

## Case Study # 4: Monopoly and Price Discrimination (75pts total)

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**Fill in Answers below in red.**

**Embed all graphs directly into appropriate question.**

### Pricing Software

You have an idea to develop and patent a new software application. You have the following options:

- Spend \$110 to develop and patent the application, and then sell it
- Do not develop the software and do something else.

**In this case study, you will:** use graphs and equations to analyze pricing strategies.

### Skills needed to complete this case study:

1. Enter data, enter formulas, and create charts in Excel
2. Use basic algebra

### Scenario:

This monopolist has an idea for a new software application. It could spend \$110 to develop and patent the software, and then sell it online. There would be zero variable costs associated with the product. Thus, the firm would have:

- $TFC = \$110$
- $MC = 0$

This monopolist sells to “type 1” and “type 2” customers that have different observable customer characteristics.

- The equation of demand 1 is  $Q = 6 - 0.5P$
- The equation of demand 2 is  $Q = 14 - 0.5P$

### Steps to complete Case Study #4

1. Open the data file for Case 4. The worksheet labeled Demand\_1 provides information about the Demand from Type 1 customers. Demand\_2 provides information about the Demand from Type 2 customers.
2. (44pts) First, assume that the firm can treat each demand separately. This means that they can sell to one group and that group cannot resell to the other group. In this case, we can find the profit-maximizing Q and P for each type and then add up the profit from both to obtain our overall profit.

- a. (1.5pts) Regardless of how demand is treated, the firm faces the same cost structure. So, find the following cost information for this firm:
- Total cost (TC)  
 $TFC = FC$   
 $VC = 0$   
 $TC = 110 + 0$   
 $TC = 110$
  - Average variable cost (AVC)  
 $Average\ variable\ cost\ (AVC) = VC/Q = 0$
  - Average total cost (ATC)  
 $Average\ total\ cost\ (ATC) = TC/Q = 110/Q$
- b. (12.5pts) In the data file for case 3, start with the Demand 1 worksheet. Find the profit-max P & Q and profit for Demand 1:
- (2pts) Find the equation for marginal revenue (MR).  
 $Q = 6 - 0.5P$   
 $0.5P = 6 - Q$   
 $P = 12 - 2Q$   
For MR, we double the slope  
 $MR = 12 - 4Q$
  - (2.5pts) Calculate the profit-maximizing level of output for this firm and find the price they will charge.

$$\begin{aligned}MR &= MC \\12 - 4Q &= 0 \\4Q &= 12 \\Q &= 3\end{aligned}$$

Substituting the value of Q in inverse demand equation

$$\begin{aligned}P &= 12 - 2Q \\P &= 12 - 2*(3) \\P &= 6\end{aligned}$$

- iii. (2pts) What is the mark up? Calculate the Lerner Index (P-MC)/P value for this firm. Do they have a lot of market power? Explain.

$$\begin{aligned}\text{Markup} &= P - MC \\&= 6 - 0 \\&= 6\end{aligned}$$

Hence, Markup = Price

The monopoly markup is the difference between price and marginal cost. We know that in a competitive market, price would be equal to marginal cost.

$$\begin{aligned}\text{Lerner Index} &= (P - MC)/P \\&= (6 - 0)/6 \\&= 1\end{aligned}$$

The Lerner index provides a concise measure of monopoly power. The index ranges from a low value of 0 to a high of 1. The higher the value of the Lerner index, the more the firm is able to charge over its marginal cost, hence the greater its monopoly power. **Since, Lerner index is 1, means very high monopoly power exists.**

- iv. (1pt) Calculate the total revenue that this firm earns from selling to type 1 customers.

$$\text{Total Revenue} = P * Q$$

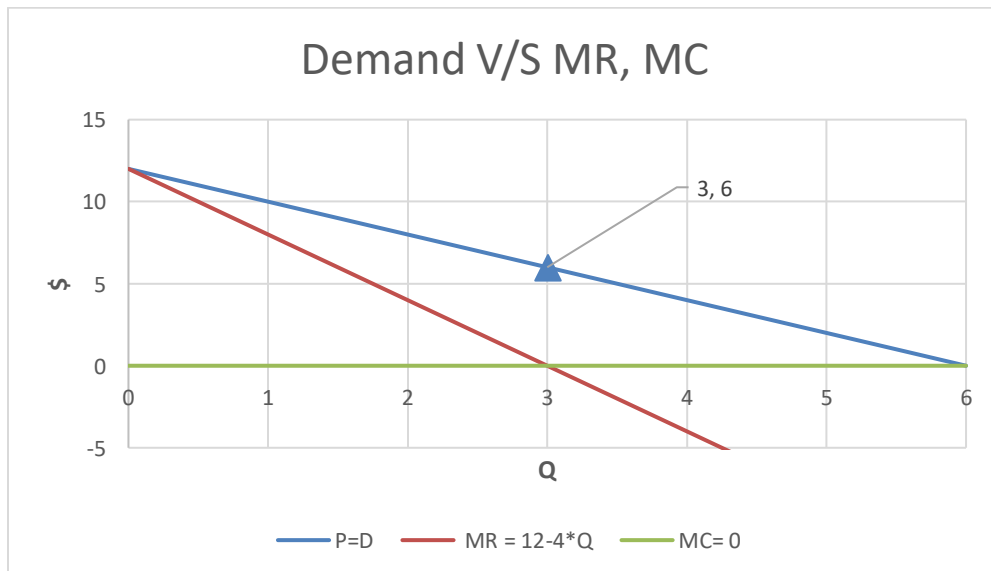
$$\text{Total Revenue} = 6 * 3$$

$$\text{Total Revenue} = 18\$$$

- v. In Excel, in column C, put in the equation for MR
- vi. In column D, put the MC=0.
- vii. (1pt) Graph the demand curve, MR, and MC. Title and label your graph. Set the min on the vertical axis to -5 and the major unit to 5. Does your graph support your calculations above? Explain.

From the graph we see that, our calculations are in line with the graph. MR and MC intersect at  $Q = 3$ . This in accordance with our calculation.

- viii. (4pts) Insert your graph here:



- c. (11pts) Consider what is happening with elasticity and total revenue associated with demand 1.

- i. (0.5pts) What is the formula for elasticity( $E^d$ )?

$$\text{Elasticity} = E^d = \left( \frac{dQ}{dP} \right) * \frac{P}{Q}$$

$$\text{Also } E^d = -1/L = \frac{P}{(MC-P)}$$

Where , L is the learners index.

- ii. (2pts) Calculate the elasticity of demand at the profit-maximizing price.

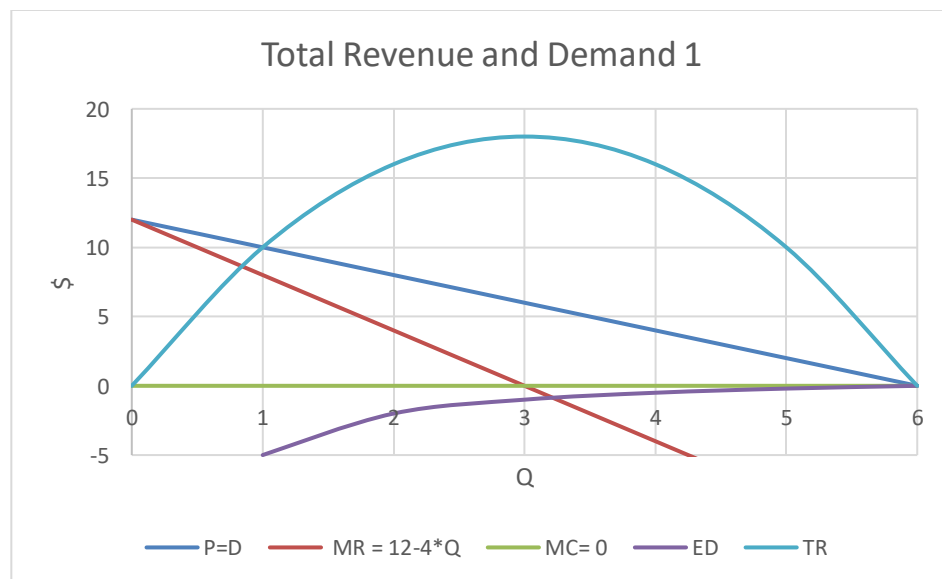
$$P = 6$$

$$Q = 3$$

$$E^d = \frac{P}{(MC-P)} = \frac{6}{(0-6)} = \frac{-6}{(6)}$$

$$E^d = -1$$

- iii. In Excel, in column E, Create a column that computes point elasticity for each price. Recall that  $E^d$  is negative – do not ignore the negative sign in this case.
- iv. In column F, compute the total revenue for each price.
- v. (2.5pts) Graph the demand, MR, MC,  $E^d$ , and TR. Title and label your graph. Set the min on the vertical axis to -5 and lengthen the graph to see it better. Insert your graph here:



- vi. (5pts) What do you notice about the relationship between TR and  $E^d$  and the relationship between MR and  $E^d$ ? What do you notice about the profit-maximizing level of Q and TR? What do you notice about the Lerner Index value calculated above and the  $E^d$ ? Does this make sense given that  $MC=0$ ? Would it be true if  $MC>0$ ? Explain.

**When  $|E_d| > 1$  and with decrease in price TR increases , because the percent change in Q < percent change in P**

**When  $|E_d| < 1$  and with decrease in price TR increases , because the percent change in Q < percent change in P**

**When  $|E_d| = 1$**

**Profit – maximizing level of Q and TR :**

At profit maximizing level of Q , TR is maximum . MC= MR. Here , Q = 3 and total revenue is maximum level at P = 18

**Lerner Index value is 1 and elasticity is -1**

$$L = (-1/E_d) = (-1/-1) = 1$$

**When MC > 0 Lerner Index decreases .**

$$L = (P-MC)/P$$

**MC > 0, L > 1**

The market power will be less compared to the market power when MC = 0

**The ratio P/MC is always greater than one. The higher the P/MC ratio, the more market power the firm possesses. As  $P_{ed}$  increases in magnitude, the P/MC ratio approaches one, and market power approaches zero.**

vii. (1pt) Fill in the following table:

	Quantity at which each event occurs:
Revenue is maximized	<b>3</b>
elasticity = -1	<b>3</b>
Marginal revenue = 0	<b>3</b>
Profit is maximized	<b>3</b>

d. (11pts) In the data file for case 3, click on the Demand 2 worksheet. Find the profit-max P & Q and profit for Demand 2:

i. (2pts) Find the equation for marginal revenue (MR). Show all work.

$$Q = 14 - 0.5P$$

$$0.5P = 14 - Q$$

$$P = 28 - 2Q$$

**For MR , we double the slope**

$$MR = 28 - 4Q$$

ii. (2pts) Calculate the profit-maximizing level of output for this firm and find the price they will charge. Show all work.

$$MR = MC$$

$$28 - 4Q = 0$$

$$4Q = 28$$

$$Q = 7$$

**Substituting the value of Q in inverse demand equation**

$$P = 28 - 2Q$$

$$P = 28 - 2*(7)$$

$$P = 14$$

- iii. (1pt) What is the mark up? Calculate the Lerner Index  $(P-MC)/P$  value for this firm. Do they have a lot of market power? Explain.

$$\text{Lerner Index} = (P-MC)/P = (14-0)/14 = 14/14$$

**Lerner Index,  $L = 1$**  Since,  $MC = 0$  and **Lerner Index = 1**, this means that they have a lot of market power, indicating the presence of monopoly.

**Markup =  $P - MC$**  Since,  $MC = 0$  and, in monopoly  $P = MC$ , We can say that, there is no markup in a monopoly.

- iv. (1pt) Calculate the total revenue that this firm earns from selling to type 2 customers.

$$\text{Total Revenue, } TR = P*Q \quad TR = 14*7 \quad TR = \$98$$

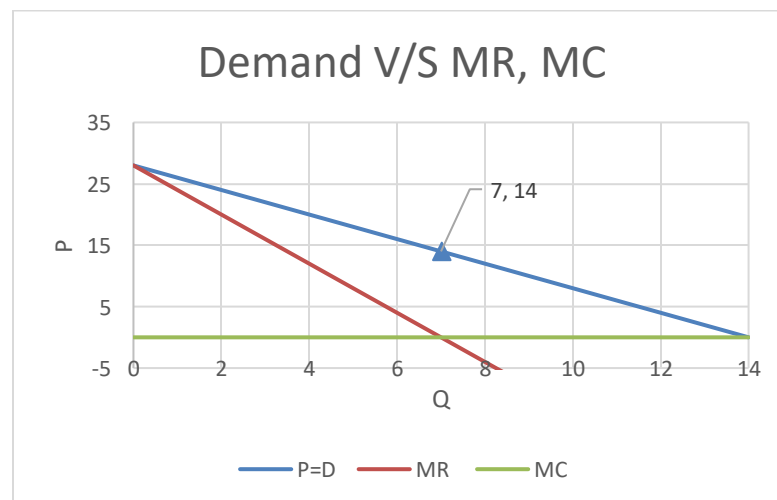
- v. In Excel, in column C, put in the equation for MR

- vi. In column D, put the  $MC=0$ .

- vii. (1pt) Graph the demand curve, MR, and MC. Title and label your graph. Set the min on the vertical axis to -5. Does your graph support your calculations above? **Explain.**

From the graph we see that, our calculations are in line with the graph. MR and MC intersect at  $Q = 7$ . This in accordance with our calculation.

- viii. (4pts) Insert your graph here:



- e. (5.5pts) Consider what is happening with elasticity and total revenue associated with demand 2.

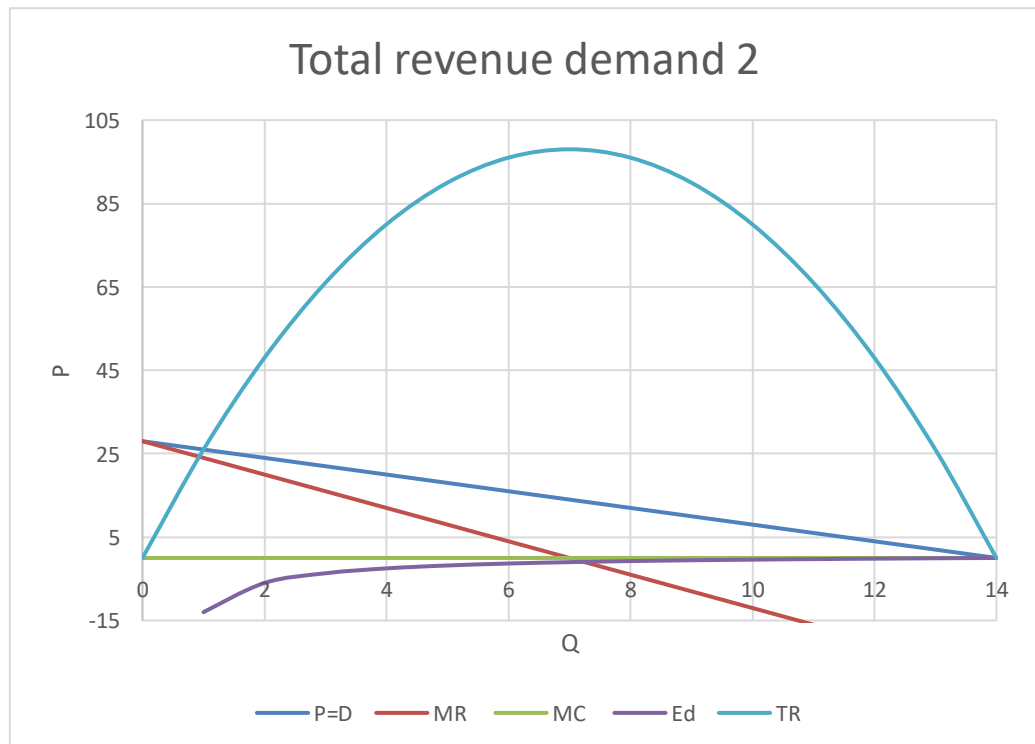
- i. (2pts) Calculate the elasticity of demand at the profit-maximizing price.

$$\text{Elasticity (E}_D\text{)} = \frac{\Delta Q}{\Delta P} * \frac{P}{Q}$$

Also,  $E_D = -1/L$  (Lerner's index)

$$\begin{aligned} \text{At } P &= 14, \\ E_D &= 14/(0-14) \\ &= -1 \end{aligned}$$

- ii. In Excel, in column E, Create a column that computes point elasticity for each price. Recall that  $E^d$  is negative – do not ignore the negative sign in this case.
- iii. In column F, compute the total revenue for each price.
- iv. (2.5pts) Graph the demand, MR, MC,  $E^d$ , and TR. Title and label your graph. Set the min on the vertical axis to -15 and the max on the horizontal axis to 14 and lengthen the graph to see it better. Insert your graph here:



- v. Notice the same relationships that were found in 2 d vi, between TR and  $E^d$  and MR and  $E^d$  and the profit-maximizing level of Q and TR and the Lerner Index value calculated above and the  $E^d$ .

- vi. (1pt) Fill in the following table:

	Quantity at which each event occurs:
Revenue is maximized	7
elasticity = -1	7
Marginal revenue = 0	7
Profit is maximized	7

- f. (0.5pts) Add up the total revenue earned from both markets.

$$\text{TR(TOTAL)} = \text{TR(Type 1)} + \text{TR (Type 2)}$$

$$\text{TR} = 18 + 98$$

$$\text{TR} = 116$$

- g. (1pt) Calculate profit using the total revenue calculated in part f.

$$\text{Profit} = \text{TR} - \text{TC}$$

$$\text{Profit} = 116 - 110$$

$$\text{Profit} = 6$$

- h. (1pt) Will the firm spend the \$110 to develop and patent the software? Explain.

The total revenue is greater than the total cost i.e. the profit for firms is positive. Therefore, the firm will **spend \$110** to develop and patent the software.

3. (13.5pts) Now suppose that the buyers in the high-price market find a way to purchase the software in the lower price market. The firm can no longer separate the two markets. It must therefore charge a single price to all buyers. You need to combine the 2 demand curves. Think about this for a minute. You need the horizontal summation of these two demand curves. That means you need to add up the quantity demanded for each price. Look at your 2 demand curves. For which prices are D1 and D2 combined and for which prices is D2 the only relevant demand? In Excel, click on the “Combined D” worksheet.

- In the column marked Q1, cut and paste the quantities from “demand 1” into the spaces that match up with the relevant prices.
- In the column marked Q2, cut and paste the quantities from “demand 2” into the spaces that match up with the relevant prices.
- In the column marked Qcomb, add Q1 and Q2.
- For the prices where D2 is the only relevant demand, the MR is the same as in 2d(i) above. Thus, in the column marked MR2, cut and paste the MR from demand 2 into the spaces that match up with the relevant prices. Use the drop



down menu on the “paste” button and click on “paste values” (this will eliminate the formulas behind the values).

- e. For the prices where D1 and D2 are combined, the MR is different. You need the inverse demand equation for the combined portion to find the new MR.

- i. (2pts) Calculate the combined demand equation. Take the demand equations listed in the “scenario” at the top of the case and simply add them together. This gives you  $Q_{\text{comb}}$  as a function of  $P$ . Show your work.

Demand 1:  $Q_1 = 6 - 0.5P$

Demand 2: is  $Q_2 = 14 - 0.5P$

Combined Demand:  $Q = Q_1 + Q_2$

$Q_{\text{comb}} = 6 - 0.5P + 14 - 0.5P$

$Q_{\text{comb}} = 20 - P$

**Combined Demand Equation:  $Q_{\text{comb}} = 20 - P$**

- ii. (1pt) Now, solve for  $P$  as a function of  $Q$ .

$Q_{\text{comb}} = 20 - P$

$20 - P = Q_{\text{comb}}$

**$P = 20 - Q$**

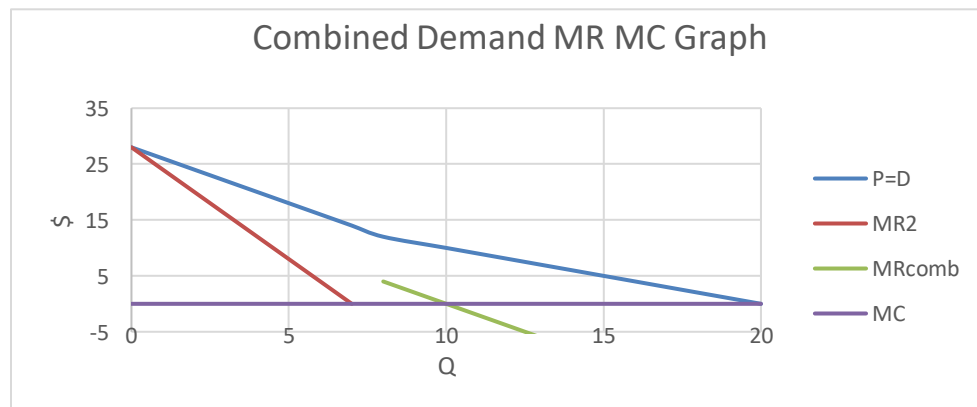
- iii. (0.5pt) What is the MR for this combined demand function?

**To find MR, double the slope  $MR = 20 - 2Q$**

- iv. In Excel, insert the MR equation into the column marked MRcomb. Only put the values in the spaces that match up with the relevant prices (your MR should be discontinuous).

- f. In column G, put in the MC

- g. (6pts) Graph the demand ( $Q_{\text{comb}}$  and  $P$ ), both MR curves, and MC. Title and label your graph. Set the min on the vertical axis to -5, set the max on the horizontal axis to 20, add major grid lines to the horizontal axis, and lengthen the graph to see it better. Insert your graph here:



- h. (3pts) Looking at the graph above, determine the firm's profit-maximizing level of output and the price will they charge. Notice that there are 2 places where  $MR=MC$ . Which one did you choose and why? [Hint: Compare the profit for each choice.]

From the graph, we see that  $MR$  and  $MC$  intersect at  $Q = 7$  and  $Q = 10$ .

At  $Q = 7$ ,  $P = 14$

$TR = \$98$

$\text{Profit} = TR - TC = 98 - 110 = -\$12$

At  $Q = 10$ ,  $P = 10$

$TR = \$100$

$\text{Profit} = 100 - 110 = -\$10$

From the above calculation, we see that profit value for  $Q=10, P=10$  is more than that at  $Q=7, P=14$ . Hence, the profit maximizing level of output and price will be 10,10. We choose this because the loss while taking  $Q=7$  is more than that at  $Q=10$ .

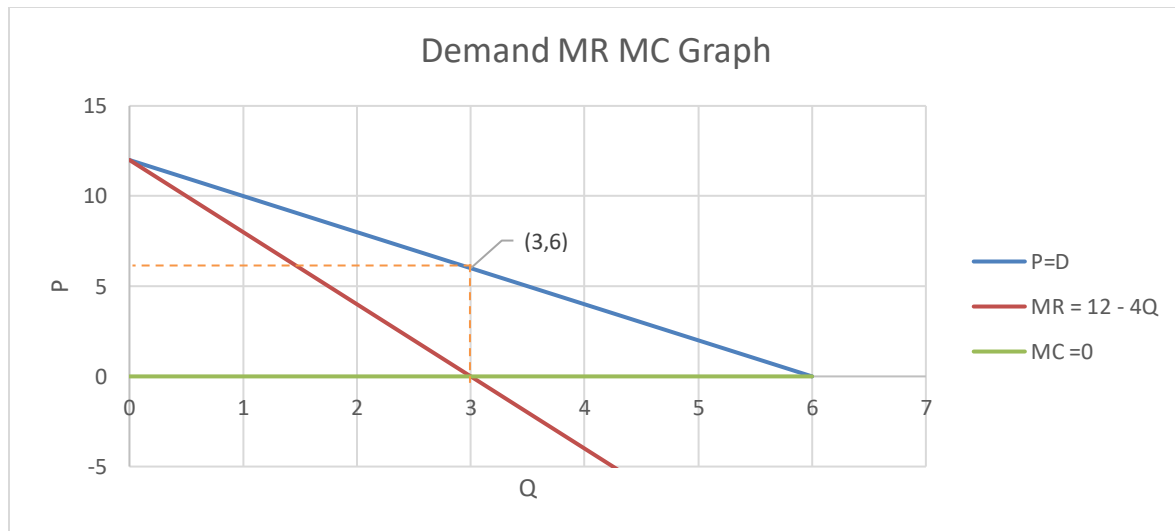
- i. (1pt) Given the profit you calculated in part 3(h) above, will the firm spend the \$110 to develop and patent this application? Explain.

The firm shouldn't spend \$110 as it will end up making loss of 10 dollars.

4. (1pt) Summarize your results in the table below.

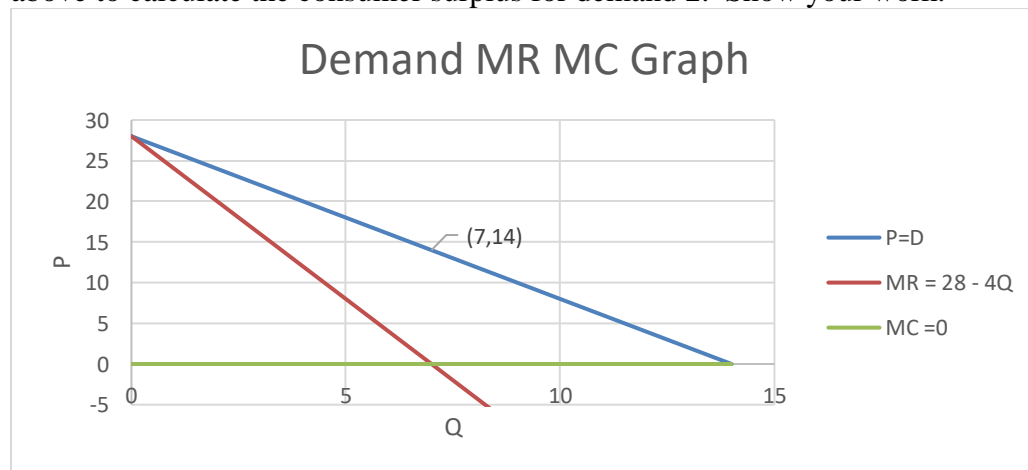
	Separate markets			Single market
	D1	D2	Total	Dcomb
Optimal P	6	14	-----	10
Quantity sold	3	7	10	10
Total Revenue	18	98	116	100
Will the firm develop/patent app?	Yes			No

5. (16.5pts) Consider the welfare effects from price discrimination.
- a. (2pts) Given that the firm price discriminates, use the information found in part 2 above to calculate the consumer surplus for demand 1. Show your work.



Consumer surplus = Maximum willing to pay price – actual price  
 Consumer surplus =  $(12-6)(3-0)/2$   
 **$CS_1 = 9$**

- b. (2pts) Given that the firm price discriminates, use the information found in part 2 above to calculate the consumer surplus for demand 2. Show your work.



- c. (0.5pts) Add the results from a and b to find the total consumer surplus with price discrimination.

Consumer surplus = Maximum willing to pay price – actual price  
 $= (28 - 14)(7-0)/2$   
 **$CS_2 = 49$**

- d. Now consider the combined demand.
- (2pts) Use the “optimal P” for the single market and plug it into demand 1 to find the number of units purchased by type 1 consumers. Use this information to calculate the consumer surplus for type 1 consumers with combined demand.

Optimal P for single market = 10

Demand 1:  $Q = 6 - 0.5P$

$$Q = 6 - (0.5)(10)$$

$$= 6 - 5$$

$$= 1$$

$$\text{Consumer Surplus} = (12-10)*(1-0)/2$$

$$= 1$$

- ii. (2pts) Plug this “optimal P” into demand 2 to find the number of units purchased by type 2 consumers. Use this information to calculate the consumer surplus for type 2 consumers with combined demand.

Optimal P for single market = 10

Demand 2:  $Q = 14 - 0.5P$

$$Q = 14 - (0.5)(10)$$

$$= 14 - 5$$

$$= 9$$

$$\text{Consumer Surplus} = (28-10)*(9-0)/2$$

$$= 81$$

- iii. (0.5pts) Add the results from i and ii to find the total consumer surplus without price discrimination

$$\text{Consumer surplus without price discrimination} = 1 + 81$$

$$= 82$$

- e. (1pt) Summarize your results in the table below.

	Price Discrimination		No Price Discrimination	
	Type 1	Type 2	Type 1	Type 2
Q consumed	3	7	1	9
P paid	6	14	10	10
Cons Surplus	9	49	1	81
Total CS	58		82	

- f. Compare the results:

- i. (1pt) How do type 1 consumers fare with price discrimination? Explain.

For type 1 customers the consumer surplus is more with price discrimination. Because, with the price discrimination, they pay 6 \$. Hence, type 1 customers get additional quantity for lesser cost and hence fare better.

- ii. (1pt) How do type 2 consumers fare with price discrimination? Explain.

For type 2 customers the consumer surplus is less with price discrimination. Because, with the price discrimination, they pay 14\$. Hence, type 2 pay more to get an addition quantity and hence doesn't fare better.

- iii. (1pt) Overall how does consumer surplus change with price discrimination? Explain. Calculate the change.

Overall consumer surplus with price discrimination changes without price discrimination. We note that for type 1 and type 2 customers ,  
**Change in Consumer Surplus = (Total Consumer Surplus)<sub>price discrimination</sub> - (Total Consumer Surplus)<sub>without price discrimination</sub>**

Change is  $58-82 = -24$

- g. (1.5pts) How much does the total profit increase with price discrimination?

$$\begin{aligned}\text{Total revenue with price discrimination} &= (6*3) + (7*14) \\ &= 116\end{aligned}$$

$$\begin{aligned}\text{Profit with discrimination} &= 116 - 110 \\ &= 6\end{aligned}$$

$$\begin{aligned}\text{Total revenue without price discrimination} &= (1*10) + (9*10) \\ &= 100\end{aligned}$$

$$\begin{aligned}\text{Profit with price discrimination} &= 100 - 110 \\ &= -10\end{aligned}$$

$$\begin{aligned}\text{Increase with price discrimination} &= 6 - 0 \\ &= 6\end{aligned}$$

Hence , Total profit increase with Price discrimination is **6**

- h. (2pts) Using the information from f and g, what is the effect on aggregate surplus? Explain and calculate the change.

In the above questions , we calculated the change in total consumer surplus with and without price discrimination to be 24 and gain = **\$16**

**Change in Consumer Surplus = (Total Consumer Surplus)<sub>price discrimination</sub> - (Total Consumer Surplus)<sub>without price discrimination</sub>**

Change is  $58-82 = -24$

The Consumers are worse off by 24\$ with price discrimination

$$\begin{aligned}\text{Aggregate surplus} &= -24 + 6 \\ &= -18 \$\end{aligned}$$

Thus , there is a loss of 18 \$ , because of price discrimination