Birla Institute of Technology & Science, Pilani
Work Integrated Learning Programmes Division
First Semester 2025-2026

Mid-Semester Test Solution
(EC-2 Makeup)

Course No. : AIML ZC418
Course Title : Introduction to Statistical Methods
Nature of Exam : Closed Book
Weightage : 30%
Duration : 2 Hours
Date of Exam : 05/10/2025 (AN)

No. of Pages = 2
No. of Questions = 7

Note to Students:
1. Please follow all the *Instructions to Candidates* given on the cover page of the answer book.
2. All parts of a question should be answered consecutively. Each answer should start from a fresh page.
3. Assumptions made if any, should be stated clearly at the beginning of your answer.

Q1. A communication channel carries messages by using only three signals, say 0, 1 and 2. Assume that, in this channel a 0 is transmitted 35% of the time, a 1 is transmitted 30% of the time and a 2 is transmitted 35% of the time.
If a 0 is transmitted, it is correctly received with probability 0.90; if an error occurs, the received signal is equally likely to be 1 or 2.
If a 1 is transmitted, it is correctly received with probability 0.85; if an error occurs, the received signal is equally likely to be 0 or 2.
If a 2 is transmitted, it is correctly received with probability 0.80; if an error occurs, the received signal is equally likely to be 0 or 1
(a) Determine the probability of a 2 being received. [3M]
The given data:

X	P(X transmitted)	P(correctly received/X transmitted)	P(each not x received/X transmitted)
0	0.35	0.90	0.10/2 = 0.05
1	0.30	0.85	0.15/2 = 0.075
2	0.35	0.80	0.20/2 = 0.10

Let T denote transmitted value and R denote received value

P(R = 2) = P(R=2|T=0)P(T=0) + P(R=2|T=1)P(T=1) + P(R=2|T=2)P(T=2) [1M]
= (0.05 * 0.35) + (0.075 * 0.3) + (0.8 * 0.35)
= 0.0175 + 0.0225 + 0. 280 = 0.32 [2M]

(b) Given a 1 is received, what is the probability that 1 was transmitted [3M]

$P(T = 1|R = 1) = \frac{P(R = 1|T = 1)P(T=1)}{P(R=1)}$ [1M]

P(R = 1) = P(R=1|T=0)P(T=0) + P(R=1|T=1)P(T=1) + P(R=1|T=2)P(T=2)
= (0.05 * 0.35) + (0.85 * 0.3) + (0.1 * 0.35)
= 0.0175 + 0.255 + 0.035 = 0.3075 [1M]

$P(T = 1|R = 1) = \frac{0.85*0.3}{0.3075} = \frac{0.255}{0.3075} = 0.8293$ [1M]

Q2. Suppose you are throwing a die twice. Let A be the event that one of the throws shows exactly 2 and the other throw shows m, where m $\leq$2. Let B be the event that one of the throws show exactly 3 and the other throw shows m, where m $\geq$ 2.

(a) Are A and B mutually exclusive events? Justify your answer. [2M]

A = {(2, 1), (2,2), (1,2)}

B = {(3, 2), (3,3), (3,4), (3,5), (3,6), (2, 3), (4,3), (5,3), (6,3)} [1M]

Since $A \cap B = \varphi$ (empty set) A and B are mutually exclusive. [1M]

(b) Are A and B independent events? Justify your answer. [2M]

P(A) = 3/36 = 1/12, P(B) = 9/36 = ¼

P(A)P(B) = 1/48

$P(A \cap B) = 0$ [1M]

$P(A \cap B) \neq P(A)P(B)$. Hence A and B are not independent. [1M]

(c) Find P(AUB). [2M]

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ [1M]

= 1/12 + ¼ = 4/12 = 1/3 [1M]

Q3. A speaks truth in 75% cases and B speaks truth in 65% cases. What percentage of cases are they likely to contradict each other in stating the same fact. [3M]

Let us use the notation

$T_A = A\ speaks\ the\ truth$

$T_B = B\ speaks\ the\ truth$

$L_A = A\ speaks\ the\ lie$

$T_B = B\ speaks\ the\ lie$

Given that $P(T_A) = 0.75$ and $P(T_B) = 0.65$

Hence $P(L_A) = 0.25$ and $P(L_B) = 0.35$ [1M]

P(A and B contradict each other on the same fact) = $P(T_A)\, P(L_B) + P(T_A)\, P(L_B)$ [1M]

= 0.75*0.35 + 0.65*0.25 = 0.2625 + 0.1625 = 0.425 or 42.5%[1M]

Q4. A rainfall event with a given intensity and duration has a probability of occurrence defined as the relative number of occurrences of the event in a long period of record of rainfall events. If a record of rainfall intensities shows that an event of 120 mm/h intensity for 10 minutes occurs 5 times in a 40-year period, what is the probability of this event occurring in any given year. [2M]

probability of this event occurring in any given year = 5/40 = 1/8. [2M]

Q5. Suppose a person decides whether to take his car to his work place based on the following features:

Traffic: Light, Moderate, Heavy

Fuel: Low, Medium, Full

Weather: Clear, Rainy

Training Data:

S.No	Traffic	Fuel	Weather	Take Car
1	Light	Full	Clear	Yes
2	Heavy	Low	Rainy	No
3	Moderate	Medium	Clear	Yes
4	Heavy	Medium	Clear	No
5	Light	Full	Rainy	Yes

6	Moderate	Full	Rainy	No
7	Heavy	Full	Clear	Yes
8	Light	Medium	Rainy	No

Using the Naïve Bayes method and the above data, predict whether the person will take the car if (Traffic = Moderate, Fuel = Medium, Weather = Rainy). [6M]

P(Yes) = 4/8 = 0.5 and P(No) = 4/8 = 0.5

P(Trafic=moderate|Yes) = ¼ = 0.25
P(Fuel=Medium|Yes) = ¼ = 0.25
P(Weather = Rainy|Yes) = ¼= 0.25 [1M]

P(Trafic=moderate|No) = ¼ = 0.25
P(Fuel=Medium|No) = 2/4 = 0.5
P(Weather = Rainy|No) = ¾ = 0.75 [1M]

$$P(Yes|Traffic = Moderate, Fuel = Medium, Weather = Rainy) \propto (Trafic = moderate|Yes)\ P(Fuel = Medium|Yes)\ P(Weather = Rainy|Yes)P(Yes) = 0.25 * 0.25 * 0.25 * 0.5 = 0.0078125$$ [1M]

$$P(No|Traffic = Moderate, Fuel = Medium, Weather = Rainy) \propto (Trafic = moderate|No)\ P(Fuel = Medium|No)\ P(Weather = Rainy|No)P(No) = 0.25 * 0.5 * 0.75 * 0.5 = 0.046875$$ [1M]

$$P(No|Traffic = Moderate, Fuel = Medium, Weather = Rainy) > P(Yes|Traffic = Moderate, Fuel = Medium, Weather = Rainy)$$

Hence the prediction is the person will not take the car. [2M]

Note:
Some students might have calculates exact probabilities of $P(Yes|Traffic = Moderate, Fuel = Medium, Weather = Rainy)$ and $P(No|Traffic = Moderate, Fuel = Medium, Weather = Rainy)$ by dividing by

$P(Traffic = Moderate, Fuel = Medium, Weather = Rainy)$ please give the marks.

Q6. A factory produces electronic components. Each component is classified based on **quality** (Good or Defective) and **size** (Small, Medium, or Large). The joint probability distribution of the random variables X (size) and Y (quality) is given below:

Y= quality / X=Size	Good	Defective	Total
Small	0.2	0.05	0.25
Medium	0.40	0.1	0.5
Large	0.15	0.10	0.25
Total	0.75	0.25	1

(a) Given that a component is defective find the probability it is large. [1M]

$$P(Large|Defective) = \frac{P(Large \cap Defective)}{P(Defective)} = \frac{0.1}{0.25} = 0.4$$ [1M]

(b) Are X and Y independent? Justify your answer mathematically. [2M]
X and Y are independent if P(X=x and Y=y) = P(X=x)P(Y=y) for all x and y.

P(X=small and Y=Defective) = 0.05

P(X=small) * P(Y=Defective) = 0.25 * 0.25 = 0.0625 [1M]

Since they are not equal X and Y are not independent. [1M]

Q7. (a) A company receives an average of 5 phone calls per minute from customers. The calls are modeled using a Poisson distribution with an average of 5 calls per minute. Calculate the probability that exactly 3 calls are received in a given minute. [2M]

$P(X = 3) = \frac{e^{-5}5^3}{3!}$ [1M]
= 0.1404 [1M]

(b) Suppose X is a continuous random variable with the probability distribution function $f(x) = \frac{1}{\sqrt{8\pi}} e^{\frac{-(x-3)^2}{8}}$. Identify what is this probability distribution function and hence find the mean and standard deviation of this probability distribution function. [2M]

First we will write f(x) in the normal distribution function form:

$$f(x) = \frac{1}{\sqrt{8\pi}} e^{\frac{-(x-3)^2}{8}} = \frac{1}{2\sqrt{2\pi}} e^{\frac{-(x-3)^2}{2*(2^2)}} \quad [1M\ ]$$

Form this form we conclude that mean = 3 and standard deviation = 2. [1M]
