Cmpt 101
Lab 8 – Examples

Example 1: Convert the following numbers from the base given into decimal.

(a) 266_8 = _____$_{10}$

Solution:

Number:	2	6	6
Positions:	2	1	0

$\mathbf{266_8}$ $= (2 * 8^2) + (6 * 8^1) + (6 * 8^0)$
$= 128 + 48 + 6$
$= \mathbf{182_{10}}$

(b) 11011.11011_2 = _____$_{10}$

Solution:

Number:	1	1	0	1	1	.	1	1	0	1	1
Positions:	4	3	2	1	0		-1	-2	-3	-4	-5

11011.11011_2 $= (1 * 2^4) + (1 * 2^3) + (1 * 2^1) + (1 * 2^0) +$
$(1 * 2^{-1}) + (1 * 2^{-2}) + (1 * 2^{-4}) + (1 * 2^{-5})$
$= 2^4 + 2^3 + 2^1 + 2^0 + (1 / 2^1) + (1 / 2^2) + (1 / 2^4) + (1 / 2^5)$
$= 16 + 8 + 2 + 1 + (1/2) + (1/4) + (1/16) + (1/32)$
$= 27 + (16 + 8 + 2 + 1)/32$
$= 27\ \ (27 / 32)$
$\mathbf{= 27.84375_{10}}$

(c) $1BD8_{16}$ = _____$_{10}$

Solution:

Number:	1	B	D	8
Positions:	3	2	1	0

$\mathbf{1BD8_{16}}$ $= (1 * 16^3) + (B * 16^2) + (D * 16^1) + (8 * 16^0)$
$= (1 * 16^3) + (11 * 16^2) + (13 * 16^1) + (8 * 16^0)$
$= (1 * 4096) + (11 * 256) + (13 * 16) + (8 * 1)$
$= 4096 + 2816 + 208 + 8$
$= \mathbf{7128_{10}}$

Example 2: Convert the following number from decimal to the base indicated.

(a) 182_{10} = _____$_2$

Solution:

182	÷ 2 =	91	rem	0	
91	÷ 2 =	45	rem	1	
45	÷ 2 =	22	rem	1	
22	÷ 2 =	11	rem	0	
11	÷ 2 =	5	rem	1	
5	÷ 2 =	2	rem	1	
2	÷ 2 =	1	rem	0	
1	÷ 2 =	0	rem	1	= **$1011\ 0110_2$**

(b) 27.84375_{10} = _____$_{16}$

Solution: **Hint:** *Split the number into two parts and solve them separately.*

Part 1: 27_{10}

27 ÷ 16 = 1 rem 11 (B)
1 ÷ 16 = 0 rem 1

= **$1B_{16}$**

Part 2: 0.84375_{10}

0.84375	* 16 =	13.5	*Whole:* 13 (D)	*Fraction:* 0.5
0.5	* 16 =	8.0	*Whole:* 8	*Fraction:* 0

= $0.D8_{16}$

Thus, **27.84375_{10}** = **$1B.D8_{16}$**

(c) 27.84375_{10} = _____$_2$

Solution: **Hint:** *Split the number into two parts and solve them separately.*

Part 1: 27_{10}

27 ÷ 2 = 13 rem 1 (B)
13 ÷ 2 = 6 rem 1
6 ÷ 2 = 3 rem 0
3 ÷ 2 = 1 rem 1
1 ÷ 2 = 0 rem 1

= **11011_2**

Part 2: 0.84375_{10}

0.84375	* 2 =	1.6875	*Whole:* 1	*Fraction:* 0.6875
0.6875	* 2 =	1.375	*Whole:* 1	*Fraction:* 0.375
0.375	* 2 =	0.75	*Whole:* 0	*Fraction:* 0.75
0.75	* 2 =	1.5	*Whole:* 1	*Fraction:* 0.5
0.5	* 2 =	1.0	*Whole:* 1	*Fraction:* 0

= 0.11011_2

Thus, **27.84375_{10}** = **11011.11011_2**

Example 3: Add the following numbers in the bases indicated.

(a)

	1	1			1		
		5	4	.	1	4	3_7
+		1	4	.	3	3	$_7$
	1	0	1	.	5	0	3_7

Start adding on the right hand side

Ensure numbers line up at the radix point

Explanation

3 + 0 = 3	3 < 7 (the base) so write **3**
4 + 3 = 7	7 (7 + 0) = 7 so write **0** and carry a 1
1(carry) + 1 + 3 = 5	5 < 7 so write **5**
4 + 4 = 8	8 (7 + 1) > 7 so write **1** and carry a 1
1(carry) + 5 + 1 = 7	7 (7 + 0) = 7 so write **0** and carry a 1
1 + 0 = 1	1 < 7 so write **1**

(b)

	1			1		
	1	5	.	4	A_{12}	
+	8	7	.	6	5_{12}	
	A	0	.	B	3_{12}	

Explanation

A + 5 = 10 + 5 = 15	15 (12 + 3) > 12 so write **3** and carry 1
1(carry) + 4 + 6 = 11	11 < 12 so write **B**
5 + 7 = 12	12 (12 + 0) = 12 so write **0** and carry 1
1(carry) + 1 + 8 = 10	10 < 12 so write **A**

Example 4: Write the decimal equivalent of **1100 0001 0001 0001** when it is treated as a

a) 16 bit **un**signed value

b) 16 bit signed magnitude value

Solution (a):

1100 0001 0001 0001 as an UNsigned value

$= (1 * 2^{15}) + (1 * 2^{14}) + (1 * 2^8) + (1 * 2^4) + (1 * 2^0)$

$= 2^{15} + 2^{14} + 2^8 + 2^4 + 2^0$

$= 32{,}768 + 16{,}384 + 256 + 16 + 1$

$= \mathbf{49{,}425_{10}}$

Solution (b):

1100 0001 0001 0001 as a signed value

$= - [(1 * 2^{14}) + (1 * 2^8) + (1 * 2^4) + (1 * 2^0)]$

$= - (2^{14} + 2^8 + 2^4 + 2^0)$

$= - (16{,}384 + 256 + 16 + 1)$

$= \mathbf{-16{,}657_{10}}$

Example 5: Convert the following values between two's complement (8 bit) representations and decimal as indicated. Show your work.

(a) 0111 1001 (Two's complement) = _____ $_{10}$

Solution: 0111 1001 $= (1 * 2^6) + (1 * 2^5) + (1 * 2^4) + (1 * 2^3) + (1 * 2^0)$
$= (64) + (32) + (16) + (8) + (1)$
$= \mathbf{+121_{10}}$

(b) 1001 0110 (Two's complement) = _____ $_{10}$

Solution: 1001 0110 $= (-1 * 2^7) + (1 * 2^4) + (1 * 2^2) + (1 * 2^1)$
$= (-128) + (16) + (4) + (2)$
$= \mathbf{-106_{10}}$

(c) 29_{10} = _____ (Two's complement (groups of 4 bits))

Solution: 29_{10} = 11101 (Binary)
= 00011101 (8 bits)
= **0001 1101** (Stop here – number was positive)

(d) -120_{10} = _____ (Two's complement (groups of 4 bits))

Solution: -120_{10} = 1111000 (Binary)
= 01111000 (8 bits)
= 1000 0111 (Complement because it was negative)
= **1000 1000** (Add 1)

Example 6: Add the two's complement (8 bit) values as indicated writing your answer under the line. Indicate either "*Overflow*" or "*No overflow*" as appropriate.

(a)

		1	1	1	1			1
	0	1	1	1	1	0	0	1
+	0	0	0	1	1	1	0	1
	1	0	0	1	0	1	1	0

Overflow

121 + 29 ≠ -106

Adding 2 positive numbers resulted in a negative number which should not happen!

(b)

	1	1	1	1	1			
	1	0	0	0	1	0	0	0
+	0	1	1	1	1	0	0	1
~~1~~	0	0	0	0	0	0	0	1

No Overflow (Truncation)

(-120) + 121 = +1

Adding a positive and a negative does not produce overflow