

Recursive Utility with Optimal Jump Intensity

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1 Basic recursion

The basic Kreps-Porteus recursion is

$$V_t = \left[(1 - \beta_\Delta) (C_t^\alpha D_t^{1-\alpha})^{1-\rho} + \beta_\Delta R_t^{1-\rho} \right]^{\frac{1}{1-\rho}}$$

where R_t is

$$R_t = \left(E_t V_{t+\Delta}^{1-\gamma} \right)^{\frac{1}{1-\gamma}}.$$

Notice that V_t is homogeneous of degree 1 in D_t .

$$\frac{V_t}{D_t} = \left[(1 - \beta_\Delta) \left(\frac{C_t}{D_t} \right)^{\alpha(1-\rho)} + \beta_\Delta \left(\frac{R_t}{D_t} \right)^{1-\rho} \right]^{\frac{1}{1-\rho}}$$

with $c = C/D$ and

$$r_t = \frac{R_t}{D_t} = \left(E_t \left(\frac{V_{t+\Delta}}{D_{t+\Delta}} \frac{D_{t+\Delta}}{D_t} \right)^{1-\gamma} \right)^{\frac{1}{1-\gamma}}.$$

Writing $v = V/D$ we can rewrite the recursion as

$$1 = \left[(1 - \beta_\Delta) \left(\frac{c_t^\alpha}{v_t} \right)^{(1-\rho)} + \beta_\Delta \left(\frac{r_t}{v_t} \right)^{1-\rho} \right]^{\frac{1}{1-\rho}}$$

and in log form

$$0 = \frac{1}{1-\rho} \log \left[(1 - \beta_\Delta) \exp \left[(1 - \rho) (\alpha \hat{c}_t - \hat{v}_t) \right] + \beta_\Delta \exp \left[(1 - \rho) (\hat{r}_t - \hat{v}_t) \right] \right].$$

Develop the continuation value recursion

$$\hat{r}_t - \hat{v}_t = \frac{1}{1-\gamma} \log E_t \exp \left[(1 - \gamma) \left(\hat{v}_{t+\Delta} - \hat{v}_t + \hat{d}_{t+\Delta} - \hat{d}_t \right) \right].$$

v only depends on time through a Brownian with jumps. Therefore v and \hat{v} do too.

2 Optimal Jump Intensities.

Let

$$d\hat{v}_t = \hat{\mu}_t^v dt + \hat{\sigma}_t^v dZ_t + (\hat{v}_t^* - \psi - \hat{v}_t) dN_t.$$

where ψ is a scalar random variable with CDF F denoting the utility cost of moving. Instead d_t is a jump process following the same counter as \hat{v} but with a different jump size

$$d\hat{d}_t = (\hat{d}_t^* - \hat{d}_t) dN_t.$$

Finally, the recursion can be written as

$$0 = \delta \frac{\left(\frac{c_t^\alpha}{v_t}\right)^{1-\rho} - 1}{1-\rho} + \hat{\mu}_t^v + (1-\gamma) \frac{(\hat{\sigma}_t^v)^2}{2} + \frac{\lambda_t}{1-\gamma} \left(E_\psi \exp \left[(1-\gamma) (\hat{v}_t^* - \psi - \hat{v}_t + \hat{d}_t^* - \hat{d}_t) \right] - 1 \right)$$

as $\rho \rightarrow 0$

$$\delta \hat{v}_t = \delta \alpha \hat{c}_t + \hat{\mu}_t^v + (1-\gamma) \frac{(\hat{\sigma}_t^v)^2}{2} + \frac{\lambda_t}{1-\gamma} \left(E_\psi \exp \left[(1-\gamma) (\hat{v}_t^* - \psi - \hat{v}_t + \hat{d}_t^* - \hat{d}_t) \right] - 1 \right)$$

and as $\gamma \rightarrow 1$

$$\delta \hat{v}_t = \delta \alpha \hat{c}_t + \hat{\mu}_t^v + \lambda_t (\hat{v}_t^* - E_\psi \psi - \hat{v}_t + \hat{d}_t^* - \hat{d}_t)$$

Therefore the decision to adjust is optimally taken only if the utility shock ψ satisfies

$$y(\hat{v}) \equiv \hat{v}_t^* - \hat{v}_t + \hat{d}_t^* - \hat{d}_t > \psi$$

or equivalently

$$\begin{aligned} D_t^* v_t^* &\geq \exp(\psi) D_t v_t \\ V_t^* &\geq \exp(\psi) V_t \end{aligned}$$

so ψ acts as a positive multiplicative utility shock to the current durable stock. In a sequence problem the decision would display as

$$\max \{V_t, \exp(-\psi) V_t^*\}$$

so it can also be interpreted as a negative utility shock to moving.

Operational HJB: From the equivalence of robustness to misspecification and recursive utility (with $\rho = 1$) we can write the HJB as a result of the following problem. Let $d\hat{v} = \mathcal{A}\hat{v}dt + \mathcal{B}\hat{v}dZ_t$, then the HJB satisfies

$$\begin{aligned} \rho \hat{v} = & \max_c \delta \alpha \hat{c} + \mathcal{A} \hat{v} + \min_h \left\{ \mathcal{B} \hat{v} h + \frac{1}{2} \frac{1}{\gamma - 1} h^2 \right\} \\ & + \kappa F(y(\hat{v})) E_\psi \left[\min_g g \{y(\hat{v}) - \psi\} + \frac{1}{\gamma - 1} (1 - g + g \log g) |\psi < y(\hat{v})| \right] \end{aligned}$$

where $y(\hat{v}) = \hat{v}_t^* - \hat{v}_t + \hat{d}_t^* - \hat{d}_t$ are the log gains from adjusting and $\kappa F(y(\hat{v})) = \kappa P(\psi \leq y(\hat{v}))$ is the optimal jump intensity at the current state. This incorporates the effects of optimal jump intensities.