NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2018-2019

EE2008 / IM1001 – DATA STRUCTURES AND ALGORITHMS

November / December 2018 Time Allowed: 2½ hours

INSTRUCTIONS

1. This paper contains 4 questions and comprises 4 pages.
2. Answer ALL questions.
3. All questions carry equal marks.
4. This is a closed book examination.
5. Unless specifically stated, all symbols have their usual meanings.

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1. (a) Determine the asymptotic upper bound for the number of times the statement "$r = r + 1$" is executed in each of the following algorithms.

(i)
```
for i = 1 to n
    for j = i to 2i
        for k = 1 to j
            r = r + 1
```

(ii)
```
i = n
while ( i ≥ 1 ) {
    r = r + 1
    i = i – 2
}
```

(10 Marks)

(b) Use mathematical induction to prove that the following equation is true.

$$\sum_{r=1}^{n} \frac{1}{(2r-1)(2r+1)} = \frac{n}{2n+1}$$

(8 Marks)

Note: Question No. 1 continues on page 2.

(c) Show that if $f(n)$ is $O(g(n))$ and $d(n)$ is $O(h(n))$, then $f(n) + d(n)$ is $O(g(n) + h(n))$.

(7 Marks)

2. (a) (i) Each element of an array A is an integer. Write a recursive algorithm to compute the product of all the elements in A.

(ii) Set up and solve the recurrence relation for the number of multiplications used by the recursive algorithm in part (i).

(9 Marks)

(b) Using pseudo-code, describe the implementation of the method *insertFirst*(e) that inserts a new element e as the first element of the LIST ADT, assuming that the LIST ADT is implemented using a doubly linked list and a pointer *start* points to the first element of the list.

(6 Marks)

(c) Write a recursive algorithm to find the product of all values greater than x in a binary search tree.

(10 Marks)

3. (a) Show and explain clearly what the following array looks like in each step of the partition algorithm.

15	20	12	7	13	23	11	6

(10 Marks)

(b) Continuing from your answer in part (a), show each step in the quicksort algorithm when applied to the array in part (a).

(8 Marks)

(c) Given an array A with n elements, write an algorithm in pseudo-code to find

$$\sum_{i=1}^{n} A[\pi(i)] \cdot A[\pi(n-i+1)],$$

where $\pi(i)$ is the index of the i-th smallest element in A. You may make use of functions discussed in lecture to construct your algorithm. What is the time complexity of your algorithm?

(7 Marks)

4. (a) Consider the graph whose adjacency list is shown in Figure 1. Determine the sequence in which its vertices are visited when the following algorithms are applied starting at vertex 1:

(i) depth-first search

(ii) breadth-first search

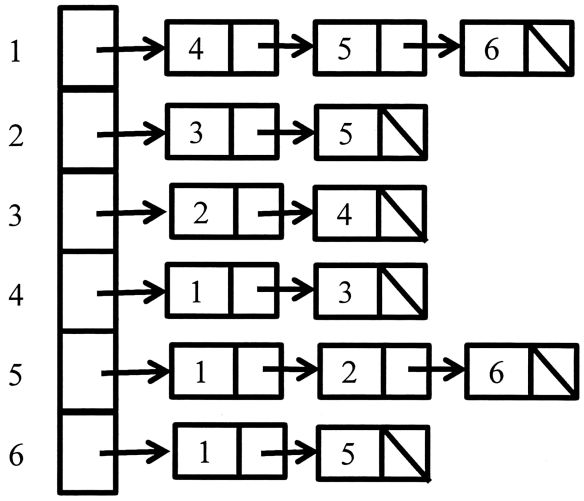

Figure 1

(6 Marks)

(b) Use Dijkstra's algorithm to find the shortest path from vertex D to vertex A in the weighted graph shown in Figure 2. Show each step clearly.

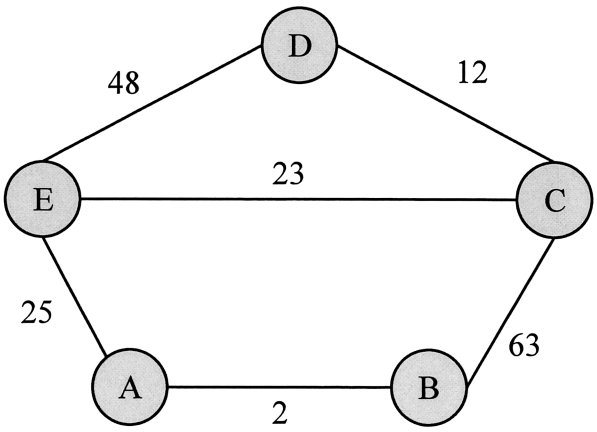

Figure 2

(7 Marks)

Note: Question No. 4 continues on page 4.

(c) Suppose that G is a weighted tree whose edge weights are integers. The distance of a vertex u from the vertex s is the length of the shortest path from s to u. Write an algorithm in pseudo-code that counts the number of vertices that are even distance from the vertex s. Assume that G is represented by an adjacency list. Hint: Modify the template bfs algorithm shown in Figure 3 to solve the problem.

```
bfs (adj, s)  {
    n = adj.last
    for i = 1 to n
        visit[i] = false
    visit[s] = true
    q.enqueue(s)
    while (!q.empty()) {
        v = q.front()
        ref = adj[v]
        while (ref != null) {
            if (!visit[ref.data]) {
                visit[ref.data] = true
                q.enqueue(ref.data)
            }
            ref = ref.next
        }
        q.dequeue()
    }
}
```

Figure 3

(12 Marks)

END OF PAPER

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Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.