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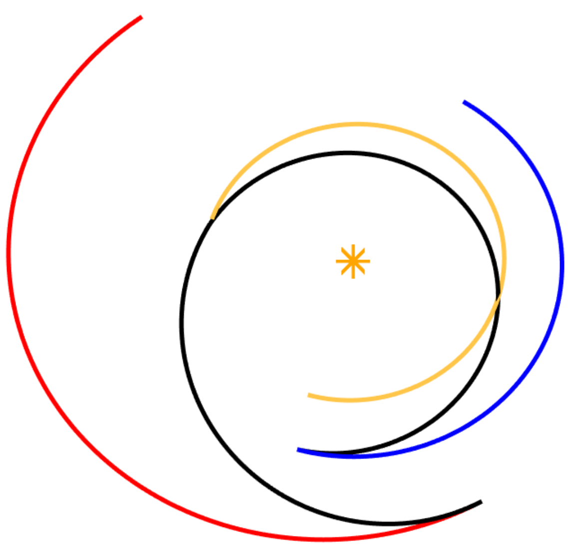
Aer E 351

Interplanetary Mission Design Report

November 2022



Introduction

In our project we were tasked with designing various orbital paths to carry a designated scientific payload from an Earth parking orbit to an orbit around Mars. To carry out this task I was provided the date of departure from Earth orbit and arrival to Mars orbit. I also knew the initial parking orbit around Earth and the final capture orbit around Mars, both being at a radius of four times their respective planets' radius. This information is enough to solve many different orbital elements for transferring the spacecraft between the two planets.

We were given multiple segments to complete for this project, each a variation on the path to go from Earth to Mars. The first part of this project was a two-impulse maneuver from Earth to Mars directly. The second part featured a three-impulse maneuver with one impulse being applied at the location of Venus before transferring to Mars. The third and final part is a two-impulse flyby of Venus. Each of these segments will be discussed in more detail in their respective sections.

In the problems, we were provided the dates and times of the departure from Earth and arrival at Mars giving us the ability to use the techniques of the Lambert problem, and its respective solution involving the Lambert algorithm. This algorithm proved to be vitally important to the calculations for determining the changes in velocities needed to complete this transfer orbit. I then could use various equations known from the characteristics of orbits such as the energy equation, and various techniques for analyzing flybys and rendezvous to determine the required amount of change in velocity for the spacecraft undergoing these orbital maneuvers.

Part 1 - Two Impulse Maneuver

The first step to make any of these calculations possible was to determine the position and velocity of the planets at the given times and dates. To accomplish this I used the '*planetEphemeris(date, center, target, model, units)*' function in the MATLAB Aerospace toolbox. This function allowed for the determination of the relative positions of many stellar bodies in relation to other bodies (in this case the sun). This function provided us with both the relative positions of the planets in three dimensions as well as the velocities of the planets at the given time and dates. The planetEphemeris function was implemented in part one to obtain the position and velocity vectors for both Earth and Mars at the given departure and arrival dates.

With the knowledge of the positions of the planets and the dates for departure and arrival, I used the aforementioned Lambert algorithm to solve for the necessary departure velocity and the velocity at arrival. The Lambert solver I used was found on GitHub but robustly solves extended-length Lambert problems, solving for both long and short paths between the points which will be important in project part 2.

The length of the mission was determined by simply finding the time between the departure from Earth and the arrival at Mars, which in this case is about 343.48 days. By then running the Lambert Solver using the position vector of Earth at departure and the position vector of Mars at arrival, I obtained the two velocity vectors for departure and arrival.

```
v1D =

    3.5516  -28.2202  -12.8019


v2A =

   -1.4036   19.9906    9.0159
```

Fig. 1. Departure from Earth and arrival at Mars velocity vectors (km/s)

It is important to note that these velocities are the velocities in the heliocentric reference frame, not in the reference frame of Earth or Mars, meaning Iwill have to find relative velocities from these heliocentric velocity vectors. To solve for the relative velocities to Earth/Mars, one must subtract the heliocentric planet velocity relative to the sun from the calculated departure and arrival velocities to obtain the relative velocities to the planets. This velocity relative to the planets is the V_∞ for the departure/arrival hyperbola when looking at the system around each of the planets.

Now that the velocities required at departure and at arrival around each planet are known, one is able to calculate the required change in the magnitude of the velocity for the departure impulse and the arrival impulse. To do this I calculated the speed on the initial parking orbit around Earth and the final capture orbit around Mars, utilizing the following calculations:

$$v_c = \sqrt{\frac{398600}{4*R_E}} = 3.953\ km/s$$

$$v_{c,M} = \sqrt{\frac{42828}{4*R_M}} = 1.777\ km/s$$

The last piece of information needed to calculate to determine the total ΔV was the vehicle velocity at perigee on the outgoing Earth and incoming Mars hyperbolic orbits. This was solved by using the energy equation for hyperbolic orbits as follows:

$$V_p = \sqrt{{V_\infty}^2 + \frac{2\mu}{r}} \qquad \text{(Eq. 1)}$$

When substituting in the proper values for the gravitational parameter for Earth and Mars, along with the condition r = 4 * planet radius, which is the given initial and final orbital distance, I obtained the velocities at perigee and therefore allowed for the calculation of the total change in velocity. To do this I calculated the absolute value of the difference between the parking orbit speed and the velocity at perigee on the planetary hyperbola.

```
--------Part 1--------
Magnitude of delta V1: 30.2527km/s
Magnitude of delta V2: 24.1481km/s
Velcity upon entering Mars SOI: 25.8032km/s


Part 1 total dV: 54.4008 km/s
```

Fig. 2. Values of ΔV for Part 1

The final portion of this part called for the plotting of the heliocentric orbit between Earth and Mars. To accomplish this, I used the ode45 function used at the beginning of class in tandem with the orbit equation to systematically plot in the heliocentric frame of whichever orbits are desired. An ode45 function call was performed for each orbit to be plotted. Part1 includes the orbit of Earth, Mars, and the spacecraft's orbit between them. This graphic is shown below.

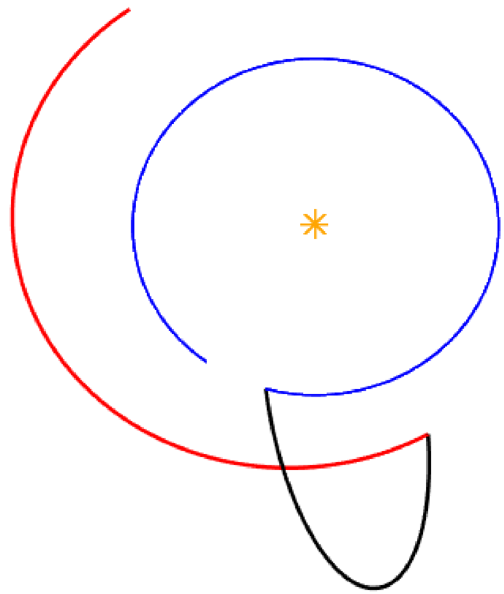


Fig. 3. Orbit between Earth and Mars through the designated dates and times. Blue orbit is that of Earth, red is Mars, and black is the spacecraft transfer orbit.

Many of the processes used throughout this part are identical when repeated in the next project parts such as initializing the Lambert algorithm, solving for perigee velocity within the planetary system, and converting between the departure and arrival vectors to the V_∞ vectors. The process remained exactly the same as described in part 1 for any future calculations.

Part 2 - Three Impulse Maneuver

This second way to go about getting to Mars is to complete an impulsive maneuver at the position of Venus at some point midway through the orbit. This involves first departing Earth and heading towards Venus, and once at Venus applying an impulse to head to Mars. This, in theory, would save energy since the departures and arrivals are closer to the planet's natural orbital directions meaning less fuel must be expended to change the velocity of the spacecraft.

The first step in solving this problem was to obtain the location of Earth at the departure date, Venus at the arrival date, and Mars at its arrival date. This was done once again by using the MATLAB function planetephemeris(), where I could obtain both the position and velocity vectors for all the necessary planets in the heliocentric frame.

```
posE =

   1.0e+08 *

   -0.4007   -1.3432   -0.5823

>> velE

velE =

   28.2334   -7.3193   -3.1725
```

Fig. 4. Earth position vector (km) and velocity vector (km/s) calculated at the departure date in the heliocentric frame. The same vectors were calculated for Venus and Mars.

The next step in calculating this three-impulse maneuver was to design an orbit that goes from Earth to Venus, arriving and departing on the designated dates. To accomplish this I used the Lambert algorithm again, as we have both the position vector of Earth and Venus upon arrival and the length of the mission between Earth and Venus. Given this information, the Lambert algorithm could be run and the necessary departure and arrival velocities would be given from this algorithm in the heliocentric frame. Important to note is that I wanted the orbit to Venus to go the "long way" since this is the direction the Earth is traveling and to go the "short way" would require a significantly larger increase in the ΔV since we would need to cancel the momentum around the sun that is naturally possessed at Earth. I ran the simulation using the short way between Earth and Venus and this resulted in a 10x increase in the necessary ΔV. This alternate version is shown in the appendix.

With the departure and arrival velocities, I once again obtained the V_∞ by subtracting the planet velocity vector, obtained from planetephemeris, from the departure/arrival vectors calculated from the Lambert algorithm. This V_∞ was then used to calculate the speed at the perigee using the energy equation (Eq. 1). The ΔV is then the velocity on hyperbolic orbit in the frame of the planet minus the velocity on the parking/capture orbit. This is the same process described in part 1 to calculate the ΔV.

The Lambert algorithm was run once again to calculate the velocity required to go from Venus at the date to Mars at the date. This process was exactly the same as the Earth to Venus calculations, or the Earth to Mars calculations from part 1. The ΔV for the Mars capture can then be calculated by the same process using V_∞ and the energy equation to find the required change in velocity.

What was remaining for the total ΔV calculations is the change in velocity required at the planet Venus. This was calculated by subtracting the Venus arrival velocity vector from the required departure velocity vector to go from Venus to Mars. Since no flyby is occurring, the difference between these two vectors is the necessary ΔV required at Venus since this would change the orbital trajectory to go from Venus to Mars. This calculation was run and the ΔV's were obtained for each of the three burns. These final results are shown below.

Magnitude of delta V1 (Earth departure): 3.3103 km/s
Part 2 dV at Venus: 3.9396 km/s
Magnitude of delta V4 (Mars capture): 4.8648 km/s

By summing all the required ΔV's for the three burns that occur during this maneuver, I obtained the total ΔV required for the entire trip, as shown below

Part 2 total ΔV: 12.1148 km/s

Finally, I plotted the heliocentric orbit for this trajectory from Earth to Venus to Mars.

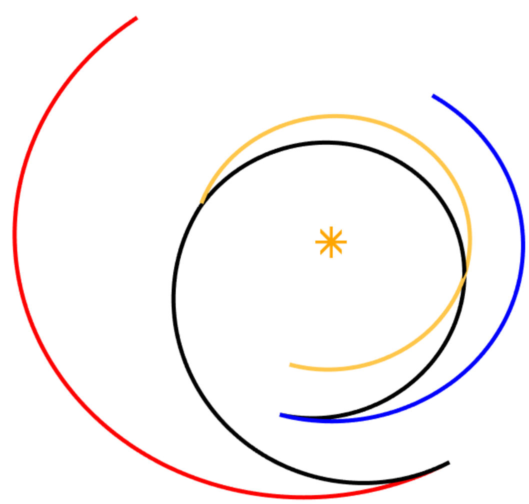


Fig. 5. Trajectory departing from Earth (blue) to Venus (yellow) to Mars (red)

One important note for this section is that the total ΔV for part 2 is significantly less than the required ΔV for part 1, even though the path heads to an internal planet before departing for Mars. This is consistent with what I expected to happen in theory, as the spacecraft is arriving at the planets at a much closer angle to the planet's natural velocity around the sun, meaning that less energy is required to change the speed of the spacecraft when arriving or departing from the planets.

Part 3 - Two impulse Flyby

The final orbit that was calculated is a two-impulse flyby of Venus, meaning that instead of applying an impulse at the location of Venus, the spacecraft would instead perform a flyby of the planet to gain the necessary change in velocity needed to travel from Venus to Mars.

For this part it was crucial to know the data I had when arriving at Venus, specifically the speed and direction of V_∞. This vector was known, as I knew the heliocentric arrival velocity of the spacecraft as well as the heliocentric velocity of Venus. The difference between these two vectors gave us V_∞. This same process was repeated to find the V_∞ departing as well. Since I knew the arrival and departure velocities relative to the planet, I easily found the ΔV required from the flyby.

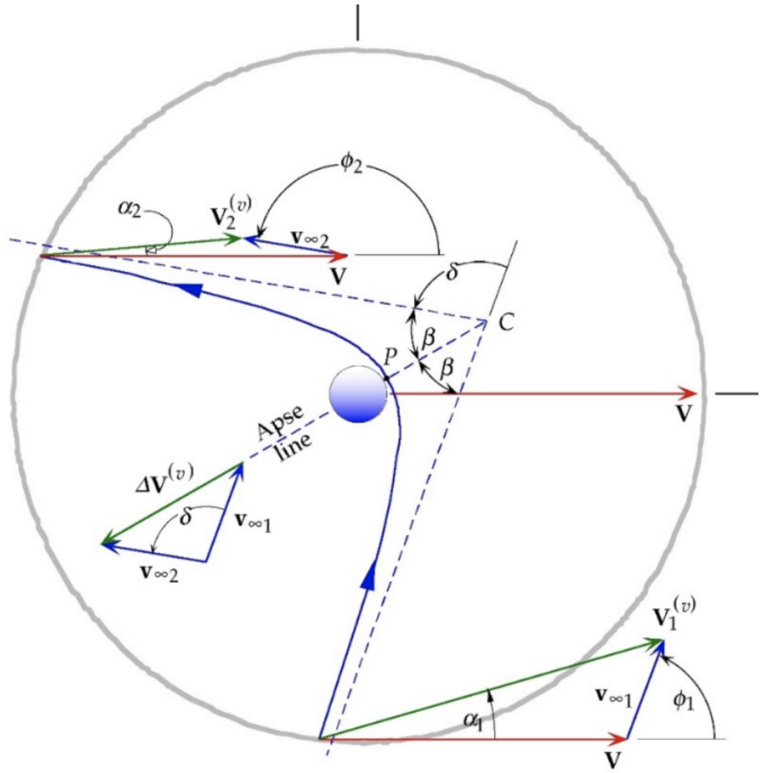


Fig. 6 Shows the various geometry associated with a planetary flyby. Obtained from Lecture 25, Slide 6

We can see from figure 6 that the total change in ΔV from the flyby is very easy to calculate. It is simply the difference between the V_∞ for arrival and V_∞ for departure. Since both of these vectors are known, we can take the difference to obtain the change in velocity brought by the flyby.

Part 3 ΔV from flyby: 3.9396 km/s

One can see that the ΔV from the flyby is exactly the same as the ΔV required from part 2 to complete the maneuver from Venus to Mars. This makes sense because ultimately we will end up on the same orbit, whether we do the flyby or just apply an impulse at the equivalent position. We can then calculate the total mission ΔV using the flyby.

Part 3 total mission dV: 8.1752 km/s

With the ΔV from the flyby known as well as both V_∞'s, any orbital parameters about the flyby orbit can be determined. I started by solving for the δ angle from figure 6. This angle is just the angle between the incoming and outgoing V_∞ vectors, meaning I can use the following formula to calculate the angle between these two vectors:

$$cos(\delta) = \frac{a \cdot b}{|a||b|}$$

By letting a and b be the V_∞ vectors and solving for delta we can obtain that the angle δ is equal to 23.2666 degrees.

Next, I solved for the eccentricity of the hyperbolic flyby trajectory by using the following formula:

$$e = 1/sin(\delta/2)$$

Substituting δ=23.2666 into this formula we can obtain the eccentricity: e = 4.9592.

Finally, and arguably most importantly the perigee radius of the flyby can be determined by using this formula, where eccentricity is known, v_∞ is known, and μ of Venus is known, therefore one can solve for the perigee radius:

$$e = 1 + \frac{r_p v_\infty^2}{\mu}$$

Solving for this perigee radius it can be ascertained that:

Perigee radius of Venus flyby: 13761.4148 km

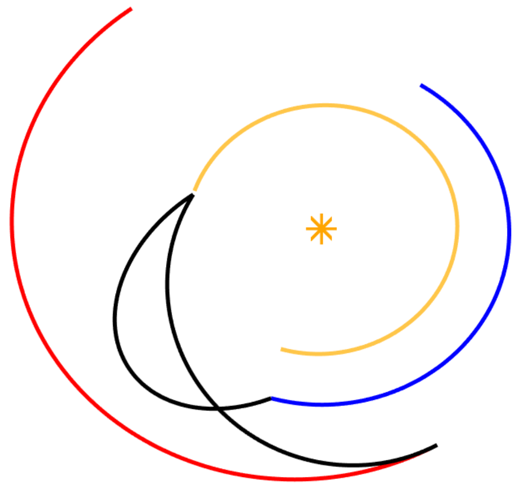
The important fact to check here is ensuring that this radius is not smaller than the radius of the planet itself. The radius of Venus is known to be roughly 6,050 km, meaning the closest approach is roughly twice the radius of Venus itself, making it perfectly safe to perform as the probe will not crash into Venus during the flyby. Also, this calculated perigee radius is very close to the recommended distance for perigee radius given in lecture, that being twice the radius of the planet.

Conclusion

When looking at all the different ways to design an orbit from Earth to Mars with the conditions of a set departure and arrival date, it is clear that there is a large difference in the total fuel expenditure required. Even though all the missions depart and arrive on the same date, they use vastly different amounts of fuel to complete their missions. The direct path from Earth to Mars has a ΔV of 54 km/s, the three-impulse maneuver has $\Delta V = 12.1$ km/s, while the final mission with the flyby has a ΔV of only 8.1 km/s.

By taking the spacecraft on the flyby trajectory instead of the direct path from Earth to Mars, there is an over 6 times reduction in required fuel usage, which is quite substantial. By carefully evaluating the various trajectories a spacecraft could take to arrive at a given destination, optimizations can be performed to significantly reduce the required fuel expenditure for mission completion.

Appendix



Magnitude of delta V1 (Earth): 48.2027km/s

Part 2 dV at Venus: 75.018 km/s

Magnitude of delta V4 (Mars capture): 4.8648km/s

Part 2 total ΔV: 128.0855 km/s

Fig. 7. Shows the orbit and ΔV used if the spacecraft was to take the “short path”

We can see that the short path travels a lesser distance but uses 10x times more fuel. This demonstrates the importance of maintaining the velocity given by the planets, as it far reduces the required change in velocity, as we must cancel the velocity of Earth/Venus since the spacecraft would be heading “backward” relative to Earth after departure and have a complete change in direction when arriving at Venus.

MATLAB code (GitHub repository):

https://github.com/jkolker02/Interplanetary-Mission-Orbital-Design