Numerical Methods Bootcamp
University of Edinburgh

Friday Assignment
Solving and simulating a model at the Zero Lower Bound
`http://www.nber.org/chapters/c12027.pdf`

For this problem set, you are asked to solve Eggertson's (2011) new-Keynesian model at the zero lower bound. The focus will be on government spending and the fiscal multiplier, so all distortionary taxes are set to zero (lump sum taxes automatically deals with any deficits).

The equilibrium is characterized by three equations (equations (10), (12), and (13) in Eggertson (2011))

$$y_t = E_t y_{t+1} - \sigma(i_t - E_t \pi_{t+1} - r_t^e) + (g_t - E_t g_{t+1}) \tag{IS}$$

$$\pi_t = \kappa y_t + \kappa\psi(-\sigma^{-1} g_t) + \beta E_t \pi_{t+1} \tag{AS}$$

$$i_t = \max\{0, r_t^e + \phi_\pi \pi_t + \phi_y y_t\} \tag{MP}$$

where σ is the elasticity of intertemporal substitution, β the discount factor, κ and ψ are composite parameters (see footnote 13 in Eggertson (2011)), and $\phi_\pi > 1$ and $\phi_y > 0$ are the coefficients on the Taylor rule. The shock, r_t^e follows the process $r_t^e = -\ln(\beta) + u_t$, where u_t is iid.

Eggertson assumes that in some period s the economy is unexpectedly hit by a negative shock to r_s^e with expected (stochastic) duration $1/\mu$. Thus, with probability μ the shock is still active in period $s+1$, but with the complementary probability $1-\mu$, the economy reverts back to it's normal state such that $r_{s+1}^e = -\ln(\beta)$. The shock is severe enough to bring the economy to a liquidity trap such that $i_s = 0$ until the reversion of the shock. Once the economy has reversed, there are no more disturbances.

Furthermore, Eggertson assumes – in the benchmark exploration – that government spending increases simultaneously with the negative shock, with an identical (stochastic) duration. Thus, in period s, g_s increases by some amount, and remains high for the duration of the liquidity trap. The question he asks is simple: What is the size of the fiscal multiplier?

This is a regime-switching linear system explored in class. Let T represent the 4×4 transition matrix of the exogenous state. Then $T_{11} = 1$, $T_{12} = 0$, $T_{21} = (1-\mu)$ and $T_{22} = \mu$. Let x_t^i $i = t, n$ denote any variable x in period t at regime t ("trap") and n ("no trap").

The two systems are then given by

$$\begin{aligned} y_t^n &= T_{11}y_{t+1}^n + T_{12}y_{t+1}^t \\ &\quad - \sigma(i_t^n - (T_{11}\pi_{t+1}^n + T_{12}\pi_{t+1}^t) - r_t^{e,n}) + (g_t^n - (T_{11}g_{t+1}^n + T_{12}g_{t+1}^t)) \end{aligned} \tag{1}$$

$$\pi_t^n = \kappa y_t^n + \kappa\psi(-\sigma^{-1}g_t^n) + \beta T_{11}\pi_{t+1}^n + T_{12}\pi_{t+1}^t \tag{2}$$

$$i_t^n = r_t^{e,n} + \phi_\pi \pi_t^n + \phi_y y_t^n \tag{3}$$

$$r_t^{e,n} = -\ln(\beta) \tag{4}$$

$$g_t^n = 0, \tag{5}$$

and

$$\begin{aligned} y_t^t &= T_{21}y_{t+1}^n + T_{22}y_{t+1}^t \\ &\quad - \sigma(i_t^t - (T_{21}\pi_{t+1}^n + T_{22}\pi_{t+1}^t) - r_t^{e,t}) + (g_t^t - (T_{21}g_{t+1}^n + T_{22}g_{t+1}^t)) \end{aligned} \tag{6}$$

$$\pi_t^t = \kappa y_t^t + \kappa\psi(-\sigma^{-1}g_t^t) + \beta T_{21}\pi_{t+1}^n + T_{22}\pi_{t+1}^t \tag{7}$$

$$i_t^t = 0 \tag{8}$$

$$r_t^{e,t} = -\ln(\beta) - u \tag{9}$$

$$g_t^t = g_{t-1}^t, \tag{10}$$

with $g_{-1}^t = g$, which is a given parameter. The variables y_t, π_t and g_t are already expressed to their "no-trap" steady state values. The nominal and real interest rates are not, which makes it easier to correctly implement the zero lower bound.[1]

Attached with this assignment are two programs: Ramsey.m and Eggertson.m. Ramsey.m solves the linearized stochastic growth model in which the productivity process can only take two values. This is another simple example of a regime switching linearized system. Run the program and familiarize yourself with it. It will be very useful for today's exercise (copy/paste is your friend).

The program Eggertson.m sets up the above two systems of equations symbolically. The *first part* of the assignment today is – following the procedure in Ramsey.m – to find the (5×5) matrices A^i, B^i, C^{ij}, and the 5×1 vector D^i, for $i = t, n$ such that the above two systems can be written as

$$A^i u_{t-1}^i + B^i u_t^i + C^{ii} u_{t+1}^i + C^{ij} u_{t+1}^j + D^i = 0, \quad i = n, t \tag{11}$$

where u_t^i is the vector $u_t^i = (y_t^i - y_{ss}, \pi_t^i - \pi_{ss}, g_t^i - g_{ss}, i_t^i - i_{ss}, r_t^{e,i} - r_{ss}^e)$, and A^i, B^i, C^{ii}, and C^{ij} are the Jacobians of system $i = t, n$ with respect to u_{t-1}^i, u_t^i, u_{t+1}^i and u_{t+1}^j, respectively. Following Eggertson (2011), we have $y_{ss} = 0$, $\pi_{ss} = 0$, $g_{ss} = 0$, $i_{ss} = r_{ss}^e = -\ln(\beta)$. The

[1]If the nominal interest rate was expressed relative to the no-trap steady state value, then we would have $i_t^t = -\ln(\beta)$ in the trap-regime, which is less clear.

vector D^i is such that system is satisfied when evaluated at the steady state (See Ramsey.m for this; you will notice that in this model $D^n = 0$, while D^t is not).

Take some time to reflect on the following questions/issues:

- Inspect the matrices A^n and A^t, and try to understand why the look like they do.
- Suppose the economy switches from the "trap" regime to the "no-trap" regime. Are there any transition dynamics towards the "no-trap" steady state? Or do output and inflation instantaneously jump to new constant values?
- Suppose the economy switches from the "no-trap" regime to the "trap" regime. Will there be any transition dynamics when the economy remains in the "trap" regime?
- As stated above, $D^n = 0$, while $D^t \neq 0$. Do you understand why?

The solution (policy functions) to the system in equation (1)-(10) takes the form

$$u_t^i = E^i + F^i u_{t-1}^i, \quad i = n, t \tag{12}$$

where E^i is a 5×1 vector, and F^i is a 5×5 matrix. The *second part* of today's assignment is to find this collection of coefficients through the iterative procedure

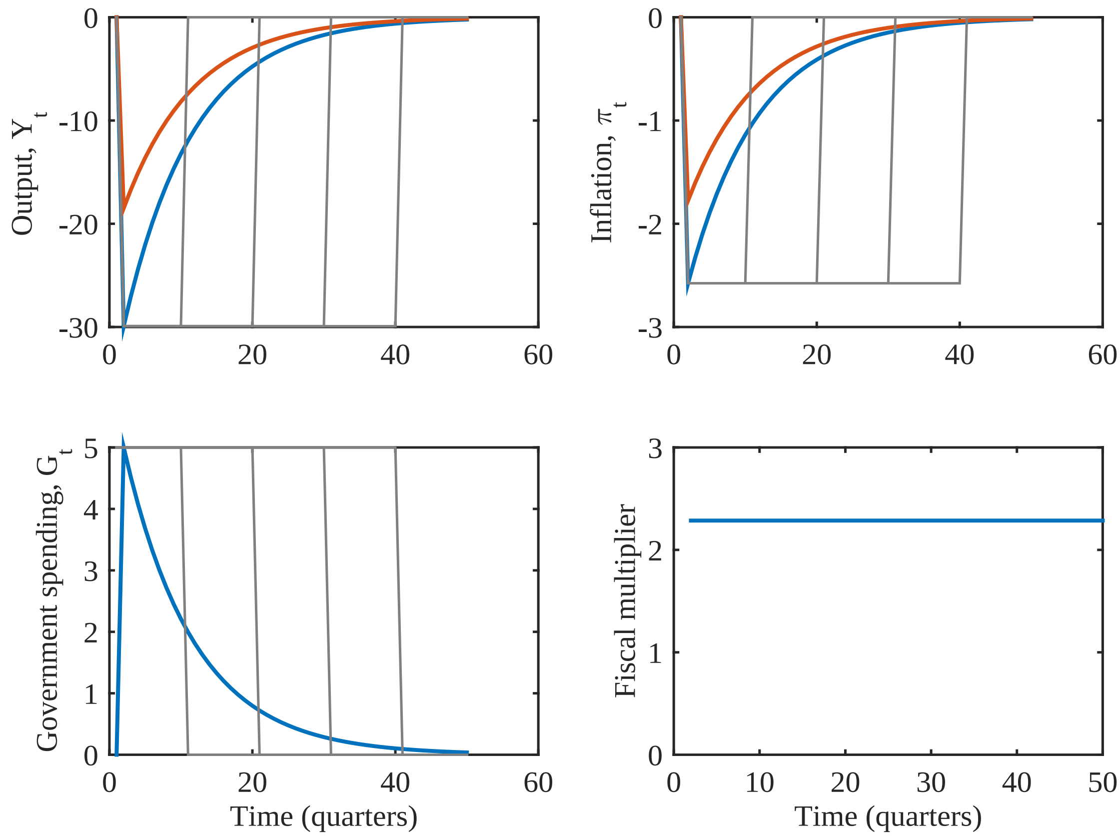
$$E_{n+1}^i = (B^i + C^{ii} F_n^i + C^{ij} F_n^j)^{-1} (-(C^{ii} E_n^i + C^{ij} E_n^j + D^i)) \tag{13}$$

$$F_{n+1}^i = (B_n^i + C^{ii} F_n^i + C^{ij} F_n^j)^{-1} (-A^i) \tag{14}$$

Upon convergence, the fiscal multiplier in a liquidity trap is given by $F_{1,3}^t$. That is, in the first row, third column of matrix F^t. How does this correspond to Eggertson's analytic result of 2.3? What happens if you increase or decrease μ a little?

The *third and last part* of your assignment is to plot the *expected* impulse responses (along with sample paths if you can), in a similar way to Ramsey.m (copy/paste is your friend, remember?). To check that you got things right, this is what mine look like; the sample paths illustrates the cases in which the economy remains in the "trap" regime for 10, 20, 30, and 40 periods.

Remember, that you should never blindly program. At each step ask yourself what you are doing. Is it possible to look at the outcomes of some intermediate products to see that you are on the right track.


Output, Y_t
0
-10
-20
-30
0
20
40
60
Inflation, π_t
0
-1
-2
-3
0
20
40
60
Government spending, G_t
5
4
3
2
1
0
0
20
40
60
Time (quarters)
Fiscal multiplier
3
2
1
0
0
10
20
30
40
50
Time (quarters)