ALL SUPPLY CURVES CAN BE FLAT LINES

1. Monopoly: Mathematically describe the supply and demand functions of a theoretical monopoly of a good (does not need to be realistic, needs to be sloping the correct directions, needs to intersect in positive space, needs to be curves that are unique among your peers/internet/your answers in the rest of the problem set). Solve the equilibrium quantity and equilibrium price and then graph your curves and your results. How much more combined utility would be gained by this market if the market were perfectly competitive instead?
2. Monopsony: Mathematically describe the supply and demand functions of a theoretical monopsony for labor (does not need to be realistic, needs to be sloping the correct directions, needs to intersect in positive space, needs to be curves that are unique among your peers/internet/your answers in the rest of the problem set). Solve the equilibrium quantity and equilibrium price and then graph your curves and your results. How much more combined utility would be gained by this market if the market for labor were perfectly competitive instead?
3. Cournot: Use your demand curve from Question 1 to now represent a market demand curve for a duopoly and create one unique supply curve for each of the two producers. Your curves do not need to be realistic, but each supply curve must intersect demand and your ultimate equilibrium must lead to each firm having positive, but different, levels of production. Solve the equilibrium quantity and equilibrium price if both firms act through choosing quantity simultaneously, then create a graph where q1 and q2 are the 2 axes and plot the best reaction functions and equilibrium quantity upon them.
4. Optional Stackelberg challenge: Do the same problem as in question 3 but with 3 players instead, and with the first acting before the second, who acts before the third.
5. Optional extreme challenge: Solve the Stackelberg and Cournot challenge for n number of players.

I use flat lines for all of my supply curves, because apparently non-flat is hard

1 | Monopoly problem

Let p equal the price of each product, and q equal the quantity produced/sold, and m equal the marginal cost to produce one more unit.

$$\begin{aligned} d(x) &= 6e - x \\ s(x) &= e \\ \pi_0 &= d(q)q - s(q)q \\ &= (6e - q)q - eq \\ &= -q^2 + 5eq \end{aligned}$$

The vertex of this parabola occurs at $-\frac{b}{2a}$, aka $q = \frac{5e}{2}$

See the Desmos here: `https://www.desmos.com/calculator/mll9lfke7z`

3 | Cournot problem

Variable	Meaning
q_X	the quantity that corporation X produces
μ_X	the marginal cost to produce one more item for corporation X

and $P(Q) = C - MQ$.

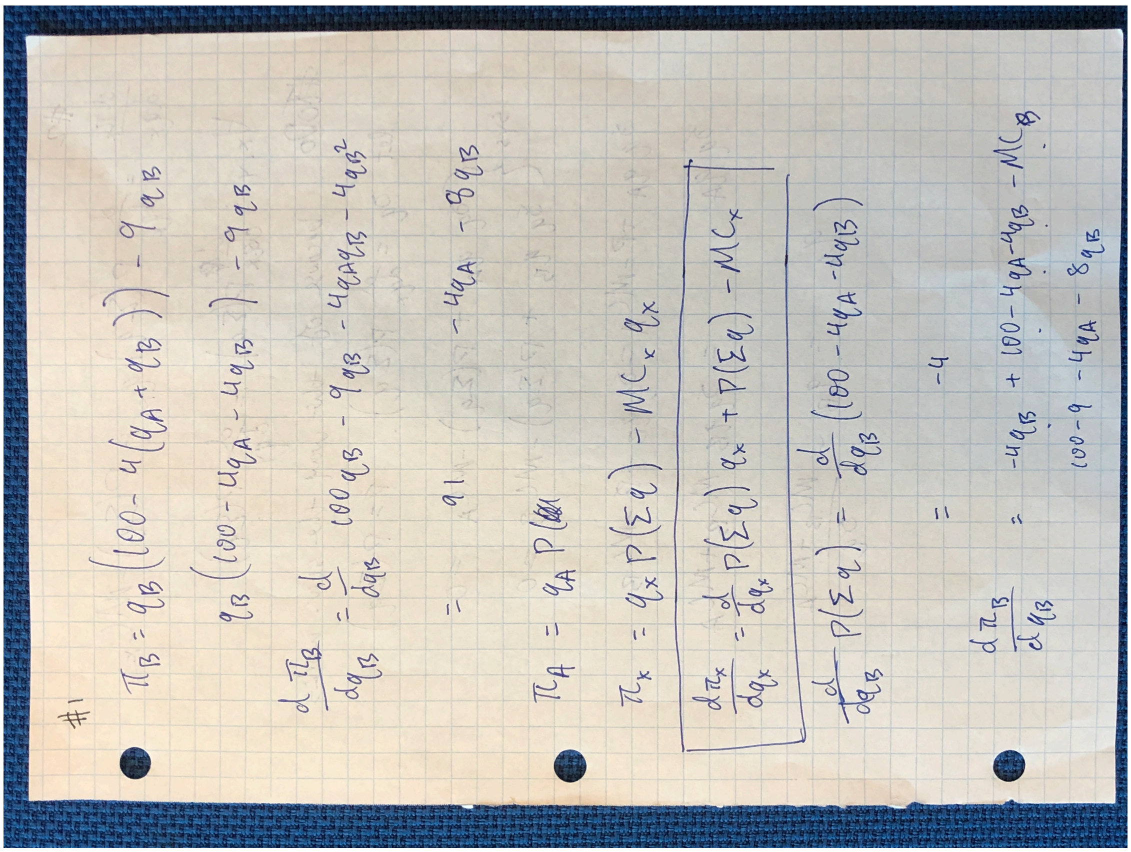
Reaction function: (A and B are symmetric)

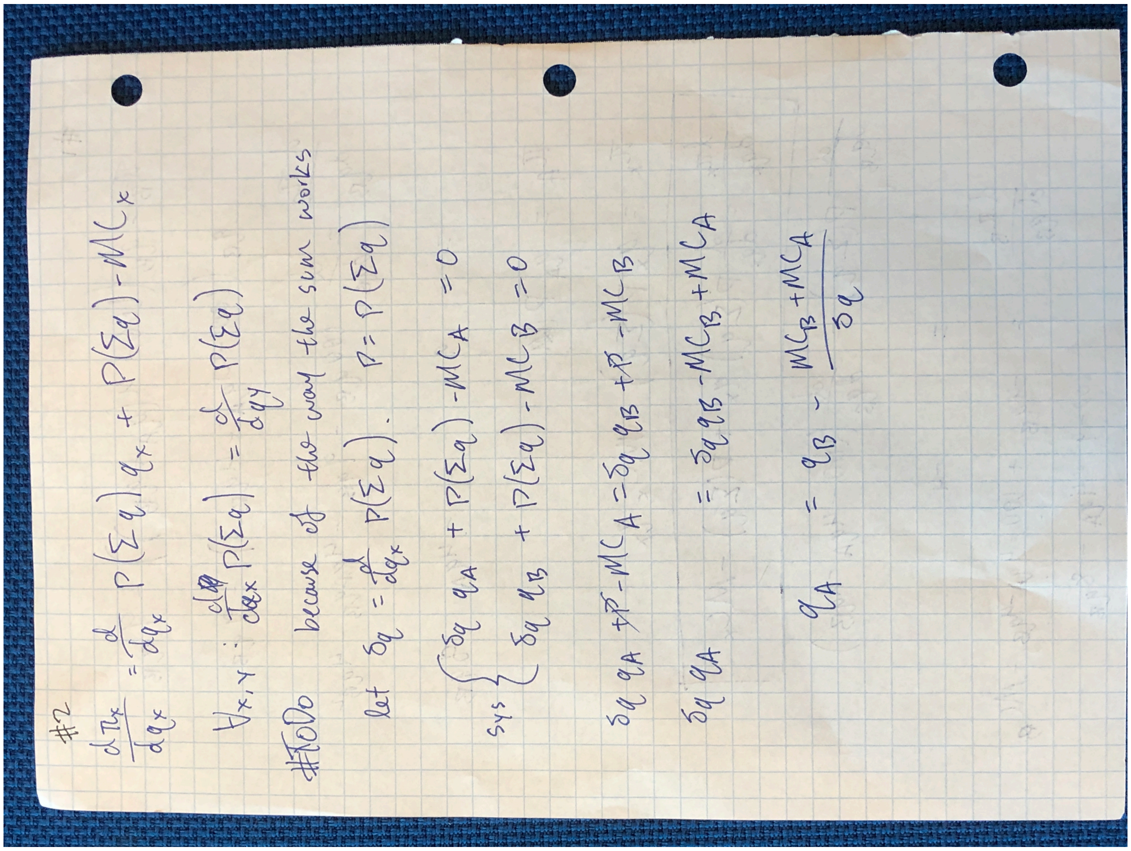
$$q_A = q_B + \frac{\mu_B + \mu_A}{M}$$

Closed form nash equillibrium:

$$q_A = \frac{C + \mu_B - 2\mu_A}{3M}$$

I made lots of algebra mistakes during my derivation:





#2

$$\frac{d\pi_x}{dq_x} = \frac{d}{dq_x} P(\Sigma q)\, q_x + P(\Sigma q) - MC_x$$

$$\forall_{x,y} : \frac{d}{dq_x} P(\Sigma q) = \frac{d}{dq_y} P(\Sigma q)$$

#TODO because of the way the sum works

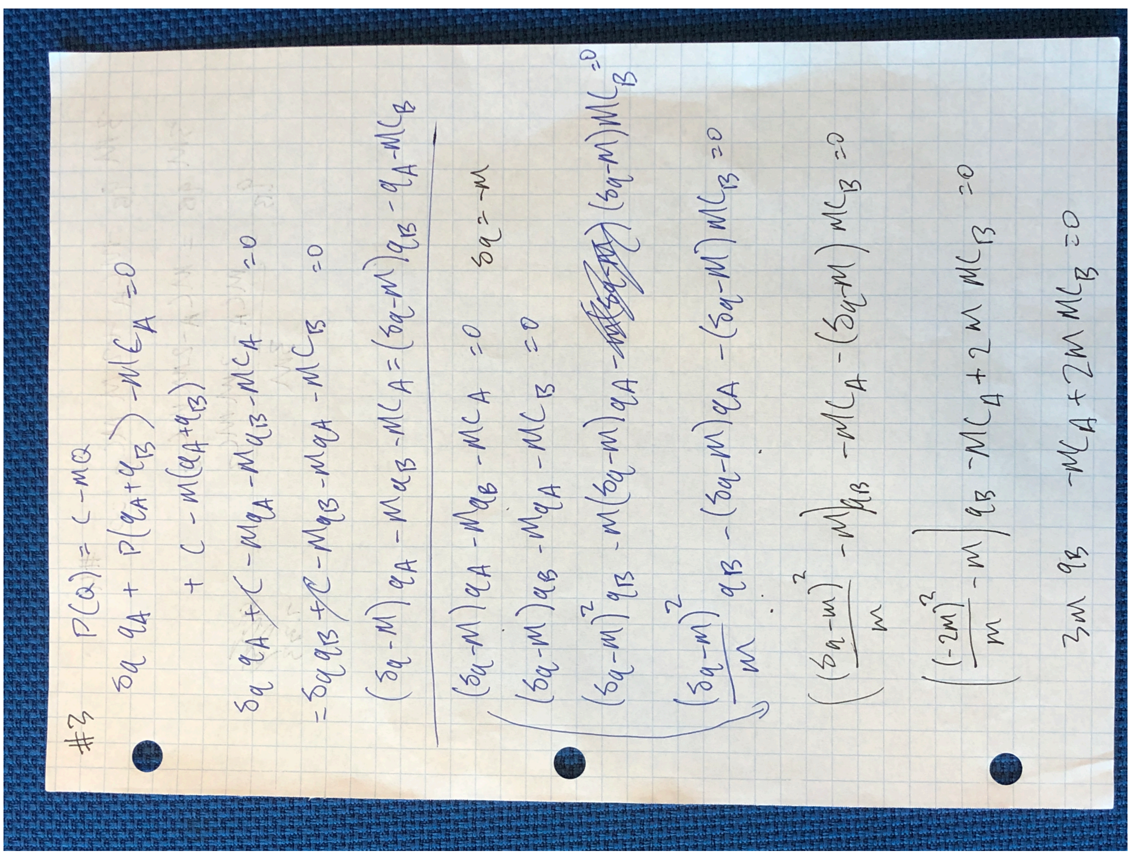
let $\delta_q = \frac{d}{dq_x} P(\Sigma q)$. $P = P(\Sigma q)$

$$\text{sys} \begin{cases} \delta_q q_A + P(\Sigma q) - MC_A = 0 \\ \delta_q q_B + P(\Sigma q) - MC_B = 0 \end{cases}$$

$$\delta_q q_A + P - MC_A = \delta_q q_B + P - MC_B$$

$$\delta_q q_A = \delta_q q_B - MC_B + MC_A$$

$$q_A = q_B - \frac{MC_B + MC_A}{\delta_q}$$



#3 $P(Q) = C - MQ$

$$\delta_q q_A + P(q_A + q_B) - MC_A = 0$$

$$+ C - M(q_A + q_B)$$

$$\delta_q q_A + \cancel{C} - Mq_A - Mq_B - MC_A = 0$$

$$= \delta_q q_B + \cancel{C} - Mq_B - Mq_A - MC_B = 0$$

$$(\delta_q - M) q_A - Mq_B - MC_A = (\delta_q - M) q_B - q_A - MC_B$$

$$(\delta_q - M) q_A - Mq_B - MC_A = 0 \qquad \delta_q = -M$$

$$(\delta_q - M) q_B - Mq_A - MC_B = 0$$

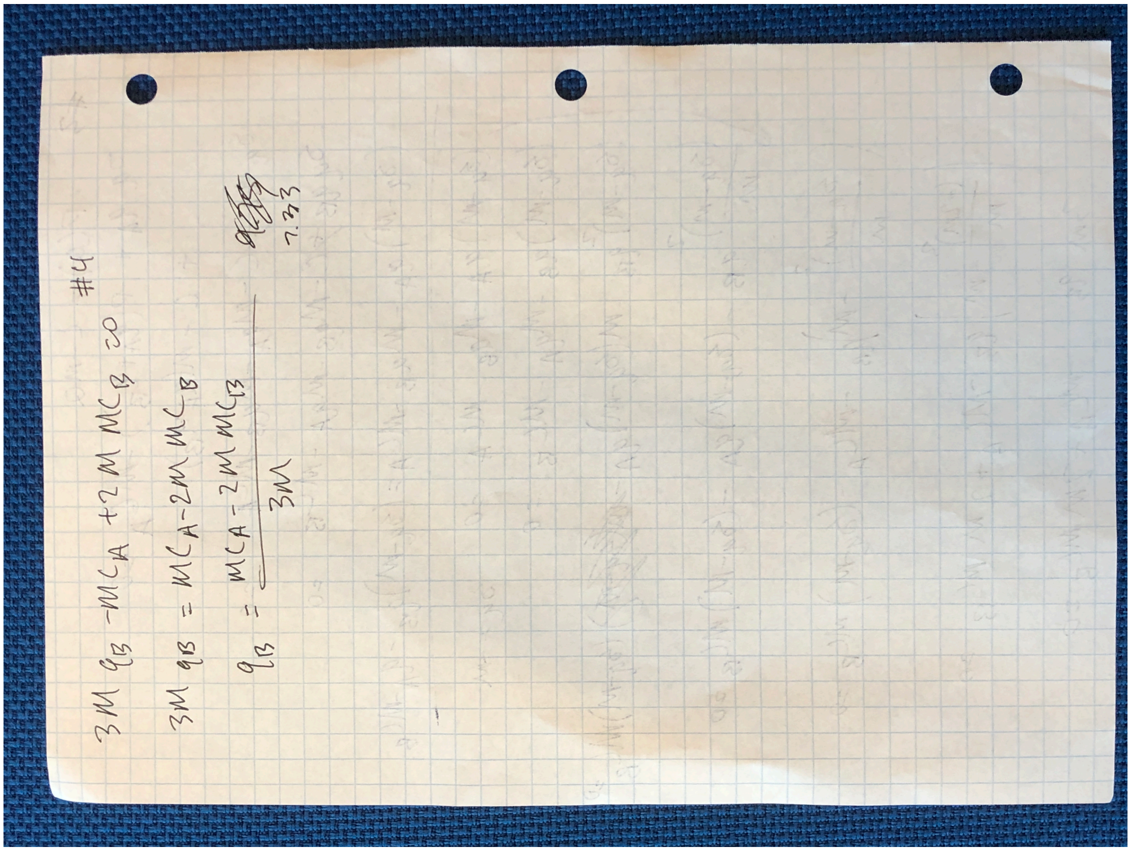
$$(\delta_q - M)^2 q_B - M(\delta_q - M) q_A - (\delta_q - M) MC_B = 0$$

$$\frac{(\delta_q - M)^2}{M} q_B - (\delta_q - M) q_A - (\delta_q - M) MC_B = 0$$

$$\left(\frac{(\delta_q - M)^2}{M} - M\right) q_B - MC_A - (\delta_q - M) MC_B = 0$$

$$\left(\frac{(-2M)^2}{M} - M\right) q_B - MC_A + 2M\, MC_B = 0$$

$$3M\, q_B - MC_A + 2M\, MC_B = 0$$



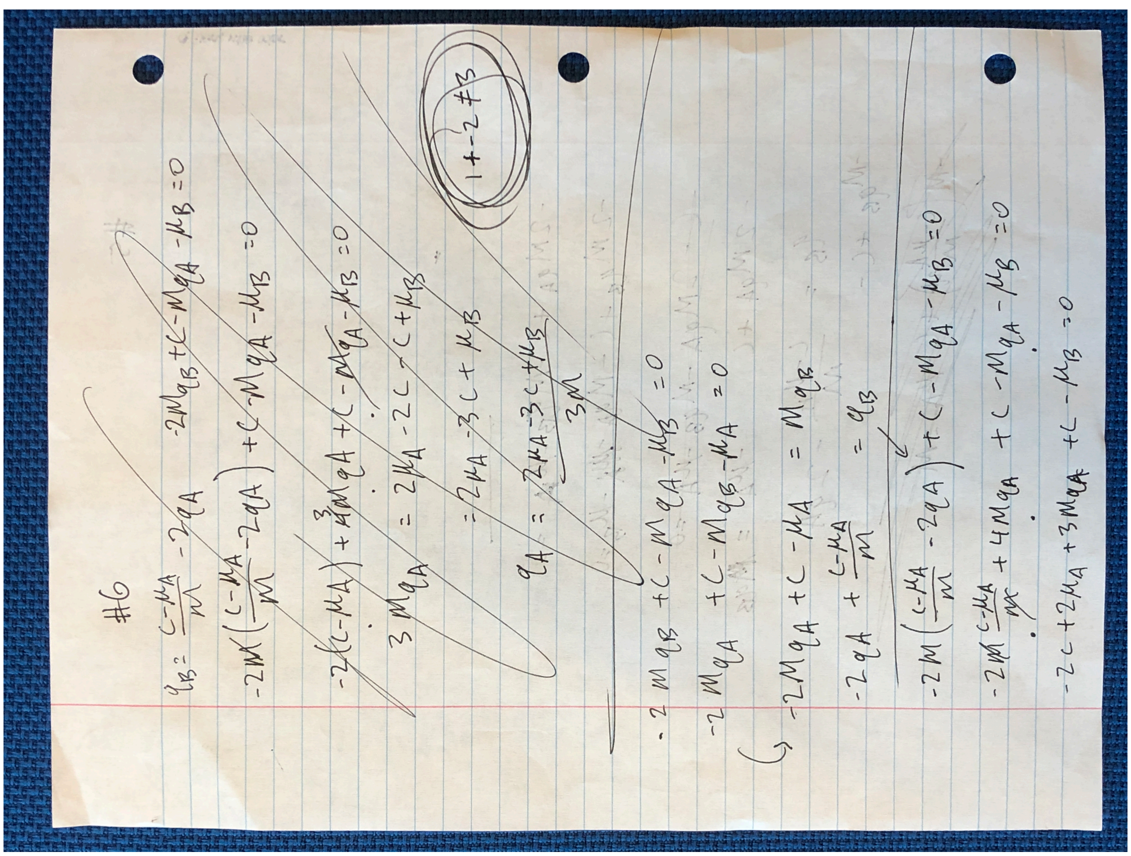
#4

$3M q_B - MC_A + 2M\, MC_B = 0$

$3M q_B = MC_A - 2M\, MC_B$

$q_B = \dfrac{MC_A - 2M\, MC_B}{3M}$

7.33



#6

$q_B = \frac{c-\mu_A}{M} - 2q_A \qquad -2Mq_B + c - Mq_A - \mu_B = 0$

$-2M\left(\frac{c-\mu_A}{M} - 2q_A\right) + c - Mq_A - \mu_B = 0$

$-2(c-\mu_A) + 4Mq_A + c - Mq_A - \mu_B = 0$

$3Mq_A = 2\mu_A - 2c - c + \mu_B$

$= 2\mu_A - 3c + \mu_B$

$q_A = \dfrac{2\mu_A - 3c + \mu_B}{3M}$

$1 + -2 \neq 3$

$-2Mq_B + c - Mq_A - \mu_B = 0$

$-2Mq_A + c - Mq_B - \mu_A = 0$

$-2Mq_A + c - \mu_A = Mq_B$

$-2q_A + \frac{c-\mu_A}{M} = q_B$

$-2M\left(\frac{c-\mu_A}{M} - 2q_A\right) + c - Mq_A - \mu_B = 0$

$-2M\frac{c-\mu_A}{M} + 4Mq_A + c - Mq_A - \mu_B = 0$

$-2c + 2\mu_A + 3Mq_A + c - \mu_B = 0$

#5

$P(x) = c - Mx$; μ_x = marginal cost for company x

$\pi_x = P(\Sigma q)\, q_x - \mu_x q_x$; N = # of players

$= (c - \Sigma q M - M q_x)\, q_x - \mu_x q_x$

$= c\, q_x - M \sum_x q\, q_x - M q_x^2 - \mu_x q_x$

$\frac{d}{dq_x}\pi_x = -2M q_x + c - M\sum_x q - \mu_x$

$\cancel{M(N-1)}$

$-2M q_A + c - M q_B - \mu_A = 0$

$-2M q_B + c - M q_A - \cancel{\mu_A} - \mu_B = 0$

$\cancel{c - 2Mq_A - Mq_B - \mu_A = 0}$

$-2M q_A + c - \mu_A = M q_B$

$q_B = \frac{c-\mu_A}{M} - 2q_A$

$-M q_B + c -$

$\cancel{-M\left(\frac{c-\mu_A}{M} - 2q_A\right) + c - \mu_A - M}$

#7

$-\cancel{2}c + 2\mu_A + 3M q_A + c - \mu_B = 0$

$-c + 2\mu_A + 3M q_A - \mu_B = 0$

$3M q_A = c - 2\mu_A + \mu_B$

$q_A = \frac{c - 2\mu_A + \mu_B}{3M}$

$q_B = \frac{c-\mu_A}{M} - 2q_A$

$= \frac{c-\mu_A}{M} - 2\left(\frac{c - 2\mu_A + \mu_B}{3M}\right)$! distribute the parens

$- \frac{(2c - 4\mu_A + 2\mu_B)}{3M}$

$= \frac{3c - 3\mu_A - 2c - 4\mu_A + 2\mu_B}{3M}$ | $\frac{3c - 3\mu_A - 2c + 4\mu_A - 2\mu_B}{3M}$

$= \frac{c - 7\mu_A + 2\mu_B}{3M}$ | $q_B = \frac{c + \mu_A - 2\mu_B}{3M}$

ok, good, it's symmetric

$q_A = \frac{c + \mu_B - 2\mu_A}{3M}$; μ_A = marginal cost of A

$q_B = \frac{c + \mu_A - 2\mu_B}{3M}$; μ_B = marginal cost of B

$P(Q) = c - MQ$

4 | **cournot problem for more players**

Let μ_i be the marginal cost for the i-th producer, q_i be the quantity produced by the i-th producer, and π_i be the profit for the i-th producer.

Additionally, the market demand curve is of the form $P = P(Q)$ where $Q = \sum q_i$.

For each player, the instantaneous reaction curve is

$$\begin{aligned} \frac{d\pi_x}{dq_x} &= \frac{d}{dq_x} P\left(\sum q\right) q_x + P\left(\sum q\right) - \mu_x \quad = 0 \\ \implies q_x &= \frac{\mu_x - P\left(\sum q\right)}{\frac{d}{dq_x} P\left(\sum q\right)} \end{aligned}$$

Unfortunately, this isn't the closed form (we would have to specify P for that).

Also, does the first player just assume that it's working in a monopoly?

Solving this is just a lot of algebra, so I'll move on.

5 | **two-player sequential**

i had some pdf corruption issues, which is why this is submitted late. here are the photos of the board from class: `https://drive.google.com/drive/u/2/folders/1al_ovroQUwrB1mKB-5ssKNubHbqku-mI`