Roofs

Nueva Multivariable Calculus

1. I love **shed roofs**. You might not be familiar with the term, but you're probably familiar with the concept: a roof that, rather than rising on both sides to a ridgeline in the middle, has a single slope, rising to one side. The library and middle school buildings at the lower school campus both have shed roofs. In case you haven't been there, here's what the interior of the library looks like (in a pre-construction architectural rendering):


[1]

Anyway, suppose you're building yourself a nice little casita with a shed roof. The casita is 20 feet wide by 30 feet long; the roof is eight feet above the back side of the house, and it rises up to twelve feet on the front side of the house (where "back" and "front" are the long, 30-foot-long sides of the house, not the short sides).

(a) Draw a picture of what this looks like. I've described it in words. But this is a *shape*, and words are a very bad medium in which to describe shapes. Pictures are a far better medium! So, translate my words into a picture. (Maybe make a nice little orthographic drawing, showing the casita from the front, the side, and the top.)

(b) We can think of the roof as being a function that takes in two numbers as input, and returns one number as output. Make a function for it.

(c) The roof has a slope! Suppose you're standing in the exact center of the roof, facing the taller side. From your perspective, what's the slope of the roof? Note that as finite, limited human beings, we can't see in two dimensions—we only see in one dimension, looking forward or whatever.

[1]http://archityperview.com/project/the-nueva-school-hillside-learning-complexissue_id638/

You don't really need calculus to figure this out (let alone multivariable calculus), but indulge me. Without bothering to use any calculus, what should the answer be?

Then, do some calculus. Set up some formalism, with a function for the roof, derivatives, and all that fun, and calculate the answer.

Also, just for fun, give the answers here both in **normal slope units** ("change in y per change in x"), and also as an **angle**.

(d) Suppose you turn ninety degrees, so that you're now parallel to the front/back/long sides of the house. From your perspective, what's the slope now? Again, you don't *need* calculus to do this—in fact, this is arguably easier than the previous problem—but for fun's sake, do the calculus, too.

(e) Now suppose you turn again, so that you're neither facing directly to the front of the house or the side of the house, but rather, to one of the top corners of the house. From your perspective, what's the slope of the roof?

2. Shed roofs are cool, but one way we can make them cooler is to make them **off-axis**! Rather than having a shed roof that rises and falls in one direction, and is flat in the other direction, we could have a shed roof that **rises and falls in two directions**!!! This is cool because, visually, it emphasizes a single point—it emphasizes one corner of the building. (Contra the shed roof from the prior problem, which emphasizes one entire side.)

Here's an example. This is the Museum of the Earth, the paleontology museum in my hometown:

[2]

It's a little hard to see, but if you look closely, you can see the roof sloping both along the short axis of the building (the left side of this picture), and also along the long axis (the right side of this picture). So the point of the roof closest to the perspective of the photographer is the high point; the point of the roof furthest from the photographer (and hidden by the building itself) is the low point.

Anyway, suppose you build a casita with this sort of design. Say that this casita is again 20 feet by 30 feet. It rises up to a high point of 15 feet. Along the long face on the front side, it rises from 10 to 15 feet; on the shorter face of the front side, it rises from 12 feet up to 15 feet.

[2] https://www.lifeinthefingerlakes.com/wp-content/uploads/2015/05/2004_summer_daytrip_museumofearth-690x394.jpg

First of all, draw this. Or try to. What are the elevations of all four corners of the roof? Can you come up with an equation for the (height of the) roof?

Then, standing on the roof, directly in the middle:

(a) What is the slope, facing towards the front (i.e., the taller/longer side)?

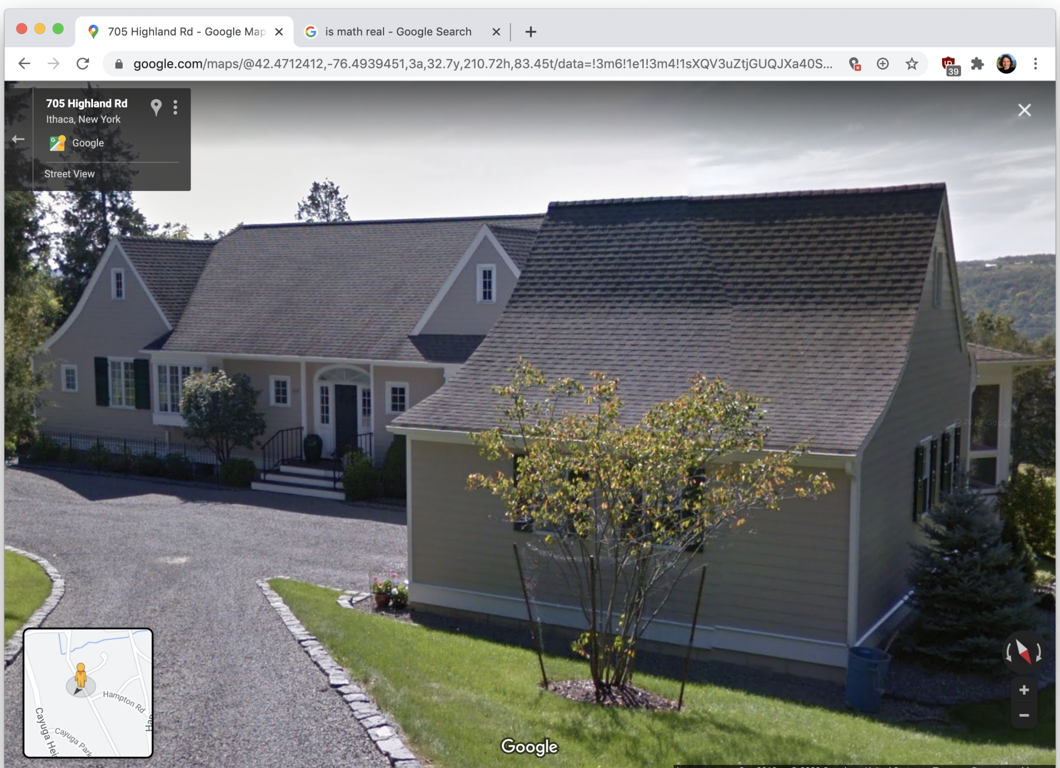
(b) What is the slope, facing towards one of the shorter sides? (Which side? Choose one, but make it clear which one you choose.)

(c) What if you face directly towards the high point of the roof (i.e., the dramatic corner)? What's the slope?

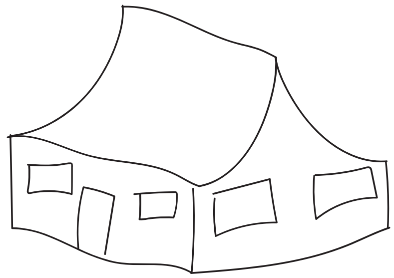
(d) What if you want to face in such a direction so as the roof is angled, from your perspective, at exactly nine degrees? Ten degrees? What direction should you turn in?

I'm deliberately not setting up any coordinate systems or giving you equations here. That's for you to do! And obviously you could all do these in slightly different ways. That's fine. The important thing is to be CLEAR about what you're doing, and why you're making the choices you're making.

3. When I was in high school, one of the last remaining empty plots of land in my hillside suburb was purchased by the recently-retired president of the local bank and his wife. They built themselves a Cape Cod-style house that was small and unassuming—except for its roof, which was subtly curved. Rather than being a normal roof at a constant angle, it bent slightly inwards:



Suppose you're building a somewhat simplified version of this house, as a single box with a curving-inwards roof, like so:



Now suppose you were standing on the roof, exactly in the middle—well, not *exactly* in the middle, since that'd be up on the ridgeline and that's kind of a boring spot to be in, but in the middle of one of the curvy *halves* of the roof. Facing towards the ridgeline, what's the slope of the roof? What about facing towards one of the peaks (i.e., the ridgeline at the edge of the house)? What if you want to face in the steepest possible direction—in which direction should you face?

This problem is intentionally somewhat open-ended. You'll have to come up with an equation for the roof—and there are lots of different possible equations you could choose. I have no idea what the equation of that roof surface is! Choose something that's more-or-less plausible—something that makes the roof nicely curve inwards, too. Oh, and the other requirement I have for you is that the house not have a square footprint—we don't want *too* much symmetry here! You could give it the same 20' by 30' casita dimensions as in the last two architecture problems, or choose different dimensions altogether—just as long as it's not square.

Since this is somewhat open-ended, and since you'll all get different answers depending on the details of your function, be sure to **write it up carefully and thoroughly**. In other words, don't just scribble some equations and calculations—actually explain what you're doing, in complete English sentences, why you're making the choices you're making, and what your hieroglyphic mathematical symbols mean.

(Use your favorite 3D graphing tool to plot/visualize your function for the house/roof, too.)

4. In the early 20th century, advances in what's called **thin shell concrete** allowed for some of the most dramatic forms of mid-century modern architecture. One popular motif was roofs in the shape of a **hyperbolic paraboloid**. Here's a stunning example—the Catalano house in Raleigh, which sadly been since demolished:


[3]

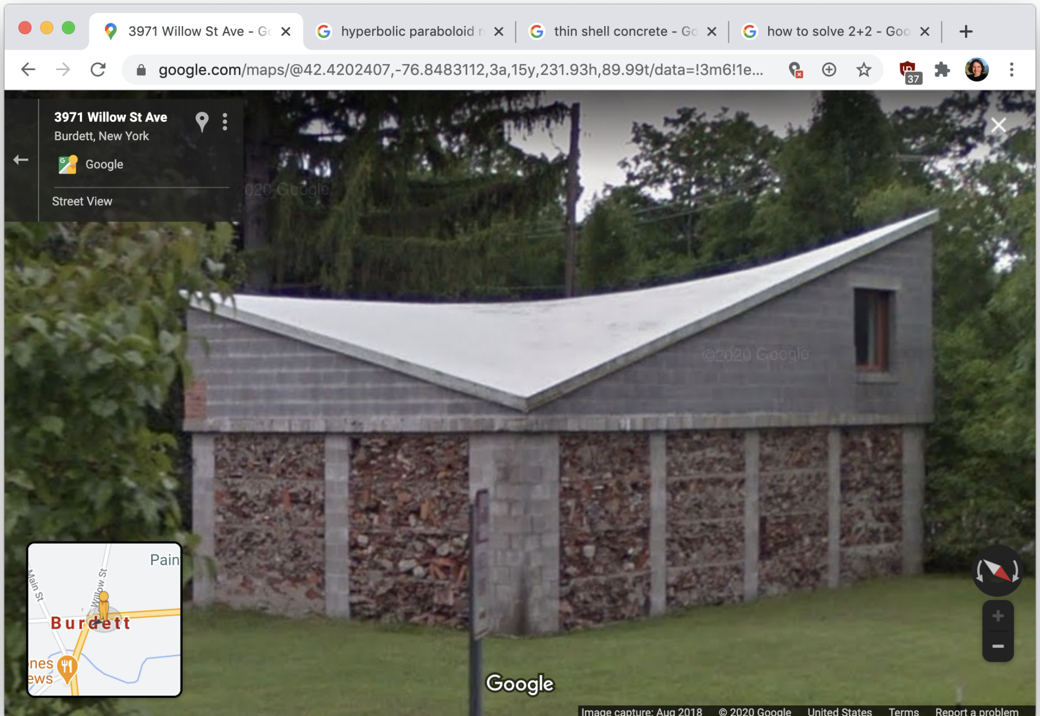
Actually, this house has a wooden roof. Hyperbolic paraboloids have a cool property[4] that make them way easier to construct than they look (even without fancy technology); I think the utility of

[3] https://soldanoluth.com/standard-blog/2016/5/17/the-iconic-hyperbolic-parabloid-roof-and-a-great-design

[4] I won't explain it here, but you can read more about it online.

thin-shell concrete is that it allowed them become really big. (But I don't actually know anything about the history of architecture! Don't listen to me.)

Anyway, like Odysseus, let's go back to my hometown of Ithaca. Half an hour outside of town, in an otherwise decrepit rural hamlet, is an incongruous example of a building with a hyperbolic paraboloid roof:



It has an interesting provenance, as one of the local papers describes:

> Years ago, a hotel stood near that spot, later supplanted by the Burdett Grange. *[Granges were 19th-century cooperative agricultural, social, and political organizations.]* Little by little, the Grange members died off, the roof leaked, the dance floor buckled, and the structure was demolished in 1980. Born of the ruins and rubble of the Grange, the current building is called the Grange Phoenix by its builder. It houses a somewhat occult and idiosyncratic collections of books along with a collection of art works, and occasionally serves as [architect Daniel] Hirtler's office.
>
> A Cooper Union graduate in architecture, he speaks deliberately, but with convincing passion for his architectural work, sustainability, and for this particular building. As sun streams into the south-facing windows separated by narrow brick piers – windows entirely invisible from the road – he tells of his own history in Burdett, how he came to work on the building, and his plans for its future. ...
>
> Hirtler purchased the property for $1,600, its assessed value. He began work on the building in 1987, having sorted through the rubble of the Grange to extract the bricks and shards of now-broken tile that had made up its foundation. After installing a slab foundation, and with help from villagers, he began creating the walls of terra cotta rubble held together by masonry cement and sand. Walls were installed in sections contained by forms. The roof is the piece de resistance, the hyperbolic parabola. "I wanted one all my

> life," he said. It's a thin shell, just one inch thick, of concrete reinforced with fiberglass, and formed on top of metal roofing and is something of an architectural wonder, its strength deriving from its surprising curve.[5]

Some questions about this building:

- **What's the slope of the roof?** Come up with a function for the slope (at any point, in any direction).
- Suppose you're standing in the exact center of this roof—or, to be a bit more precise, on a spot that's directly over the center of the footprint of the building. **What's the slope of the roof at that point?**
- Suppose you're standing in that same spot, facing directly towards the middle of one of the shorter-length sides. **What's the slope at that point, in that direction?**
- Suppose you're standing in that same spot, facing directly towards one of the corners of the roof. **What's the slope at that point, in that direction?** (Make sure you make it clear to *which* corner you're pointing!
- Suppose you're standing in that same spot, and you want to face in the direction such that the roof is flat. What direction do you face in? (Give your answer both as a direction/unit vector, and also as an angle on the xy-plane.)

This is also somewhat open-ended. I don't want to be too detailed about how you should figure this out. You'll need to make a function for the roof itself (the surface of the roof?). To do that you'll need to estimate the length and width and height (height*s*) of the building. You can go on Google Maps to get a better look at it, and/or Google it to find the architect's website, which also has some pictures. Note that this roof isn't symmetric in the way you might think it is—all four corners are at a different height (whoa!). So you'll need to take the equation for a hyperbolic paraboloid (golly, I guess you'll need to look that up, too!) and futz with it until you get something that basically works. (It takes a bunch of futzing. Depending on how you set up your coordinates, you'll probably need to rotate it. Use your favorite 3D visualizamajigger to help. And include some pictures and graphs!)

Some ideas for further problems: what if you want to buy a powerful, clean-room-quality air filtration system to protect your house from wildfire smoke, viral fomites, and strongly-flavored cooking fumes? Such systems are sold/rated/marketed by how many cubic units of air they can filter per unit time. What capacity systems ought we buy for each of these four houses? Suppose we buy a system of some given capacity: what's the **flux** *across the filtration membranes? And: what's the surface area of all of these roofs, in contrast to the surface area/footprint of the houses themselves?*

[5] "Library and Gallery Built From Rubble of Burdett Grange," *Ithaca Times*, 31 July 2013