What drives estuarine circulation?

Jody M. Klymak

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Introduction

Oddly, the texts don't give a nice description of *how* estuarine circulation actually happens. How estuarine circulation occurs is slightly subtle, but not terribly difficult to understand. It is, however, not amenable to simple mathematical description, which may explain its omission from basic texts.

Pressure differences

Before we start, it is helpful to think about how pressure differences drive flows. If the sea-surface is tilted, then the pressure will be greater under the deeper than under the shallower. Thus in figure 1 $P_1 > P_2$. This will drive the flow from left to right.

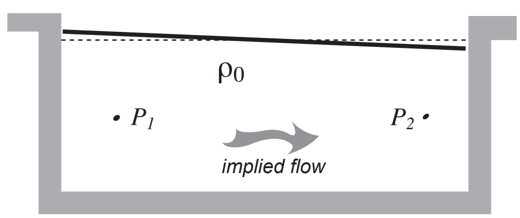

Figure 1: A body of water of uniform density with a tilted upper surface.

In general, it is a good rule to think that the water will move in the direction that will lead to a flattening of the surface. The resting state of the body of water is to have a flat surface, as indicated with a dashed line.

The situation is more complicated if we add density variations. Consider just tipping the interface of a two-layer fluid (figure 2). (This is actually hard to do, but just imagine). At first, the pressure difference in the upper layer is zero ($P_3 = P_4$) because the upper interface is not tilted. However, there is more dense water on the left side

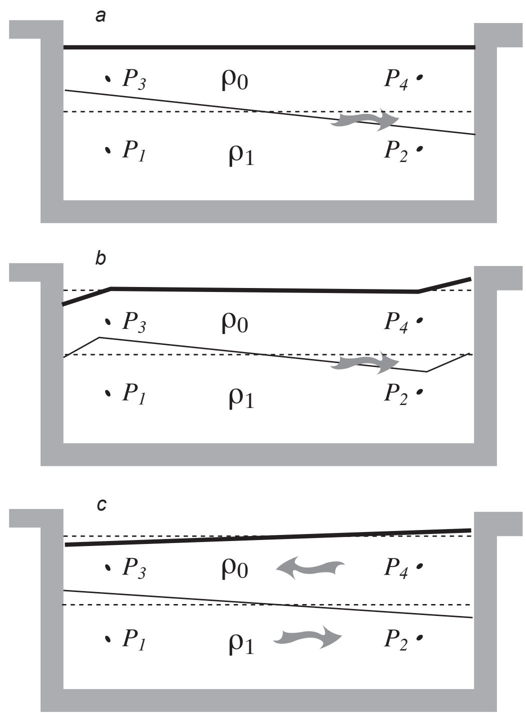

Figure 2: A two-layer body water with an interface that is initially tilted. See text for the description.

than the right, so there is a pressure gradient below the interface, $P_1 > P_2$, driving deep water to the right.

This leads to more water, in total, on the right hand side than the left side (figure 2b). This makes a surface pressure gradient that tends to drive the upper layer to the left ($P_4 > P_3$). This surface pressure gradient is set up *very* quickly, and only involves a tiny amount of water in order to drive the surface-layer flow to the left.

The water column will slosh around for a while, but friction will eventually damp the motions and the interfaces will be flattened as indicated with the dashed lines (figure 2c).

Again, the general idea is that the body of water has pressure gradients in it that drive a flow that will tend to flatten the layers.

Q: in figure 3, what direction does each layer flow? Q: If the density differences between the layers are equal, which layer will initially have the strongest flow?

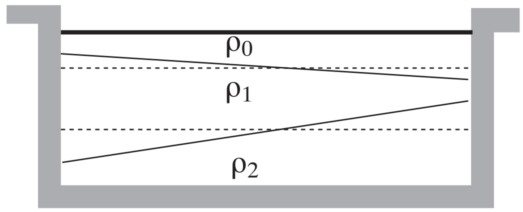

Figure 3: A three-layer body water with tilted interfaces.

Estuarine Flow

Salt-wedge estuary

Estuaries are bodies of water with opposing buoyancy sources at their ends, usually a river at the head and the ocean at the mouth. The simplest case is the river spills into the ocean, spreads out and becomes a buoyant plume (figure 4). If we quantify the river volume transport as R, then in steady state the transport out the mouth of the estuary will be $Q_o = R$. The deep ocean layer is typically stagnant.

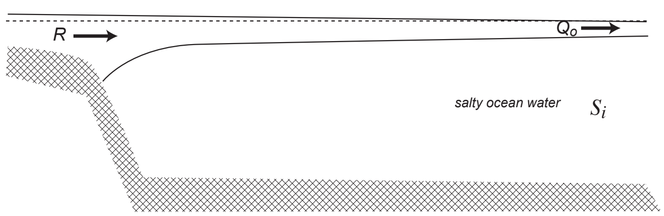

Figure 4: A salt-wedge estuary and river plume. The river discharges into the salt water without substantial mixing.

This type of estuary is called a salt-wedge estuary. The mouth of the Fraser river is a classic example of a salt-wedge estuary with a spreading plume (figure 5). Fresh water runs over salty with a small amount of mixing between the two layers. The salinity goes from 2 psu to 24 psu in just a few meters vertically. This sharp front moves back and forth with the tide, a typical feature of most salt-wedge estuaries.

Fjord-type estuary

Most estuaries have an exchange flow which is much stronger than what is driven by just the river flow. This flow is driven by vertical mixing creating tilted isopycnals.

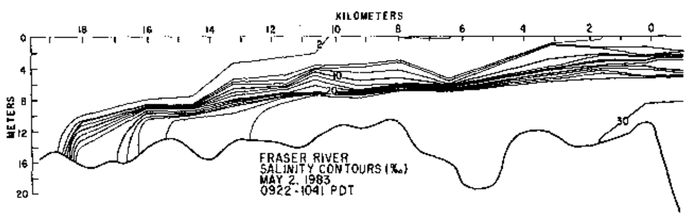

Figure 5: Salinity contours in mouth of the Fraser River. The river is on the left, the ocean on the right. (Geyer and Farmer, 1989)

These tilted isopycnals want to slump due to the pressure gradients that are set up, just like we saw above.

As an example, imagine the situation pictured in figure 6. Here a river flows into salty water. If turbulent mixing is “turned on” by tidal flow over the topographic bump in the middle of the estuary, then intermediate-salinity water will be formed. This intermediate-salinity water will be denser than the river water, but lighter than the ocean water, and will set up a flow that is pictured in figure 6.

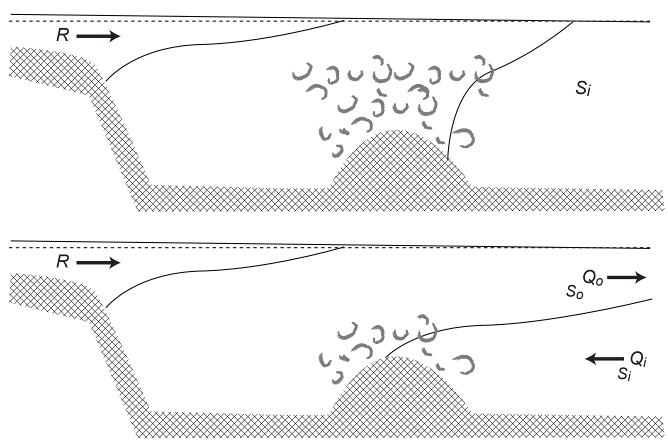

Figure 6: The effect of weak local mixing on flow in a fjord-type estuary. Intermediate salinity water is formed by the mixing which drives a flow out to sea.

Determining the steady-state for such a flow is not trivial, and not very conducive to theoretical formulation. However, general tendencies can be determined. If there is more mixing, more intermediate water is formed. In steady state this intermediate water must be flushed away, so there is a stronger circulation. Conversely, if the mixing is weaker, less intermediate water is produced, weakening the circulation.

Flood-plain estuaries

The more mixing that takes place, the more vertical the isopycnals, and the stronger the exchange flow that is driven. The salt and density fields also become much more complicated. In shallow estuaries, of which the Chesapeake Bay is an excellent example, mixing from bottom stress and the winds tends to affect the whole water column, not just near topographic features. This sets up large along-estuary salinity and density gradients (figure 7). These types of estuaries tend to have much stronger amplification of the river flow (*R*) because of the extra mixing.

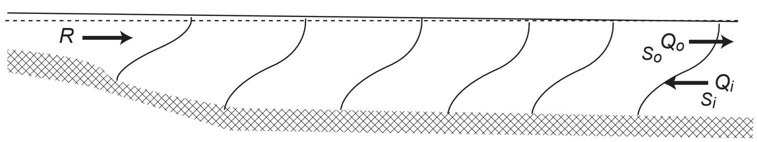

Figure 7: A flood-plain estuary schematic. Salinity increases from the head to the mouth, with weak vertical gradients.

An example from an arm of the Chesapeake demonstrates how such an estuary may look (figure 8). Up inlet of Maryland Point, the estuary is really a river with no salinity. However, unlike the salt-wedge estuary, there is a gradual horizontal gradient of many kilometers before salty water is found. Note that even at this point the water is still only 12 psu, and that the Chesapeake Estuary continues for hundreds more kilometers before open-ocean water is found.

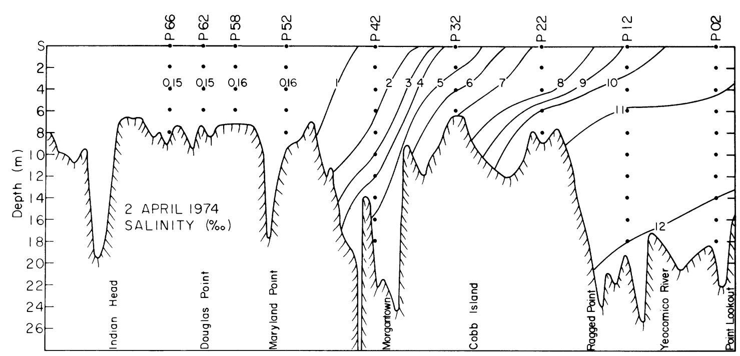

Fig. 8. Salinity section, April 1974.

Figure 8: A salinity cross-section from the Upper Potomac estuary, part of the Chesapeake Bay estuarine system (Elliott, 1976).

The whole estuary has been measured and simulated numerous times. A recent example is shown in figure 9. Here the upper layer of fresher water can be seen flowing south and seaward. There is a strong return flow in the bottom 20 m. Note that the volume transport of the return flow is not accurately represented by these velocities, since they do not take into account the fact that the Bay widens considerably as it travels further south.

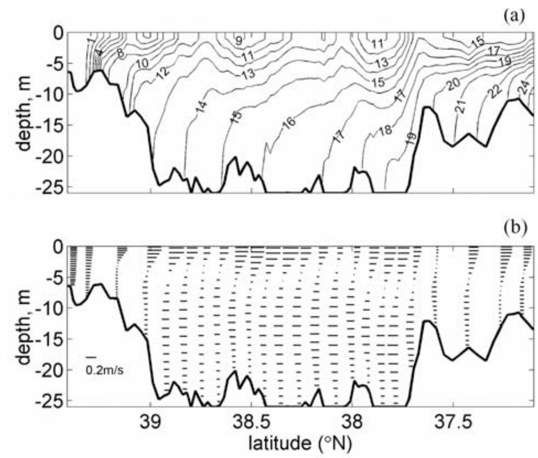

Figure 9: A salinity and mean velocity sections from a numerical model of the whole Chesapeake Bay (Li et al., 2005)

Also note the lenses of fresher water that appear along the estuary. These occur because of rivers along the length of the estuary. This complicates the simple picture given above, but the general trend of the flow remains unchanged.

Time-dependence

Time dependence changes the steady-state balance in estuaries in three main ways. First, the fresh-water input at the head of the estuary can change (figure 10). This significantly changes the salinity content of the estuary and the estuarine circulation. Somewhat paradoxically, the increased river flow can lead to a decreased estuarine flow. For instance in the Apr 1968 panel the isopycnals are much less tilted than in the Oct panel, indicating enhanced flow. The balance here, however, is not a perfect one, and determining the exact response if the river forcing changes depends on the mixing response.

Similarly, the ocean water at the mouth of the estuary can change salinity due to seasonal changes in the open ocean. This tends to be a smaller effect because the salinity differences, even in coastal waters, tend to be less pronounced.

Finally, there is a time dependence to the mixing that ultimately drives the enhanced estuarine circulation. The most regular source of time-dependence is the tidal forcing. This varies with the fortnightly modulation of the tide, and with the long-term perigree/apogee variations of the moon-earth-sun system. More mixing during the spring tides leads to isohalines that are more tilted (figure 11) which therefore drive more circulation.

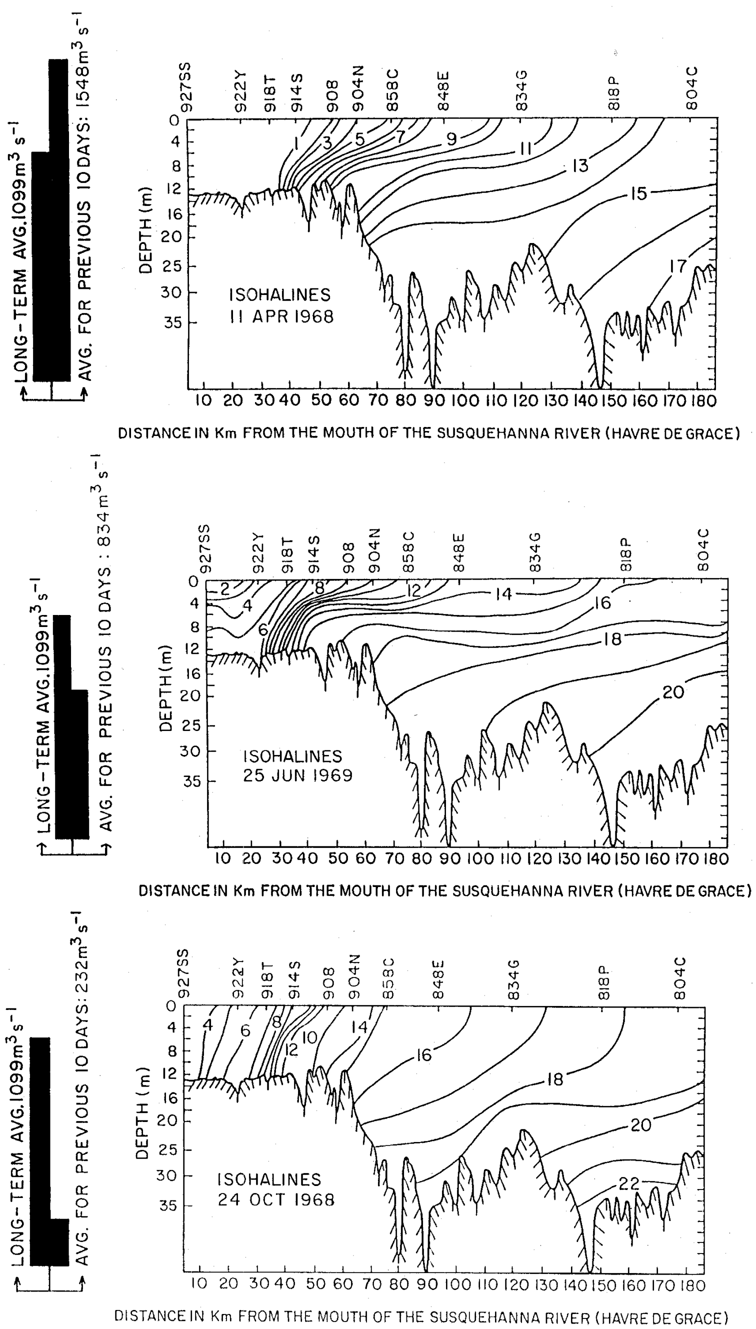

Figure 10: Salinity in the Chesapeake as it varies due to river discharge over the year (bars on left) (Schubel and Pritchard, 1986).

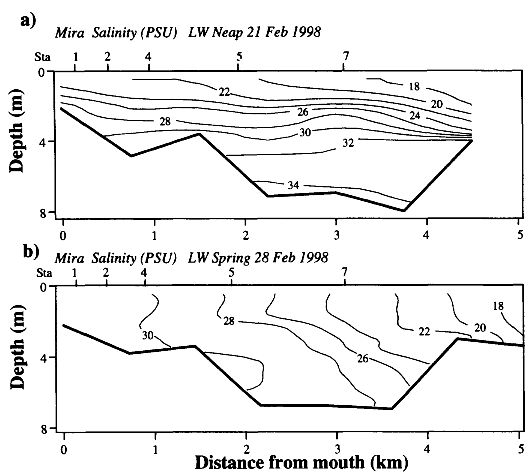

Figure 11: Salinity in the Mira estuary (Portugal) during neap and spring tides (Blanton et al., 2000).

Topographic blocking (mostly in fjords)

One final topic is helpful in understanding the dynamics of fjords, and that is topographic blocking. Fjords are often demarcated from the ocean by underwater sills (terminal moraines left over from glaciations). These sills can be quite tall and sometimes the tides are not strong enough to bring the densest water from the ocean-side of the sill over the sill (figure 12a). The estuarine circulation carries on as normal above this depth, but the deep ocean layer does not make it into the landward basin. This is called "blocking".

However, occasionally strong tides perhaps enhanced by atmospheric forcing will conspire to provide enough energy for the densest water to get over the sill. This dense water will spill over the sill and, if it does not mix, will reach the bottom of the fjord (figure 12b). For some fjords this happens every spring-neap cycle. For others only when the spring tide is particularly large.

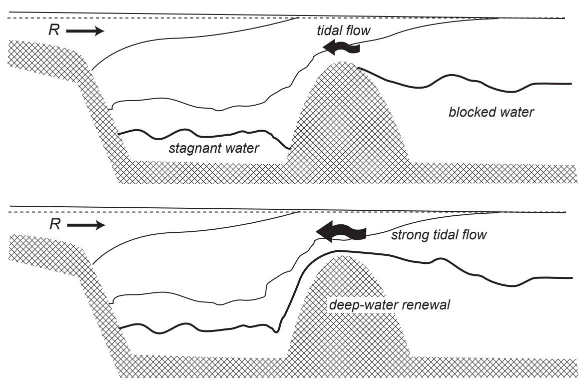

Figure 12: A narrow-silled fjord with blocking of deep water, and a deep-water renewal event.

Conversely, if the sill is broad, or has a lot of horizontal constrictions, deep-water renewal can happen on the exact opposite phase of the spring-neap cycle (figure 13). During spring tides the incoming dense water is subjected to so much mixing that it is mixed away before it can reach the inner basin. It is only during neap tides with enough velocity to push the dense water into the inner basin, but not so much as to cause excessive mixing that deep-water renewal can take place.

Quantifying the Circulation

See also, Talley et. al. 5.1-5.2.

Estuaries are a great example of mass and volume conservation. However, these concepts are very important for all of oceanography (and probably most of science in

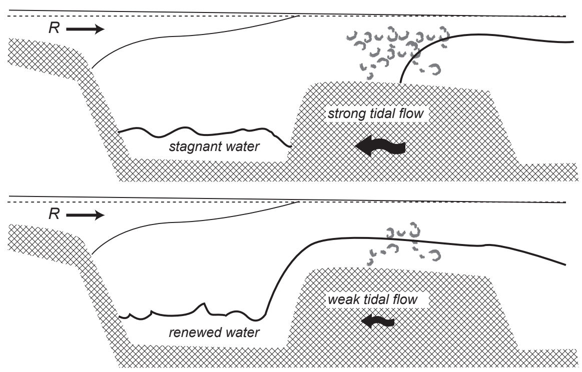

Figure 13: A broad-silled fjord with blocking of deep water during spring tides, and a deep-water renewal during neaps.

general). Most people are familiar with the concepts, but laying out the algebra is a bit harder.

In an estuary, water flows in the river mouth, and in and out at the ocean end of the estuary. If everything is in "steady-state" we can write equations that relate the incoming mass of water and mass of salt to the outgoing, and make assumptions about the flow based on those measurements. The discussion below indicates how that is done.

The Conservation of Volume

Suppose we are interested in the dynamics of a body of water, say a rectangular swimming pool. The pool has a hose pumping water into it at a rate of volume transport of $q[\mathrm{m^3 s^{-1}}]$. If its surface area A, how fast is the water rising in the pool?

Here the answer is relatively easy. The speed at which water rises in the pool is $w = q/A$, the volume transport divided by the surface area. Here we have applied the conservation of volume:

$$\frac{\mathrm{d}V}{\mathrm{d}t} = -\sum_i q_i + \sum_j s_j. \tag{1}$$

where V is the volume of fluid in the body we are studying, q_i are individual transports out of (positive) or into (negative) the volume. The last term is the sum of the sources (positive) and sinks (negative) inside the volume, and is included for completeness. For the conservation of volume of water, a sink could be evaporation, and a source rainfall. We differentiate such surface transports from the "advective" transports because the physics is fundamentally separate.

Formally, this should really be conservation of mass in the volume. In which case:

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_V \rho \mathrm{d}V = -\sum_i \rho_i q_i + \sum_j \rho_j s_j. \tag{2}$$

where the integration over the volume is summing all the mass in the volume. However, the density of water only changes by a few percent, so equation (1) is a very good approximation.

Calculating volume transport

Volume transports only arise because water is moving from point a to point b. Sometimes we are given the volume transport (in $\mathrm{m^3\,s^{-1}}$), but sometimes we want to report the velocity. i.e. in the swimming pool example above we wanted to know how fast the water was rising (i.e. in $\mathrm{ms^{-1}}$). In the simple case:

$$q_i = A_i u_i \tag{3}$$

where A_i is the area the volume flux is flowing through, and u_i is a velocity *perpendicular* to the area. In vector calculus we write this as:

$$q_i = \int_A \mathbf{u} \cdot \mathrm{d}\mathbf{A} \tag{4}$$

where the dot product makes it clear that the flux is perpendicular to the area.

Conservation of a mass of a substance

The conservation of a substance (or heat) has essentially the same terms:

$$\overbrace{\frac{\mathrm{d}}{\mathrm{dt}} \int_V S\,\mathrm{d}V}^{\text{change of amount of stuff}} = \overbrace{\sum_i q_S}^{\text{transport of stuff in}} + \overbrace{\sum_j s_S}^{\text{internal sources}} . \tag{5}$$

Here the substance, S, is expressed in stuff-per-volume. i.e. $\mathrm{g\,m^{-3}}$. We often discuss the *mean concentration in a volume* as $\overline{S} = \int_V S\ \mathrm{d}V/V$, in which case the equation simplifies to:

$$V \frac{\mathrm{d}\overline{S}}{\mathrm{d}t} = \sum_i q_S + \sum_j s_S \tag{6}$$

The *transport* of stuff q_S is expressed in units of stuff per time (i.e. $\mathrm{g\,s^{-1}}$).

Using the concentration and volume transport to calculate advective transport and flux

In fluid mechanics we often measure the concentration of a substance (S) separately from the volume transport q or velocity u. For instance if Chlorine is mixed in a vat at a concentration of $S = 200\ \mathrm{g/m^3}$ and flows through a pipe at $q_v = 0.01\mathrm{m^3 s^{-1}}$ then the rate of chlorine transport into the pool is:

$$q_S = S q_v \tag{7}$$

and has units $\mathrm{gs^{-1}}$. This is called an *advective transport* of chlorine.

If instead we knew the velocity of the water u and the concentration, then we might have expressed this as a *flux* of chlorine:

$$F_S = Su \tag{8}$$

Because velocity is a vector, this flux is more properly written as a vector:

$$\mathbf{F}_S = S\mathbf{u} \tag{9}$$

and the flux flows in the same direction as the water. Note that the units of the flux are $\mathrm{g\,s^{-1}\,m^{-2}}$. The transport of the material through an area A is thus simply:

$$q_S = F_S A = SuA \tag{10}$$

if u is a constant in the area A.

This can all get even more confusing if the concentration is expressed as parts per thousand, or some other volume unit like that (i.e. mL/L). This can be dealt with in the same ways, but it requires some extra care with the units.

Examples mass balance calculation

Q: Suppose we have a vat, with volume $V = 10\ \mathrm{m}^3$ of water in it. A hose flows into it at a rate of $0.1\ \mathrm{m}^3/\mathrm{s}$ and a second hose flows out at the same rate. If the concentration of Caffeine in the vat is initially $0g/m^3$, and $10g/m^3$ in the hose, what is the rate that caffeine is being added to the tank?

A: it is simply $qC = 1\ g/s$.

Q: What rate is caffeine leaving the tank initially?

A: The initial concentration in the tank is zero, so $qC = 0g/s$.

Q: Assuming the tank is well mixed, what is the rate of change of the concentration in the vat, initially?

A: This is just the rate of change of the amount of caffeine, $1g/s$ divided by the volume of water in the vat, so $0.1gm^{-3}s^{-1}$.

Q: New vat, with three hoses. One hose has $C_1 = 10g/m^3$ of caffeine and flows at $q_1 = 0.1\ \mathrm{m}^3/\mathrm{s}$. The second hose flows in with $C_2 = 5g/m^3$, and $q_2 = 0.4\ \mathrm{m}^3/\mathrm{s}$. If the vat is well-mixed and in steady state, what is the flow out the third hose, and what is the concentration of the caffeine in the water in the vat?

A: First, in steady state, the volume fluxes equal zero (there can be no net flow into the vat), so $q_1 + q_2 + q_3 = 0$. We know q_1 and q_2, so we know $q_3 = -0.5\ \mathrm{m}^3/\mathrm{s}$, where the negative sign means water is flowing out.

We also know that there can be no net flow of caffeine, so $C_1q_1 + C_2q_2 + C_3q_3 = 0$. We know everything except for C_3, so we solve and get $C_3 = \frac{C_1q_1+C_2q_2}{-q_3} = 6g/m^3$.

Application to Estuaries: the Knudsen Relation

An application to estuaries is to conserve both volume in the estuary, and salt. Imagine that the flow at the mouth is two layers, one in with a volume transport Q_i $\mathrm{m}^3\mathrm{s}^{-1}$, and one out with volume transport Q_o, and that the river has a volume transport of R.

Suppose the salinity of the lower layer flowing out is S_i in units of parts-per-thousand, and the salinity of the layer flowing out is S_o.

Q: What is the salinity of the river?

Q: What is the salt transport into the fjord in g/s*?*

Q: Assuming the fjord is in steady state, write out an equation for the volume budget, and a second equation for the salt budget. i.e the volume transport in equals the volume transport out and the salt transport in is equal to the salt transport out.

Q: Assume R, S_i, and S_o are known. Show that:

$$Q_i = \frac{S_o R}{S_i - S_o}. \tag{11}$$

Q: If there is no mixing of the river water, what is S_o, and how big is Q_i?

Q: If there is lots of mixing, what happens to $S_i - S_o$ and thus Q_i?

Exercise

We will practice some of what we have learnt based on a paper about local waters by Masson and Cummins (2004). As part of your final project you will read about similar processes in Saanich Inlet (Gargett et al., 2003).

Q: Consider figure 14, which shows the salinity observed and modeled in the Strait of Juan de Fuca. Based on these plots, where do you think the mixing is the strongest in the Strait?

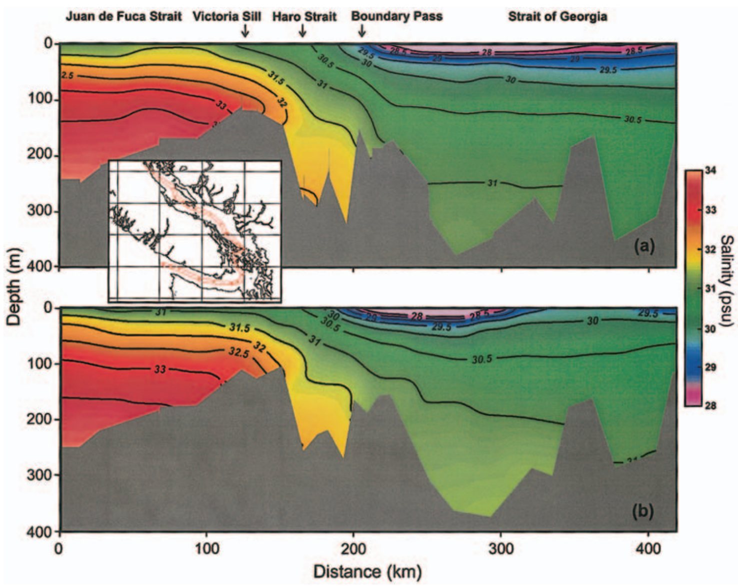

Figure 6. Along-strait section of annual mean salinity from observations (a) and the reference experiment (b).

Figure 14: Observed and modeled salinity in the Straits of Juan de Fuca and Georgia (Masson and Cummins, 2004). The ocean is on the left, and the Fraser River is at Distance = 270 km. The upper plot (a) are the observations and the lower (b) a numerical simulation.

Q: For the flow in figure 14, sketch where you think the water is flowing. What happens north of the Fraser river (Distance = 250 km)?

Q: Consider figure 15, which compares two numerical model runs, one with tides and one without. Which do you think is which, and why? Which has the stronger horizontal circulation?

Q: A seasonal time series of the salinity in Haro Strait is given in figure 16. What features of the flow can you identify that indicate the time dependence of the estuarine flow? Pay particular attention to the modeled timeseries (which has better temporal resolution)?

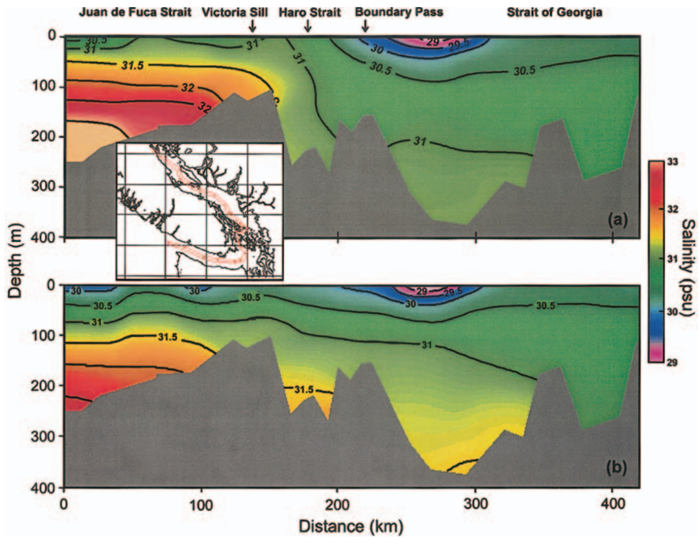

Figure 15: Salinity for two model runs, one with and the other without tides (Masson and Cummins, 2004).

Q: Use the Knudsen relation on the two data plots in figure 15 to *estimate* the exchange flow if the river input is 10^4 m^3 s^{-1}. Which case has a stronger exchange, a) or b)?

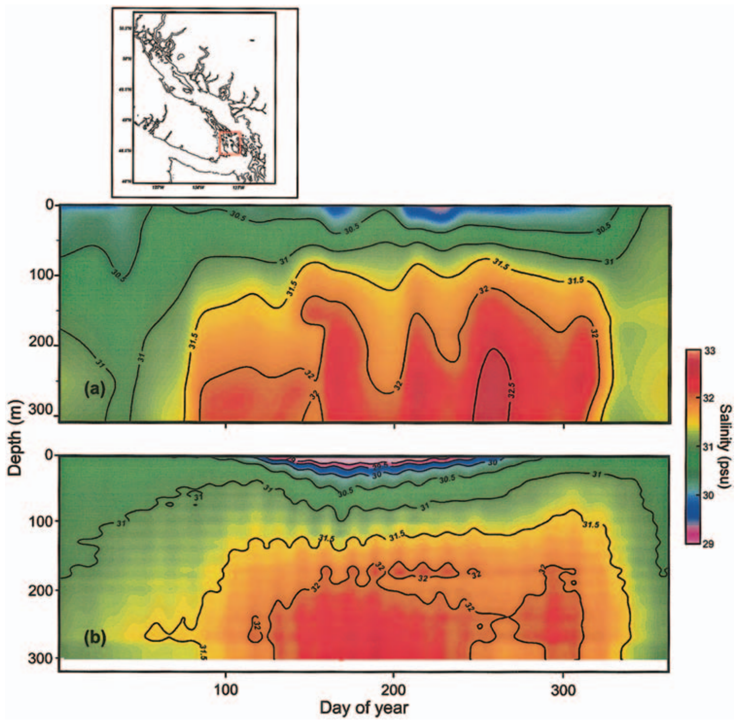

Figure 16: Time evolution of observed and modeled salinity in Haro Strait (Masson and Cummins, 2004).

Practice questions

Some students are not comfortable with volume and mass balances. If that is the case, try some extra practice questions below. Please ask me if you need more help with these.

Suppose the water level of a straight-sided pool is observed to rise at 0.01 m/s, and has a surface area of 50 m^2. There is one hose filling it with speed 4 m/s and cross-sectional area 0.01m^2, and a second hose filling it with a cross sectional area of 0.02m^2. What must the flow speed be in the second hose?

Suppose a pipe has a diameter of 0.1 m and a flow speed of 1 m/s. If the pipe narrows to 0.05 m what is the flow speed at that part of the pipe (in steady state)?

Suppose there is a vat with 200 g/m^3 solution of chlorine being fed into the pool through a hose with diameter 0.1 m at a rate fast enough to replace evaporative losses in the pool of 0.01 m^3/s. What is the transport of chlorine into the pool?

If the average concentration of chlorine in the pool is initially 50 g/m^3 and 1000 s later is measured to be 55 g/m^3, what is the volume of the pool?

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