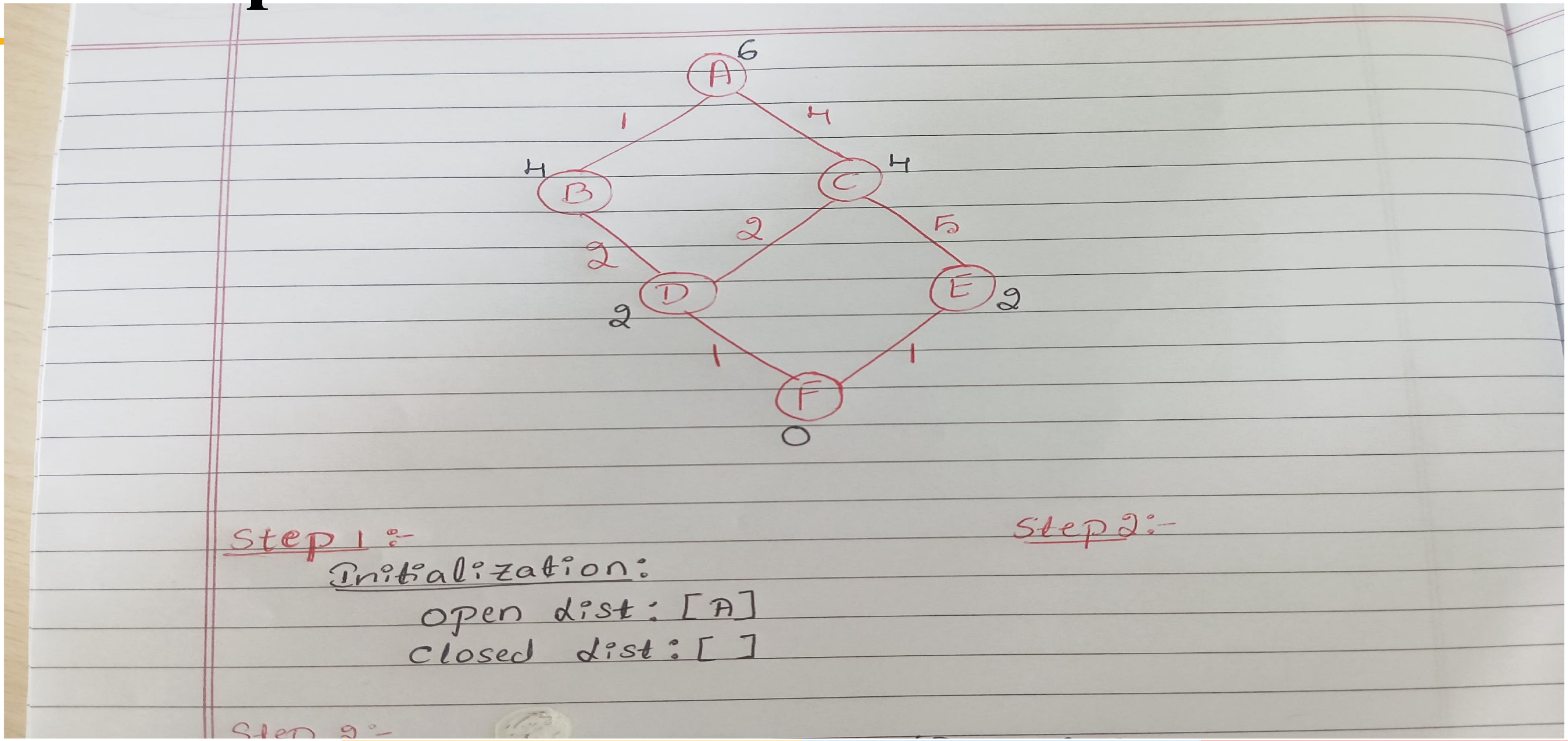
Complete

- If the number of nodes with cost <=C* is finite
- If the branching factor is finite
- A* expands no nodes with f(n) > C*, known as pruning

Time Complexity - $\mathcal{O}(b^{\Delta})$ where the absolute error $\Delta = h^* - h$

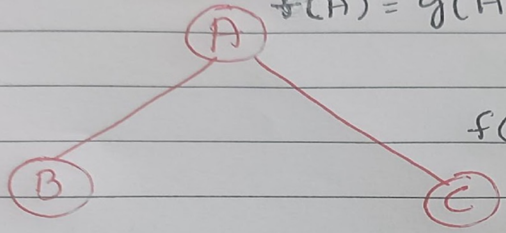
Example


A 6
1
4
B 4
C 4
2
2
5
D 2
E 2
1
1
F 0
Step 1:-
Initialization:
open list: [A]
closed list: [ ]
Step 2:-

Cont…

Step 2:-

$f(A) = g(A,A) + h(A) = 0 + 6 = 6$



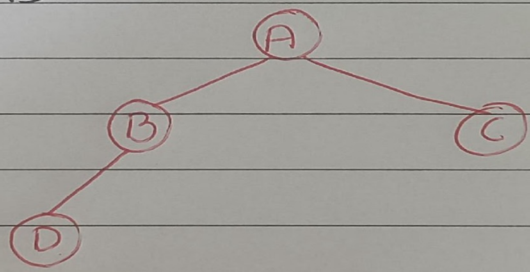
$f(B) = g(A,B) + h(B) = 1 + 4 = 5$

$f(C) = g(A,C) + h(C) = 4 + 4 = 8$

Open list : [B, C]

Closed list : [A]

Step 3:-



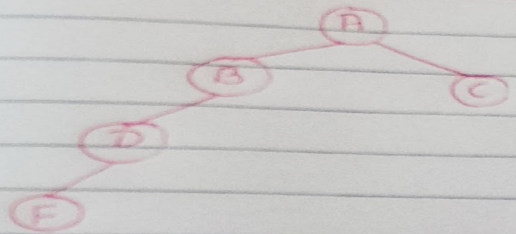
$f(D) = g(B,D) + h(D) = (1+2) + 2 = 5$

Open list : [C, D]

Closed list : [A, B]

Cont…

Step 4:-

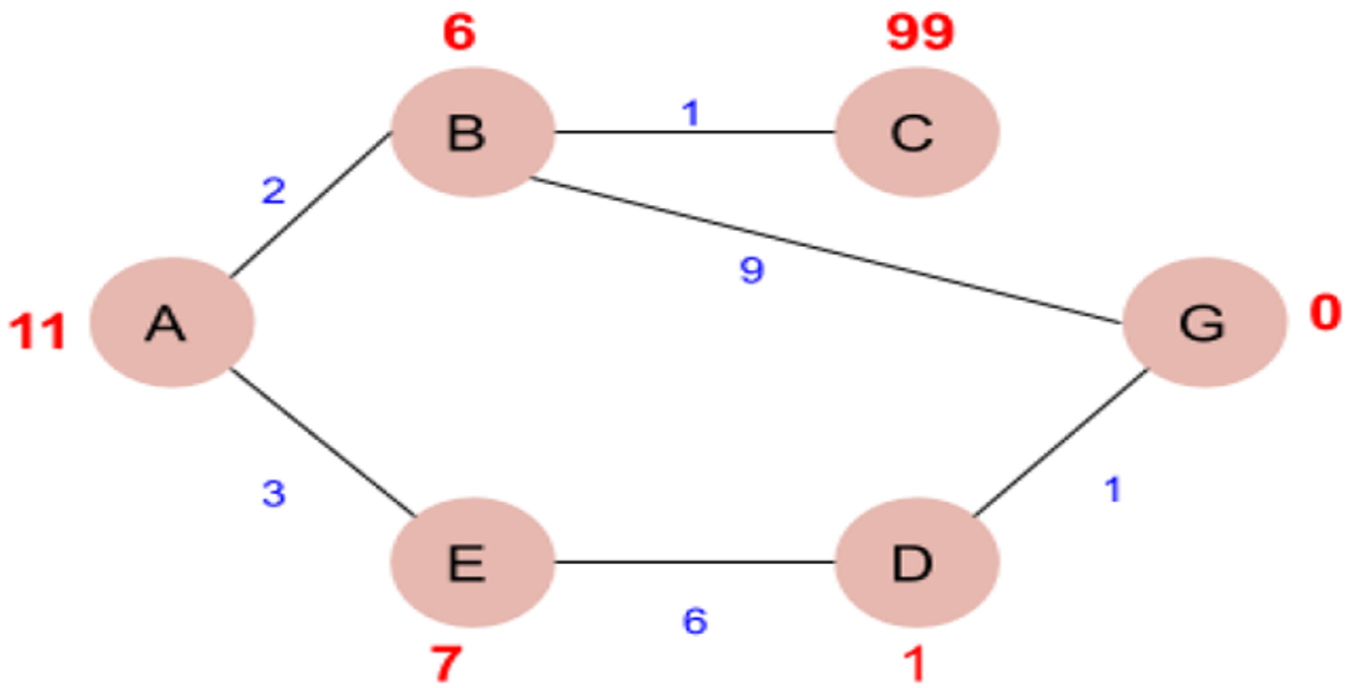


Open list: [C, F]
Closed list: [A, B, D]

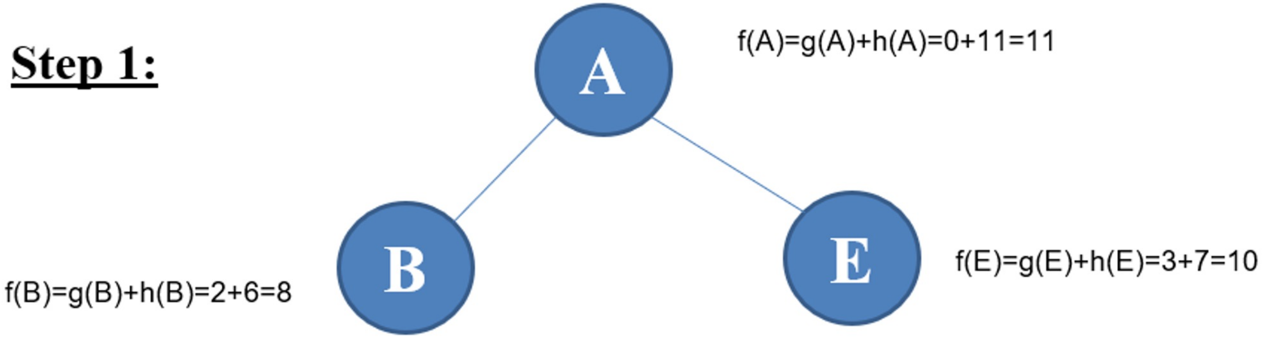
$f(F) = g(D, F) + h(F)$
$= (1+2+1) + 0 = 4$

$A \rightarrow B \rightarrow D \rightarrow F$

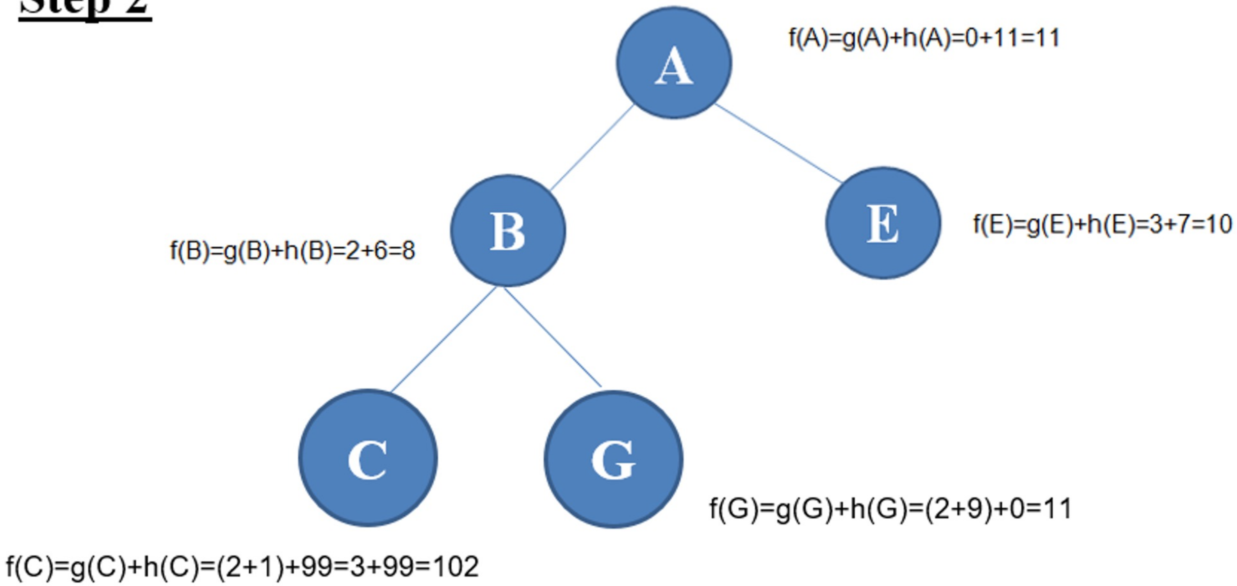
Example 1



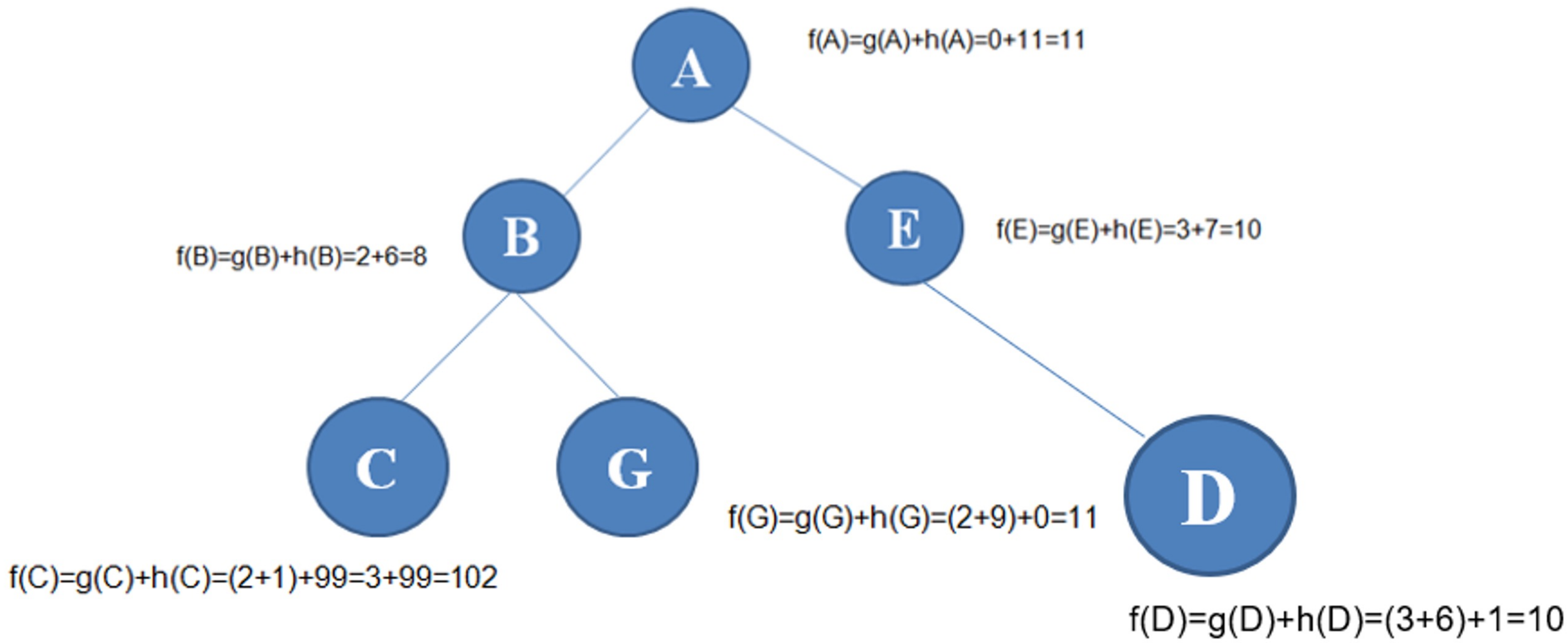
Step 1:


A
f(A)=g(A)+h(A)=0+11=11
B
f(B)=g(B)+h(B)=2+6=8
E
f(E)=g(E)+h(E)=3+7=10

Step 2

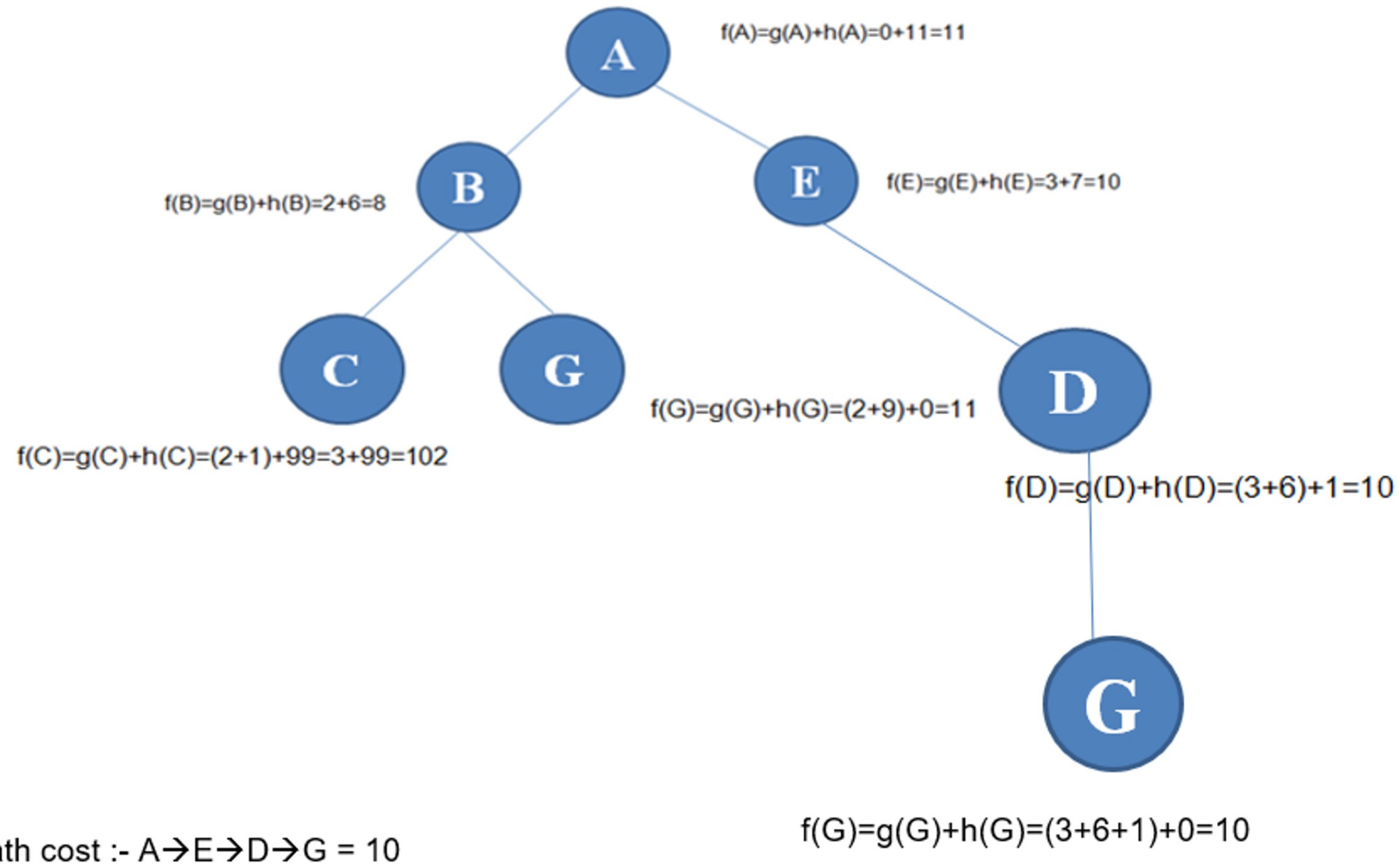

A
f(A)=g(A)+h(A)=0+11=11
B
f(B)=g(B)+h(B)=2+6=8
E
f(E)=g(E)+h(E)=3+7=10
C
G
f(G)=g(G)+h(G)=(2+9)+0=11
f(C)=g(C)+h(C)=(2+1)+99=3+99=102

Step 3:



A
f(A)=g(A)+h(A)=0+11=11
B
f(B)=g(B)+h(B)=2+6=8
E
f(E)=g(E)+h(E)=3+7=10
C
G
D
f(G)=g(G)+h(G)=(2+9)+0=11
f(C)=g(C)+h(C)=(2+1)+99=3+99=102
f(D)=g(D)+h(D)=(3+6)+1=10

Step 4:



Path cost :- A→E→D→G = 10