Random Variables

1 *Bernoulli Random Variable*

A *Bernoulli random variable* is the simplest kind of random variable. It can take on two values, 1 and 0. It takes on a 1 if an experiment with probability p resulted in success and a 0 otherwise.[1] Some example uses include a coin flip, a random binary digit, and whether a disk drive crashed. If X is a Bernoulli random variable, denoted[2] $X \sim \text{Ber}(p)$:

$$
\begin{aligned}
\textit{Probability mass function:} \quad & P(X=1) = p && (1)\\
& P(X=0) = (1-p) && (2)\\
\textit{Expectation:} \quad & \mathbb{E}[X] = p && (3)\\
\textit{Variance:} \quad & \text{Var}(X) = p(1-p) && (4)
\end{aligned}
$$

Bernoulli random variables and *indicator variables* are two aspects of the same concept. A random variable I is an indicator variable for an event A if $I=1$ when A occurs and $I=0$ if A does not occur. $P(I{=}1){=}P(A)$ and $\mathbb{E}[I]{=}P(A)$. Indicator random variables are Bernoulli random variables, with $p{=}P(A)$.

A *Bernoulli random variable* maps "success" to 1 and "failure" to 0. Support for *Bernoulli*: $\{0, 1\}$

[1] The Bernoulli random variable is the simplest random variable (i.e. an *indicator* or *boolean* random variable)

[2] Sampling x from a distribution D can also be written $x \sim D$, where $\sim$ is read as "is distributed as".

2 *Binomial Random Variable*

A *binomial random variable* is random variable that represents the number of successes in n successive independent trials of a Bernoulli experiment. Some example uses include the number of heads in n coin flips, the number of disk drives that crashed in a cluster of 1000 computers, and the number of advertisements that are clicked when 40,000 are served.

If X is a Binomial random variable, we denote this $X \sim \text{Bin}(n, p)$, where p is the probability of success in a given trial. A binomial random variable has the following properties:[3]

$$
\begin{aligned}
\textit{Probability mass function:} \quad & \begin{cases} P(X=k) = \binom{n}{k} p^k (1-p)^{n-k} & \text{if } k \in \mathbb{N},\ 0 \le k \le n \\ 0 & \text{otherwise} \end{cases} && (5)\\
\textit{Expectation:} \quad & \mathbb{E}[X] = np && (6)\\
\textit{Variance:} \quad & \text{Var}(X) = np(1-p) && (7)
\end{aligned}
$$

A *binomial random variable* is the number of successes in n trials. Note that $\text{Ber}(p) = \text{Bin}(1, p)$.

Support for *binomial*: $\{0, 1, \ldots, n\}$

[3] A binomial random variable is the sum of Bernoulli random variables.

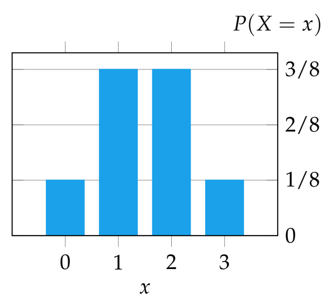

Figure 1. *Probability mass function* of a *binomial random variable*; number of heads after three coin flips.