Portfolio Choice and Aggregate Durable Consumption

Ken Miyahara

University of Chicago

July 15, 2025

Introduction

- Are aggregate durable consumption responses pro or countercyclical?
- Today: Analytically tractable model to recover primitives of adj. frictions
  - New: Use the empirical distribution of durable adjustment
  - New: Use equilibrium mapping from hazards to distribution of gaps
- Existing approaches (Berger and Vavra, 2014) input gaps, hazards and distribution of gaps without using data on adjustment sizes
  - They use data on assets, house size and adjustment indicators
- Do not use eqbm. one-to-one mapping from hazards to distribution of gaps
- Goal: Estimate primitive adjustment frictions and study responses to aggregate shocks

Model description

- Households derive utility from durable and non-durable consumption

Liquid account

- Liquid income: bonds and risky asset with idiosyncratic risk (private equity)
- Liquid expenditure: non-durable consumption and mortgage payments

Illiquid durable

- Durable: can adjust subject to a transaction cost and (random) utility cost
- Transfers to liquid acct.: sales proceeds in proportion to equity in durable

Sequential Problem

Value Function:

$$V(W,D) = \max_{C(t),\theta(t)} E_0\Big[\int_0^{\tau} e^{-\rho t}\frac{\big(C(t)^{\alpha}D^{1-\alpha}\big)^{1-\gamma}}{1-\gamma}dt$$
$$+ e^{-\rho\tau}\int_0^{\infty}\max_{\hat{D}}\{V(W(\tau),D), V[W(\tau)-fD+\epsilon\big(D-\hat{D}\big),\hat{D}]-\psi D^{1-\gamma}\}dG(\psi)\Big]$$

- No depreciation in D
- Opportunities to adjust at stopping times τ arriving with Poisson intensity κ
- Pay fixed cost f, proceeds in proportion to home equity ϵ
- Pay utility cost $\psi > 0$ drawn from G
- Uncontrolled dynamics:
  $dW = [rW + (r^e - r)\theta W - C - (1-\epsilon)(r+s)D]dt + \theta W\sigma dZ$
- At adjustment dates: $W_{t^+} = W_{t^-} - fD + \epsilon\big(D - \hat{D}\big)$

Sequential Problem: Homothetic formulation

- V is homogeneous of degree $1-\gamma$ in D, i.e. $V(W,D) = D^{1-\gamma} V(W/D, 1)$
- New state variable $w = W/D$ and value function $v(w) \equiv V(w,1)$

$$v(w) = \max_{c(t),\theta(t)} E_0 \Big[\int_0^{\tau} e^{-\rho t} \frac{c(t)^{\alpha(1-\gamma)}}{1-\gamma} dt + e^{-\rho\tau} \int_0^{\infty} \max_{\hat{w}} \{ v(w(\tau)), \Big(\frac{w(\tau) - f + \epsilon}{\hat{w} + \epsilon}\Big)^{1-\gamma} v(\hat{w}) - \psi \} dG(\psi) \Big]$$

- where $c \equiv C/D$
- Uncontrolled dynamics: $dw = [rw + (r^e - r)\theta w - c - (1-\epsilon)(r+s)]dt + \theta w \sigma dZ$
- At adjustment dates: $\hat{w}\hat{D}_t = w_t D_t - fD_t + \epsilon\big(D_t - \hat{D}_t\big)$ or equivalently

$$\hat{D}_t = \frac{w_t - f + \epsilon}{\hat{w} + \epsilon} D_t$$

Recursive Formulation

- Value function satisfies the HJB equation:

$$\rho v(w) = \max_{c,\theta} \frac{c^{\alpha(1-\gamma)}}{1-\gamma} + \mu_w(w,c,\theta)v'(w) + \frac{\sigma_w(w,\theta)^2}{2}v''(w) + H[y(w)]$$

where $\mu_W(w,c,\theta) = [rw + (r^e - r)\theta w - c - (1-\epsilon)(r+s)]$ and $\sigma_W(w,\theta) = \theta\sigma w$

- Benefit of adjustment $y(w)$ is

$$y(w) \equiv (w - f + \epsilon)^{1-\gamma} \max_{\hat{w}} \frac{v(\hat{w})}{(\hat{w}+\epsilon)^{1-\gamma}} - v(w)$$

- Impulse Hamiltonian H and hazard λ_w

$$H(y) = \kappa\left[G(y)\,y - \int_0^y \psi dG(\psi)\right], \qquad \lambda_w(w) = H'(y(w)) = \kappa G(y(w)).$$

- Policy functions: $c(w)$, $\theta(w)$, $\lambda_W(w)$

Aggregation

- The cross-sectional distribution of wealth-to-house ratios m satisfies the KFE

  $$\lambda_w(w)m_w(w) = -\frac{d}{dw}\left(\mu_w[w, c(w), \theta(w)]\, m(w)\right) + \frac{1}{2}\frac{d^2}{dw^2}\left(\sigma_w[w, \theta(w)]m_w(w)\right)$$

  at non re-injection points $w \neq \hat{w}$

- Given policies c, θ, there is a one-to-one mapping between the hazard λ_w and m_w

Mapping model to observables

- Define the size of *desired* log adjustments

$$\Delta d = \log\left(\frac{\hat{D}}{D}\right) \implies w = g(\Delta d) \equiv \exp(\Delta d)(\hat{w} + \epsilon) - \epsilon + f$$

- Define $m \equiv m_w \circ g$ and $\lambda \equiv \lambda_w \circ g$
- Frequency of adjustments

$$N \equiv \int \lambda(\Delta d) m(\Delta d) \mathrm{d}\Delta d,$$

- Distribution of sizes of adjustments

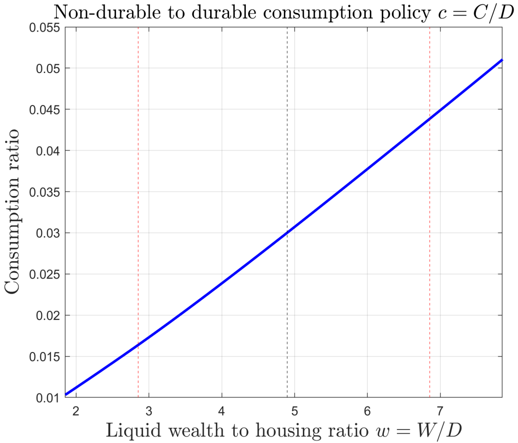
$$q(\Delta d) \equiv \frac{\lambda(\Delta d)\, m(\Delta d)}{N}.$$

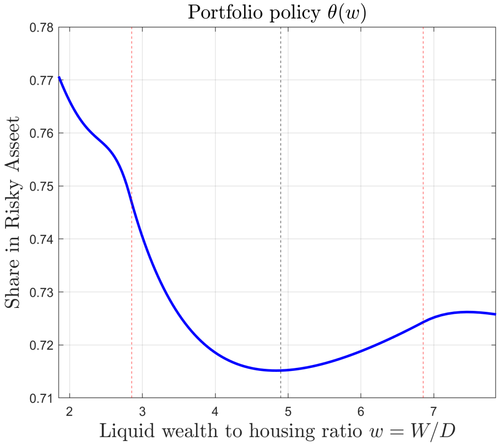
Model calibration

Parameter	Value	Description
ρ	2.5%	Discount rate
γ	3.1	Risk aversion
α	0.8	Share in non-durables
r	1%	Risk-free interest rate
$r^e - r$	6%	Risk premium
σ	16.55%	Idiosyncratic risk
s	2%	Mortgage rate spread
ϵ	20%	Home equity requirement
f	2%	Transaction cost
κ	1 per week	Arrival of adjustment opportunities
Ψ	15000	Upper bound of random utility cost: $G \sim U([0, \Psi])$

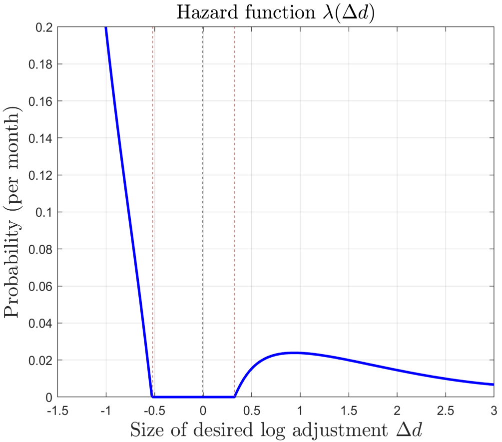
Table: Baseline parameters

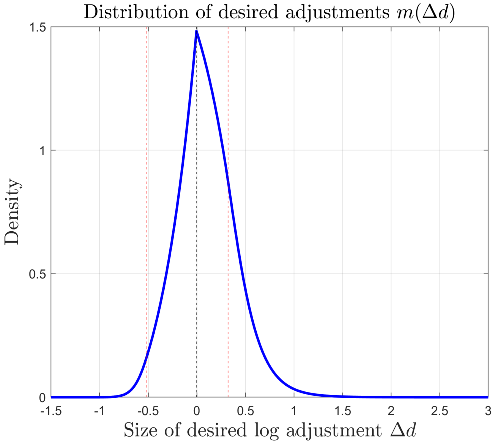
Policy function: c, θ



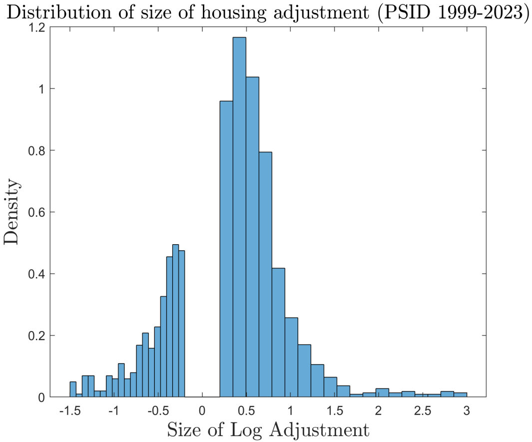


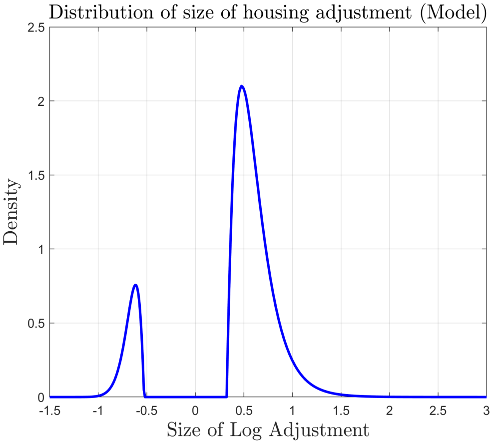
Hazard λ and distribution m





Distribution of adjustments: PSID Data and Model





Identifying κ, G

- Iterate on $\{\psi_i, G_i\}_i, \kappa$
- Given parameters $\{\alpha, \sigma^2, r, r^e, s, \epsilon, f, \gamma\}$, each $\{\psi_i, G_i\}_i, \kappa$ yields a unique λ_w
- λ_w and policies c, θ yield m_w and q (numerically) through the KFE

Next Steps

- Broad idea: Data histograms have 40 bins. Use it to identify a discrete distribution $\{\psi_i, G_i\}$ such that $G(\psi_i) \equiv G_i$
- Think harder if there is another way to have cleaner identification
- Compute the frequency of adjustments in the data
- The frequency of adjustments in the model depends on the μ_w, σ_w processes and the adjustment frictions. There should be an homogeneity property of time-scaling for the system (perhaps, $\{\kappa, \text{returns}, \sigma^2\}$).
- What does $\Psi = 15000$ mean?