Homework 3 - Frame Analysis by Stiffness Method

This is an individual homework assignment in association with Lab 9 frame analysis and testing. In this assignment, you are tasked to write a computer program that calculates the deformations and load distributions of a 2-D rigid frame structure using the stiffness method. We will start with the foundation we built when studying beam deflection earlier this semester, and add more capabilities step by step.

As was in beam deflection analysis, here you need to thoroughly study stiffness method for frame structure using Week 12 lecture notes or any other materials you prefer. The worked examples in the lecture would be particularly helpful. You also need to bring your programming skill up to speed if not already. Like before, you may choose whatever computer language you like among the current popular ones such as Matlab, C/C++, Java, Python and Fortran. You will need to write vector/matrix operation and equation solver routines if they are not provided by your language or toolkit. You may discuss with and get conceptual help from your classmates, but **you must write your own code**!

If you would choose the approach described in Week 12 lecture notes, we strongly recommend you to follow the procedure outlined on pages 20-25 in the lecture as your programming guide, and use the worked example on pages 26-42 to validate and debug your code. Once you can reproduce exactly the same outputs as in the worked example, you can then easily modify the structure geometry and load distribution to fit with this homework problem.

Fig. 1 shows the frame structure setup of the homework problem. Note the 15 lb weight is applied to joint 6 and the 50lb-in moment is applied to joint 2. For grading convenience, we ask you to stick with the same joint and member numbering system shown in the figure. Also, we recommend you use the node numbering technique described in the lecture notes. This node number technique should help you greatly in tackling the tricky part of programming task, i.e. how to keep track of the contributions of individual member to the entire structure, and to assemble the global frame stiffness matrix of the structure. By using the node numbering technique, you can trace that, for example, S_{11} component (refer to lecture page 7 for notation summary) in lecture's worked example #1 (page 30) is actually attributed to K_{44} of member 1 (page 28) and K_{11} of member 2 (page 29). See that all three components have the same node number index (1,1)?

As suggested, your code writing may want to start with identifying the global unknown displacements d's, followed by assembling the local and global member stiffness matrices k's and K's, the transformation matrices T's and the global structure stiffness matrix S. Together with the global joint loads P's, you now have a matrix equation in S to solve for the d's. You then reverse the process to match the global member displacements v's with d's, calculate local displacement u's, local member forces Q's and the normal stresses and strains. Logically you should write corresponding functions/modules along this line. If you recall, extensive discussions about these points were already described earlier in the beam deflection lab. Unless you are a software guru, don't expect your code to work the very first time, especially at this level of complexity. So make good use of the debugger if you can access one, or screen-dump as many intermediate variables as you need to make sure your code is doing what you think it is – plus a lot of patience. Once your code is fully functional, go do the following:

1. (65 points) calculate the "usual suspects" for the homework problem setup, similar to the worked examples in lecture: the global displacement unknowns d's, the global member matrices K's, the global joint loads P's, the global frame stiffness matrix S, the local member displacements u's and forces Q's, the global forces F's and the axial stresses σ_a and strains ε_a (see page 32). Also identify the reactions at the fixed ends. Display all numbers in clear vector/matrix format like in the lecture.
2. (25 points) following lecture page 35, calculate the normal stresses σ_b due to bending moments along member length (x direction) in local coordinates for each member on both top and bottom surfaces where σ_b reaches maxima. Combine σ_b with the normal stresses due to axial loading σ_a and calculate the combined strains on the surfaces. Plot the combined strain on top surface vs. position in x direction for member 5 and repeat for bottom surface in the same chart. Do the same for member 6. Note that the assignment of top and bottom surfaces depends on how you pick the beginning and ending joints (see where "b" and "e" are assigned on page 35?)
3. (10 points) In addition to the calculation outputs above, submit a printout of your working code. Summarize your code structure using a flow chart or similar, comment your code throughout and highlight the key segments.
4. (**extra** 50 points) following lecture pages 10-11, derive the member's transverse deflection y as a function of member length x and the local displacements u's (hint: u's are the boundary conditions). Do the same for axial deformation due to axial forces Q_1 and Q_4. Note that axial deformation is linear, since Q_1 and Q_4 constitute a constant tension or compression. Sample a number of positions along x and you have a full picture of the member's deformation due to the combined loads. Transform all these (x, y) pairs to global coordinates and plot the deformation for each member, similar to Fig. 2 where the deformation is plotted for the case of worked example #1 in lecture.

This homework assignment is due 11:59pm, Sunday, April 24 (all students).

Happy programming and good luck!

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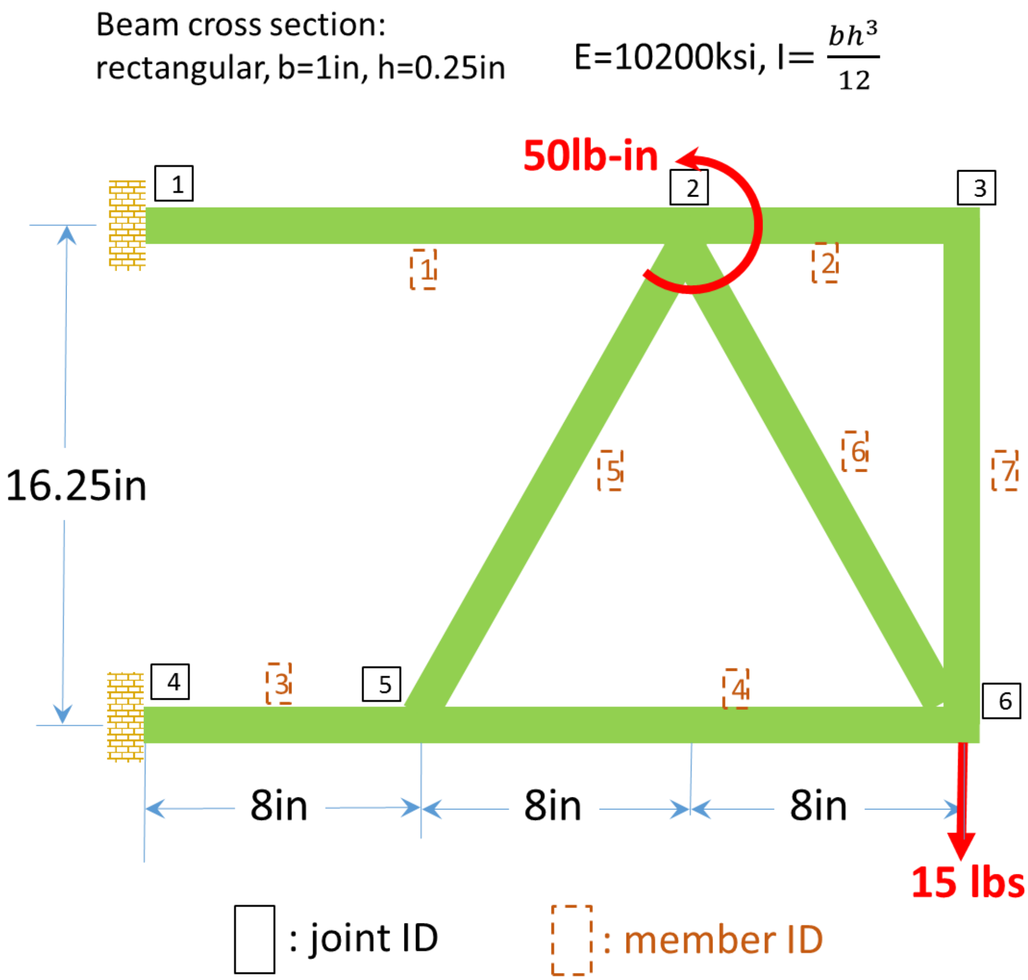

Figure 1. Frame structure setup for the homework problem

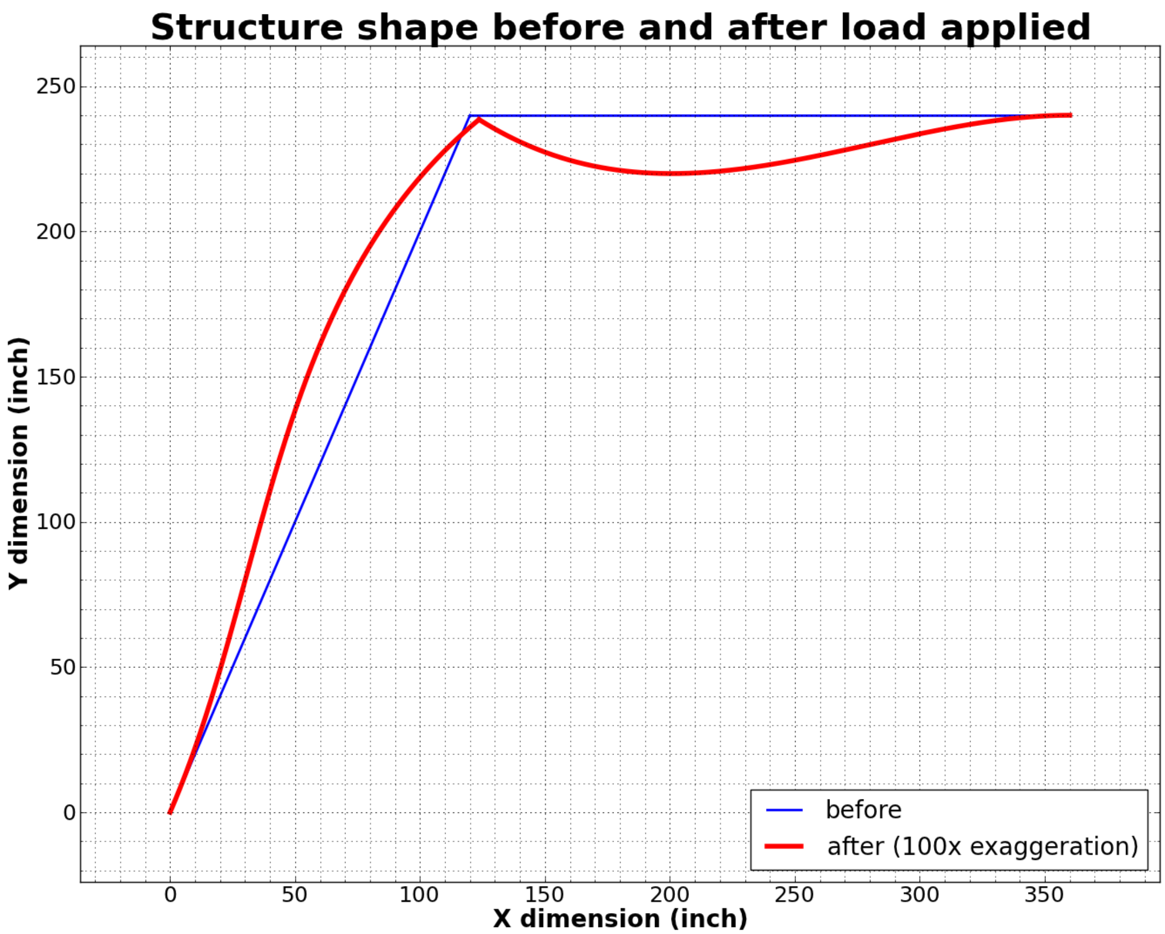

Figure 2. Structure deformation plot corresponding to the lecture worked example #1