Section 3

$$\lambda_1 \quad X_1 = 0 b_1 - 2 b_2 + 1 b_3 - 1 b_4$$

$$\lambda_2 \quad X_2 = -4 b_1 - 2 b_2 + 4 b_4$$

$$\lambda_3 \quad X_3 = 2 b_1 + 3 b_2 - b_3 - 3 b_4$$

$$\lambda_4 \quad X_4 = 17 b_1 - 10 b_2 + 11 b_3 + b_4$$

b_i are L In

$$\underline{\lambda_1} x_1 + \underline{\lambda_2} x_2 + \underline{\lambda_3} x_3 + \underline{\lambda_4} x_4 = 0$$

$$(\lambda_1 - 4\lambda_2 + 2\lambda_3 + 17\lambda_4) b_1 + (-2\lambda_1 - 2\lambda_2 + 3\lambda_3 - 10\lambda_4) b_2$$

$$+ (\ 2 \ ) b_3 + (\ 2 \ ) b_4 = 0$$

b_i are LI

$\Rightarrow$ each coeff should be 0

$$\left\{ \begin{array}{l} \lambda_1 - 4\lambda_2 + 2\lambda_3 + 17\lambda_4 = 0 \\ -2\lambda_1 - 2\lambda_2 + 3\lambda_3 - 10\lambda_4 = 0 \\ \_\_\_\_\_\_\_\_ = 0 \\ \_\_\_\_\_\_\_\_ = 0 \end{array} \right.$$

$$\Lambda \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

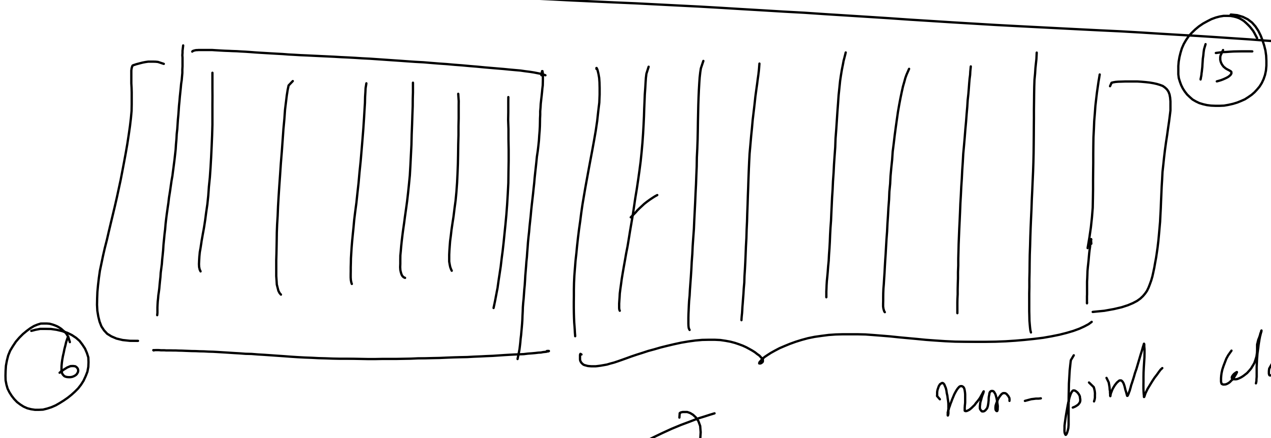
$$\hat{\lambda} = \begin{bmatrix} 1 & -4 & 2 & 17 \\ -2 & -2 & 3 & -10 \\ - & - & - & \\ - & - & - & - & - \end{bmatrix}$$

$$\hat{\lambda} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

If $\hat{\lambda}$ has rank 4 $\Rightarrow$ $\hat{\lambda}$ has LI

$$\hat{\lambda}^{-1} \hat{\lambda} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{bmatrix} = \hat{\lambda}^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow \lambda_i = 0 \ \forall i \Rightarrow$ LI



non-pivot columns

(ex) $\begin{bmatrix} 1 & 3 & 8 \\ 2 & 4 & 9 \end{bmatrix}$ rank $= 2$

$$\begin{bmatrix} 8 \\ 9 \end{bmatrix} = \alpha_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \alpha_2 \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$A = \begin{bmatrix} \underline{\text{rank}(6)} & \cancel{A \text{ is invertible}} \end{bmatrix}$$

6

rank $\in \{0,1,2,3,4,5,6\}$

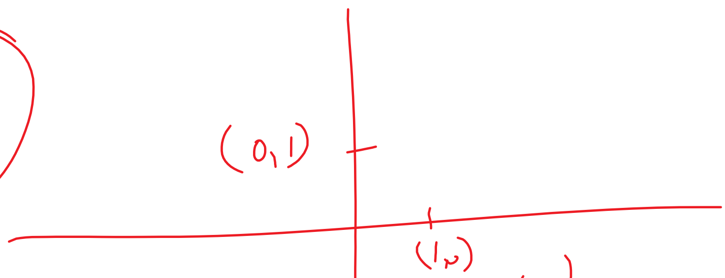
$$B = \begin{bmatrix} \quad \end{bmatrix}_{6\times 6} \quad \text{rank } \{0,1,2,3,4,5,6\}$$

B has full rank 6 $\Rightarrow$ B is invertible.

Generating set $S = \{v_1, v_2 \ldots v_m\}$

$LS(S) = V$

(eg) $\mathbb{R}^2$



$\{(1,0)^T, (0,1)^T\}$ is a gen set of $\mathbb{R}^2$

$\{(1,0)^T, (0,1)^T, (7,8)^T\}$ is also a gen set.

$(1,0)^T$

$$\mathbb{R}^2 \ni \begin{pmatrix} 3 \\ 6 \end{pmatrix} = 3\begin{pmatrix} 1 \\ 0 \end{pmatrix} + 6\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$\mathbb{R}^{2}\left(\begin{pmatrix}3\\6\end{pmatrix}\right) = 3\begin{pmatrix}\underline{\ 0\ }\end{pmatrix} + 6\begin{pmatrix}\underline{\ 1\ }\end{pmatrix}$

Q. Set for coffee $\{$ Coff. pow., milk, water, sugar, green chilies, $\}$ Sugar ...

Basis $\{v_1, v_2 \ldots v_n\}$ is a basis of V if

$\quad$ (i) $\operatorname{Span}\{v_1 \ldots v_n\} = V$

$\quad$ & $\{v_1 \ldots v_n\}$ are LI

$\{a, b, c, \ldots \ldots z\}$

Cat

$1 \cdot c + 1\, a + 1 \cdot t$

Carrot

$1 \cdot c + 1 \cdot a + 2 \cdot r + 1 \cdot o + 1 \cdot t$

$S = \{0, v_1, v_2, v_3\}$ V

is LD

Can S be a basis?

Is S LI?

$\alpha_1 \cdot 0 + \alpha_2 v_1 + \alpha_3 v_2 + \alpha_4 v_3 = 0$

$\alpha_1 = 5, \quad \alpha_2 = 0, \quad \alpha_3 = 0, \quad \alpha_4 = 0$

$\begin{bmatrix} 2 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \end{bmatrix}$ are LI & span $\mathbb{R}^2$

$\begin{bmatrix} 3 \\ 8 \end{bmatrix} \begin{bmatrix} 12 \\ 16 \end{bmatrix}$ also does this?

(ex)

Vector Space.	Field	Dimension
$\mathbb{R}$	$\mathbb{R}$	1.
$\mathbb{C}$	$\mathbb{R}$	$\{1, i\}$ 2
$5+7i$		
$\mathbb{C}$	$\mathbb{C}$	1
	$a+ib$	

Any no of the form $a+ib$ where $i = \sqrt{-1}$ is called a complex no (a, b) are real nos

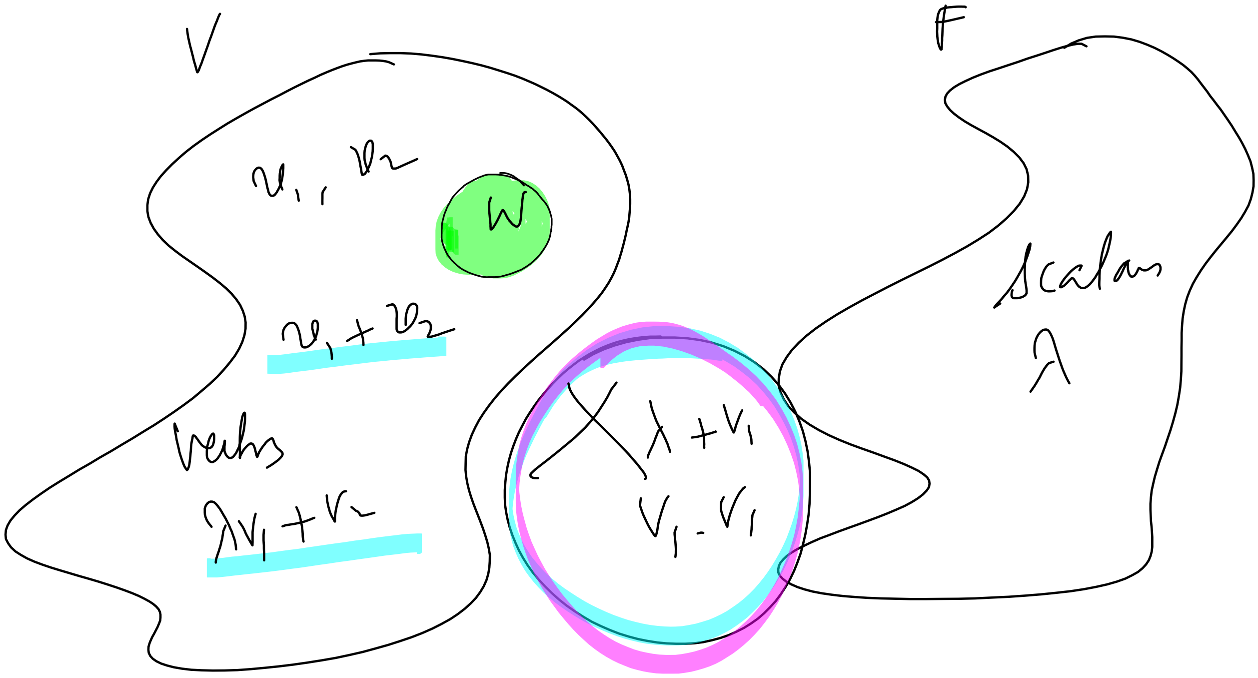
$3+7i$ $\qquad$ $\mathbb{R}$ $\qquad$ $\{1, i\}$ $\{3, 8i\}$

$\mathbb{R}$ $\qquad$ $\mathbb{C}$ $(a+ib)$ $\qquad$ 1 (3) $\quad$ $(3+7i)$

$\overline{5}$ $(a+ib)$ (3) $\overline{(3+7i)}$

$$(3+7i)\cdot z = 5$$

$$z = \frac{5}{(3+7i)}$$



{Coffee, cake, milk, sugar, ~~green tea~~}

milk with sugar

Manhattan norm: $\|x\|_1 = \sum_{i=1}^{n} |x_i|$

$$x^{L} = (x_1, x_2, \ldots, x_n)^T$$

(ex) $x = (-1, 2, -3, 5)^T$

$\|x\|_1 = 1 + 2 + 3 + 5 = 11$

$\frac{d}{dx}\left(f(x) + g(x)\right) = \frac{d}{dx} f(x) + \frac{d}{dx} g(x)$

$\frac{d}{dx}\left(\lambda f(x) + \mu g(x)\right) = \lambda \frac{d}{dx} f(x) + \mu \frac{d g(x)}{dx}$

① $\Omega\left(\underline{\lambda x + \mu y}, \underline{z}\right)$

$= \lambda \, \Omega(x, z) + \mu \, \Omega(y, z)$

② $\Omega\left(x, \mu y + \lambda z\right) = \mu \Omega(x, y) + \lambda \Omega(x, z)$

$\underbrace{\begin{bmatrix} b_1 \end{bmatrix}, \begin{bmatrix} b_2 \end{bmatrix}, \begin{bmatrix} b_3 \end{bmatrix}}$ $\longrightarrow$ basis $\begin{bmatrix} b_1 \end{bmatrix} \begin{bmatrix} b_2 \end{bmatrix} \begin{bmatrix} b_3 \end{bmatrix}$

$\underline{x} = \begin{bmatrix} 1 \\ -1 \\ 23 \end{bmatrix} = \underline{\alpha_1} \begin{bmatrix} b_1 \end{bmatrix} + \underline{\alpha_2} \begin{bmatrix} b_2 \end{bmatrix} + \underline{\alpha_3} \begin{bmatrix} b_3 \end{bmatrix}$?

$\begin{bmatrix} -85 \end{bmatrix}$

$\hat{x} = \begin{bmatrix} -1 \\ 23 \end{bmatrix}$

$\hat{x} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix}$, $y = \begin{bmatrix} -857 \\ 254 \\ 103 \end{bmatrix} = \beta_1 y_1 + \beta_2 y_2 + \beta_3 y_3$

$\hat{y} = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix}$

Scalar $\langle x, y \rangle = \hat{x}^T A \cdot \hat{y}$

A is a sym. +ve definite matrix

$1\times 3 \quad 3\times 3 \quad \times 3 \times 1$

$1\times 1 =$ scalar

Q Can a sym +ve def. matrix have less than full rank? (1 mark)

Answer Let A be a sym, +ve def matrix

$\underline{Ax = 0}$ nullspace of $A = \{x \mid Ax = 0\}$

Rank nullity theorem

rank(A) + dim null space of A = no of col of A (n)

$A_{m \times n}$

Let us assume that $X \neq 0$ & $AX = 0$

$$\underbrace{X^T A X}_{>0} = X^T 0 = 0$$

No $X \neq 0$ exists such that $AX = 0$

Null space of $A = \{0\}$ (ϕ)

dim null space of $A = 0$

$A_{n \times n}$

rank(A) + dim null space of $A = n$

rank$(A) = n$

1. So A will always have full rank.