Macroeconomics Summer School
Part II: Advanced Tools
Instructor: Pontus Rendahl

Wednesday Assignment
Solving and simulating the Diamon-Mortensen-Pissarides model

For this problem set, you are asked to solve the Diamon-Mortensen-Pissarides model in continuous time with risk-averse firm owners. For simplicity, I will assume that real wages are sticky, such that profits, π_t, are exogenously given. In Δ-units of time, the model is given by the equations

$$J_t = \Delta\pi_t + (1-\Delta\rho)\frac{u'(c_{t+\Delta})}{u'(c_t)} J_{t+\Delta}(1-\Delta\delta), \tag{1}$$

$$\Delta\kappa = \Delta h(\theta_t) J_t, \tag{2}$$

$$\Delta c_t = \Delta(n_t - \kappa\theta_t(1-n_t)), \tag{3}$$

$$n_{t+\Delta} = (1-n_t)\Delta f(\theta_t) + (1-\Delta\delta)n_t, \tag{4}$$

where $f(\cdot)$ and $h(\cdot)$ refer to the job-finding rate and job-filling rate, respectively.

Part A. Derive the continuous time equivalents to equations (1)-(4).

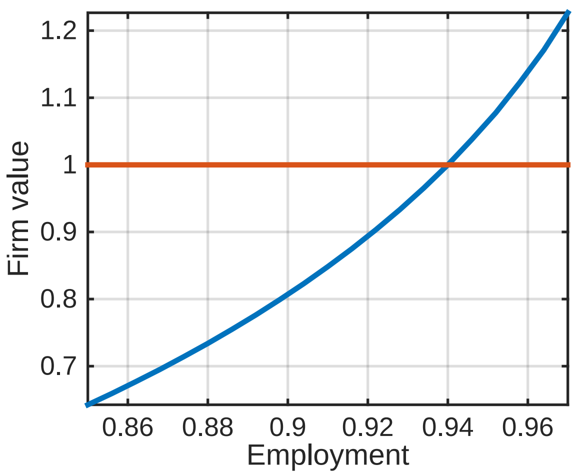
Part B. Attached with this assignment are three programs: `HJB_Euler.m`; `Solow.m`; and `DMP_incomplete.m`. You numerical task for the day is to complete the `DMP_incomplete.m` code using your answer from Part A. The codes `HJB_Euler.m` and `Solow.m` are not important to complete the task, but can provide some guidance and help regarding how to approach this problem.

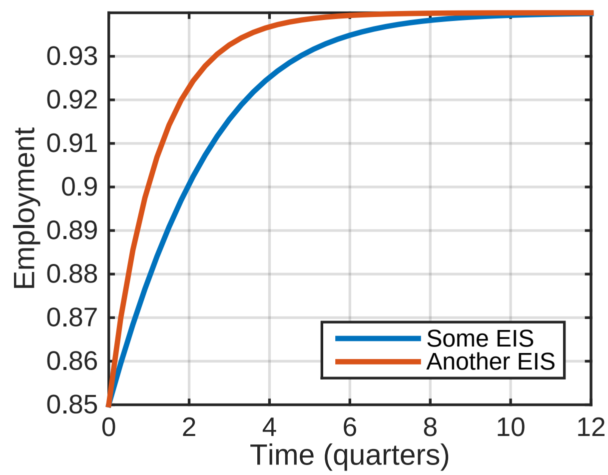
When the model is solved

(i) Provide a graph illustrating the firm value, J on the y-axis, and employment, n on the x-axis. Compare your result with the equivalent graph for the $\gamma = 0$ case (i.e. infinite EIS case). Mortensen and Nagypal (2007) state that "the unemployment rate is not an information-relevant state variable". For which values of γ is this true?

(ii) Given your solution, use the `ODE45` command in Matlab and calculate the transitional dynamics from time 0 to 12, starting from the lowest value of n on the grid. Compare your results again with the $\gamma = 0$ case. How does a low EIS (i.e. high γ) affect the transitional dynamics?

If your computations are correct, your graphs should look something like those below.





Lastly, if you would happen to have some time left over and are in for a challenge, consider the stochastic version of the above model:

$$J_t^g = \Delta\pi_t^g + (1-\Delta\rho)\left[(1-\Delta q)\frac{u'(c_{t+\Delta}^g)}{u'(c_t^g)}J_{t+\Delta}^g(1-\Delta\delta) + \Delta q\frac{u'(c_{t+\Delta}^b)}{u'(c_t^g)}J_{t+\Delta}^b(1-\Delta\delta)\right],$$

$$\Delta\kappa = \Delta h(\theta_t^g)J_t^g,$$

$$\Delta c_t^g = \Delta(n_t - \kappa\theta_t^g(1-n_t)),$$

$$n_{t+\Delta} = (1-n_t)\Delta f(\theta_t^g) + (1-\Delta\delta)n_t,$$

where g represents the "good state". An equivalent set of equations holds for the "bad state", b. Repeat Part A above.