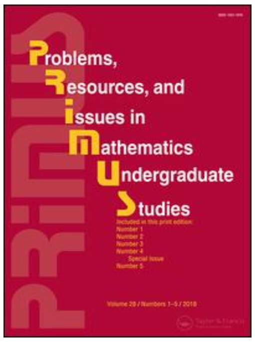

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Team-Based Inquiry Learning

Drew Lewis, Steven Clontz, and Julie Estis

Abstract: Team-Based Learning (TBL) is a cooperative learning strategy blending elements of flipped learning, inquiry-based learning, and problem-based learning. Although used quite frequently in other disciplines, use of this strategy in mathematics has been limited. In this article, we describe how TBL can be implemented in math courses with adherence to essential elements of TBL and introduce modifications specific to mathematics instruction. In particular, we introduce a particular style of TBL, which we term Team-Based Inquiry Learning, that satisfies the defining pillars of inquiry-based learning.

Keywords: Team-based learning, inquiry-based learning

1. INTRODUCTION

Decades of literature supports the need to transition mathematics instruction to more active forms of teaching and learning [10]. However, the (context-dependent) question of how to do so optimally is still very much unanswered. In this article, we propose Team-Based Learning (TBL) as an active, collaborative instructional method for mathematics, and describe the success we have experienced in our current instructional context.

Evolving from earlier forms of collaborative and active learning, TBL is a distinctive, highly structured approach to small-group instruction that supports application of course content, requires team problem-solving, and leads to significant learning [22–24]. Students acquire conceptual and procedural knowledge as they complete independent preparation work, and then spend the majority of class time solving problems in strategically organized, permanent teams. Although TBL is increasingly used in other

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disciplines, particularly the health sciences, implementation in math courses is limited.

TBL has been shown to improve student achievement by encouraging more scientific thinking, developing a deeper understanding of course content, and increasing student reasoning, problem-solving and critical thinking skills [5, 13, 14, 21, 36–38]. A meta-analysis of TBL effectiveness [12] showed several trends across studies. Teams consistently performed better than individuals, communication was improved, and class participation was improved with TBL. Although student enjoyment was reportedly lower for TBL, student perceptions of self-efficacy and interest were higher. Two studies showed successful transfer of interpersonal skills gained in TBL to job performance. One key finding across multiple studies was that students tended to benefit from TBL, regardless of achievement or demographic profiles; however, students at the lower end of class achievement showed the greatest benefit. A more recent meta-analysis [20] showed that TBL improves student learning gains by an average of 0.5 standard deviations, compared with the other methods explored across studies.

TBL has been shown to be an effective pedagogy in several STEM disciplines in particular. For example, Carmichael [2] reported that TBL increased exam scores in an introductory biology course. Kreie et al. [15] showed that TBL increased retention in an introductory information systems course, and Dinan and Frydrychowski [4] found increased final exam scores in a chemistry implementation.

However, the literature on TBL in mathematics is quite limited. We found only one article discussing student learning gains: Nanes [27] described using a modified TBL methodology in linear algebra. He found this approach to improve final exam scores and course grades versus a historical control. More recently, Peters et al. [30] used a similar modified methodology in calculus. A more faithful TBL implementation in upper-level courses such as combinatorics is discussed in [28, 29].

In this paper, we provide an introduction to TBL, including its relation to other forms of active learning. In our view, TBL (as applied to math education) is a structure that facilitates instructors introducing collaborative inquiry into their classrooms; in Section 3, we describe our particular implementation, which we term *Team-Based Inquiry Learning*, and describe how it satisfies the four pillars defining inquiry-based learning. In Section 4, we describe the changes we saw take place as a result of implementing TBL.

2. TBL

In this section, we briefly describe TBL; we refer the reader to [25, 33] for further details.

2.1. General Aspects of TBL

TBL is designed to engage prepared teams of students in working together to apply course content. Differing from other collaborative learning approaches, TBL is a comprehensive instructional strategy with a specific sequence of learning activities rather than independent group activities within a course. A TBL course is comprised of modules for each content area. Each TBL instructional module begins with individual *preparation*. Students prepare for a content module by reading, viewing videos, practicing problems, and/or completing other instructor-assigned preparation activities. The *Readiness Assurance Process*, which includes an individual test, a team test, appeals, and corrective instruction, is conducted during the first class of a module. Through *preparation* and the *Readiness Assurance Process*, students become prepared to answer deeper questions and solve problems collaboratively during in-class *Application Activities*. These activities are rich tasks in which students work in permanent teams, using knowledge and skills from the preparation phase to solve problems, create explanations, or make predictions that become increasingly difficult as they expand their knowledge of the content.

TBL consists of four essential elements [26]. First, teams must be properly formed and managed. Typically, this is accomplished through the use of permanent, diverse teams so that student resources are distributed across teams. When permanent teams are formed based on criteria, rather than self-selection, barriers to team cohesion are reduced, and teams develop skills from working together over time. Second, students need to be accountable for their individual preparation and for contributing to their teams. Individual *Readiness Assurance Tests* provide accountability for preparation, and peer evaluation is used to assess individual contributions to the team. Third, students must receive frequent and immediate feedback. Through the team Readiness Assurance Test, students receive immediate feedback on the accuracy of their responses. Team activities are also structured in a manner that provides frequent and immediate feedback. Finally, teams spend the bulk of class time working collaboratively on *Application Activities*, which are designed in a specific manner summarized by the 4-Ss: (i) activities involve a **S**ignificant problem that is meaningful and relevant to students, (ii) all teams work on the **S**ame problem, (iii) students solve the problem by making a **S**pecific choice, and (iv) teams **S**imultaneously report their choices. Creating application activities within this structure and completing the activities in class promotes active student engagement, eliminates a “divide and conquer” approach, and promotes team development. By combining the essential elements of TBL in this structured sequence, the instructor facilitates deep, enduring understanding of the course content with application of content and higher-order problem-solving.

2.2. Contrast with Other Forms of Active Learning

The preparation activities referenced in the previous section bring to mind Flipped Learning, a pedagogy that is increasingly used in math education. Talbert [35] gives the following definition of flipped learning:

> Flipped Learning is a pedagogical approach in which first contact with new concepts moves from the group learning space to the individual learning space in the form of structured activity, and the resulting group space is transformed into a dynamic, interactive learning environment where the educator guides students as they apply concepts and engage creatively in the subject matter.

For the unfamiliar reader, "group learning space" is used in this context as a generalization of classroom time; in many face-to-face classes, it may be synonymous with the classroom. The "individual learning space" refers to time students spend working alone; traditionally, this is out-of-class work. TBL clearly satisfies the second part of this definition; indeed, TBL instructors' primary role is to facilitate students through interactive team activities.

However, TBL does not inherently satisfy the first criterion. Quite often, TBL instructors will structure the Readiness Assurance Process to initiate first contact with some new concepts in the individual space; however, this is not required (and in fact, we argue below for a different approach). Moreover, TBL does not require **structured** activity in the individual space; in fact, we think many TBL instructors would do well to improve their readiness assurance materials along these lines by, for example, adding interactive quizzes to their static videos or readings.

TBL also has some similarities with Process Oriented Guided Inquiry Learning (POGIL), a particular structure of collaborative inquiry learning. POGIL is much more recent than TBL, originating in the last 20 years in chemistry education [9] and is now being applied to math courses [1]. In both TBL and POGIL classrooms, the instructor functions as a facilitator, supporting and guiding as teams independently work on activities. Unlike TBL, POGIL involves assigning specific roles to team members and explicitly focuses on teaching process skills in addition to content. Also, the 4-S structure of TBL application activities is not required in POGIL activities, limiting the ability to compare student thinking across teams. Moreover, POGIL lacks a Readiness Assurance Process, a TBL feature we find very helpful in ensuring students are ready to solve challenging problems.

Another related pedagogy is Peer Led Team Learning (PLTL), which also originated in chemistry education (see [11] for a general reference, and [19] for a mathematics implementation). Like TBL, PLTL involves students working in groups, but in a recitation format led by a peer leader, usually an undergraduate who previously has taken and performed well in the course. However, like POGIL, PLTL lacks both the Readiness Assurance Process, as well as the 4-S structure of activities.

Many readers will also be familiar with Inquiry-Based Learning (IBL). Although not entirely well-defined, IBL is commonly described as having two [7, 8], three [31], or more recently, four pillars [16]:

(1) students engage deeply with coherent and meaningful mathematical tasks,
(2) students collaboratively process mathematical ideas,
(3) instructors inquire into student thinking,
(4) instructors foster equity in their design and facilitation choices.

TBL is well suited for integrating each of these four pillars into instruction. It is clear that TBL inherently provides (indeed, mandates) opportunities to collaborate. TBL also provides ample opportunity for engaging students in rich mathematics through well-designed application activities, but does not require doing so. The inter-team reporting phase of TBL allows instructors to habitually inquire into and make visible student thinking. And finally, a thoughtful and effective TBL facilitator can foster equity in their facilitation choices. In the next section, we describe an approach that combines TBL and IBL as a means of introducing inquiry learning into a structured collaborative classroom.

3. TEAM-BASED INQUIRY LEARNING

One of the difficulties the authors have encountered in attempting to implement IBL, particularly in intermediate-level service courses such as calculus, linear algebra, and differential equations, is a lack of structure inherent to IBL. For example, in some IBL implementations students are given broad latitude to determine the direction of the class period by picking and choosing the problems on which to work. Although this has the advantage of allowing students the flexibility to solve difficult problems and ask interesting questions perhaps not anticipated by the instructor, we questioned how we could ensure students learned all of the topics we intended for them to learn. In many cases, particularly advanced mathematics courses, we see the course objectives as "more about the journey than the destination": the problem-solving techniques themselves are more important than the specific content, and thus we are not concerned about students not mastering a particular content standard. However, we also teach a number of courses whose content goals are strongly influenced by their applicability to post-requisite courses in other disciplines such as engineering. In these courses, where our instructional context requires us to train students to utilize a particular skill-set to enable success in future coursework, we encountered difficulty in balancing specific content goals with open-ended inquiry. In addition, when using a less-structured IBL implementation, we found it more difficult to gain buy-in from colleagues and administrators who seek a more concrete outline of standards of learning and a more rigid timeline for course content.

Another common difficulty we found in implementing IBL was obtaining student collaboration and buy-in [8]. Although ideally students in a IBL course will share ideas and work together to learn the material, we sought a mechanism to guarantee this outcome. In addition, we wanted to move away from the more individual-driven structures, such as a single student at the blackboard lecturing on their solution to a particular problem. In such an environment, we were concerned that students with advanced levels of understanding may not have a strong incentive to support the instruction of their peers, whereas other students may become frustrated by the lack of direct instruction.

Additionally, we had concerns that the differing mathematical backgrounds of students could pose a challenge when using IBL approaches. Although to some extent tasks can be modified to have multiple entry points, some students may spend a great deal of cognitive load on topics taught in previous courses (or earlier in the course) that are needed to engage in the present task. Thus, we sought some additional structure to minimize this "pre-requisite cognitive load," and allow all of our students to fully engage in the rich tasks we posed to them during class.

To address these concerns, we developed what we coined Team-Based *Inquiry* Learning (TBIL) with the goal of marrying the structure and ensured collaboration found in TBL with the opportunities for mathematical inquiry found in IBL. Although the structure of TBIL is nearly identical to the canonical structure of TBL, we consider TBIL to be a distinct extension of TBL that utilizes several components of TBL to foster inquiry by our students.

In [27], Nanes discusses aspects of TBL that he feels require modification for math courses: most significantly, the Readiness Assurance Process, and certain aspects of the 4-S structure of application activities. In the subsequent sections, we describe how these components of TBL fit into the TBIL framework.

3.1. Readiness Assurance Process

Nanes asserts that the cumulative nature of mathematics courses requires more frequent Readiness Assurance Processes, and describes his use of daily Rediness Assurance Processes; similarly, Peters et al. [30] describe using a Readiness Assurance Policy approximately every second day. Indeed, this interpretation of TBL tracks more closely with flipped learning, in which more frequent readiness checks can be effective (see [34], for example).

However, we take a slightly different approach. Instead of assigning readings and videos on new material and giving a test to ensure students have read and/or watched these materials, we take the tautological view that the purpose of the Readiness Assurance Process is to ensure that our students are ready for the application activities we have planned. In TBIL, we devote class time to

Before beginning this module, students should be able to

- Add Euclidean vectors and multiply Euclidean vectors by scalars.
- Add complex numbers and multiply complex numbers by scalars.
- Add polynomials and multiply polynomials by scalars.
- Perform basic manipulations of augmented matrices and solve linear systems.

Figure 1. Readiness assurance outcomes.

students discovering new material by themselves through guided mathematical inquiry. This frees us to use the Readiness Assurance Process to alleviate a different problem: in our instructional context, we have found that many of our students are under-prepared, in that they do not retain material from pre-requisite courses such as calculus, pre-calculus, and even high school algebra. Thus, we focus the Readiness Assurance Process on ensuring students recall this necessary pre-requisite material. For example, in a module containing a discussion of determinants, one of the outcomes of the Readiness Assurance Process will be for students to be able to find the area of a parallelogram, something students have (in our experience) previously learned but almost always forgotten.

As discussed in [34], it is essential that the readiness assurance materials are well structured. We provide students with a list of "readiness assurance outcomes," which explicitly state what students will need to be able to do in order to be ready for the application activities of that module. We then provide resources (often videos) to refresh students on that pre-requisite material. We note that several free and commercial products even allow instructors to embed practice problems in these videos, an approach we find highly effective.

Figure 1 provides a list of appropriate readiness assurance outcomes from a module introducing vector spaces, span, and linear independence. Students are responsible for (re)mastering these pre-requisite skills so that they are ready to engage with the scaffolded inquiry that leads to the discovery of the new mathematical ideas to be mastered in the course. Students are provided this list in advance, along with resources (in this case, videos) to remind them how to do these tasks. Note that although the first three items are all concepts covered in pre-requisite mathematics courses, the final outcome listed here is based upon the material covered in the previous module on systems of linear equations. This earlier module includes three learning standards, all of which will be necessary for success in this module, so they are explicitly specified for review by students.

As with other interpretations of TBL, students are held accountable for this out-of-class preparation by means of individual and team Readiness

Assurance Tests (iRAT/tRAT). These tests are a two-stage assessment covering 10 questions. For example, we often have a question intended to assess mastery of the readiness assurance outcome related to polynomials. This also serves to expose students to the notion of a linear combination of abstract vectors, which will be defined and investigated during the module. After completing this assessment individually, students repeat the exact same assessment, working as a team, and immediately receiving feedback on the correctness of their answers.

This approach to the Readiness Assurance Process also increases the interleaving of crucial pre-requisite material through the course. Students will be reminded of the material before the module begins, and then use it throughout the module. Moreover, when a readiness assurance outcome is a learning objective from earlier in the course, this serves to interleave that learning objective further throughout the course. This interleaving serves to create more durable learning [32].

Some critics of the Readiness Assurance Process have suggested that there is little value in the team portion, as students will simply vote on the most popular answer. To test this, we compared teams' actual tRAT scores with what they would have scored through a voting strategy. Teams' actual scores were 9.7% higher on average (a *t*-test showed this difference to be significant, $p < 0.0001$), indicating that team discussions were actually driving teams towards the correct answer instead of simply the most popular answer. Additionally, our observations as instructors are that the tRATs usually produce rich discussions within teams on the more difficult questions.

3.2. 4-S Activities

As described above, a typical TBL implementation will have teams spending class time working on 4-S application activities: teams work on the **S**ame problem, which is a **S**ignificant problem; and they **S**imultaneously report a **S**pecific choice to the class.

One of the benefits of TBL is the rich inter-team discussion that takes place after a simultaneous report. Indeed, the reasoning behind requiring a specific choice is to force teams to make a specific claim, and be prepared to argue for their answer. In the context of a math class, this need not mean that students must answer a multiple choice question (though as we describe below, this is often helpful). It may mean that they have done a specific computation (e.g., computed a determinant) and simultaneously report that result. Inter-team discussion is then facilitated by asking teams to explain their reasoning; this is wonderfully effective on tasks with many possible appropriate techniques. Moreover, this puts student thinking at the (metaphorical) front and center of the classroom.

One of the motivations for the Readiness Assurance Process in TBL is to free up the group space (e.g., class time) for interactions between students as they complete application activities to bring theoretical knowledge into practice. However, in mathematics, it is the theoretical knowledge itself with which we are concerned, so our application activities must be designed to develop this process.

To this end, application activities in TBIL are designed around building upon the background knowledge refreshed during the Readiness Assurance Process to (dis)cover new material. By carefully scaffolding questions that get students moving through a particular line of inquiry, students are enabled to conjecture or even prove new mathematical ideas for themselves, rather than having these ideas dictated to them via a lecture or reading. These activities can broadly be sorted into three categories.

(1) **Scaffolded exploration and discovery.** Students are guided to a new concept through a sequence of carefully scaffolded activities. These activities include exploratory activities to motivate an entire line of thinking; working a series of examples to have students realize the need for a new definition; and working a carefully scaffolded problem to develop a general algorithm for solving similar problems, for example.
(2) **Fluency builders.** Once a new mathematical concept has been established, students need an opportunity to put it into practice and build fluency in procedural tasks. These activities are often similar or identical to exercises that would be asked on assessments, but are solved within teams in order to provide students a model for how to organize their thoughts and writing when demonstrating mastery on quizzes or exams.
(3) **Flexible extension.** Once a concept is established and students gain some fluency working with it, we ask students to apply this new concept to slightly different contexts. This can take various forms, such as checking if something satisfies a new definition; extending an idea from its "natural" setting to more generality; or a true application to another field such as balancing chemical equations by solving systems of linear equations. In these activities, the emphasis is on helping students develop a flexible mindset when faced with new problems.

We note that the same task could be in different categories depending on the context; for example, the first encounter with a particular question might be an extension, but once a model for answering such a question is established, asking a similar question again might be a fluency builder.

In Figures 2 and 3, we provide approximately one class day's worth of activities to exemplify the three categories of activities. The activities are projected one at a time (and "parts" are successively uncovered). Activity 2.1 in Figure 2 gives an example of a scaffolded exploration activity; students work through the steps to discover an algorithm to answer these questions, using

Activity 2.1 The vector $\begin{bmatrix}-1\\-6\\1\end{bmatrix}$ belongs to span $\left\{\begin{bmatrix}1\\0\\-3\end{bmatrix}, \begin{bmatrix}-1\\-3\\2\end{bmatrix}\right\}$ exactly when there exists a solution to the vector equation $x_1\begin{bmatrix}1\\0\\-3\end{bmatrix} + x_2\begin{bmatrix}-1\\-3\\2\end{bmatrix} = \begin{bmatrix}-1\\-6\\1\end{bmatrix}$.

Part 1: Reinterpret this vector equation as a system of linear equations.

Part 2: Find its solution set, using technology to find RREF of its corresponding augmented matrix.

Part 3: Does $\begin{bmatrix}-1\\-6\\1\end{bmatrix}$ belong to span $\left\{\begin{bmatrix}1\\0\\-3\end{bmatrix}, \begin{bmatrix}-1\\-3\\2\end{bmatrix}\right\}$?

Figure 2. Scaffolded exploration activity.

their prior knowledge of how to solve systems of linear equations (learned in a previous module). After completing this activity, we present the algorithm for reference in Observation 2.2 (Figure 3); students then work two procedural activities (2.3 and 2.4) for practice and to build fluency. Then, an extension activity (2.5) asks them to apply their knowledge of how to answer questions about span for Euclidean vectors to an abstract vector space (in this case, polynomials).

In order to promote a fruitful class-wide discussion, these activities follow TBL's 4-S structure: students work on a **Significant problem**, so that they are engaged; teams work on the **Same problem**, so that different approaches and reasoning can be contrasted across teams; students make a **Specific commitment** (e.g., to a particular solution) which they may be asked to defend to the class; and teams **Simultaneously report** their response.

While teams are working on an activity, the instructor circulates the room to ensure each team is progressing towards a solution. Sometimes this involves a short conversation with a team; sometimes a single question posed to a struggling team will suffice; and sometimes an impromptu mini-lecture might be needed.

After each team is given a chance to complete the activity, the instructor facilitates a class-wide discussion by asking a team to explain their reasoning. Students often take different approaches to the same question, so we typically ask multiple teams to share their reasoning (even when all teams report the same correct "answer"). As the teams all worked on the **Same problem**, students can compare their team's reasoning to that shared by other teams. Teams

Observation 2.2 A vector $\vec{b}$ belongs to $\operatorname{span}\{\vec{v}_1, \ldots, \vec{v}_n\}$ if and only if the linear system corresponding to $[\vec{v}_1 \, \ldots \, \vec{v}_n \,|\, \vec{b}]$ is consistent.

Put another way, $\vec{b}$ belongs to $\operatorname{span}\{\vec{v}_1, \ldots, \vec{v}_n\}$ exactly when $\operatorname{RREF}[\vec{v}_1 \, \ldots \, \vec{v}_n \,|\, \vec{b}]$ doesn't have a row $[0 \cdots 0 \,|\, 1]$ representing the contradiction $0 = 1$.

Activity 2.3 Determine if $\begin{bmatrix} 3 \\ -2 \\ 1 \\ 5 \end{bmatrix}$ belongs to span $\left\{ \begin{bmatrix} 1 \\ 0 \\ -3 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \\ 2 \\ 2 \end{bmatrix} \right\}$ by row-reducing an appropriate matrix.

Activity 2.4 Determine if $\begin{bmatrix} -1 \\ -9 \\ 0 \end{bmatrix}$ belongs to span $\left\{ \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \\ 2 \end{bmatrix} \right\}$ by row-reducing an appropriate matrix.

Activity 2.5 Does the third-degree polynomial $3y^3 - 2y^2 + y + 5$ in $\mathcal{P}^3$ belong to $\operatorname{span}\{y^3 - 3y + 2, -y^3 - 3y^2 + 2y + 2\}$?

Part 1: Reinterpret this question as an equivalent exercise involving Euclidean vectors in $\mathbb{R}^4$. (Hint: What four numbers must you know to write a $\mathcal{P}^3$ polynomial?)

Part 2: Solve this equivalent exercise, and use its solution to answer the original question.

Figure 3. A sequence of activities on span.

have made a **Specific commitment**, which prepares them to defend their reasoning to the class. And since teams **Simultaneously report** their response, teams with a minority response are not tempted to switch their response to a majority response.

We note that some aspects of the 4-S structure can be somewhat relaxed, provided the instructor keeps in mind the goal of displaying and contrasting student thinking through the class-wide discussion phase. For example, most TBL literature refers to a "specific choice" unlike the "specific commitment" to which we refer. Since the rationale for "specific choice" is that students must be prepared to defend their reasoning, we can also achieve this by having students perform some specific task such as solving a particular problem. Another way the 4-S structure can be modified slightly is a loosening of "simultaneous reporting." We have teams work on vertical whiteboards to promote collaboration and reduce the amount of time it takes to get students working [18]. Thus, as we circulate the room as teams are working, we simply make a mental note of each team's solution as they finish the activity. This allows the instructor to collect all the responses without the teams being biased by seeing other teams' responses. Again, the guiding philosophy here is to promote a class-wide discussion that highlights and contrasts students' reasoning.

3.3. Implementation Details

The first and second authors initially developed activities for a linear algebra course. These materials [3] have undergone several semesters of revision, and have now been used by five different instructors at the University of South Alabama.

The linear algebra course discussed in this article is a three-credit sophomore-level course consisting primarily of engineering majors, with a minority of math majors and minors. We divide the course into five modules (Systems of Linear Equations, Vector Spaces, Algebraic Properties of Linear Maps, Algebraic Properties of Matrices, and Geometric Properties of Linear Maps), and thus spend five class days (about 250 minutes) on the Readiness Assurance Process. Each Readiness Assurance Process Day begins with the iRAT, which we give as a 10-question multiple choice test, having students fill out a bubble sheet with their answers. After approximately 20 minutes, students then immediately take the same test with their teams, using scratch-off cards to gain immediate feedback. While the students are taking the team portion, we grade the individual part by scanning the bubble sheet on our phone, providing us with near-instant feedback on which questions caused students difficulties. Then, we eavesdrop on the team discussions to ensure students are gaining understanding of these troublesome questions. If this does not happen, we then hold a brief class-wide discussion about these questions to ensure everyone leaves the Readiness Assurance Process understanding all of the concepts.

The remainder of class days are spent on activities such as those in Figures 2 and 3. Teams spend classtime working the activities one at a time on a vertical whiteboard, followed by a class discussion of the activity. Outside of class, students work on homework problems similar to courses using other pedagogies. Student learning is assessed as in other classes; in particular, the majority of a student's grade should still be based upon their individual mastery of course standards. As such, the authors prefer to use Standards-Based Grading (see [6] for details, and [3] for a list of the standards). Using this method, each student must individually demonstrate full understanding of as many of the course standards as possible on proctored assessments such as quizzes or tests, and their letter grade is then based primarily upon how many standards have been fully mastered by the end of the course. In order to incentivize preparation and participation, one approach is to have a separate "participation score": as with other TBL implementations, the class chooses by consensus how to compute this score as a weighted average of criteria such as iRATs, tRATs, class attendance, and peer evaluations. Importantly, this participation score is completely separate from content mastery; students must both master a certain number of standards and have a certain participation score to earn a given letter grade.

4. IMPACT ON STUDENT LEARNING

In this section, we discuss the perceptions of students and the authors on the impact of TBIL on students' learning. A follow-up paper [17] will more rigorously examine the impact of TBIL on student learning.

To gather students' persepctives, we administered surveys to students in six sections of TBIL linear algebra taught by the first two authors at a large, public university; the third author also conducted two focus groups (nine total students). These sections ranged from 9–30 students each and included a mixture of math majors and other STEM (primarily engineering) majors. In the survey, we asked students to rate their agreement on a six-point Likert scale to several statements, and then explain their responses in an open-ended format. Table 1 shows they generally agreed that TBL was valuable, helped them learn more, and improved their problem-solving skills. When asked an open-ended question "What aspect of this course did you enjoy the most?", 40% of students mentioned the team activities.

Several themes emerged from the focus groups and survey responses. First, students recognized the value provided from peer instruction in their teams. This was mentioned by 38% of students explaining their response on the survey question on the value of TBL. Students spoke at length about this in the focus groups as well; as one student said "It has really given me confidence because I might not be able to get all the way to the right answer, but ... you can sort of work together to get further on the problem." Survey responses indicated that students found value in hearing both alternate techniques and alternate explanations (e.g., in novice language rather than expert language) for the same concept from their teammates.

Several students indicated that they understood the material better and that TBL increased their learning. They indicated that the learning was occurring in class primarily during application activities: "The best experience I had was doing it and seeing what needs to be done for each topic and concept." One student also noted that "I learned a bit more by having to explain the concepts to my team members."

Table 1. Quantitative survey responses

Statement	Responses	Agreement (%)
"The use of Team-Based Learning during class time was a valuable learning experience."	48	77
"The use of Team-Based Learning in this course helped me to learn more than in a traditional course."	55	71
"This course helped me improve my problem solving skills."	55	87

Students also noticed that class time is focused on what matters most, because they came to class prepared. Individual preparation and review outside of class led to increased class time for new topics and challenging problems during class. Students recognized that their knowledge and skills developed over the course of the module with distributed practice: "I think it helped that you are not trying to learn it all in one day, but you are trying to build off what you think you know."

When describing the TBIL class environment, students discussed actively working together to solve problems and reported increased engagement. One student said, "Zoning out is easier in lectures." They frequently stated that they "worked it out together." Other comments were "I feel I spend more time actively engaged with learning in class, and less time distracted or bored" and "When you are able to work together and discuss problems, you get a better understanding of the solution." They recognized that when solving problems together with the support of teammates and the instructor they persisted with challenging problems.

Students in the focus group recognized that the TBIL structure provides a safe environment for productive struggle. As one said, "If everything is easy you are not retaining it." Another noted that "to really genuinely learn you have to struggle." Although sometimes they did not know how to solve problem initially, "[TBL] gets you brainstorming and actually struggling to learn."

Instructors and students both observed reduced anxiety in students. One student said:

> With me, it's been 15 years since I've taken a math class so I was really nervous about coming into a math class, but it's been really relaxed. We are working in teams it's been great, so I didn't think I needed to be nervous.

4.1. Instructor Reflections

In a TBIL environment, the role of the instructor shifts from that of a lecturer to that of a facilitator. Since students are working in teams the vast majority of class time, the instructor is able to spend most of their time observing students' thinking. Eavesdropping on team discussions can provide a much richer picture of how students are making sense of the material than one is able to discern in a lecture. In our first few TBIL class sessions, we had concerns that students were not picking up concepts as quickly as they might seem to do in lecture. However, as the course progressed, it became clear that this confusion also would have existed in a comparable lecture, but in TBIL it was exposed during team interactions. More importantly, TBIL allowed us to identify and remedy such issues during class.

Students in our TBIL classes seem to develop more flexible problem-solving skills than when we have taught via lectures. Although this idea is quantified and explored deeper in [17], we will provide an anecdote here: when computing determinants, our students in lectures would most often resort to using Laplace expansion, likely because it is an algorithm they can follow. However, in our TBIL classes, students are more likely to perform a clever row operation or two to simplify the resulting calculation, rather than blindly following an algorithm. We hypothesize that the team-reporting phase of class is the crucial feature supporting this behavior; the instructor can highlight distinct approaches by different teams, all resulting in the same correct answer, but some approaches involve less computation than others.

We have also noticed that students tend to ask more questions in TBIL classes. In a lecture, a question by a student draws the attention of the entire class, and many shy or introverted students are hesitant to ask. However, while the instructor is circulating the room during team activities, these same students seem to be more willing to ask a question. Moreover, we observe students are much more willing to pose questions to their classmates; as one student put it in a survey response, "I feel that the team based learning curriculum helps students that are afraid to ask a question, get their question answered by a classmate."

Finally, we note that we found the TBIL structure to be quite helpful as we were developing the application activities for the course. Our initial reaction (matched by many other mathematicians to whom we have presented TBIL) was that the 4-S structure of application activities was too rigid for mathematical tasks, but in practice we found this to not be the case. Indeed, in our first attempt at writing an activity, we often disregarded one of the 4-S components, such as Specific choice. However, after using the activity in class, we would often realize that the class discussion was improved by modifying the activity to be more closely aligned with the 4-S structure. In particular, not requesting a specific choice in an activity would often leave students sitting silently as they pondered how to proceed. However, by defining the specific choice to be made (even sometimes going as far as to make the activity multiple-choice), the increased scaffolding helped students be more willing to take risks with the problem and engage with their team. Regardless of how many individual teams might come to the correct conclusion, the following class-wide discussion clarifying the correct conclusion was typically deeper and more productive.

5. CONCLUSION

Our experience with this initial exploration of implementing TBL in the undergraduate mathematics curriculum leaves us optimistic that TBIL is a promising format for introducing inquiry learning into a heterogeneous sophomore mathematics classroom serving both mathematics and other STEM majors. The strong structure of TBIL seems to lessen the culture shock and potential loss of buy-in risked by other active learning formats, making it an appealing option for instructors looking to increase collaboration and inquiry in their classrooms. Although the present paper provides mostly student perceptions of TBIL classes, we point the reader to the forthcoming paper [17] for more concrete evidence of the anecdotal claims of this paper, including comparing concrete measures of student learning between TBIL and other classes.

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