Birla Institute of Technology & Science, Pilani
Work Integrated Learning Programmes Division
First Semester 2025-2026

Mid-Semester Test
(EC-2 Regular)

Course No.	: AIML ZC418
Course Title	: Introduction to Statistical Methods
Nature of Exam	: Closed Book
Weightage	: 30%
Duration	: 2 Hours
Date of Exam	: 20/09/2025 (AN)

No. of Pages = 2
No. of Questions = 6

Note to Students:
1. Please follow all the *Instructions to Candidates* given on the cover page of the answer book.
2. All parts of a question should be answered consecutively. Each answer should start from a fresh page.
3. Assumptions made if any, should be stated clearly at the beginning of your answer.

Q1. A subscription-based video streaming service has data on its customers. 85% of customers remain subscribed after the first year, and 15% cancel their subscriptions. A marketing campaign is targeted at customers who remain subscribed. If a customer remains subscribed, there is a 90% chance they will watch at least 5 movies per month. However, if a customer cancels their subscription, there is a 30% chance they will watch at least 5 movies per month. If a customer is observed to watch at least 5 movies per month, what is the probability that they remain subscribed? [3M]

Let us use the following Notation for the events:
R: remaining subscribed after the first year
C: Cancelling subscription after the first year
W: Watching five movies per month
Given: P(R) = 0.85 and P(C) = 0.15
P(W|R) = 0.9 and P(W|C) = 0.3
We have to find P(R|W) =?

$$P(R|W) = \frac{P(W|R)P(R)}{P(W|R)P(R)+P(W|C)P(C)} \quad [1M]$$

$$= \frac{0.9 * 0.85}{(0.9 * 0.85) + (0.3 * 0.15)}$$

$$\frac{0.765}{0.765+0.045} = \frac{0.765}{0.81} = 0.9444 \quad [2M]$$

Q2. A box contains 3 red balls and 2 blue balls. Two balls are drawn without replacement. Let the random variable X = the number of red balls drawn and the random variable Y = the number of blue balls drawn.
(a) Construct the joint probability distribution of (X, Y). [2M]

P(X=0, Y=2) = $\frac{\binom{3}{0}\binom{2}{2}}{\binom{5}{2}} = \frac{1}{10}$

P(X=1, Y = 1) = $\frac{\binom{3}{1}\binom{2}{1}}{\binom{5}{2}} = \frac{6}{10}$

P(X=2, Y=0) = $\frac{\binom{3}{2}\binom{2}{0}}{\binom{5}{2}} = \frac{3}{10}$ [1M]

X	Y	P(X, Y)
0	2	0.1
1	1	0.6
2	0	0.3

[1M]

(b) Find the marginal distributions of X and Y.

Marginal distribution of X:

P(X=0) = $\frac{1}{10}$

P(X=1) = $\frac{6}{10}$

P(X=2) = $\frac{3}{10}$ [0.5M]

Marginal distribution of Y:

P(Y=0) = $\frac{3}{10}$

P(Y=1) = $\frac{6}{10}$

P(Y=2) = $\frac{1}{10}$ [0.5M]

(c) Compute P(X=1, Y=1) = 0.6 [1M]

Q3. In a garden, there are 7 plants. The heights (in cm) of the plants are 38, 51, 46, 79, 63, 47 and 57 respectively. Calculate the inter quartile range and standard deviation of their heights. [4M]

First arrange them in order: 38, 46, 47, 51,57, 63, 79.

Median = 51

Q1 = First quartile = 46

Q3 = 3rd quartile = 63 [0.5M]

Inter quartile range = Q3 – Q1 = 17[0.5M]

Mean = (38 + 46 + 47 + 51 + 57 + 63 + 79)/7 = 381/7 = 54.43

x	X - Mean	$(x – \text{Mean})^2$
38	-16.43	269.9449
46	-8.43	71.0649
47	-7.43	55.2049
51	-3.43	11.7649
57	2.57	6.6049
63	8.57	73.4449
79	24.57	603.6849

Variance = 155.9592 [2M\

Standard Deviation = 12.4883 [1M] [if 2 decimal approximation is correct give full mark]

Q4. A person applies for a job in Ola and Uber. The probability of his being selected in Ola is 0.6 and being rejected at Uber is 0.5. The probability of at least one of his applications being rejected is 0.6. What is the probability that he will be selected by one of these companies? [4M]

A : getting a job in Ola
B: getting a job in Uber
Given that $P(A) = 0.6$ and $P(B^c) = 0.5$. $P(B) = 0.5$
Let S be the sample space. Given that $P(S - (A\cap B)) = 0.6$ [2M]
$P(A\cap B) = 1 - 0.6 = 0.4$
[If the students interpreted the question as prob of selected by at least one of the company the following answer is correct. Award full mark but they should clearly write this assumtion]
The probability that he will be selected by at least one of these companies = $P(A \cup B)$
$= P(A) + P(B) - P(A\cap B)$
$= 0.6 + 0.5 - 0.4 = 0.7$ [2M]
But what we really asked here is prob of selected by exactly one of the companies
The probability that he will be selected by exactly one of these companies = $P(A \cup B)$
$= P(A) + P(B) - 2P(A\cap B)$
$= 0.6 + 0.5 - 0.8 = 0.3$ [2M]

Q5. The events A and B are independent with $P(A) = 0.4$ and $P(B)=0.7$.
(a) Find the probability that neither of the events occurs. [2M]
$P(A\cap B) = P(A)\, P(B) = 0.4 * 0.7 = 0.28$
$P(A \cup B) = P(A) + P(B) - P(A\cap B) = 1.1 - 0.28 = 0.82$ [1M]
probability that neither of the events occurs = $1 - P(A \cup B) = 1 - 0.82 = 0.18$ [1M]

(b) $P(A|B) = P(A\cap B)/P(B) = 0.28/0.7 = 0.4$ [1M]

(c) Are A and B mutually exclusive? Why? [1M]
No, A and B are not mutually exclusive since $P(A\cap B) = 0.28 \neq 0$. [1M]

(d) Are A and B mutually exhaustive? Why? [1M]
No, since $P(A \cup B) = 0.82 \neq 1$.

(e) Is it possible to have an event C such that $A\cap C \subseteq B$, $P(A\cap C) = 0.4$ and $P(B\cap C) = 0.2$. [2M]
Suppose there is an event C such that $A\cap C \subseteq B$, $P(A\cap C) = 0.4$ and $P(B\cap C) = 0.2$.
Since $A\cap C \subseteq B$, $A\cap C \subseteq B\cap C$. $P(A\cap C) \leq P(B\cap C)$. [1M]
This implies $0.4 \leq 0.2$ which is a contradiction. [1M]

Q6. After a national Science symposium, the organizing committee decides to form a review panel consisting of seven researchers selected from a team of eighteen. Out of these eighteen researchers, eight are Senior Researchers (SR) and 10 are Junior Researchers (JR). If researchers are selected at random to form the review panel, what is the probability that
i) the panel consists of 4 SR and 3 JR [1M]

P(the panel consists of 4 SR and 3 JR) = $\frac{\binom{8}{4}\binom{10}{3}}{\binom{18}{7}} = \frac{70 * 120}{3184} = \frac{8400}{31824} = 0.264$ [1M]

ii) the panel consists of only SR [1M]

P(the panel consists of only SR) = = $\frac{\binom{8}{7}\binom{10}{0}}{\binom{18}{7}} = \frac{8}{31824} = 0.00251$ [1M]

Q7. A city experiences major storms with a 10-year return period. Using the Poisson distribution, calculate the probability that exactly 3 such storms occur in a 20-year period. Provide the necessary steps and formulae. [3M]

Here p = 1/10 = 0.1 and n = 20

Therefore $\lambda = np = 0.1 * 20 = 2$ [1M]

The Poisson distribution P(X=x) = $\frac{e^{-\lambda}\lambda^{x}}{x!}$ [1M]

P(X = 3) = $\frac{e^{-2}2^{3}}{3!} = \frac{(2.7183)^{-2} * 8}{6} = \frac{0.1353 * 8}{6} = 0.1804$ [1M]

Q8. A city's annual average PM2.5 concentration follows a normal distribution with a mean of 12 μg/m³ and a standard deviation of 3 μg/m³. Calculate the probability that the concentration exceeds 18 μg/m³ in a given year. (Use the value $P(Z \leq 2) = 0.9772$). [3M]

Given that Mean μ = 12 and standard deviation σ = 3.

We have the formula $Z = \frac{X-\mu}{\sigma}$ [1M]

When X = 18, $Z = \frac{18-12}{3} = \frac{6}{3} = 2.$ [1M]

P(X>18) = P(Z>2) $= 1 - P(Z \leq 2) = 1 - 0.9772 = 0.0228$ [1M]
